Path-integral representation for the relativistic particle propagators and BFV quantization

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The path-integral representations for the propagators of scalar and spinor fields in an external electromagnetic field are derived. The Hamiltonian form of such expressions can be interpreted in the sense of Batalin-Fradkin-Vilkovisky quantization of one-particle theory. The Lagrangian representation as derived allows one to extract in a natural way the expressions for the corresponding gauge-invariant (reparametrization- and supergauge-invariant) actions for pointlike scalar and spinning particles. At the same time, the measure and ranges of integrations, admissible gauge conditions, and boundary conditions can be exactly established.

I. INTRODUCTION

The theory of the relativistic pointlike particle is a prototype of string theory and therefore different versions of such kinds of theories have been discussed in recent years. To formulate the first-quantized theory of the relativistic particle, it is necessary to construct a corresponding classical action possessing some gauge symmetry or supersymmetry. Such actions are being constructed and investigated in many works [1–5]. Then the next step is to perform the quantization of the action. In this way one has to prove that the Hamiltonian operator quantization leads to the Schrödinger equation which is equivalent, in a certain sense, to one-particle equations of the corresponding field theory, for example, to the Klein-Gordon or Dirac equations. On the other hand, the quantization by means of a path integral must lead to the propagator of the particle of the corresponding quantum field theory. The latter procedure can be treated as an initial step to the second quantization of the theory. Solving both these problems one has to take into account both the usual difficulties with the first-class constraints and the zero Hamiltonian due to reparametrization invariance, in particular. As to the Hamiltonian operator quantization, the problem was solved in the canonical gauge consistently in Ref. [6]. Here one can also find the criticism of the previous works in this direction. Path-integral quantization has been discussed in different papers (see Ref. [5] and survey [7]). Concerning this problem, one needs to point out that the representation of the Dirac propagator by means of an appropriate (over additional Grassmann variables) path integral of $\exp(iS)$ was given by Fradkin [8]. Thus, the action S of the spinning particle has arisen for the first time. This action was nondegenerate and well adjusted to the concrete evaluations. The Berezin-Marinov action [1] is, in fact, its reparametrization and supergauge extension.

In this work, which can be regarded as a development of a previous work [9], we want to turn to the problem of path-integral quantization of the theory of the relativistic particle from the point of view of the ideas of the work [8]. Namely, we start from well-known expressions for the propagators of the corresponding quantum field theories to get a special path-integral representation with effective classical actions being already reparametrization invariant and supergauge invariant. This way of action allows us to kill two birds with one stone. Indeed, here the actions of the relativistic particle arise in a natural manner and therefore this way can be treated as a general method of the derivation of such kinds of Lagrangians. In addition, here one can more deeply comprehend the role of the supplementary fields of integration making the actions gauge invariant. The restrictions on the ranges of their functional integrations can be regarded as gauge conditions, so we get the admissible gauge conditions and, moreover, the ability to automatically put the integration measure into concrete terms. Finally, it turns out that the resulting path integral can be interpreted in the frame of the Batalin-Fradkin-Vilkovisky (BFV) method of quantization.

II. SCALAR PARTICLE

The propagator of the scalar particle, interacting with an external electromagnetic field $\mathcal{A}_\mu(x)$, is the causal Green's function $D^c(x,y)$ of the Klein-Gordon equation:

$$(\mathcal{P}^2 - m^2 + i\epsilon) D^c(x,y) = -\delta^4(x-y) , \tag{1}$$

where $\mathcal{P}_\mu = i\partial_\mu - g\mathcal{A}_\mu(x)$ and the Minkowski tensor is $\eta_{\mu\nu} = \mathrm{diag}(1,-1,-1,-1)$.

