String theory and black holes

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An exact conformal field theory describing a black hole in two-dimensional space-time is found as an SL(2,ℝ)/U(1) gauged Wess-Zumino-Witten model. For $k=9/4$, the conformal field theory can be regarded as a classical solution of the same system that is probed in the $c=1$ matrix model. The conformal field theory governing the space-time is regular at the Riemannian singularity, but it appears that generic perturbations blow up there. It is argued that the end point of the Hawking black-hole evaporation is the *standard* space-time of the $c=1$ matrix model, which should be regarded as an analog of the extreme Reissner-Nordström black hole of four-dimensional general relativity. The $c=1$ model is thus a model of the quantum mechanics of matter interacting with a black hole.

Two-dimensional current algebra with a noncompact symmetry group is nonunitary, but nonetheless it has been known for some time that some cosets [1,2] of noncompact groups are unitary at least in that one can find unitary representations of the appropriate chiral algebras. This was first found for SL(2,ℝ)/U(1) cosets [3] which have been studied considerably along with their generalizations [4–6]. (It was proposed in [4] that such a model would have an interpretation in two-dimensional target space-time.) There has been, however, little success at forming modular-invariant combinations of left- and right-moving SL(2,ℝ)/U(1) representations, nor has there been much insight about what would be the content of a such a hypothetical theory.

Gauged Wess-Zumino-Witten (WZW) models are a natural framework [2,7–10] for giving a Lagrangian (and hence manifestly modular-invariant) realization of coset models. In this paper we will construct a modular-invariant SL(2,ℝ)/U(1) coset model as a gauged WZW model. What we will get is a conformal field theory describing a black hole in a two-dimensional target space-time. Depending on which subgroup we gauge, we obtain a Euclidean black hole or its Lorentzian continuation. Although it is not necessarily easy, the conformal field theory governing the black hole is presumably more or less exactly soluble because of the extended chiral algebra of SL(2,ℝ)/U(1).

Different forms of the solution we will be discussing have been obtained (without the black-hole interpretation) by Bardakci *et al.* and by Rocek *et al.* (who found another real form of the solution in studying SU(2)/U(1) and SU(2)×U(1) WZW models, respectively [11,12]), and by Mandal *et al.* [13] (who found the part of the space-time exterior to the horizon by studying the $O(\alpha')$ β-function equations for string theory with a two-dimensional target space). The black hole is also reminiscent of instanton and soliton solutions found by Callan, Harvey, and Strominger [14]. Black-hole solutions of limiting low-energy field theories derived from string theory in four (or more) dimensions have been analyzed in several papers [15–18].

Although we will consider in this paper only the bosonic theory, there is also a superconformal version of the black-hole solution. It can be found by gauging a supersymmetric SL(2,ℝ) WZW model. (One starts with an $N=1$ WZW model, but the resulting coset model turns out to have $N=2$ supersymmetry, as in the construction of Kazama and Suzuki [19].) The superconformal black hole is a kind of analytical continuation of the $N=2$ discrete series, which can be described by an analogous supersymmetric SU(2)/U(1) coset.

The central charge of the SL(2,ℝ)/U(1) model is [3]

$$c=\frac{3k}{k-2}-1=2+\frac{6}{k-2}\ . \tag{1}$$

[This formula originates as follows. The central charge of SU(2) current algebra at level k is $3k/(k+2)$. Analytically continuing from SU(2) to SL(2,ℝ) would by itself not change the value of c extracted from Feynman diagrams, since minus signs in vertices cancel against minus signs in propagators. However, in passing from SU(2) to SL(2,ℝ), one takes $k\to-k$ since we want the SL(2,ℝ) manifold to have signature $(-++)$ and not $(+--)$. This gives the $3k/(k-2)$. Finally, one subtracts 1 for the gauged U(1).] There are two interesting regions. For $k\to\infty$, the σ model describing the black hole is weakly coupled and can be understood semiclassically (in the world-sheet sense). To obtain a bosonic string background, one must then adjoin additional matter of c near 24. Alternatively, for $k=\frac{9}{4}$, the black hole has $c=26$ and can be considered as a bosonic string background in its own right. As we will see, it is asymptotic at spatial infinity to the two-dimensional space-time that appears in the standard $c=1$ matrix model [22–25]. (The latter model superficially has a one-dimensional target space, but in fact is naturally understood [20,21] in terms of a two-dimensional target space-time, the second dimension being the Liouville mode.) We will suggest later that the black hole plays a key role in the physics of the $c=1$ model. This may be related to the fact that Liouville theory and some of its generalizations arise as other SL(2,ℝ) cosets [26–28].

The ungauged SL(2,ℝ) WZW action is

$$L(g)=\frac{k}{8\pi}\int_\Sigma \sqrt{h}\; h^{ij}\mathrm{Tr}(g^{-1}\partial_i g g^{-1}\partial_j g)+ik\Gamma\ . \quad (2)$$

Here Σ is a Riemann surface with metric tensor h, $g:\Sigma\rightarrow SL(2,\mathbb{R})$ is the field variable of the model, Tr is the trace in the two-dimensional representation of $SL(2,\mathbb{R})$, k is a positive real number [as noted above, the sign of the Lagrangian has been reversed compared to the SU(2) case], and Γ is the Wess-Zumino term [29]. The latter can be described as follows [30]. If B is a three-manifold with boundary Σ, and we pick an extension of g to a map from B to $SL(2,\mathbb{R})$, which we also call g, then

$$\Gamma(g)=\frac{1}{12\pi}\int_B \mathrm{Tr}\, g^{-1}dg\wedge g^{-1}dg\wedge g^{-1}dg\ . \quad (3)$$

As $H^3(SL(2,\mathbb{R}),\mathbb{R})=0$, Γ is independent of the choices that have been made. Because of the indefinite signature of the $SL(2,\mathbb{R})$ manifold, (2) does not lead to a unitary conformal field theory. It is tempting to regard it as a string solution in a three-dimensional Lorentzian world, but the absence of an analog of the no-ghost theorem in this situation [31,4] discourages this interpretation.

