Time variation of fundamental constants. II. Quark masses as time-dependent parameters

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Consistent bounds for the simultaneous variations of fundamental constants in the standard model of fundamental interactions are obtained from astronomical, astrophysical, geophysical, and laboratory data. Improving the analysis made in our previous work, we treat quark masses as time-dependent variables. QCD chiral invariance breaks down and the general scaling of strong energies has to be reanalyzed. We also discuss the existence of preferred reference systems in a world with time-varying fundamental parameters. As in our previous work, the bounds obtained exclude the Dirac large-number hypothesis and, in general, any theory demanding a large variation of the fundamental constants.

I. INTRODUCTION

The observation of any time or position dependence of the fundamental constants has been considered as an important way of detecting the low-energy consequences of the grand unified theory of all interactions [1–4]. Unifying schemes such as Kaluza-Klein theories [1] or superstring theories [4,5] provide a very general framework to study the time variation of fundamental constants. Indeed, it has been shown that Kaluza-Klein theories have cosmological solutions where the fundamental constants do vary [2], and the same occurs in superstring theories [4-6].

Partially inspired by these theoretical results, many attempts have been made to set observational or experimental bounds on the time variation of fundamental constants. In Ref. [7] (hereafter SV1) we mentioned some of them. They all assume the given constant is the only one which varies in time. However, any conspiration between the variation of the constants that cancels the observational effects we consider makes the upper bound thus obtained meaningless.

The standard model (SM) of fundamental interactions together with general relativity (GR) provides a consistent description of all known low-energy phenomena [i.e., low compared with the grand unified (GU) energy scale], in good agreement with experiment. The set of fundamental "constants" on which these theories depend are supposed to be universal parameters, i.e., time, position, and reference-frame invariant. Indeed, the Einstein equivalence principle, on which GR is based, implies such an invariance.

However, the time variation of fundamental constants can preserve consistency with the inner symmetries of any fundamental theory. For instance, any time variation of Newton's gravitational constant G_N measured in atomic units is consistent with the general covariance of GR if G_N and the mass of any test body become constant in gravitational units [8,9].

In SV1 we analyzed short-term local phenomena: astronomical and geophysical data based on time intervals much shorter than the age of the Universe, and so set bounds on the variability of the fundamental constants today in the solar system. We analyzed the time variation of fundamental constants without the assumption of no conspiracy among the time variation of the QCD scale parameter Λ, Fermi constant G_F, fine-structure constant α, electron mass m_e and G_N. However, the u-, d-, and s-quark masses m_u, m_d, and m_s were assumed to be zero, and so we were able to derive a simple general scaling law of all strong energies with Λ. In this paper we lift such an assumption, and so derive a new approximate scaling law.

Those phenomena analyzed in SV1 are considered again in this work, together with Eötvös-type experiments, spectra of medium-distance radio sources and mass splitting of the $K^0\bar{K}^0$ system [10].

Our paper is organized as follows. In Sec. II we review the phenomenological framework of SV1 together with the modifications due to the new choice of fundamental parameters. In Sec. III we discuss the observational evidence available from astronomical, geophysical, and laboratory phenomena, and in Sec. IV we state our conclusions. Appendix A discusses the quark-mass dependence of the nucleon and meson masses, and Appendix B analyzes Eötvös experiments as a way of finding preferred reference systems.

II. A PHENOMENOLOGICAL MODEL

In this section we briefly discuss the phenomenological model presented in SV1 for the analysis of the consequences of the time variation of fundamental constants, together with the modifications introduced here. The model is based on the adiabatic hypothesis: i.e., that the main changes in observable quantities are due to the time variation of the parameters, neglecting the necessary modifications of the SM. With this assumption we are neglecting any local coupling of the fundamental constants with any other field. If we restrict our attention to local phenomena (local with respect to any cosmological scale such as the Hubble time or the particle horizon), we do not need the knowledge of the dynamics of the fundamental constants on a cosmological scale. As was stated in SV1, within such a procedure and without a deeper analysis, one would not be able to relate the rate of change to interesting quantities, such as the Hubble constant or the contraction rate of extra dimensions.

In order to choose a definite system of units, one must consider certain parameters as time independent. We choose $\hbar$ and c constants, since these quantities fix the length-to-time and energy-to-frequency units ratio. At this point one can use a finite, time-dependent renormalization-group transformation to select any dimensional quantity as a time independent energy unit [11]. In SV1 we introduced the Salam-Weinberg system of units (SWU), where the mass of the intermediary vector meson W, M_W, is taken as the time-independent energy unit. Our entire analysis will be carried out in SWU.

The rest of this section is devoted to presenting the set of hypotheses that specifies our phenomenological model.

A. Time-varying parameters

The choice of time-varying parameters is the same as in SV1, with the inclusion of the u-, d-, and s-quark masses as variables. Recent estimates of the quark content of the nucleons show that the hypothesis of a simple scaling law for the nucleon masses with Λ is not correct. In fact, it is presently believed that the chiral (zero-quark-mass) limit for the masses of the nucleons hardly exceeds 50% of the total nucleon mass [12-14]. Thus, we need to include the quark masses (which belong to the Higgs sector) as time-dependent parameters in order to improve the scope of our analysis.

As in SV1, we shall assume the validity of the fundamental relations between the parameters of the Salam-Weinberg theory of electroweak interactions, as a consequence of the adiabatic hypothesis and the experimental support for it.

B. Thermodynamical considerations

The time variation of fundamental parameters will produce changes in the equation of state of macroscopic bodies that can be computed using simple thermodynamic considerations of a very general nature. In SV1 we obtained the following expression for the time variation of the free energy:

$$\dot{F}=\sum \dot{C}_i \left[\frac{\partial\langle T\rangle}{\partial C_i}+\frac{\partial\langle U\rangle}{\partial C_i} \right] , \tag{2.1}$$

where both $\langle T\rangle$ and $\langle U\rangle$ can be written in terms of observable quantities using energy conservation and the virial theorem.

C. Time units transformation

As stated in SV1, the time derivatives in SWU will be related to the reported ones through the equations

$$\frac{da}{dt}=\frac{dt_1}{dt}\frac{da}{dt_1}=(1+\theta t)\frac{da}{dt_1} , \tag{2.2a}$$

$$\frac{d^2a}{dt^2}=\theta\frac{da}{dt_1}+(1+2\theta t)\frac{d^2a}{dt_1^2} , \tag{2.2b}$$

the last term being generally negligible. The quantity θ is some linear combination of the time derivatives of the fundamental constants, which stands for the relation between the observation time (atomic, ephemeris, etc.) t_1 and the SWU time t, which can be written as

$$t_1=t+\tfrac{1}{2}\theta t^2 . \tag{2.3}$$

D. Renormalization-group equations

Following SV1, we shall limit ourselves to show how any model-dependent parameter can be computed in a grand unified theory (GUT) which can be a low energy limit of a Kaluza-Klein or superstring model. We assume that at the grand unification scale Λ_U all the running coupling constants have a common value α_U. The renormalization-group equations are the same as in SV1.