Following Schwinger [10], we present $D^c(x,y)$ as a matrix element of an operator D^c

$$D^c(x,y) = \langle x | D^c | y \rangle , \tag{2}$$

where $|x\rangle$ are eigenvectors for some self-conjugated operators of coordinates X^μ; the corresponding canonical-conjugated operators of momenta are P_μ so that

$$
\begin{aligned}
&[P_\mu, X^\nu] = -i\delta_\mu^\nu ,\\
&X^\mu |x\rangle = x^\mu |x\rangle ,\\
&P_\mu |p\rangle = p_\mu |p\rangle ,\\
&\langle x|y\rangle = \delta^4(x-y) ,\\
&\langle p|p'\rangle = \delta^4(p-p') ,\\
&\int |x\rangle\langle x| dx = I ,\\
&\langle x|p\rangle = \frac{1}{(2\pi)^2}\exp(ipx) ,\\
&\langle x|P_\mu|y\rangle = -i\partial_\mu\delta^4(x-y) ,\\
&[\Pi_\mu, \Pi_\nu]_- = -igF_{\mu\nu}(X) ,\\
&\Pi_\mu = -P_\mu - g\mathcal{A}_\mu(X) .
\end{aligned}
\tag{3}
$$

Equation (1) implies the equation for the operator D^c:

$$(\Pi^2 - m^2 + i\epsilon)D^c = -I$$

or

$$D^c = (m^2 - \Pi^2 - i\epsilon)^{-1} .$$

The inverse for the Bose operator can be written by means of an integral over proper time λ,

$$
\begin{aligned}
&D^c = i\int_0^\infty e^{-i\mathcal{H}(\lambda,X,P)} d\lambda ,\\
&\mathcal{H}(\lambda,X,P) = \lambda(m^2 - \Pi^2 - i\epsilon) .
\end{aligned}
\tag{4}
$$

In what follows we omit the infinitesimal quantity ϵ. According to Eqs. (2), and (4), one can write

$$D^c(x_{\text{out}}, x_{\text{in}}) = i\int_0^\infty \left\langle x_{\text{out}} \left| e^{-i\mathcal{H}(\lambda,X,P)} \right| x_{\text{in}} \right\rangle d\lambda . \tag{5}$$

Now we present the matrix element in (5) by means of a path integral in the phase space, taking into account the Weyl-ordering procedure for operators:

$$
\begin{aligned}
D^c(x_{\text{out}}, x_{\text{in}}) &= i\int_0^\infty d\lambda_0 \lim_{N\to\infty} \int_{-\infty}^{\infty} dx_1 \cdots dx_{N-1} d\lambda_1 \cdots d\lambda_N \langle x_{\text{out}} | e^{-i\mathcal{H}(\lambda_N,X,P)/N} | x_{N-1}\rangle \delta(\lambda_N - \lambda_{N-1}) \cdots \\
&\qquad \times \left\langle x_i \left| e^{-i\mathcal{H}(\lambda_i,X,P)/N} \right| x_{i-1} \right\rangle \delta(\lambda_i - \lambda_{i-1}) \cdots \\
&\qquad \times \left\langle x_1 \left| e^{-i\mathcal{H}(\lambda_1,X,P)/N} \right| x_{\text{in}} \right\rangle \delta(\lambda_1 - \lambda_0) \\
&= i\int_0^\infty d\lambda_0 \lim_{N\to\infty} \int_{-\infty}^{\infty} dx_1 \cdots dx_{N-1} \frac{dp_1}{(2\pi)^4} \cdots \frac{dp_N}{(2\pi)^4} d\lambda_1 \cdots d\lambda_N \\
&\qquad \times \frac{d\pi_1}{(2\pi)} \cdots \frac{d\pi_N}{(2\pi)} \exp\left[i \sum_{k=1}^{N} \left[p_k \frac{\Delta x_k}{\Delta\tau} - \mathcal{H}(\lambda_k, \bar{x}_k, p_k) + \pi_k \frac{\Delta\lambda_k}{\Delta\tau} \right] \Delta\tau \right] ,
\end{aligned}
$$

where we used the notation

$$\Delta\tau = \frac{1}{N}, \quad \bar{x}_k = \frac{x_k + x_{k-1}}{2}, \quad \Delta x_k = x_k - x_{k-1}, \quad \Delta\lambda_k = \lambda_k - \lambda_{k-1} .$$

So we get

$$D^c(x_{\text{out}}, x_{\text{in}}) = i\int_0^\infty d\lambda_0 \int Dx\, Dp\, D\lambda\, D\pi \exp\left[i\int_0^1 [\lambda(\mathcal{P}^2 - m^2) + p\dot{x} + \pi\dot{\lambda}] d\tau \right] , \tag{6}$$

where $\mathcal{P}_\mu = -p_\mu - g\mathcal{A}_\mu(x)$ and the integration goes over the trajectories $x^\mu(\tau)$, $p_\mu(\tau)$, $\lambda(\tau)$, $\mu(\tau)$, parametrized by some parameters $\tau \in [0,1]$ and obeying the boundary conditions

$$x(0) = x_{\text{in}}, \quad x(1) = x_{\text{out}}, \quad \lambda(0) = \lambda_0 . \tag{7}$$