The Lagrangian (2) has a global $SL(2,\mathbb{R})\times SL(2,\mathbb{R})$ symmetry corresponding to $g\rightarrow agb^{-1}$, with $a,b\in SL(2,\mathbb{R})$. Usually it is possible to gauge an arbitrary subgroup of the global symmetry group of a theory, but for WZW models, that is not possible, because of the peculiar nature of the Wess-Zumino term. Only subgroups that obey a certain condition of anomaly cancellation can be gauged. We first wish to consider the gauging of an anomaly-free subgroup chosen to remove the negative signature mode of g so as to get a Euclidean signature conformal field theory. We consider the U(1) subgroup generated infinitesimally by

$$\delta g=\epsilon\left[\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}g+g\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}\right]\ . \quad (4)$$

To gauge this symmetry we introduce an Abelian gauge field A with

$$\delta A_i=-\partial_i\epsilon\ . \quad (5)$$

The gauge-invariant generalization of the WZW action is in local complex coordinates $z,\bar{z}$ (here d^2z denotes the measure $|dz\,d\bar{z}|$)

$$L'(g,A)=L(g)+\frac{k}{2\pi}\int d^2z\left\{A_{\bar{z}}\mathrm{Tr}\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}g^{-1}\partial_z g+A_z\mathrm{Tr}\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}\partial_{\bar{z}}g g^{-1}\right.$$
$$\left.+A_zA_{\bar{z}}\left[-2+\mathrm{Tr}\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}g\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}g^{-1}\right]\right\}\ . \quad (6)$$

Because the gauge group has been chosen to act freely, one can conveniently fix the gauge by gauging away one component of g (such a gauge choice is often called a unitary gauge). The gauge invariance can be precisely fixed by setting

$$g=\cosh r+\sinh r\begin{bmatrix}\cos\theta & \sin\theta\\ \sin\theta & -\cos\theta\end{bmatrix}\ . \quad (7)$$

As the Lagrangian is quadratic in A, and the quadratic piece is invertible and nonderivative, A can be integrated out, to give

$$I(r,\theta)=\frac{k}{2\pi}\int d^2z(\partial_z r\partial_{\bar{z}}r+\tanh^2 r\,\partial_z\theta\partial_{\bar{z}}\theta)$$
$$=\frac{k}{4\pi}\int d^2x\sqrt{h}\; h^{ij}(\partial_i r\partial_j r+\tanh^2 r\,\partial_i\theta\partial_j\theta)\ . \quad (8)$$

(The Wess-Zumino term is a total derivative in this gauge and has been dropped.)

As $\tanh r\rightarrow 1$ for large r, it is easy to see that the target space metric of this theory,

$$ds^2=\frac{k}{2}d\sigma^2\ ,\text{with } d\sigma^2=[(dr)^2+\tanh^2 r(d\theta)^2]\ , \quad (9)$$

has the form (Fig. 1) of a semi-infinite cigar, asymptotic for $r\rightarrow\infty$ to $\mathbb{R}\times S^1$ with a flat metric. In general, a d-dimensional Euclidean black hole is asymptotic to $\mathbb{R}^{d-1}\times S^1$, so this space-time is a candidate for interpretation as such a black hole. Before discussing this further, let us note the following conundrum: the metric (9) is certainly not flat, and therefore it is not Ricci flat (as the two concepts coincide in two dimensions), so how can the β function vanish even in the one-loop approximation, which is valid for large k? In problems of roughly this type [32] (and in this precise problem for a different real form of the metric [12]) it is known that a finite correction coming from the measure in the integration over A gives rise to a target space dilaton field, a more accurate representation of the classical action for r and θ being

$$I(r,\theta)=\frac{k}{4\pi}\int d^2x\sqrt{h}\; h^{ij}(\partial_i r\partial_j r+\tanh^2 r\,\partial_i\theta\partial_j\theta)$$
$$-\frac{1}{8\pi}\int d^2x\sqrt{h}\;\Phi(r,\theta)R^{(2)}\ , \quad (10)$$

for some function Φ on the target space. ($R^{(2)}$ is the curvature of the world-sheet metric h.) Without imitating

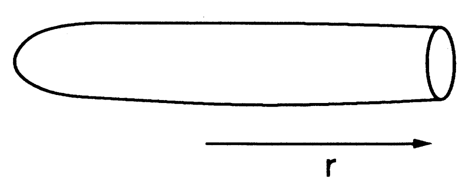

FIG. 1. A semi-infinite cigar.

the analysis of [32], Φ can be determined by noting that it must obey the one-loop equation [33]

$$R_{ab} = D_a D_b \Phi \ . \tag{11}$$

(R_{ab} is the Ricci tensor of the target space.) This gives

$$\Phi = 2 \ln \cosh r + \text{const} \ . \tag{12}$$

The possibility of adding a constant to Φ amounts to the freedom to change the string coupling constant. This additive constant plays an important role, as we will see later. Φ grows linearly for large r, with a sign such that the string coupling constant vanishes for $r \to \infty$.

If one assumes that (10) represents an exact conformal field theory for all k, then the central charge can be computed by going to the asymptotically flat region of large r. Including in the standard fashion [34] the shift in c due to the correction in the stress tensor that comes from the asymptotically linear dilaton field (12), one gets $c = 2 + 6/k$. This agrees with the exact answer (1) only to within an error of order $1/k^2$. This must mean that there are further corrections to the Lagrangian (10), presumably coming from additional contributions to the result obtained in integrating out A. Our point of view is that the exact black-hole quantum field theory is represented by the gauged WZW Lagrangian (6). The transformation to unitary gauge is difficult to carry out exactly, but is very useful for getting a qualitative picture of the physics. Unfortunately, the value $k = \frac{9}{4}$ at which $c = 26$ is in the strong-coupling region, and without a better understanding, it is difficult to determine such properties as the radius of the circle at infinity.

We will express some of the formulas in terms of not k but $k' = k - 2$. There is no loss in making this substitution, since perturbation theory in $1/k$ can be rearranged as perturbation theory in $1/k'$. One advantage in expressing the formulas in terms of k' rather than k is that this gives the right results for things that are controlled by the asymptotic behavior of the dilaton field (since this is known in view of its relation to the central charge). The simple substitution of k by k' in (10), is not, however, guaranteed to give the right value of the asymptotic radius of the cigar.

Let us now compare to Liouville theory. Liouville theory coupled to $c = 1$ matter has a two-dimensional space-time interpretation [20,21] with coordinates ϕ, θ and a world-sheet action that reads, in part,

$$I = \frac{1}{4\pi\alpha'} \int d^2x \sqrt{h} \, (h^{ij} \partial_i \phi \partial_j \phi + h^{ij} \partial_i \theta \partial_j \theta) - \frac{1}{8\pi} \int d^2 x \sqrt{h} \, 4\alpha'^{\,-1/2} \phi R^{(2)} \ . \tag{13}$$

ϕ is the Liouville field; the string coupling constant is weak for $\phi \to \infty$ and strong for $\phi \to -\infty$. (To compare to Ref. [20], one should set $\alpha' = 2$, while in the rest of this paper we will set $\alpha' = \frac{1}{2}$.) It is convenient (and, according to matrix model results, possibly necessary) to add a tachyon potential

$$\Delta I = \mu \int_\Sigma d^2x \sqrt{h} \, \exp(-2\phi/\sqrt{\alpha'}) \tag{14}$$

to the action to suppress the strongly coupled region at $\phi \to -\infty$. Comparing (10) and (13), we see that, for large r, r can be identified with the Liouville field ϕ. But a cutoff to dynamically suppress the region of $r \to -\infty$ is unnecessary, since this region is missing from the black hole space-time. Because of this, the region $k > \frac{9}{4}$, which corresponds to what is usually regarded as the forbidden region of $c > 1$ in Liouville theory, makes perfect sense for the Euclidean black hole. In a sense, replacing standard Liouville theory with the Euclidean black hole could be regarded as a way of getting past the Liouville theory barrier at $c = 1$.