E. Nuclear binding energies

The strong nuclear binding energy plays an important role in the observations to be analyzed. It depends on the fundamental constants nontrivially. The potential that describes reasonably the nucleon-nucleon interaction for typical nuclear distances is the Yukawa potential. In the case of pseudoscalar-meson exchange (e.g., π meson) between nucleons it reads [15]

$$V_P=g_P^2\frac{m_P^3}{m^2}\frac{e^{-r^*}}{r^*} , \tag{2.4}$$

where m_P and m are the pseudoscalar meson and nucleon masses, respectively, g_P is the corresponding coupling constant, and $r^*=rm_\pi$ is a natural nondimensional distance parameter.

When the nucleons exchange scalar mesons expression (2.4) becomes

$$V_S=g_S^2 m_S\frac{e^{-r^{**}}}{r^{**}} , \tag{2.5}$$

where now g_S and m_S are the coupling constant and scalar meson mass, respectively, and $r^{**}=rm_S$ is another natural nondimensional distance parameter. The potential for the exchange of a vector meson has the same form (2.5) with a different coupling constant g_V. We define g_S^{eff} as the sum g_S+g_V.

The processes that take place between nucleons include interchanges of two, three, and more either scalar or pseudoscalar mesons. At short distances the interchange of more than one meson becomes more relevant.

If the masses or the coupling constants in Eqs. (2.4) and (2.5) change, each potential will scale differently. If we make the transformations

$$m_S\to\lambda_S m_S ,$$

$$m\to\lambda_m m, \quad r^*\to r^* , \tag{2.6}$$

the potential and kinetic energies scale as follows:

$$V_S \to \lambda_S V_S \ , \quad T \to \lambda_m^{-1} \lambda_S^2 T \ . \tag{2.7}$$

Now, if we change the scalar-vector parameters

$$m_P \to \lambda_P m_P \ ,$$
$$m \to \lambda_m m, \quad r^{**} \to r^{**} \ , \tag{2.8}$$

we have

$$V_p \to \lambda_P^3 \lambda_m^{-2} V_S \ , \quad T \to \lambda_m^{-1} \lambda_P^2 T \ . \tag{2.9}$$

In neither case does the total Hamiltonian get multiplied by a constant, so there is no simple scaling law for a set of transformations involving all the parameters. However, the nuclear potential is a superposition of scalar, pseudoscalar, and vector exchanges. We assume that such a superposition introduces a new scaling law, a sort of "mean scaling," involving a "mean potential" with "mean parameters," which reads

$$\bar{V} = \bar{g}^2 \frac{\bar{m}^2}{\bar{m}} \frac{e^{-r/\bar{m}}}{r/\bar{m}} \ , \tag{2.10}$$

where $\bar{g}$ and $\bar{m}$ are the mean coupling constant and mean meson mass, respectively. Now, under the transformations

$$\bar{m} \to \lambda_{\mathrm{mp}} \bar{m} \ ,$$
$$m \to \lambda_m m \ , \quad r \to \lambda_{mp}^{-1} r \ , \tag{2.11}$$

the energies scale as follows:

$$\bar{V} \to \lambda_{mp}^2 \lambda_m^{-1} V_P \ , \quad T \to \lambda_m^{-1} \lambda_{mp}^2 T \ , \tag{2.12}$$

so the total Hamiltonian scales in the same way. In particular, the strong-binding energy of any nucleus will scale in the same way. This result sets us free from any model dependence, even though we assume a very particular hypothesis for the effective potential (2.10).

We can then write

$$\frac{\dot{E}}{E} = -\frac{\dot{m}}{m} + 2\frac{\dot{\bar{m}}}{\bar{m}} \ , \tag{2.13}$$

$$\frac{\dot{R}}{R} = -\frac{\dot{\bar{m}}}{\bar{m}} \ , \tag{2.14}$$

where E is the total binding energy, m is the (isotopically symmetric) nucleon mass, $\bar{m}$ is the mean meson mass (which we still have to define in terms of the quark masses and the scale parameter Λ), and R is the nuclear radius. We will neglect any contribution to the variation of the nuclear radius from Coulomb or weak interactions, so (2.14) will be the expression for the total time variation of R.

The time variation of the nuclear mass will be given by Eqs. (2.13) and (2.14) together with the expressions found in SV1 for the time variation of the Coulomb and weak-binding energies. It will be valid in the approximation $Z \simeq N$, where Z and N are the number of protons and neutrons, respectively. The final expression is

$$\frac{\dot{M}_{Z,A}}{M_{Z,A}} = \left[-\frac{E_s}{M_{Z,A}} + \frac{Am}{M_{Z,A}} \right] \frac{\dot{m}}{m} + \frac{E_c}{M_{Z,A}} \frac{\dot{\alpha}}{\alpha} + 0.83 \frac{E_w}{M_{Z,A}} \frac{\dot{G}_w}{G_w} + \left[2\frac{E_s}{M_{Z,A}} + \frac{E_c}{M_{Z,A}} + 3\frac{E_w}{M_{Z,A}} \right] \frac{\dot{\bar{m}}}{\bar{m}} \ . \tag{2.15}$$

F. Preferred reference systems

The existence of space- and time-dependent parameters introduce preferred Minkowskian reference systems. For instance, these can be defined to be reference systems where a definite set of fundamental parameters are space independent. One might say that it is not appropriate to talk about time-dependent parameters, but rather time dependent *fields*. In our model any of these fields have no dynamics, or at least, if there is such a dynamics, its coupling with local phenomena must be weak enough to neglect it and consider its time evolution as predefined. It is important to note that our model predicts the existence of *local* preferred reference systems, but nothing is said about *global* preferred reference systems, as we are not concerned about the cosmological dynamics of the fundamental constants.

The simplest assumption, which will be made in this work, considers these parameters as Lorentz scalars, as most of Kaluza-Klein and superstring models predict. it is natural to assume that there is a set of preferred reference systems where all fundamental constants are at most time dependent. These reference systems are at rest with respect to the background cosmic radiation (BCR), and we call them "locally comoving reference systems" (LCRS).

G. *CPT* invariance and transformation laws for time-varying parameters

The *CPT* theorem is based on the very general assumptions [16] of Hermiticity, Lorentz invariance, and weak locality of interactions. No matter how "physical" these assumptions may seem, physicists would prefer to test them through *CPT* invariance experimentally. On the other hand, it can be easily shown that *CPT* invariance of mass and minimal gauge coupling Lagrangian terms is granted if masses and gauge coupling constants, considered as fields, are scalars under Lorentz transformations. These conditions hold in our model, so we will extend these results to every sector of it, and so assume that *CPT* is an exact symmetry of nature.

III. ANALYSIS OF OBSERVATIONS

In this section we shall discuss the modifications introduced in the analysis of geophysical, astronomical, and geochemical phenomena made in SV1. The observational data are listed in Table I. In this analysis we shall consider as fundamental variable parameters the nucleon mass m, the isotopic breaking parameter $\Delta m = M_n - M_p$, the mean meson mass $\overline{m}$, G_N, G_F, m_e, and α. In fact, these are our directly observable variable parameters. At this level, the only differences between the analysis in SV1 and the one made here are the nuclear radii and strong binding energies scaling laws.

The scale parameter Λ does not appear until the nucleon and pion masses are replaced by the u-,d-,s-quark masses and Λ as fundamental parameters. The modifications can be summarized as follows.