The path integral (6) can be interpreted in the frame of generalized Hamiltonian quantization method (BFV method) [11–13] for a spinless pointlike particle, described by the well-known action

$$S = -\int_0^1 [m\sqrt{\dot{x}^2} + g\dot{x}\mathcal{A}(x)] d\tau . \tag{8}$$

It is obvious that the functions $x(\tau)$ and $x(f(\tau))$, $df/d\tau > 0$, $f(0) = 0$, $f(1) = 1$ described the same path. So, there are many trajectories which correspond to one and the same physical state; therefore, the theory is a gauge theory. The action (8) is invariant under the above-mentioned transformations (reparametrizations) which are, in fact, gauge transformations:

$$x(\tau) \to x(f(\tau)) . \tag{9}$$

Their infinitesimal form is

$$\delta x = \dot{x}\epsilon, \quad \epsilon(0) = \epsilon(1) = 0 .$$

Going over to the Hamiltonian formulation, according to the general rules [14] one can find that the Hamiltonian on the constraints surface equals zero and there is only one constraint,

$$\mathcal{P}^2 - m^2 = 0 ,$$

which is first class. In the generalized Hamiltonian formulation, one needs to introduce new canonical pairs of variables (λ,π), $(c,\bar{\mathcal{B}})$, $(\mathcal{B},\bar{c})$. The first one is even and the two latter are odd. The BFV fermion generating operator Ω (see Refs. [11–13]) obeys the equation $\{\Omega,\Omega\}=0$, where $\{\ ,\ \}$ is the Poisson brackets, and can be chosen in the form $\Omega=-c(\mathcal{P}^2-m^2)+\pi\mathcal{B}$. On account of the ordinary Hamiltonian being equal to zero, the corresponding Hamiltonian of the generalized formulation has the form $H=\{\Omega,\Psi\}$, where Ψ is the gauge fermion. The latter can be written in the form $\Psi=\chi_1\bar{\mathcal{B}}+\chi_2\bar{c}$, where χ_σ, in the general case, may depend on all canonical variables being even and having the ghost number equaling zero. The functions χ_σ play the role of gauge conditions. In the simplest case one can choose $\chi_1=\lambda$ and $\chi_2=0$. Then the Hamiltonian H has the form

$$H=-\lambda(\mathcal{P}^2-m^2)\ ,$$

and the corresponding action S equals

$$S=\int_0^1[\lambda(\mathcal{P}^2-m^2)+p\dot{x}+\pi\dot{\lambda}+\bar{\mathcal{B}}\dot{c}+\bar{c}\dot{\mathcal{B}}]d\tau\ . \tag{10}$$

The generalized Hamiltonian formulation presents the nondegenerate theory, in the case under consideration, without constraints. Therefore, the corresponding path integral has to be taken over all trajectories x, p, λ, π, c, $\bar{\mathcal{B}}$, $\mathcal{B}$, $\bar{c}$. It is seen that the ghosts do not interact with other variables and one can integrate over them. Thus, we get to the path integral (6). The boundary conditions (7) and the necessity to integrate over λ_0, which is, in fact, the zero mode of λ, are connected with the choice of concrete Green's functions of the Klein-Gordon equation.

Expression (6) is the Hamiltonian form of the path integral for the propagator of spinless particles. To go over to the Lagrangian form one needs to integrate over the momenta p. To this end we make the shift $-p_\mu\to p_\mu+(\dot{x}_\mu/2\lambda)+g\mathcal{A}_\mu(x)$ and the replacement $e=2\lambda$. Thus we get

$$D^c(x_{\rm out},x_{\rm in})=\frac{i}{2}\int_0^\infty de_0\int Dx\,De\,D\pi\,M(e)\exp\left[i\int_0^1\left[-\frac{\dot{x}^2}{2e}-\frac{e}{2}m^2-g\dot{x}\mathcal{A}(x)+\pi\dot{e}\right]d\tau\right]\ , \tag{11}$$

where the boundary conditions $x(0)=x_{\rm in}$, $x(1)=x_{\rm out}$, $e(0)=e_0$ are supposed and the measure $M(e)$ has the form