To understand the relation between the black hole at $k = \frac{9}{4}$ and the $c = 1$ matrix model, it is useful to first understand the general relation between critical strings with a D-dimensional target space and noncritical strings with a $(D-1)$-dimensional target space. A noncritical string in $D-1$ dimensions (i.e., two-dimensional gravity without conformal invariance, coupled to a $(D-1)$-dimensional σ model) can always be interpreted as a critical string in D dimensions with the Liouville field as the Dth dimension. It is obvious though not always emphasized that the reverse mapping is possible only under special circumstances. This is evident from any clear statement of what the reverse map means when it exists. Consider a D-dimensional target space model with

$$I_0 = \frac{1}{2\pi} \int d^2x \sqrt{h} \; h^{ij} \partial_i X^a \partial_j X^b g_{ab}(X) \ . \tag{15}$$

If one considers the naive conformal transformation law

$$\delta h_{ij} = \epsilon h_{ij}, \quad \delta X^a = 0 \ , \tag{16}$$

then one finds at the one-loop level that the effective action is not invariant; rather,

$$\delta I_{\text{eff}} = -\frac{1}{8\pi} \int d^2x \sqrt{h} \; h^{ij} \partial_i X^a \partial_j X^b R_{ab} \ . \tag{17}$$

Suppose, however, that there is a function Φ on the target space such that $R_{ab} = D_a D_b \Phi$. Then [33] one adds to the action a counterterm

$$I_1 = -\frac{1}{8\pi} \int_\Sigma d^2x \sqrt{h} \; R^{(2)} \Phi \ , \tag{18}$$

and one finds that, purely at the classical level,

$$\delta I_1 = \frac{1}{8\pi} \int d^2x \sqrt{h} \; \epsilon (h^{ij} \partial_i X^a \partial_j X^b D_a D_b \Phi + D_a \Phi \cdot h^{ij} D_i D_j X^a) \ . \tag{19}$$

Combining (17) and (19) we see that if we start with the action $I = I_0 + I_1$, then up to this order the effective action is conformally invariant modulo the equation of motion $h^{ij} D_i D_j X^a = 0$. Instead of speaking of conformal invariance modulo the equations of motion, it is much better to modify the conformal transformation laws to achieve invariance off shell. The modified transformation laws, which are adequate up to this order, are

$$\delta h_{ij} = \epsilon h_{ij}, \quad \delta X^a = \frac{\epsilon}{8} g^{ab} D_b \Phi \ . \tag{20}$$

There are presumably further corrections in higher order.

From (20) one can determine the conditions under which a D-dimensional critical string theory can be deduced from a $(D-1)$-dimensional noncritical string theory. If the gradient of the dilaton is everywhere timelike, then one can use conformal transformations to gauge away a time coordinate, by fixing a gauge in which, say, $X^0=0$. In this way time is eliminated in favor of the determinant of the two-dimensional metric. Similarly, if the gradient of the dilaton is everywhere spacelike, then one can use conformal transformations to fix a gauge such as $X^1=0$, eliminating one of the space coordinates in favor of the determinant of the metric. However, this identification of a D-dimensional critical string theory with a $(D-1)$-noncritical string theory will fail in the (typical) case in which the gradient of the dilaton is neither everywhere spacelike nor everywhere timelike.

For instance, in the case of the Euclidean black hole, the transformation laws in this approximation are

$$\delta r=\epsilon\frac{1}{2k'}\tanh r, \quad \delta\theta=0 \ . \tag{21}$$

For $r \gg 0$ this is approximately the Liouville transformation law $\delta r=\epsilon\times\text{const}$ which would assert that a power of e^r can be identified with the determinant of the metric. In that region, r can be gauged away, and the system can be described as noncritical two-dimensional gravity coupled to a single scalar field θ. Near $r=0$, however, the attempt to gauge away r in favor of the determinant of the metric is not valid, since $\nabla\Phi=0$ at $r=0$. Hence, though for $k=\frac{9}{4}$ the Euclidean black hole is a classical solution of the same theory that is usually studied in the $c=1$ matrix model, it cannot actually be regarded as a theory of $c=1$ matter coupled to two-dimensional gravity. Later, when we construct the Lorentz signature black hole, it will be apparent that the situation is much worse: the gradient of the dilaton is spacelike outside the horizon, and timelike inside, so if one tried to represent this system as noncritical two-dimensional gravity coupled to $c=1$ matter, one would obtain a dreadful mess.

Now let us discuss the analytic continuation of the black hole to a Lorentz signature. Naively, one simply sets $\theta=it$, whereupon

$$d\sigma^2=dr^2-\tanh^2 r\, dt^2 \ . \tag{22}$$

Notice that in this continuation, it is θ, not r, that is rotated, so we are committed to interpreting the "Liouville" coordinate r as a spatial coordinate. (For the $c=1$ model considered in isolation, the Liouville mode can be given a spatial interpretation favored in [20] or a time interpretation assumed by some other authors. The black-hole physics corresponds to the spatial interpretation. The black hole does not have another analytic continuation in which r would be rotated instead of ϕ.) Equation (22) appears to have a singularity at $r=0$, but this must be purely a coordinate singularity, since the scalar curvature

$$R=\frac{4}{\cosh^2 r} \tag{23}$$

is regular at $r=0$. The analytic continuation past the coordinate singularity can be found by imitating the similar procedure for the Schwarzschild solution [35]. After setting

$$r'=r+\ln(1-e^{-2r}) \ , \tag{24}$$

we get

$$d\sigma^2=\tanh^2 r[(dr')^2-dt^2] \ . \tag{25}$$

With

$$2v=e^{r'+t}, \quad 2u=-e^{r'-t} \ , \tag{26}$$

so that

$$\cosh^2 r=1-uv, \quad \sinh^2 r=-uv \ , \tag{27}$$

we get

$$d\sigma^2=-\frac{du\, dv}{1-uv} \ . \tag{28}$$

Now we can easily see the essential features of the space-time. The original r,t asymptotically flat half-space corresponds to region I in Fig. 2. The coordinate singularity at $r=0$ corresponds to the two lines $u=0$ and $v=0$. The physical singularity is at $uv=1$ [where the curvature blows up according to (23)] and consists of a past and future branch. The future branch is the black-hole singularity, from which no signal can cross the horizon to an observer in region I. The past branch is a naked singularity. There is also a second asymptotically flat half-space, region III. No signal can propagate from region I to region III or vice versa. Regions I–IV, and the causal relations between them, have precise analogs in the (positive mass) Schwarzschild solution in four dimensions. Finally, if one considers the space-time (28) for $uv>1$, one finds two additional asymptotically flat half-spaces, regions V and VI. Time flows sideways in these spaces [because the factor $1/(1-uv)$ changes sign in crossing $uv=1$]. The singularity at $uv=1$, rather than being in the past or future, as appears to be the case to an ob-

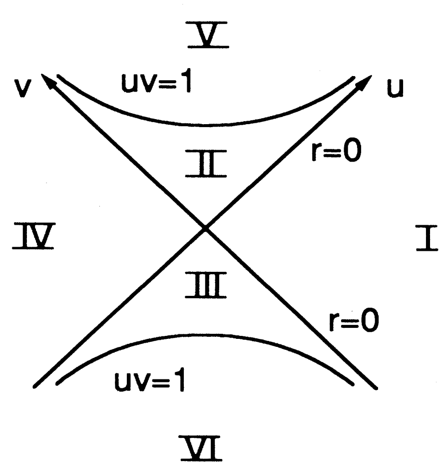

FIG. 2. The analytically continued black-hole space-time.

server in one of regions I–IV, appears at the end of the spatial world to an observer in region V or VI and is a naked singularity. From the analog of the ADM definition of mass that we will compute later, the reader can verify that regions V and VI are space-times with negative mass; indeed, they are analogs of the negative mass Schwarzschild solution in four dimensions. As has occasionally been noted [36], such regions with negative mass naked singularities appear in the four-dimensional Schwarzschild solution if one continues past the singularity.