(i) Planetary paleoradius and orbital perturbations. In SV1 the parameter $(\dot{\Lambda}/\Lambda)_{\mathrm{SV1}}$ stems from $\dot{M}_{Z,A}/M_{Z,A}$. We can use expression (2.15) in every case except for the solar and pulsar masses. In this case neutron and proton masses M_n and M_p appear explicitly in a separate way, as their relative abundances differ considerably. The solar composition is estimated [17] essentially to be 29% helium and 71% hydrogen. It is easy then to write for the time variation of the solar mass the expression

$$\frac{\dot{M}_S}{M_S}=0.855\frac{\dot{M}_p}{M_p}+0.145\frac{\dot{M}_n}{M_n} . \tag{3.1}$$

It is not clear whether pulsars are neutron stars [18] or strange stars [19]. In either case the expressions for their mass-time variation will differ considerably from the Earth-type planetary case. In this work we shall conservatively adhere to the neutron-star pulsar hypothesis. If we neglect the binding energy between neutrons, the expression for the time variation of the mass M_{NS} of any neutron star will essentially be

TABLE I. Observational data. The columns show the data number (correlated with the conditional equation number in Table III), a simple data description, the observed value and the corresponding standard deviations (in units of 10^{-11} yr^{-1}), and the system of units of the observation and the reference.

Eq. description	Value (10^{-11} yr^{-1})	Unit	Ref.
Planetary paleoradius $\dot{R}/R$			
(1) Mercury	0.0±0.012	SW	[45]
(2) Moon	0.0±0.015	SW	[45]
(3) Mars	0.0±0.03	SW	[45]
Lunar secular acceleration: $\dot{n}/n$			
(4) Mercury transits	−15.0±1.2	ET	[46]
(5) Ancient eclipses	−17.3±1.8	ET	[47]
(6) Growth rhythms	−14.2±2.4	AT	[48,49]
(7) LLR	−13.7±1.0	AT	[50]
(8) Tidal models	−15.2±3.0	SW	[51,48]
(9) Satellite data	−14.4±1.7	SW	[51]
Earth's secular acceleration: $\dot{\Omega}/\Omega$			
(10) Ancient solar eclipses	24.3±2.0	ET	[47]
(11) Ancient lunar eclipses	−20.6±2.6	ET	[52]
(12) Ancient equinoxes	−23.6±2.3	ET	[47]
13) Growth rhythms	−22.5±1.0	AT	[48,49]
Viking ranging data			
(14) $\dot{G}_N/G_N$	0.0±1.2	AT	[53]
(15) $\dot{\beta}$	0.0±2.4	AT	[53]
Binary pulsar data			
(16) $\dot{n}/n$	1.0±1.2	AT	[54]
Long-lived β decayers: $\dot{\lambda}/\lambda$			
(17) ^{187}Re	0.0±0.15	αU	[55,56]
(18) ^{40}K	0.0±0.29	αU	[57]
(19) ^{87}Rb	0.0±0.29	αU	[57]
Oklo phenomenon: $\dot{\sigma}/\sigma$			
(20) ^{149}Sm	0.0±69.0	SW	[21]
(21) ^{157}Gd	0.0±123	SW	[21]
(22) ^{151}Eu	0.0±630	SW	[21]
(23) ^{113}Cd	0.0±280	SW	[23]
Eötvös experiments			
(24) η(A1-Pt)	0.0±0.06	SW	[29]
Absorption lines of QSD			
(25) Hyp. Hydr/Mg II dobl	0.0±0.002	SW	[30]
(26) Mg^+-Mg II	0.0±0.002	SW	[30]
Laboratory data:			
(27) Clock rate difference	−0.2±1.2	SW	[58]
(28) a-factor difference	0.0±0.06	SW	[33,34,59]
(29) K^0-$\bar{K}^0$ mass difference	$0.0\pm3.30\times10^{-8}$	SW	[38]

$$\frac{\dot{M}_{\rm NS}}{M_{\rm NS}}=\frac{\dot{M}_n}{M_n} \ . \tag{3.2}$$

The treatment of the time variation of Mars' radius is very similar to Mercury's SV1 analysis, as the best accepted models propose a two-layer composition for it [20].

(ii) Long-lived β decayers. The difference with SV1 arises only when we express the time variation of the energy released in the transition in terms of the variation of fundamental constants. The mass difference between neutron and proton will appear as a new directly observable variable parameter. The analysis for the time variation of the nuclear binding energies is the same as in Sec. II. The equation for the time variation of the released energy W_0 is then

$$\frac{\dot{W}_0}{W_0}=-\frac{\Delta E_s}{W_0}\frac{\dot{m}}{m}+\frac{\Delta M}{W_0}\frac{\Delta\dot{M}}{\Delta M} +\frac{\dot{\alpha}}{\alpha}\frac{\Delta E_c}{W_0}+\frac{\dot{G}_w}{G_w}0.83\frac{\Delta E_w}{W_0} +\frac{\dot{\overline{m}}}{\overline{m}}\left[2\frac{\Delta E_s}{W_0}+\frac{\Delta E_c}{W_0}+3\frac{\Delta E_w}{W_0}\right] , \tag{3.3}$$

where ΔE_i is the energy difference of type (i) between both nuclei.

Finally, the equation for the time variation of the decay constant for α decay is obtained by making in the corresponding equation of SV1 the replacement

$$\frac{\dot{\Lambda}_{\rm SV1}}{\Lambda}\rightarrow\frac{\dot{M}_{\rm He}}{M_{\rm He}}\simeq\frac{\dot{m}}{m} \ . \tag{3.4}$$

(iii) The Oklo equation. Using Eqs. (2.3)–(2.15) the analysis of the modifications is similar to that made above. We improve the analysis by considering the effect of the time variation of ΔM, due to the difference between the number of protons and neutrons in the absorver and compound nuclei. This manifests in the kinetic energy of the nucleus, which can be written as

$$E_K=\frac{3}{5}(ZE_F^Z+NE_F^N) , \tag{3.5}$$

where

$$E_F^{N(Z)}=\frac{1}{2}\left[\frac{9\pi}{4}\frac{N(Z)}{A}\right]^{2/3}\frac{\hbar^2}{M_{p(n)}r_0^2} , \tag{3.6}$$

and $r_0=1.25\times10^{-13}$ cm (we refer the reader to the Fermi gas model of the nucleus used in SV1).

The main contribution to the time variation of E_K is provided by the general scaling law [Eqs. (2.13) and (2.14)]. However, there is an additional contribution due to the difference between the number of protons and neutrons in the nucleus, which is proportional to the time variation of ΔM. We take the time derivative of Eq. (3.6) and obtain the expression

$$\dot{E}_K=0.029(N-Z)\frac{\Delta\dot{M}}{\Delta M}\ \text{MeV} . \tag{3.7}$$

This last expression will provide the contributions of $\Delta\dot{M}/\Delta M$ to the time variation of the resonance and binding energies E_0 and W_0, and finally to the equation for the time variation of the neutron absorption cross sections of strong absorvers. In SV1 we only consider ^{149}Sm. Here we extend the analysis to ^{157}Gd, ^{151}Eu, and ^{113}Cd, which are also strong thermal neutron absorvers. The neutron capture cross sections of these nuclei during the Oklo working period have not been previously calculated explicitly. However, we can estimate them from the isotopic abundances of these strong absorvers in the Oklo mines, and then compare them with the abundances predicted by the modern values of the capture cross sections. We use the values for the predicted and observed isotopic abundances that are shown in Ref. [21]. Their ratios provide a good estimate of the Oklo TACS (thermal average cross section) to present TACS ratios.