$$M(e)=\int Dp\,\exp\left[\frac{i}{2}\int_0^1 ep^2d\tau\right]\ . \tag{12}$$

The exponent in the integrand in (11) can be treated as a Lagrangian action of the relativistic spinless particle, consisting of two parts. The first of them

$$S=-\int_0^1\left[\frac{\dot{x}^2}{2e}+\frac{e}{2}m^2+g\dot{x}\mathcal{A}(x)\right]d\tau \tag{13}$$

is a gauge-invariant (reparametrization-invariant) action of the relativistic spinless particle. The corresponding gauge transformations have the form

$$\begin{aligned} &x(\tau)\to x(f(\tau))\ ,\\ &e(\tau)\to e(f(\tau))\dot{f}(\tau)\ ,\\ &f(0)=0\ ,\\ &f(1)=1\ , \end{aligned} \tag{14}$$

or in the infinitesimal form

$$\delta x=\dot{x}\epsilon,\quad \delta e=\frac{d}{d\tau}(e\epsilon),\quad \epsilon(0)=\epsilon(1)=0\ . \tag{15}$$

The second term in the action, which is

$$S_{\rm GF}=\int_0^1\pi\dot{e}\,d\tau\ , \tag{16}$$

can be treated as a gauge-fixing term. In fact, here we have the gauge

$$\dot{e}=0\ . \tag{17}$$

It is easy to understand why one cannot use a gauge condition which fixes the variable e fully. Indeed, the gauge condition is intended to fix the arbitrary function $f(\tau)$ in (14), taking into account two boundary conditions. It can be done only if the gauge condition leads to the differential equation of second order for the function $f(\tau)$, or, according to (14), to the differential equation of first order for the variable e. The condition (17) is the simplest gauge condition of such a kind. One can use the gauge conditions of a more general form. In this case the ghosts may become nonfree and the integration over them can give nontrivial measure. Moreover, the expression (11) can depend on a gauge condition choice, but the physical quantities (scattering amplitudes and so on), calculated by means of the propagator, do not depend on this choice.

The measure $M(e)$ appearing in (11) plays an important role. On the first view, it seems that this measure gives a divergency to the path integral (11) on account of the gauge $\dot{e}=0$. However, it is needed to compensate the same divergency which goes from the path integral over x. To comprehend the mechanism of this compensation, it is useful to consider the simple case when an external electromagnetic field is switched off. Taking into account the origin of the path integrals over x and p, namely, the existence of the one additional integration over p, we can make the change of variables

$$\sqrt{e}p\to p,\quad \frac{x-x_{\rm in}-\tau(x_{\rm out}-x_{\rm in})}{\sqrt{e}}\to x\ ,$$

where new trajectories x already have zero values at the end points, $x(0)=x(1)=0$. After integrating over π we get

$$D^c(x_{\rm out},x_{\rm in}) = C\int_0^\infty \frac{de}{e^2}\exp\left[-\frac{i}{2}\left[em^2+\frac{(x_{\rm out}-x_{\rm in})^2}{e}\right]\right], \quad (18)$$

where

$$C=\frac{i}{2}\int Dx\,dp\,\exp\left[\frac{i}{2}\int_0^1(p^2-\dot{x}^2)\right]d\tau = 1/2(2\pi)^2 .$$

One can easily recognize the causal Green's function of the Klein-Gordon equation in expression (18).

III. SPINNING PARTICLE

In this section we are going to write the path-integral representation for the propagator of the spinning particle interacting with an external electromagnetic field. This propagator is the causal Green's function $S^c(x,y)$ of the Dirac equation

$$(\mathcal{P}\!\!\!/-m)S^c(x,y)=-\delta^4(x-y) ,$$
$$\mathcal{P}\!\!\!/=\mathcal{P}_\mu\gamma^\mu , \quad (19)$$
$$[\gamma^\mu,\gamma^\nu]_+=2\eta^{\mu\nu} .$$

For our purpose, it is convenient to deal with the transformed by $\gamma^5=\gamma^0\gamma^1\gamma^2\gamma^3$ function $\tilde{S}^c(x,y)=S^c(x,y)\gamma^5$. The latter obeys the equation

$$(\mathcal{P}\!\!\!/-\gamma^5 m)\tilde{S}^c(x,y)=\delta^4(x-y) ,$$
$$[\gamma^m,\gamma^n]_+=2\eta^{mn} ,$$
$$m,n=0,\ldots,4 , \quad (20)$$
$$\eta^{mn}=\mathrm{diag}(1,-1,-1,-1,-1) .$$