As in the case of the four-dimensional Schwarzschild solution, this space-time (or more exactly regions I–IV) can be considered to develop from smooth initial data on a nonsingular Cauchy hypersurface such as $u=-v$. In contrast with the four-dimensional case in which such a hypersurface necessarily has an exotic topology, here the initial value surface is just a copy of $\mathbb{R}$ and so is not distinct topologically from a possible initial value surface in the standard two-dimensional space-time of the $c=1$ model. What is exotic about the initial data is the following. The dilaton field $\Phi=\ln\cosh^2 r=\ln(1-uv)$ *grows* at each end of the spatial world. If, therefore, we parametrize the initial value surface $u=-v$ by an arc-length coordinate r'' [so that its metric is $(dr'')^2$], then for $r''\gg 0$, r'' looks like the Liouville field, while for $r''\ll 0$, it looks like $-r''$ is the Liouville field. Obviously, there is no simple way to straighten out the disagreement between observers at the two ends about whether r'' or $-r''$ is the approximate Liouville field. Collapse to a black hole is the result. It seems plausible that any initial conditions with the dilaton growing at each end will lead to a black-hole collapse.

Now that we have exhibited the two asymptotically flat regions of the system, perhaps it is worth mentioning that if one wishes to have a conformal field theory describing a world with only one such region, one can do this by taking an orbifold of this space-time, dividing by the $\mathbb{Z}_2$ group $u\leftrightarrow v$.

Instead of obtaining the Lorentz signature black hole by a purely formal analytic continuation from Euclidean space, one can obtain it directly as a conformal field theory by gauging a different subgroup of $SL(2,\mathbb{R})$. We consider the noncompact one parameter symmetry group generated by

$$\delta g=\epsilon\left[\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}g+g\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\right], \tag{29}$$

and we parametrize the group manifold by

$$g=\begin{pmatrix}a & u\\ -v & b\end{pmatrix},\quad \text{with } ab+uv=1 . \tag{30}$$

The gauged WZW action turns out to be

$$\begin{aligned}L=&-\frac{k}{4\pi}\int d^2z(\partial_z u\partial_{\bar z}v+\partial_{\bar z}u\partial_z v+\partial_z a\partial_{\bar z}b+\partial_{\bar z}a\partial_z b)\\&+\frac{k}{2\pi}\int d^2z[A_{\bar z}(b\partial_z a-a\partial_z b+u\partial_z v-v\partial_z u)+A_z(b\partial_{\bar z}a-a\partial_{\bar z}b-u\partial_{\bar z}v+v\partial_{\bar z}u)+A_zA_{\bar z}(4-4uv)\\&\qquad+\ln a(\partial_z u\partial_{\bar z}v-\partial_{\bar z}u\partial_z v)]\end{aligned} \tag{31}$$

with gauge invariance

$$\begin{aligned}&\delta a=2\epsilon a,\quad \delta b=-2\epsilon b ,\\&\delta u=\delta v=0,\quad \delta A_i=-\partial_i\epsilon .\end{aligned} \tag{32}$$

The peculiar term involving $\ln a$ comes from the Wess-Zumino term and can be rewritten in various ways using $ab+uv=1$. For example,

$$\int d^2z\,\ln a(\partial_z u\partial_{\bar z}v-\partial_{\bar z}u\partial_z v)=\int d^2z\,\ln u(\partial_z a\partial_{\bar z}b-\partial_{\bar z}a\partial_z b) \tag{33}$$

after discarding a total derivative.

Further development requires fixing a gauge. In the region with $1-uv>0$, it is natural to fix the gauge $a=b$. [If $1-uv>0$, then $ab+uv=1$ implies $a,b>0$ or $a,b<0$. Both possibilities occur on the $SL(2,\mathbb{R})$ manifold, so if we take the $SL(2,\mathbb{R})$ picture literally, the full space-time contains two copies of each of regions I–IV, and, for a similar reason, two copies of regions V, VI. If one considers the universal cover of the $SL(2,\mathbb{R})$ manifold, one would get infinitely many copies. It is not clear if these facts have any real relevance to the physics of the black hole.] In that gauge, after eliminating that auxiliary field A, one gets

$$L=-\frac{k}{4\pi}\int d^2x\sqrt{h}\,\frac{h^{ij}\partial_i u\partial_j v}{1-uv}, \tag{34}$$

thus exhibiting regions I–IV of the Lorentzian black hole directly from a gauged WZW model. We will call this gauge choice (i). In the region $1-uv<0$, we pick gauge (ii) with $a=-b$. Upon eliminating A we get back the same formula (34), but now in regions V and VI. The question now arises of what happens near $uv=1$. The Lagrangian (33) has absolutely no pathology there. The problem comes entirely from the gauge choices $a=b$ and $a=-b$ which are both invalid at $uv=1$. Near $uv=1$ one has $v=u^{-1}+\cdots$. If we write $u=e^w$, $v=e^{-w}$, then the Lagrangian near what in general relativity appears to be the singularity at $uv=1$ takes the form

$$\begin{aligned}L=&-\frac{k}{4\pi}\int d^2x\sqrt{h}\;h^{ij}D_iaD_jb\\&+\frac{ik}{2\pi}\int d^2x\sqrt{h}\;w\epsilon^{ij}F_{ij}+\cdots ,\end{aligned} \tag{35}$$

where the ellipsis denotes higher-order terms in an expansion near $uv=1$, and $F_{ij}=\partial_i A_j - \partial_j A_i$. This Lagrangian is perfectly well behaved. The Lagrangian for the w-A system by itself is a topological field theory which can be regarded as the dimensional reduction of three-dimensional Abelian Chern-Simons theory (with w as the third component of A). If the Lagrangian were precisely as written, the integration over w (as explained, for instance, in [37]) would give a δ-function setting $F=0$, after which A could be gauged away locally and (except for global effects) the a-b system would be free. The higher-order terms not written in (35) make such a procedure awkward. Instead, one can study the theory in a gauge such as gauge (iii), $\partial_i A^i=0$, and easily verify that there is nothing singular or pathological about the propagators or vertices near $uv=1$. Gauge (iii) makes the fact that the conformal field theory is nonsingular perfectly manifest. It can also be used elsewhere in the space-time. Its only drawback is that it fails to exhibit the fact that away from the singularity the theory has an interpretation with a *two*-dimensional target space [as opposed to the three independent scalars and one gauge field present in (34)].