In the case of ^{113}Cd it is convenient to use Bateman's formula [22]

$$N_{113}=\frac{N_{235}\sigma_F\gamma_{113}}{\sigma_{113}}(1-e^{-(\Phi\sigma_{113}t)}) , \tag{3.8}$$

where N_{113} is the number of atoms of ^{113}Cd, N_{235} is the amount of U_{235} present at the beginning of the fission process, γ_{113} is the yield of the mass 113 fission chain, Φ is the thermal neutron flux, σ_F is the fission cross section

TABLE II. (a) Nuclear parameters used to evaluate Oklo equations. (b) Measurements of g factors. ET and AT stand for ephemeris and atomic time, respectively. LLR denotes lunar laser ranging.

(a)

(meV)	^{149}Sm	^{157}Gd	^{151}Eu	^{113}Cd
E_0	98	30	327	178
Γ_γ	63	100	70	113
Γ_n	0.5	0.030	0.065	0.650
Q_{33} (barns)	0.06	2	1.2	0

(b)

t	$g(e^-)$ (10^{-12})	$g(e^+)$ (10^{-12})	Reference
1984	1 159 652 193 (4)	1 159 652 222(50)	[30]
1987	1 159 652 188.4(4.3)	1 159 652 187.9(4.3)	[31]

for ^{235}U and σ_c is the neutron capture cross section for ^{113}Cd. Reference [23] provides us with the abundances of ^{113}Cd for three samples, together with the parameters necessary to use Eq. (3.8). Table II(a) shows the nuclear parameters used to evaluate the entries of the equations for the time variation of the cross sections of ^{149}Sm, ^{157}Gd, ^{151}Eu, and ^{113}Cd. There we see that the quadrupole moment of ^{113}Cd is zero, so we evaluate its magnetic dipole energy in order to find the Coulomb contribution to the resonance energy. For more details, see Ref. [27].

The last two subsections account for three additional observations with respect to SV1 which are considered in this work.

(iv) Eötvös experiments. These experiments are designed to study violations of the weak equivalence principle (WEP) [28]. In modern versions the parameter to be measured is the difference in the gravitational acceleration g_S induced by the Sun on two laboratory objects of different chemical composition. If a non-null result is obtained, then the violation of the WEP is inferred.

As we stated in Sec. I, the time dependence of the fundamental constants will provide preferred inertial reference systems. In any nonlocal comoving reference system (LCRS) the fundamental constants will be spatial as well as time dependent.

In Appendix B we obtain the equation

$$\frac{|a_i-a_j|}{g_S}=\frac{1}{2}\frac{\omega}{g_S}\left[\frac{\dot{m}_i}{m_i}-\frac{\dot{m}_j}{m_j}\right] , \tag{3.9}$$

where a_i and m_i are the acceleration and mass of body i, and ω is the velocity of the laboratory reference system relative to the comoving system. The results obtained by Braginsky and Panov [29] for aluminum and platinum allows us to evaluate the left-hand side of Eq. (3.9).

(v) Redshifts of absorption lines of quasistellar objects (QSO's) Nearly all QSO's show a redshift in both their continuum and line radiation. We can obtain information on local atomic transition processes, and hence the values of various fundamental parameters which govern these transitions, over a time interval corresponding to a significant fraction of the age of the Universe such as 35%. This may be in conflict with the linear approximation used both here and in SV1 for the time dependence of the fundamental constants. However, any look-back time less than 50% of the age of the Universe will be acceptable in such approximation.

The detection of Mg II fine-structure and hydrogen hyperfine absorption lines toward the radio source AD 0.235+164 allowed Wolfe, Brown, and Roberts [30] to place upper limits on three products of the fine-structure constant, the nuclear g factor of the proton and the electron and proton masses m_e and M_p, viz. $\alpha^2 g_p m/M_p$, $g_p m/M_p$, and α. The upper bounds to the redshift differences of those lines for the same object provide upper bounds for the time variation of the products mentioned above. As the analysis of Ref. [30] applies exactly in our case, we just write the two relevant equations for our work.

(vi) Time variation of $K^0\bar{K}^0$ masses. In Ref. [10] we obtained an equation for the time variation of the mass difference between K^0 and $\bar{K}^0$ mesons, making use of *CPT* invariance. We obtain, using the mass values for u, d, and s quarks given in Appendix A, the equation

$$\frac{\Delta M_K}{\Delta T}\frac{1}{M_K}=\frac{\dot{\Lambda}}{\Lambda}+0.047\frac{\dot{m}_d}{m_d}+0.953\frac{\dot{m}_s}{m_s} . \tag{3.10}$$

A modern value for ΔM is [31]

$$\frac{\Delta M_K}{M_K}=(0.0\pm6.0)\times10^{-19} , \tag{3.11}$$

while a 10-yr-old value is [32]

$$\frac{\Delta M_K}{M_K}+(0.0\pm3.3)\times10^{-18} . \tag{3.12}$$

We estimate the time derivative $\Delta\dot{M}$ as

$$\frac{\Delta M(t')-\Delta M(t)}{t'-t}=(0.0\pm3.30)\times10^{-19}\ \mathrm{yr}^{-1} , \tag{3.13}$$

and the final equation reads

$$\frac{\dot{\Lambda}}{\Lambda}+0.047\frac{\dot{m}_d}{m_d}+0.953\frac{\dot{m}_s}{m_s}=(0.0\pm3.30)\times10^{-19}\ \mathrm{yr}^{-1} . \tag{3.14}$$

(vii) Time variation of electron and positron g factors. One of the most exacting tests of charged particle-antiparticle symmetry to date is the modern high-precision measurements of the magnetic moment or g factor (which equals twice the magnetic moment in units of Bohr magnetons) of a single electron and a single proton.

In general, the charged particle is introduced in a Penning trap, where electric and magnetic fields generate a rapid cyclotron rotation. The parameters that are measured are the cyclotron frequency, which depends on the charge and mass of the particle, and the spin-precession frequency, which depends on the magnetic moment of it.

However, in our model the fundamental constants change with time and their transformation properties are such that *CPT* symmetry is preserved. Then, a *C* violation is to be expected.

The magnetic interaction term of an electron with an external magnetic field **B** is

$$H_I=-\boldsymbol{\mu}\cdot\mathbf{B} , \tag{3.15}$$

where $\boldsymbol{\mu}(e^-)=(g_e e/m_e 2)\boldsymbol{\sigma}/2$, $\boldsymbol{\sigma}$ denotes the Pauli matrices and $g_e\simeq2$ is the electron g factor. H_I is *CPT* invariant if the following condition holds:

$$g^{CPT}(-x,e^-)\frac{e(-x)}{m_e(-x)}=g(-x,e^-)\frac{e(x)}{m_e(x)} , \tag{3.16}$$

where $g^{CPT}(-x,e^-)$ is the *CPT*-transformed "g field" of the electron, and we have used the transformation properties of the fundamental constants discussed in Sec. III G. If g is a scalar field, like the rest of the fundamental constants, we can write

$$g^{CPT}(x)=Cg^{PT}(x)=Cg(-x) . \tag{3.17}$$

Then we have

$$Cg(x,e^-)=g(x,e^-)\frac{e(x)}{e(-x)}\frac{m_e(-x)}{m_e(x)}=g(x,e^+) ; \tag{3.18}$$

so if we use the linear approximation

$$g(t,e^{\pm})=g_0+\dot{g}^{\pm}(t-t_0) , \tag{3.19}$$

we obtain the result

$$\frac{\dot{g}^+}{g_0}-\frac{\dot{g}^-}{g_0}=\left[\frac{\dot{e}}{e}-\frac{\dot{m}_e}{m_e}\right] . \tag{3.20}$$

The difference between the electron and positron g factor can be written as

$$g(t,e^-)-g(t,e^+)=(\dot{g}^- -\dot{g}^+)(t-t_0) , \tag{3.21}$$

or, in terms of the anomalous g factor $a=g/2-1$, we obtain the final result

$$\frac{\dot{m}_e}{m_e}-\frac{1}{2}\frac{\dot{\alpha}}{\alpha}=\frac{a(e^-)-a(e^+)}{g_0}\frac{1}{t_2-t_1} , \tag{3.22}$$

where t_2 and t_1 are two arbitrary epochs in which the g factors are measured.