Then, as in Sec. II, we present $\tilde{S}^c(x,y)$ as a matrix element in coordinate space:

$$\tilde{S}^c_{\alpha\beta}(x,y)=\langle x|\tilde{S}^c_{\alpha\beta}|y\rangle , \quad (21)$$

where the spinor indices are written explicitly. For the matrix operator $\tilde{S}^c_{\alpha\beta}$ it follows, from (20), that

$$(\Pi\!\!\!/-\gamma^5 m)\tilde{S}^c=I$$

or

$$\tilde{S}^c=(\Pi\!\!\!/-\gamma^5 m)^{-1} .$$

The operator $(\Pi\!\!\!/-\gamma^5 m)$ is already the pure Fermi one. Nevertheless, there exists a possibility to present it by means of integration of an exponential with an even exponent. To this end we use the general formula (see also Ref. [15]). Namely, let A be an odd (Fermi) operator. Then

$$A^{-1}=\int_0^\infty d\lambda\int e^{i[\lambda(A^2+i\epsilon)+\chi A]}d\chi , \quad (22)$$

where λ is an even variable and χ is an odd (Grassmann) variable, anticommuting by definition with A. The exponent in the integral (22) is already even. The representation (22) is an analog of the Schwinger proper-time representation for an inverse operator convenient in the Fermi case. The pair (λ,χ) can be treated as a superproper time. Using (22), we can write, for the operator $\tilde{S}^c$, taking into account that $(\Pi\!\!\!/-\gamma^5 m)^2=\Pi\!\!\!/^2-m^2$,

$$\tilde{S}^c=\int_0^\infty d\lambda\int e^{-i\mathcal{H}}d\chi , \quad (23)$$

where

$$\mathcal{H}=\lambda(m^2-\Pi\!\!\!/^2)+(\Pi\!\!\!/-\gamma^5 m)\chi = \lambda\left[m^2-\Pi^2+\frac{ig}{2}F_{\mu\nu}\gamma^\mu\gamma^\nu\right]+(\Pi_\mu\gamma^\mu-\gamma^5 m)\chi .$$

Thus, the propagator (21) can be presented in the form

$$\tilde{S}^c(x_{\rm out},x_{\rm in})=\int_0^\infty d\lambda\int\langle x_{\rm out}|e^{-i\mathcal{H}}|x_{\rm in}\rangle d\chi . \quad (24)$$

The next step is to present the matrix element entered in (24) by means of an appropriate path integral. In spite of the operator $\mathcal{H}$ having the γ-matrix structure, it can be done, as usual, by dividing the interval $[x_{\rm in},x_{\rm out}]$ into N parts and inserting $(N-1)$ unit decompositions $\int|x\rangle\langle x|dx$. To gather then all the expressions belonging to each interval $[x_i,x_{i-1}]$, $i=1,\ldots,N$, it is necessary, because of the noncommuting γ matrices, to attribute formally to all the γ matrices their own time τ, according to their disposition, and to introduce the T product of γ matrices, which allows us to deal with them similarly as Grassmann variables. In addition, we transform the ordinary integral over λ into a path integral over trajectories $\lambda(\tau)$ by means of inserting the corresponding δ functions [see (6)], and in the same manner, we transform the integral over a Grassmann variable χ into a Grassmann path integral over trajectories $\chi(\tau)$. [In this way we use the representation for a one-dimensional δ function of a Grassmann variable, $\delta(\chi)=i\int\exp(i\nu\chi)d\nu$, where ν is also a Grassmann variable.] So we get

$$\tilde{S}^c(x_{\text{out}},x_{\text{in}})=T\int_0^\infty d\lambda_0\int d\chi_0\int Dx\,Dp\,D\lambda\,D\pi\,D\chi\,D\nu \times\exp\left\{i\int_0^1\left[\lambda\left[\mathcal{P}^2-m^2-\frac{ig}{2}F_{\mu\nu}\gamma^\mu\gamma^\nu\right]+(\gamma^5 m-\mathcal{P}_\mu\gamma^\mu)\chi+p\dot{x}+\pi\dot{\lambda}+\nu\dot{\chi}\right]d\tau\right\}$$