It is amusing to compare this situation with the motivation for studying topological field theories that was proposed some time ago [38]. It was felt that space-time and world-sheet topological field theories might describe an "unbroken phase" of space-time and world-sheet general relativity, with a breakdown of the Riemannian concepts, and that such a phase might be the key to understanding quantum gravity and string theory. The singularity of a black hole is one place where one might expect Riemannian notions to fail, and we have indeed found a conformal field theory in which the "physics" at the singularity cannot be understood in two-dimensional Riemannian geometry, and in which a certain trivial-looking topological gauge theory is an essential piece.

At this point, one might be tempted to think that the "singularity" of the black hole is completely spurious and purely a result of a bad gauge choice. I will, however, argue that this is not the right interpretation. First of all, let us make an analogy with black holes in four dimensions. The Schwarzschild solution in four dimensions has all of the six regions in Fig. 2. For a more realistic astrophysical black hole which forms from a spherically symmetric collapsing star, some of the regions are missing, but one still has parts of regions I, II, and V. In particular, one still has the question of whether signals incident on the singularity in region II should somehow be continued beyond the singularity into region V. What makes this seem implausible is that (as in our two-dimensional model), time is flowing sideways in region V, and consequently region V cannot be considered to be in the future of region II. As a result, regardless of which way one considers to be the flow of time in region V, if signals can flow back and forth across the border between regions II and V, a future-going signal in region II could cross into region V and return as a past-going signal in region II, as shown in Fig. 3. This is somewhat analogous to having a closed timelike loop. As is usual with closed timelike loops, consideration of such phenomena will lead to pathologies unless one assumes the absence of life (and of macroscopic structures of all kinds). [In a space-time with closed timelike curves, solving physical equations requires putting restrictions on the initial data that are analogous to eigenvalue equations (a wave propagating about a closed timelike loop must be single valued) and are likely to be incompatible with the existence of complex structures. This is why, in the presence of closed timelike loops, if one assumed the existence of life one reaches contradictions like the possibility of killing one's own remote ancestors.] Indeed, given the initial data in region I (far in the past before a trapped surface forms), one cannot without solving the problem of free will predict whether a black hole will form; this could be prevented by a determined human effort to disperse the infalling matter to infinity. Thus a possible complex civilization in region V would have its very existence depend on a decision made in the "future" in region I.

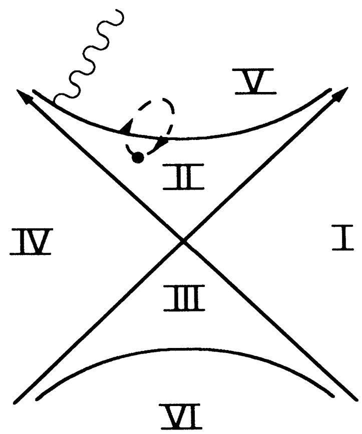

FIG. 3. The region of initial (or final) data in region V is indicated, along with a hypothetical signal from region II to region V and back.

The contradiction cannot be stated so sharply in two dimensions, since, for instance, the formation of the black hole in region II is probably determined by the "topology" of the initial conditions (the clash as to whether r'' or $-r''$ is the Liouville mode). Nevertheless, and despite the fact that the conformal field theory is nonsingular at $uv=1$, I believe that trying to use the continuation to regions V and VI as a way to resolve the physical questions associated with black holes is unattractive, for the reasons indicated above.

To try to give a technical justification for not taking the continuation across $uv=1$ too seriously, let us consider the propagation of a small "tachyon" (the expression is a misnomer in $D=2$) disturbance in the black-hole background. Because of the underlying $SL(2,\mathbb{R})/U(1)$ chiral algebra, it may be possible to solve exactly for the tachyon vertex operators, but we will simply consider perturbation theory in $1/k$. This approximation is not necessarily reliable, as it might break down in the region of high curvature near $uv=1$. We will comment on that later. The tachyon field $T(u,v)$, to lowest order, is governed by an effective action in space-time:

$$L(T)=\int d^2X\sqrt{g}\; e^{\Phi}(g^{ij}\partial_i T\partial_j T-8T^2)\ . \qquad (36)$$

In the black-hole space-time, this is

$$L(T)=\int du\; dv \left[(1-uv)\partial_u T\partial_v T-\frac{16}{k'}T^2\right]\ . \qquad (37)$$

The tachyon field equations are therefore

$$\partial_u((1-uv)\partial_v T)+\partial_v((1-uv)\partial_u T)+\frac{16}{k'}T=0\ . \qquad (38)$$

Using the symmetry of the space-time under $u\to e^t u, v\to e^{-t}v$, one can look for solutions in the form $T(u,v)=(u/v)^\gamma f(uv)$. Inserting this ansatz in (38), one finds that f obeys

$$\left[x\frac{d^2}{dx^2}+\frac{d}{dx}+\cdots\right]f(x)=0\ , \qquad (39)$$

where $x=1-uv$ and the ellipsis denotes terms that are softer (more powers of x or less powers of d/dx) near $x=0$. This second-order equation has two linearly independent solutions. One is regular near $x=0$ and can be expanded as a power series, $f(x)=1+\sum_{n=1}^{\infty}a_n x^n$, and the other has a logarithmic singularity, $f(x)=\ln x+O(x\ln x)$.

If this result is not an artifact of the $1/k$ expansion, it means that generic initial data posed on a smooth initial value surface will propagate forward to a singularity at $uv=1$. In this case, the region $uv=1$, though nonsingular in the original black-hole solution, will be unstable and will become a physical singularity under a generic perturbation. Such phenomena are known in general relativity; for instance, the "inner horizon" of the Reissner-Nordström solution is a nonsingular locus where it is believed that a singularity would form under a generic perturbation.

Naively, the dilaton formula $\Phi=\ln(1-uv)$ appears to suggest that the string coupling constant, which is weak at spatial infinity, blows up at $uv=1$. For the exact black-hole solution, this is certainly false; in gauge (iii) it is obvious that the string coupling constant is not anomalously strong at $uv=1$. It must be that in gauge (i) or (ii) the determinant of fluctuations in u and v blows up for $uv\to 1$ in such a way as to cancel the effects of the dilaton. It may be that under a generic perturbation of the black hole, this cancellation is ruined and the string coupling constant becomes strong near the singularity.

The claim that a generic perturbation blows up at $uv=1$ can be checked in gauge (iii), where there is no reason for the $1/k$ expansion to be breaking down at $uv=1$. In gauge (iii), the equation for the tachyon would be $\partial_a\partial_b T+\cdots=0$, which permits in addition to a regular solution the singular solution $T=\ln(ab)=\ln(1-uv)$ that we found earlier. This, together with the potential pathologies cited earlier, is fairly convincing evidence that the physical problems of black holes should not be solved by trying to exploit the fact that the conformal field theory is nonsingular at $uv=1$.