In Table II(b) we quote the values from Refs. [33] and [34], corresponding to two different measurements of g factors. We can now estimate an upper bound for the anomalous g-factor difference throughout the last seven years:

$$a(e^-)-a(e^+)\leq 5.9\times 10^{-12} . \tag{3.23}$$

As in Secs. III–VI, if we choose 1984 as t_0 and assume that ΔT is four years, a rigorous bound results and the final equation reads

$$\frac{\dot{m}_e}{m_e}-\frac{1}{2}\frac{\dot{\alpha}}{\alpha}\leq 6\times 10^{-13}\ \mathrm{yr}^{-1} . \tag{3.24}$$

IV. RESULTS AND CONCLUSIONS

The equations are solved for the time logarithmic derivatives of m, ΔM, $\overline{m}$, m_e, α, G_N, G_F, n (tidal), and Ω (nontidal). In Appendix A we derive expressions for the time variation of m, $\overline{M}$, and ΔM in terms of the time variation of the quark masses and Λ. We need one more independent equation relating the time variation of the quark masses and Λ with some other observable (four independent observables fix the time variation of the four parameters m_q and Λ). This equation comes from the measurement of $K^0\overline{K}^0$ mass splitting, and is derived in Ref. [10]. The changes (or their absence) refer to time scales ranging from the age of the solar system (meteorites) to the duration of an atomic experiment (12 days). Nevertheless, the time independence of the time logarithmic derivatives of the fundamental parameters, which is assumed in this work as being a good approximation for time scales much shorter than the age of the Universe, permits us to solve the conditional equations all together. Thus we have, as in SV1, an overdetermined set of constraints (Table III) that the observational data, which are shown in Table I, must satisfy.

We consider these observational data uncorrelated random variables, whose standard deviations are shown in Table I. The estimates of $\dot{m}$, $\dot{\overline{m}}$, $\dot{\alpha}$, and $\dot{m}_e$ are highly correlated, and, since most of the data are upper bounds, they are certainly not Gaussian. Indeed, the corresponding "jackknife" estimate shows that the distributions are highly leptokurtic. The least-squares estimates of errors are not reliable under these circumstances. However, in order to compare the results of the jackknife and the (Gaussian) adjustment by elements [35], we show in Table IV the least-squares solution, together with the correlations between the estimates. Such correlations enhance the error of the estimates, which are not clearly distinguished by the observational data. The correlations between some of the dependent parameters [Tables IV(e) and IV(f)] are still higher because of the particular structure of their coefficient matrices (see especially Table V).

In the jackknife process [36-38], each of the 28 equations was deleted one by one of the data set and 28 "pseudovalues" [36] were obtained for each of the fundamental constants. The jackknife estimates are the mean and standard error for the pseudovalues. 95% confidence limits were obtained from the t distribution, and the larger was taken as an upper bound for the time variation of the constant. In Table VI it can be clearly seen that the errors are much lower than in the least-squares estimation, because of the inadequacy of the Gaussian hypothesis, both for the null observations and for the estimates of the time variation of the fundamental constants.

Despite the increment in the number of unknowns with respect to SV1, the increment in the number of observations considered and the more realistic jackknife treatment allowed us to obtain stronger bounds in the present work. Similar bounds have been obtained by Shlyakhter [21], coupling the Oklo phenomenon with probabilistic arguments.

As in SV1, we can find upper bounds for the time variation of the fundamental constants at the grand unification scale. These results are shown in Table IV(c). We conclude that no parameter is allowed to vary with time at a rate comparable to the Hubble constant H_0, and so the large-number hypothesis is wholly refuted.

We can impose stronger constraints on the time variation of the size of the extra dimensional space R_I in both Kaluza-Klein and superstring theories than in SV1. In the case of Kaluza-Klein theories, $\alpha_U \propto R_I^{-2}$ and from this relation we find the result

$$\left|\frac{\dot{R}_N}{R_N}\right|\leq 3\times 10^{-13}\ \mathrm{yr}^{-1} , \tag{4.1}$$

for the present contraction rate. In the superstring theory cosmological solution of Wu and Wang [4], $G_N \propto R_I^{-6}$ and so we obtain

$$\frac{\dot{R}_\sigma}{R_\sigma}\leq 7.8\times 10^{-12}\ \mathrm{yr}^{-1} . \tag{4.2}$$

TABLE III. Coefficients of conditional equations. The columns show (1) the equation number (the same as that of Table I), (2) $\dot{m}/m$, (3) $\Delta\dot{M}/\Delta M$, (4) $\dot{M}_{mp}/M_{mp}$, (5) $\dot{G}_N/G_N$, (6) $\dot{\alpha}/\alpha$, (7) $\dot{G}_F/G_F$, (8) $\dot{m}_e/m_e$, (9) $\dot{n}/n$, (10) $\dot{\Omega}_N/\Omega_N$.

Eq. (1)	M_N (2)	ΔM (3)	M_{mp} (4)	G_N (5)	α (6)	G_F (7)	m_e (8)	n (9)	Ω_N (10)
1	-1.62	0.0	0.053	-0.02	1.17	0.0	1.32		
2	-1.28×10^{-2}	0.0	4.17×10^{-4}	-4.1×10^{-3}	0.63	0.0	.9		
3	-1.3	0.	0.04	-0.03	0.83	0.0	1.32		
4	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0
5	-0.29	0.0	9.40×10^{-3}	-0.11	0.0	0.0	0.0	1.0	0.0
6	-5.2	0.0	0.17	-2	-2	0.0	-1	1.0	0.0
7	-4.13	0.0	0.13	-2	-4	0.0	-2	1.0	0.0
8	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	1.0	0.0
10	-11.5	0.0	0.40	-3.83	-1.61	0.0	-1.44	1.85	1.0
11	-11.5	0.0	0.40	-3.83	-1.61	0.0	-1.44	1.85	1.0
12	-11.5	0.0	0.40	-3.83	-1.61	0.0	-1.44	1.85	1.0
13	-6.5	0.0	0.22	-1.83	-2	0.0	-1	1.85	1.0
14	1.0	-4.88×10^{-4}	-0.034	1.0	-8	0.0	-4		
15	1.02	0.0	-3.36×10^{-2}	0.0	-4	0.0	-2		
16	4.13	0.0	-0.13	2	4	0.0	2		
17	-7.16×10^{3}	4.84×10^{2}	7.27×10^{3}	0.0	-2.28×10^{4}	2.00	589		
18	-3.30×10^{2}	-70.6	2.59×10^{2}	0.0	766	-7.82	7.06		
19	-35.5	3.38	-74.3	0.0	9.28×10^{2}	2	-7.8		
20	9.17×10^{6}	-1.56×10^{5}	-9.17×10^{6}	0.0	9.17×10^{6}	176	0.0		
21	-3.51×10^{8}	-2.83×10^{5}	3.51×10^{8}	0.0	-3.51×10^{8}	334	0.0		
22	-1.25×10^{8}	-1.71×10^{5}	1.25×10^{8}	0.0	-1.25×10^{8}	194	0.0		
23	1.67×10^{7}	-2.00×10^{5}	-1.67×10^{7}	0.0	1.67×10^{7}	210	0.0		
24	-2.1×10^{-3}	-5.2×10^{-5}	1.7×10^{-3}	0.0	-2.51×10^{-3}	0.0	0.0		
25	-1.0	6.87×10^{-4}	0.0	0.0	2	0.0	1.0		
26	-1.0	6.87×10^{-4}	0.0	0.0	0.0	0.0	0.0		
27	-0.951	0.0	2.3×10^{-2}	0.0	3	0.0	1		
28	0.0	0.0	0.0	0.0	-0.5	0.0	1.0		