$$=\int_0^\infty d\lambda_0\int d\chi_0\int Dx\,Dp\,D\lambda\,D\pi\,D\chi\,D\nu \times\exp\left\{i\int_0^1\left[\lambda\left[\mathcal{P}^2-m^2-\frac{ig}{2}F_{\mu\nu}\frac{\delta_l}{\delta\rho_\mu}\frac{\delta_l}{\delta\rho_\nu}\right] +\left[m\frac{\delta_l}{\delta\rho_5}-\mathcal{P}_\mu\frac{\delta_l}{\delta\rho_\mu}\right]\chi+p\dot{x}+\pi\dot{\lambda}+\nu\dot{\chi}\right]d\tau\right\}T\exp\left[\int_0^1\rho_n(\tau)\gamma^n d\tau\Big|_{\rho=0}\right], \tag{25}$$

where

$$x(0)=x_{\text{in}},\quad x(1)=x_{\text{out}},\quad \lambda(0)=\lambda_0,\quad \chi(0)=\chi_0\ ,$$

and five odd (Grassmann) sources $\rho_n(\tau)$, $n=0,\ldots,4$ are introduced, anticommuting with γ matrices by the definition. The quantity $T\exp\,[\int_0^1\rho_n(\tau)\gamma^n d\tau]$ can be written with the help of a Grassmann path integral. To this end, we use the formula, which is one version of the weak theorem (see, for example, Ref. [16]). Let $\hat{\varphi}(\tau)$ be some operators and $F(\hat{\varphi})$ some functional of them. Then

$$TF(\hat{\varphi})=\text{Sym}\exp\left[\frac{1}{2}\int\frac{\delta r}{\delta\varphi(\tau_1)}K(\tau_1,\tau_2)\frac{\delta r}{\delta\varphi(\tau_2)}d\tau_1 d\tau_2\right]F(\varphi)\Big|_{\varphi=\hat{\varphi}}\ ,$$

where

$$K(\tau_1,\tau_2)=T\hat{\varphi}(\tau_1)\hat{\varphi}(\tau_2)-\text{Sym}\hat{\varphi}(\tau_1)\hat{\varphi}(\tau_2)=\tfrac{1}{2}\epsilon(\tau_1-\tau_2)[\hat{\varphi}(\tau_1),\hat{\varphi}(\tau_2)]_-\ ,$$

$$\epsilon(\tau)=\text{sgn}\tau\ .$$

In the case under consideration, $F(\hat{\varphi})=\exp(\int_0^1\hat{\varphi}(\tau)d\tau)$, $\hat{\varphi}(\tau)=\rho_n(\tau)\gamma^n$, and $K(\tau_1,\tau_2)=-\epsilon(\tau_1-\tau_2)\rho(\tau_1)\rho(\tau_2)$. Consequently,

$$T\exp\left[\int_0^1\rho_n(\tau)\gamma^n d\tau\right]=\exp\left[-\frac{1}{2}\int\int_0^1\rho(\tau_1)\epsilon(\tau_1-\tau_2)\rho(\tau_2)d\tau_1 d\tau_2\right]\exp\left[\int_0^1\rho_n(\tau)\gamma^n d\tau\right], \tag{26}$$

because of the concrete structure of the functional, the symbol Sym may be omitted.

The quadratic exponential can be presented by means of the Gaussian path integral over Grassmann variables:

$$\exp\left[-\frac{1}{2}\int\int_0^1\rho(\tau_1)\epsilon(\tau_1-\tau_2)\rho(\tau_2)d\tau_1 d\tau_2\right]=\frac{J(\rho)}{J(0)}\ ,$$
$$J(\rho)=\int\exp\left[\int_0^1[\tfrac{1}{4}\xi(\tau)\dot{\xi}(\tau)-i\rho(\tau)\xi(\tau)]d\tau\,D\xi\right], \tag{27}$$

where $\xi^n(\tau)$, $n=0,\ldots,4$ are odd trajectories, anticommuting with γ matrices and obeying the boundary conditions

$$\xi(0)+\xi(1)=0\ . \tag{28}$$

The latter are necessary to make the path integral invariant under the shifts of integration variables. In addition, we present $\exp[\int_0^1\rho_n(\tau)\gamma^n d\tau]$ in the form