Before proposing another, speculative, interpretation, we should address the following question. What is the mass of the black hole? To answer this we recall that to lowest order in world-sheet perturbation theory, the graviton-dilaton system can be described by a space-time effective action

$$L=\int d^2X\sqrt{g}\; e^{\Phi}\left[R+g^{ik}\partial_i\Phi\partial_j\Phi+\frac{8}{k'}\right]\ . \qquad (40)$$

(The overall normalization could be changed by adding a constant to Φ, and it may be that a different choice would be more convenient.) The additive constant $8/k'$ originates from the familiar $D-26$ of the bosonic string. The gravitational and dilaton field equations derived from this action are, respectively,

$$0=-D_iD_l\Phi+g_{il}\left[D_kD^k\Phi+\frac{1}{2}D_k\Phi D^k\Phi-\frac{4}{k'}\right]\ ,$$
$$0=2D_kD^k\Phi+D_k\Phi D^k\Phi-R-\frac{8}{k'}\ . \qquad (41)$$

These equations have a flat solution in a world with space and time coordinates ρ and τ with

$$ds^2=d\rho^2-d\tau^2,\quad \Phi=\rho\sqrt{8/k'}\ . \qquad (42)$$

This is the standard Liouville solution. Of course, the black hole is asymptotic to this solution with $r\sim\rho\sqrt{2/k'}$ and $t\sim\tau\sqrt{2/k'}$. As the Liouville solution is invariant under time translations, there is a conserved energy in the fluctuations about it. Actually, in general relativity a conserved energy can be defined in each asymptotically flat end of space; the idealized flat solution has two ends. We are only interested in the $\rho\to+\infty$ end, since only this one appears in the black-hole space-time. (The other end of the black hole is a second copy of $\rho\to+\infty$.) We need to find the analog of the Arnowitt-Deser-Misner (ADM) formula for energy in general relativity at the $\rho\to+\infty$ end.

There is a standard computational procedure for finding the ADM formula. Denote the right-hand side of the gravitational field equation [the first equation in (41)] as Q_{ij}. Consider a solution that is asymptotic to the flat-space solution, with

$$\Phi=\rho\sqrt{8/k'}+\varphi,\quad g_{ij}=\eta_{ij}+h_{ij}\ , \qquad (43)$$

where φ, h vanish for $\rho\to\infty$. (Here η is the flat-space metric, corresponding to the line element $d\rho^2-d\tau^2$. It is used to raise and lower indices in the linearized expressions.) Let q_{ij} be the linearization of Q_{ij}, so $Q_{ij}=q_{ij}+$terms of higher order. The gravitational Bianchi identities imply that if the vector field v^i generates an asymptotic symmetry of the space-time, then $S_i=q_{ij}v^j$ is a conserved current asymptotically. The corresponding conserved charge density S^0 is always a total divergence, so its integral, the conserved charge, can be measured as a surface term at $\rho=\infty$. (This in turn implies that the linearized approximation to the equations is good enough to use in proving that the charge is conserved.) The conserved momenta and angular momenta of general relativity can be obtained in this way.

In the case at hand, the only asymptotic symmetry is the time translation generator, $v^i=\delta^{i0}$, so the only conserved quantity will be the total energy or mass. Carrying out this procedure, the conserved current of the linearized theory is

$$S_i=q_{i0}=e^{\rho\sqrt{8/k'}}\left[-\partial_i\partial_0\varphi-\delta_{i0}(\partial_k\partial^k\varphi+\sqrt{8/k'}\partial_1\varphi)+\frac{4}{k'}\delta_{i0}h_{11}+\sqrt{2/k'}[\partial_i h_{01}+\partial_0 h_{i1}-\partial_1 h_{i0}+\delta_{i0}(2\partial^k h_{k1}-\partial_1 h_{kl}\cdot\eta^{kl})]\right] . \tag{44}$$

(Indices 0 and 1 refer to τ and ρ, respectively.) The energy density is

$$S_0=-\frac{\partial}{\partial\rho}\left\{e^{\rho\sqrt{8/k'}}\left[\partial_1\varphi+\left[\frac{2}{k'}\right]^{1/2}h_{11}\right]\right\} . \tag{45}$$

The total mass measured at the $\rho=\infty$ end is therefore

$$M=\left\{-e^{\rho\sqrt{8/k'}}\left[\frac{\partial}{\partial\rho}\varphi+\left[\frac{2}{k'}\right]^{1/2}h_{11}\right]\right\}_{\rho=\infty} . \tag{46}$$

Now we can calculate the mass of the black hole. For a black hole with $ds^2=(k'/2)(dr^2-\tanh^2 r\, dt^2)$ and $\Phi=2\ln\cosh r+a$, to ensure that $\Phi=\sqrt{8/k'}\rho+\varphi$, where φ vanishes at infinity, we take $r=\rho\sqrt{2/k'}-(a/2)+\ln 2$, and then one finds

$$\varphi=\frac{e^a e^{-\rho\sqrt{8/k'}}}{4},\quad h_{11}=0 . \tag{47}$$

The mass then comes out to be

$$M=\sqrt{2/k'}\exp(a) . \tag{48}$$

Thus, we see the significance of the trivial-looking possibility of adding a constant to the dilaton field: this parameter determines the mass of the black hole.

It is important to stress that adding a constant to Φ does not change the physical state at $\rho\to\infty$ (since asymptotically a constant added to Φ can be absorbed in a translation of ρ). This is why the black hole with $\Phi=2\ln\cosh r+a$, and variable a, can be regarded as a family of objects of variable mass inserted in a *fixed* space-time background.

We also see that for a black hole of mass M the value of the dilaton field on the horizon is

$$\Phi(r=0)=a=\ln(M\sqrt{k'/2}) . \tag{49}$$

Now, let us think in a speculative way about the fate of the black hole, taking into account the effects of Hawking radiation [39]. The black hole has a nonzero temperature, given by the inverse radius of the circle at infinity in the Euclidean black-hole solution. [Unfortunately, since the world-sheet action of the Euclidean black hole in $r-\theta$ coordinates is given correctly by (10) only to within an error of order $1/k$, we cannot easily compute this temperature at $k=\frac{9}{4}$.] As a result it will radiate, and lose mass. This mass loss means according to (49) that the value of the dilaton field on the horizon will diminish (and the string coupling constant will get stronger there).

Now, consider the physics as seen by an observer at a great distance, who ensures that he or she is at a fixed physical location by ensuring that the dilaton field that he or she observes remains fixed while the black hole evaporates. Suppose that the observer sits at a position where the dilaton field (which the observer measures by measuring the string coupling constant) is fixed at a value Φ_0. From (49) we see that when the black hole has mass M, its distance from the observer is approximately

$$\left[\frac{k'}{8}\right]^{1/2}[\Phi_0-\ln(M\sqrt{k'/2})] . \tag{50}$$

Thus, the distance from the observer to the horizon diverges logarithmically as the hole evaporates.

This is analogous to the case of an electrically or magnetically charged black hole in four dimensions. The mass M of such a hole in general relativity is always at least as great as the charge Q, and as one approaches the limiting value $M=Q$, the distance from an outside observer to the horizon diverges.