As we have mentioned before, the Einstein equivalence principle implies that all nongravitational constants of nature must be time and position independent. The strong equivalence principle extends that statement to gravitational phenomena. Our results show that both forms of the principle of equivalence are very well satisfied, within a small fraction of the Hubble rate. Since the unrestricted validity of the principle of equivalence leads to general relativity as the only low-energy theory of gravitation, our results should be considered as an accurate verification of general relativity.

Following Barrow and Tipler [39], we mention that any experimental evidence against the time dependence of the fundamental constants leaves the large value of Dirac's dimensionless numbers unexplained. We do not know whether any explanation is desirable for such values. Were this the case we could look for theories including ensembles of worlds, such as chaotic inflationary scenarios [40] or Everett's many world interpretation of quantum mechanics [41], on which anthropic arguments could be scientifically based.

Finally, Broadhurst, Ellis, Koo, and Szalay [42] report-

TABLE IV. Coefficients of QCD parameters. The columns show the coefficients relating the time variation of M_N, ΔM, M_{mp}, and ΔM_K with the time variation of Λ and u, d, and s quark masses.

Observable	Λ (QCD)	m_u	m_d	m_s
M_N	0.53	0.022	0.038	0.40
ΔM	0.53	-4.34	8.04	0.40
M_{mp}	0.50	0.065	0.11	0.32
$M(K^0)-M(\bar{K}^0)$	1.0	0.0	0.047	0.95

TABLE V. Least-squares results. The table shows the name of the parameter, the value and the standard deviation for the fundamental parameters of our model, the 95% confidence limits as upper bounds and the same quantities in units of the Hubble constant. In order to get upper bounds a low value of $K_0 \geq 55$ km sec^{-1} Mpc^{-1} has been used. The last part of the table shows correlation matrices necessary to obtain bounds on the time variation of dependent parameters. (a) Values and bounds for the fundamental parameters. (b) Model independent bounds for SM parameters. (c) Bounds for GUT and model-dependent parameters. (d) Correlation matrix for directly observable parameters [with the numeration of (b)]. (e) Correlation matrix of Λ and quark masses. (f) Correlation matrix of SM-dependent parameters.

(a)

$\dot{M}_N/M_N$	$(0.0\pm2.1)\times10^{-4}$	2.4×10^{-4}	3.9×10^{-5}
$\Delta M/\Delta M$	$(0.0\pm7.6)\times10^{-4}$	1.5×10^{-3}	2.7×10^{-4}
$\dot{M}_{mp}/_{mp}$	$(0.0\pm1.9)\times10^{-4}$	3.8×10^{-4}	6.8×10^{-5}
$\dot{m}_e/m_e$	$(0.0\pm1.3)\times10^{-3}$	2.6×10^{-3}	4.6×10^{-4}
$\dot{\alpha}/\alpha$	$(0.3\pm5.9)\times10^{-5}$	1.2×10^{-4}	2.1×10^{-5}
$\dot{G}_F^{ef}/G_F^{ef}$	$(0.0\pm2.4)\times10^{-2}$	4.8×10^{-2}	8.5×10^{-3}
G_N/G_N	$(-0.9\pm1.9)\times10^{-1}$	4.7×10^{-1}	6.8×10^{-2}
$\dot{n}/n$	-14.73 ± 0.44		
$\dot{\Omega}_N/\Omega_N$	4.74 ± 0.90		

(b)

$\dot{\Lambda}/\Lambda$	5.0×10^{-3}	9.0×10^{-4}
$\dot{m}_u/m_u$	4.0×10^{-3}	7.2×10^{-3}
$\dot{m}_d/m_d$	2.4×10^{-3}	4.4×10^{-4}
$\dot{m}_s/m_s$	5.2×10^{-3}	9.4×10^{-4}
$\dot{\theta}_c$	6.4×10^{-3}	1.2×10^{-3}
$\dot{\alpha}_1/\alpha_1$	4.8×10^{-2}	8.8×10^{-3}
$\dot{\alpha}_2/\alpha_2$	1.1×10^{-1}	2.0×10^{-2}
$\dot{\theta}_w$	1.3×10^{-2}	2.4×10^{-3}
$\dot{M}_Z/M_Z$	1.4×10^{-2}	2.6×10^{-3}
$\dot{v}/v$	2.4×10^{-2}	4.4×10^{-3}

(c)

$\dot{\alpha}_U/\alpha_U$	6.2×10^{-2}	1.1×10^{-2}
$\dot{\alpha}_3/\alpha_3$	4.0	7.2×10^{-1}
$\dot{\mu}/\mu$	9.2×10^{-1}	1.7×10^{-1}
$\dot{\Lambda}_U/\Lambda_U$	2.0	3.6×10^{-1}

(d)

1.00	−0.094	0.96	−0.47	0.12	−0.058	−0.011	-0.34×10^{-2}	-0.16×10^{-2}
	1.00	−0.89	0.49	−0.11	0.11	0.012	0.36×10^{-2}	0.17×10^{-2}
		1.00	−0.21	−0.15	−0.065	−0.045	−0.017	−0.010
			1.00	−0.91	-0.43×10^{-2}	−0.11	−0.044	−0.027
				1.00	0.11	0.13	0.052	0.032
					1.00	0.016	0.67×10^{-2}	0.40×10^{-2}
						1.00	0.40	0.25
							1.00	−0.66
								1.00

(e)

Λ	m_u	m_d	m_s
1.000 00	−0.954 33	−0.963 68	−0.999 98
	1.000 00	0.999 05	0.952 50
		1.000 00	0.962 03
			1.000 00

(f)

θ_C	G_F	θ_W	α_2	μ
1.000 000	−0.118 669	0.117 840	−0.118 201	−0.112 264
	1.000 000	−9.999 97	0.999 999	0.999 961
		1.000 000	−0.999 999	−0.999 971
			1.000 000	0.999 967
				1.000 000

TABLE VI. Jackknife results. The table shows the name of the parameter, its jackknife mean and standard error, the absolute value of the larger of the 95% confidence limits for the t distribution and the same quantity in units of the Hubble constant.