$$\exp\left[\int_0^1\rho_n(\tau)\gamma^n d\tau\right]=\exp\left[i\gamma^n\frac{\partial_l}{\partial\theta^n}\right]\exp\left[-i\int_0^1\rho_n(\tau)\theta^n d\tau\Big|_{\theta=0}\right], \tag{29}$$

where θ^n, $n=0,\ldots,4$ are odd variables anticommuting with γ matrices. Gathering (25)–(29) and making the change of variables

$$\xi(\tau)+\theta=2\psi(\tau)\ ,$$

we get

$$\tilde{S}^c(x_{\rm out},x_{\rm in}) = \exp\left[i\gamma^n \frac{\partial_l}{\partial\theta^n}\right]\int_0^\infty d\lambda_0 \int d\chi_0 \int \exp\left[i\int_0^1 [\lambda(\mathcal{P}^2 - m^2 + 2igF_{\mu\nu}\psi^\mu\psi^\nu) - 2i(m\psi^5 - \mathcal{P}_\mu\psi^\mu)\chi - i\psi_n\dot{\psi}^n + p\dot{x} + \pi\dot{\lambda} + \nu\dot{\chi}]d\tau + \psi_n(1)\psi^n(0)\right] \times Dx\, Dp\, D\lambda\, D\pi\, D\chi\, D\nu\, D\psi\Big|_{\theta=0}, \quad (30)$$

$$x(0)=x_{\rm in},\quad x(1)=x_{\rm out},\quad \lambda(0)=\lambda_0,\quad \chi(0)=\chi_0,\quad \psi(1)+\psi(0)=\theta\,.$$

This is the Hamiltonian path-integral representation for a spinning particle propagator. Integrating over momenta p, we get the corresponding path integral in the Lagrangian form

$$\tilde{S}^c(x_{\rm out},x_{\rm in}) = \exp\left[i\gamma^n \frac{\partial_l}{\partial\theta^n}\right]\int_0^\infty de_0 \int d\chi_0 \int \exp\left[i\left\{\int_0^1\left[-\frac{\dot{x}^2}{2e} - e\frac{m^2}{2} - g\dot{x}\mathcal{A}(x) + iegF_{\mu\nu}(x)\psi^\mu\psi^\nu + i\left[\frac{\dot{x}_\mu\psi^\mu}{e} - m\psi^5\right]\chi - i\psi_n\dot{\psi}^n + \pi\dot{e} + \nu\dot{\chi}\right]d\tau + \psi_n(1)\psi^n(0)\right\} M(e)Dx\, De\, D\pi\, D\chi\, D\nu\, D\psi\Big|_{\theta=0}\right], \quad (31)$$

where the measure $M(e)$ is defined in (12) and the boundary conditions hold:

$$x(0)=x_{\rm in}\,,$$

$$x(1)=x_{\rm out}\,,$$

$$e(0)=e_0\,,$$

$$\chi(0)=\chi_0\,,$$

$$\psi(1)+\psi(0)=\theta\,.$$

The path integrals (30) and (31) can be interpreted in the same manner as in the spinless case. So, the exponent in the Lagrangian path integral (31) we treat as an action of the relativistic spinning particle. Separating the gauge-fixing term, we get the gauge-invariant (reparametrization- and supergauge-invariant) action of the relativistic spinning particle, proposed in Refs. [1,2]:

$$S = \int_0^1\left[-\frac{\dot{x}^2}{2e} - e\frac{m^2}{2} - g\dot{x}\mathcal{A}(x) + iegF_{\mu\nu}(x)\psi^\mu\psi^\nu + i\left[\frac{\dot{x}_\mu}{e}\psi^\mu - m\psi^5\right]\chi - \psi_n\dot{\psi}^n\right]d\tau\,.$$

Thus, using the method under consideration, one can exactly formulate the prescription for the path-integral quantization of the relativistic particle and at the same time to extract the explicit form for the corresponding Lagrangians and for admissible gauge conditions. So, the measure (12), the ranges of integration over zero modes, which are, in fact, e_0 and χ_0, the boundary terms $\psi_n(1)\psi^n(0)$, and the antiperiodic conditions for the spin variables ψ_n, some of which one was forced to introduce in Refs. [5,7], here appear automatically and in a natural way.

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