What is the end point of the black-hole evaporation? For $M\to 0$, the physics as measured at a fixed R approaches more and more the idealized flat-space solution (42). [For instance, one can see this in (49). $M\to 0$ means $a\to-\infty$, and for $a\to-\infty$, one has $\varphi\to 0$ at any fixed R.] Thus, this flat space solution, which is studied in the $c=1$ matrix model (except that we have not yet incorporated the cosmological constant) would appear to be the end point of the black-hole evaporation. Evaporation will be occurring at both ends of the hole, so the end point of the evaporation would appear to produce *two* copies of the standard space-time.

The space-time action (40) does not have a Poincaré-invariant solution. The flat-space solution (42) seems to be its most symmetric solution. Hitherto this solution has been regarded as a somewhat mangled, anisotropic version of Minkowski space. My point is that the solution (42) is better regarded as an analog of the extreme Reissner-Nordström solution than as an analog of Minkowski space. This of course would make the anisotropy natural. The fact that (42) describes a space-time with $M=0$ is no obstruction to this interpretation since the two-dimensional theory under consideration does not have a Poincaré-invariant solution to which the mass of the spacetime described by (42) could be compared.

The basis for regarding the idealized flat space-time as an analog of the extreme Reissner-Nordström solution rather than as an analog of Minkowski space is not limited to the fact that this space-time is, naively, the end point of the Hawking process. More importantly, this space-time is on the edge of being a real black hole, with a singularity shielded by a horizon, in the following sense. Consider perturbing the flat space-time (42) by sending in a particle of energy ϵ from $R=+\infty$ (Fig. 4). To the right of the particle trajectory, we must describe the result by a conformal field theory describing a space-time of mass ϵ; presumably, this will be the black-hole solution.

To really see black-hole physics, it is necessary for the particle to reach the analog of the Schwarzschild radius. What plays that role in this situation? If we do form a black hole of mass ϵ, the value of the dilaton field on the horizon will be, according to (49),

$$\Phi=\ln(\epsilon\sqrt{8/k'})\ . \tag{51}$$

The dilaton field as a function of ρ is $\Phi=\rho\sqrt{8/k'}$, so the Schwarzschild "radius," or more exactly the Schwarzschild value of ρ, for a particle of energy ϵ, is

$$\rho(\epsilon)=\left[\frac{k'}{8}\right]^{1/2}\ln(\epsilon\sqrt{8/k'})\ . \tag{52}$$

If a particle (or collection of particles) of total energy ϵ propagates into a position given approximately by $\rho=\rho(\epsilon)$, then, according to the classical equations, a horizon will form and the incoming particle or particles will be unable to escape.

The issues that arise are just those that one might contemplate for an extreme Reissner-Nordström black hole in four dimensions. Either the particle tossed in from the right will be reflected before reaching its Schwarzschild radius or, if it reaches the Schwarzschild radius, a horizon will appear to form but presumably the energy involved will ultimately be reemitted as some form of Hawking radiation. Either way, what we expect to see is a quantum-mechanical S matrix, with particles coming in from the right and ultimately being ejected, in one form or another. Precisely such an S matrix has been computed recently by several groups in the $c=1$ matrix model [40–45]. To determine whether what is being seen in that S matrix is a reflection outside the Schwarzschild radius or Hawking radiation of particles that have fallen inside the Schwarzschild radius, we must incorporate a key

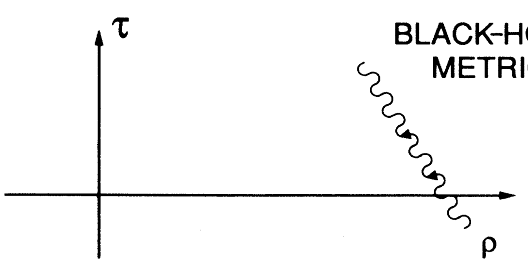

FIG. 4. Tossing in a particle to the "extreme black hole." To the right of the particle trajectory, we will be left with a black-hole space-time.

feature of the $c=1$ model which has not so far appeared in our discussion. This is what in the context of Liouville theory is usually called the cosmological constant. The cosmological constant is a tachyon field with $T(\rho,\tau)=\mu e^{-\sqrt{8}\rho}$ that can be added to the flat solution (42) and has the effect of suppressing string propagation into the region $\rho\to-\infty$ where the string coupling constant would otherwise become strong. Many matrix model formulas are singular for $\mu\to 0$, suggesting that there may be an instability that requires the inclusion of the cosmological constant (and perhaps will cause it to turn on spontaneously during black-hole evaporation, if not initially present). Whether or not this is so, μ plays an important role in the $c=1$ matrix model, and one would like to know how it appears in the black-hole physics.

We will now argue that the "cosmological constant" enters the black-hole physics even prior to the Hawking process as a failure of the analog of the "no-hair theorem." In addition to the mass, there is at least one more parameter in the most general static black-hole solution. In fact, we can find this parameter in the equations for a Euclidean black hole. The parameter in question involves the tachyon field, that is, the possibility of adding to the world-sheet action a term $\int d^2x\sqrt{h}\ T(x)$. The tachyon field is governed in lowest order by the following space-time effective action (which we discussed before in analyzing propagation of signals near $uv=1$):

$$L(T)=\int d^2X\sqrt{g}\ e^{\Phi}(g^{ij}\partial_i T\partial_j T-8T^2)\ . \tag{53}$$

In the field of a Euclidean black hole, with $ds^2=d\rho^2+\tanh^2(\rho\sqrt{2/k'})d\tau^2$, $\Phi=\ln\cosh^2(\rho/\sqrt{2/k'})+a$, the equation for a static (τ-independent) mode comes out to be

$$\left[\frac{1}{\sinh(\rho\sqrt{8/k'})}\frac{d}{d\rho}\sinh(\rho\sqrt{8/k'})\frac{d}{d\rho}+8\right]T(\rho)=0\ . \tag{54}$$

The two linearly independent solutions behave for $\rho\to+\infty$ as

$$T\sim e^{\lambda_{\pm}\rho}\ , \tag{55}$$

with

$$\lambda_{\pm}=-\left[\frac{2}{k'}\right]^{1/2}\pm\left[\frac{2}{k'}-8\right]^{1/2}\ . \tag{56}$$

The important point is that both λ_+ and λ_- have a negative real part, so both solutions decay as $\rho\to+\infty$. On the other hand, at $\rho=0$ one of the two solutions blows up (as $\ln\rho$) and one is regular. If one starts at $\rho=0$ with the regular solution and integrates outward, one will automatically get a solution that decays exponentially for $\rho\to\infty$, since all solutions have this property. Therefore, we have found a marginal deformation of the Euclidean black hole.