$\dot{M}_N/M_N$	8.2×10^{-4}	6.5×10^{-4}	2.1×10^{-4}	3.9×10^{-5}
$\Delta\dot{M}/\Delta M$	4.4×10^{-6}	4.3×10^{-6}	1.3×10^{-5}	2.4×10^{-6}
$\dot{M}_{mp}/_{mp}$	9.1×10^{-4}	7.1×10^{-4}	2.4×10^{-3}	4.4×10^{-4}
$\dot{m}_e/m_e$	-7.2×10^{-4}	1.1×10^{-3}	3.0×10^{-3}	5.5×10^{-4}
$\dot{\alpha}/\alpha$	9.7×10^{-5}	6.9×10^{-5}	2.4×10^{-4}	4.4×10^{-5}
$\dot{G}_F^{ef}/G_F^{ef}$	3.4×10^{-3}	3.7×10^{-3}	1.1×10^{-2}	2.0×10^{-3}
G_N/G_N	-3.0×10^{-2}	3.0×10^{-1}	6.6×10^{-1}	1.2×10^{-1}
$\dot{n}/n$	-1.49×10^{1}	7.0×10^{-1}		
$\dot{\Omega}_N/\Omega_N$	5.16×10^{0}	1.7×10^{0}		
$\dot{\Lambda}/\Lambda$	8.8×10^{-3}	7.0×10^{-3}	2.4×10^{-2}	4.3×10^{-3}
$\dot{m}_u/m_u$	-4.4×10^{-3}	3.6×10^{-3}	1.2×10^{-2}	2.2×10^{-3}
$\dot{m}_d/m_d$	-2.5×10^{-3}	2.1×10^{-3}	6.6×10^{-3}	1.2×10^{-3}
$\dot{m}_s/m_s$	-9.1×10^{-3}	7.3×10^{-3}	2.4×10^{-2}	4.4×10^{-3}
$\dot{\theta}_C$	1.4×10^{-2}	1.6×10^{-2}	5.3×10^{-2}	9.7×10^{-3}
$\dot{\alpha}_1/\alpha_1$	-3.5×10^{-3}	2.6×10^{-3}	9.3×10^{-3}	1.7×10^{-3}
$\dot{\alpha}_2/\alpha_2$	-7.3×10^{-4}	5.6×10^{-4}	1.9×10^{-3}	3.5×10^{-4}
$\dot{\theta}_w$	-1.2×10^{-2}	1.4×10^{-2}	4.1×10^{-2}	7.4×10^{-3}
$\dot{M}_Z/M_Z$	4.1×10^{-4}	3.1×10^{-4}	1.0×10^{-3}	1.9×10^{-4}
$\dot{v}/v$	1.8×10^{-3}	1.3×10^{-3}	7.7×10^{-3}	1.4×10^{-3}
$\dot{\alpha}_U/\alpha_U$	-9.9×10^{-4}	7.5×10^{-4}	2.6×10^{-3}	4.7×10^{-4}
$\dot{\alpha}_3/\alpha_3$	6.6×10^{-3}	5.0×10^{-3}	1.7×10^{-2}	3.1×10^{-3}
$\dot{\mu}/\mu$	4.9×10^{-2}	3.6×10^{-2}	1.3×10^{-1}	2.3×10^{-2}
$\dot{\Lambda}_U/A_u$	1.3×10^{-1}	1.0×10^{-1}	3.4×10^{-1}	6.2×10^{-2}

ed that the pair correlation function between galaxies as a function of the comoving separation shows a (damped) periodicity of 128 h^{-1} Mpc. Hill, Steinhardt, and Turner [43] proposed that this fact could be explained if the gravitational constant or the Rydberg constant oscillated with time. The "modern" values (entries 1–5, 7–12, 14–18, 24, and 27–29 in Table I) force the phase of the oscillation to be near zero. On the other hand, long-lived β emitters relate nowadays fundamental constants to their values at the solar-system formation age, 4500 Myr ago; and the Oklo phenomenon to their value 1800 Myr ago. These two epochs exclude periodic variation of fundamental constants except for a window near the 450-Myr period. The perversity of unanimated objects makes this window very near the observed value of the period. On the other hand, bounds on a periodic variation of G_N can be obtained only through paleontological data. Visual inspection of the data set *does* show a periodic variation with a period near 400 Myr superimposed on a linear trend. Whether this is a real effect or an artifact of the data set requires a delicate separate analysis.

Thus, our analysis cannot exclude the time variation of fundamental constants as the origin of a periodicity in the structure of the Universe, but makes it unlikely. If our solitude should gladden with this elegant hope [44], we should seek its origin elsewhere.

APPENDIX A: TIME VARIATION OF NUCLEON AND MESON MASSES

Nucleon masses

In SV1 we worked with chiral QCD, that is, QCD with massless quarks. In that limit all observables with dimension of energy are proportional to Λ, and the theory possesses the internal symmetry $SU(N_f)\times SU(N_f)$, where N_f is the number of quark flavors. The theory does not distinguish between quarks of different flavors nor between left and right components. Any member of a given multiplet has the same mass. When we introduce in the Lagrangian quark mass terms, the $SU(N_f)$ multiplets become nondegenerate. Although N_f (the number of quark flavors) is six, the heavy-quark degrees of freedoms are frozen. The decoupling theorem [12] asserts that infinitely heavy quarks decouple from all quantities of physical interest. For instance, for any member of the baryon octet ($M\leq1$ GeV) the effective number of quark flavors is three (there is not enough energy to produce any appreciable amount of quark-antiquark pairs with masses exceeding 500 MeV). Indeed, we can calculate their masses perturbatively around the chiral mass value, taking as small parameters the quotients m_q/Λ. Following Gasser and Leutwyler [12], the expansion of the square of the mass $M_n^2(\Lambda,m_q)$ ($q=u$, d, s, etc.) in powers of the quark masses takes the form

$$M_n^2=A_n+m_uB_n^u+m_dB_n^d+m_sB_n^s+\cdots\ , \tag{A1}$$

where A_n denotes the (mass)^2 of the level in the chiral limit. The expansion coefficients B_n^u, B_n^d, and B_u^s are the matrix elements of the operators $\bar{u}u$, $\bar{d}d$, and $\bar{s}s$ in the unperturbed, symmetric state $|p,n\rangle$:

$$B_n^q=\langle p,n|\bar{q}q|p,n\rangle,\quad q=u,d,s\ . \tag{A2}$$

According to the general scaling which stands in the chiral limit, A_n is a pure number multiplied by Λ^2 and the coefficients B_n^q are proportional to Λ. Had we calculated B_n^q using the complete Hamiltonian, any of the pure numbers mentioned above would be functions of the quantities m_q/Λ.

If A_n does not vanish, the square root of the expansion (A1) yields a new expansion for the nucleon mass:

$$M_n = a_n + \sum_q m_q b_n^q + \cdots \ , \tag{A3}$$

where $a_n = A_n^{1/2}$ and $b_n^q = 1/2\, A_n^{-1/2} B_n^q$.