Although I will not attempt to work out all of the details, one could expect to prove along the following lines that this marginal deformation is tangent to an actual deformation. The basic point is that the existence of a black-hole solution of the graviton-dilaton field equations (41) is stable in the sense that if one adds to the equations any perturbation of rapid decay at infinity, a solution still exists, perhaps after changing the value of the asymptotic constants in the solution—the radius of the circle and the coefficient of the linear term in Φ. [If one did not have the exact $SL(2,\mathbb{R})/U(1)$ theory, one would use an argument along precisely these lines to show that the existence of the Euclidean black hole, though not the details of the metric, is stable against corrections to the large-k' equations. The possible need to change the radius of the circle means that the Hawking temperature may depend on the cosmological constant.] Similarly, the argument by which we predicted the existence of a regular solution of (54) with exponential decay at infinity showed that the existence of this solution is stable against small nonlinearities or changes in the dilaton-graviton background that vanish at infinity. This stability of the equations means that adding of terms quadratic and higher order in T will not affect the existence of a family of solutions labeled at least in an open set by the parameters seen in the linear analysis, so that the marginal deformation we have found will be tangent to an actual deformation.

For the value $k'=\frac{1}{4}$ that corresponds to the $c=1$ model, $\lambda_+=\lambda_-=-\sqrt{8}$. [These values are reliable as they depend only on the known asymptotic behavior of the dilaton field and not the unknown details of the black-hole metric. The fact that $\lambda_+=\lambda_-$ at $k'=\frac{1}{4}$ is related to the fact that in Liouville theory $k'=\frac{1}{4}$ is regarded as the largest allowed value. For $k'>\frac{1}{4}$, $\lambda_\pm$ have imaginary parts which mean that the regular solution of (54) has an oscillatory sign. This is perfectly sensible in the black-hole physics, but is considered pathological in Liouville theory since one wants a positive exponential that could represent the determinant of the metric.] The two solutions of (54) at large ρ behave as $e^{-\rho\sqrt{8}}$ and $\rho e^{-\rho\sqrt{8}}$, as noted in [20]. The latter will dominate for large ρ, so the marginal deformation of the black hole looks like $T\sim\rho e^{-\rho\sqrt{8}}$ for large ρ. This marginal deformation can be analytically continued to Minkowski space where, outside the horizon, it looks much like the usual tachyon background of the $c=1$ model. Our earlier analysis of (38) suggests that the tachyon background corresponding to the marginal deformation probably diverges at $uv=1$. It would be interesting to verify this, even to lowest order, by a more extensive study of (38) and (54). If it is true that the tachyon background diverges at $uv=1$, this would mean that in the presence of a cosmological constant, the "singularity" really is a singularity in the conformal field theory.

Let us now consider a weak tachyon signal of frequency ω propagating inward from $\rho=+\infty$ on the limiting black hole with a linear dilaton field of a strength corresponding to the $c=1$ model, that is, $k'=\frac{1}{4}$. (Or, what would be much the same thing, consider propagation toward an $M>0$ black hole in the region outside the horizon where the dilaton is approximately linear.) Following [20] we assume a T^3 term in (53) with a coefficient that we will call $-2g/3$. The linearized tachyon equation for small disturbances is

$$\left[e^{-2\rho\sqrt{8}}\frac{d}{d\rho}e^{2\rho\sqrt{8}}\frac{d}{d\rho}+8-\frac{d^2}{d\tau^2}-g\mu e^{-\sqrt{8}\rho}\right]T(\rho,\tau)=0\ . \tag{57}$$

If we set $T(\rho,\tau)=e^{-\rho\sqrt{8}}e^{-i\omega\tau}w(\rho)$ we get

$$\left[\frac{d^2}{d\rho^2}+\omega^2-g\mu e^{-\sqrt{8}\rho}\right]w(\rho)=0\ . \tag{58}$$

The classically forbidden region is the region with

$$\omega^2-g\mu e^{-\sqrt{8}\rho}<0\ , \tag{59}$$

and $w(\rho)$ vanishes exponentially at smaller ρ. We may very crudely say, therefore, that the distance to which the wave reaches is given by

$$\rho=\ln(g\mu/\omega^2)/\sqrt{8}\ . \tag{60}$$

If our wave of frequency ω has an amplitude corresponding to N elementary quanta, then the energy in this wave is $\epsilon=N\omega$. Let us now recall that the Schwarzschild distance for a system of energy ϵ was [according to (52)]

$$\rho(\epsilon)=\left[\frac{k'}{8}\right]^{1/2}\ln(\epsilon\sqrt{8/k'})\ . \tag{61}$$

The wave will fall into the black hole if the value of ρ that it reaches is less than the critical value $\rho(\epsilon)$. At $k'=\frac{1}{4}$, the criterion is

$$\tfrac{1}{2}\ln(N\omega\sqrt{32})\gg\ln(g\mu/\omega^2) \tag{62}$$

or (dropping an untrustworthy constant)

$$\omega\gg\mu^{2/5}N^{-1/5}\ . \tag{63}$$

This criterion is not obeyed in perturbative computations in the $c=1$ model, where one considers ω fixed, $N\sim1$, and $\mu\to\infty$. Therefore, the usual perturbative approximation to the $c=1$ model apparently corresponds to a situation in which the incident particles are repelled by the tachyon potential, without really probing the black-hole physics. If the matrix model framework can be taken literally, it is certainly possible to perform nonperturbative computations (some of which in fact have been done in [40]) in the region where (63) is obeyed. The result (63) is, however, puzzling, since it is not obvious that anything interesting happens in the matrix model when ω reaches a crucial value proportional to $\mu^{2/5}N^{-1/5}$.

Altogether, the existence of the black-hole solution seems to mean that the dynamics of the target space geometry must be taken seriously in the $c=1$, $D=2$ matrix model. Once one does this, it is evident that there must be a relation between two of the strangest features of the model. The first strange feature, manifest in the matrix model description by free fermions, is the existence of infinitely many conserved quantities, associated with conserved currents of higher spin such as $\bar{\psi}_+\partial_+^n\psi_+$. The second strange feature is the existence of discrete states of the string with definite energy and momentum; these states showed up in the computations of [43] and were interpreted in [44] as quasitopological modes of fields of higher spin. Despite their exponential fall-off (which reflects the role of the dilaton and the fact that the string coupling constant vanishes exponentially at infinity), the topological nature of these fields means that they are analogous to long-range fields in more conventional theories. The reason that there must be a relation between the two strange facts we have cited is that in the presence of space-time gravity, conserved currents of high spin do not give rise to conserved charges unless they couple to long-range fields. For a high spin current, say a stress tensor $T^{\mu\nu}$ or a higher spin current $Q^{\mu\nu\alpha}$, a covariant conservation law $D_\mu T^{\mu\nu}=0$ or $D_\mu Q^{\mu\nu\alpha}=0$ in curved space-time will not lead to the existence of a conserved quantity, unless the current couples to a long-range field (in which case, as in our above computation of the ADM mass of the black hole, the conserved quantity is determined by the asymptotic behavior of the long-range field and is then exactly conserved even by the Hawking process). In the $c=1$, $D=2$ model, the candidate long-range fields are the discrete topological modes, and (apart perhaps from the fermion number, which is associated with a spin-1 current $\bar{\psi}_+\psi_+$) whatever quantum numbers of the fermions are exactly conserved in the model must couple to them.

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