Now we have $a_n \propto \Lambda$ and $b_n \propto$ constant. In the chiral limit the baryon multiplet is degenerate, so $a_n = a$ for a given multiplet. Furthermore, SU(3) symmetry imposes some constraints between the coefficients b_n^q. It can be shown [12] that the expansion (A3) for the nucleon masses takes the forms

$$M_p = a + M_p^q \ \ (\text{proton}) \ , \tag{A4}$$

$$M_n = a + M_n^q \ \ (\text{neutron}) \ , \tag{A5}$$

where

$$M_p^q = m_u b^u + m_d b^d + m_s b^s \ , \tag{A6}$$

$$M_n^q = m_u b^d + m_d b^u + m_s b^s \ . \tag{A7}$$

There is an experimental indication that strange quarks in the proton are present already at large distances. The determination of the πN Σ term

$$\Sigma^{\pi N} = \tfrac{1}{2}(m_u + m_d)\langle p|\bar{u}u + \bar{d}d|p\rangle \ , \tag{A8}$$

is a factor of about 2 larger than the value expected from the Gell-Mann–Okubo mass formula and the assumption $\langle p|\bar{s}s|p\rangle = 0$. The value of $\Sigma^{\pi N}$ indicates instead that [13]

$$\frac{\langle p|\bar{s}s|p\rangle}{\langle p|(\bar{u}u + \bar{d}d + \bar{s}s|p\rangle} \simeq 0.21 \ . \tag{A9}$$

Similar results have been obtained for the proton matrix elements of the pseudoscalar quark densities $\bar{q}i\gamma_5 q$ [14].

Donoghue and Nappi [60] have estimated, based both on Skyrme and bag models, the following quark content for the nucleons:

$$b^u/b = 0.45, \quad b^d/b = 0.36, \quad b^s/b = 0.29 \ . \tag{A10}$$

where $b = b^u + b^d + b^s$. Following Ref. [60], the value for b is 11.6 ± 3.8.

Dominquez and de Rafael estimate the following values for the light-quark masses [61]:

$$m_u = 5.6 \pm 1.2 \ \text{MeV}, \quad m_d = 9.9 \pm 1.1 \ \text{MeV} \ ,$$
$$m_s = 199 \pm 33 \ \text{MeV} \ . \tag{A11}$$

Taking time-logarithmic derivatives in Eqs. (A6) and (A7) and using the values (A10) and (A11) we obtain the results

$$\frac{\dot{M}_p}{M_p} = 0.53\frac{\dot{\Lambda}}{\Lambda} + 0.025\frac{\dot{m}_u}{m_u} + 0.033\frac{\dot{m}_d}{m_d} + 0.40\frac{\dot{m}_s}{m_s} \ , \tag{A12}$$

$$\frac{\dot{M}_n}{M_n} = 0.53\frac{\dot{\Lambda}}{\Lambda} + 0.019\frac{\dot{m}_u}{m_u} + 0.044\frac{\dot{m}_d}{m_d} + 0.40\frac{\dot{m}_s}{m_s} \ . \tag{A13}$$

It is remarkable that 40% of both nucleon masses is accounted by the strange-quark mass. Indeed, the strange content of the nucleon is the principal cause of breakdown of the general scaling found in SV1.

Meson masses

We first analyze the π-meson masses. The square of the charged pions can be written as [12]

$$M_{\pi^\pm}^2 = (m_u + m_d)B + O(m_q^2 \ln m_q) \ , \tag{A14}$$

where $B = -(2/f_\pi^2)\langle 0|uu|0\rangle$ is proportional to Λ. The expression for the neutral pion reads

$$M_{\pi^0}^2 = 2\check{m}B - \frac{4}{3}(m_s - \check{m})B\frac{\sin^2\theta}{\cos\theta} \ , \tag{A15}$$

where $\check{m} = (m_u + m_d)/2$. The mixing angle is proportional to the ratio of the SU(2)-breaking mass difference $m_s - \check{m}$ to the SU(3)-breaking mass difference $m_s - \check{m}$. Since this ratio is a small number, the mixing angle is small, so up to terms of order θ^2 the state π^0 is degenerate with $\pi^\pm$.

The final formula for the time variation of the pion masses is then

$$\frac{\dot{M}_\pi}{M_\pi} = 0.5\frac{\dot{\Lambda}}{\Lambda} + \frac{1}{2}\left[\frac{m_u}{m_u + m_d}\frac{\dot{m}_u}{m_u} + \frac{m_d}{m_u + m_d}\frac{\dot{m}_d}{m_d}\right] \ . \tag{A16}$$

In a similar way, the time variation of the K^0 mesons can be written as

$$\frac{\dot{M}_K}{M_K} = 0.5\frac{\dot{\Lambda}}{\Lambda} + \frac{1}{2}\left[\frac{m_d}{m_d + m_s}\frac{\dot{m}_d}{m_d} + \frac{m_s}{m_d + m_s}\frac{\dot{m}_s}{m_s}\right] \ , \tag{A17}$$

and for charged kaons the formula is similar to Eq. (A17) but with the d quark replaced by a u quark.

Nucleons exchange Goldstone mesons (pseudoscalar octet) as well as vector mesons. As was mentioned above, the strange content of the proton is important, so we cannot neglect the exchange of mesons with strangeness different from zero. On the other hand, the wave functions of the vector octet are identical to the pseudoscalar octet ones in their flavor indices [62]. All this suggests the expression as an approximation for the time variation of the mean nucleon exchange meson,

$$\begin{aligned}\frac{\dot{\bar{m}}}{\bar{m}} = 0.5\frac{\dot{\Lambda}}{\Lambda} + \frac{1}{6}\Bigg[& m_s\left[\frac{1}{m_s + m_u} + \frac{1}{m_s + m_u}\right]\frac{\dot{m}_s}{m_s} \\ & + m_s\left[\frac{1}{m_s + m_u} + \frac{1}{m_s + m_u}\right]\frac{\dot{m}_s}{m_s} \\ & + m_s\left[\frac{1}{m_s + m_u} + \frac{1}{m_s + m_u}\right]\frac{\dot{m}_s}{m_s}\Bigg] \ ,\end{aligned} \tag{A18}$$

or, using Eq. (A11) for the quark masses,

$$\frac{\dot{m}}{m}=0.5\frac{\dot{\Lambda}}{\Lambda}+0.321\frac{\dot{m}_s}{m_s}+0.114\frac{\dot{m}_d}{m_d}+0.065\frac{\dot{m}_u}{m_u} . \tag{A19}$$

APPENDIX B: EÖTVÖS EXPERIMENTS AND PREFERRED REFERENCE SYSTEMS

As proved experimentally [63], the solar system is moving with respect to the BCR at a velocity of 350 km/sec. It is reasonable to expect a space dependence of the fundamental constants. In order to determine this dependence we write, for any fundamental parameter η, the equation

$$\eta(t)=\eta_0+\dot{\eta}t , \tag{B1}$$

which holds in the comoving reference system.

The change of a scalar field under post-Galilean transformations [28] leads to the expression

$$\eta'(x',t')=\eta_0+\dot{\eta}t'(1+w^2/2)+\dot{\eta}(1+w^2/2)x'w . \tag{B2}$$

The action S for a point particle in a gravitational field is

$$S=-\int d\tau\, m[x(\tau)]g_{\mu\nu}[x(\tau)]\left[\frac{dx^\mu}{d\tau}\frac{dx^\nu}{d\tau}\right] , \tag{B3}$$

and the geodesic equation becomes

$$\frac{d^2x^\sigma}{d\tau^2}+\Gamma^\sigma_{\alpha\mu}\frac{dx^\alpha}{d\tau}\frac{dx^\mu}{d\tau}+\frac{1}{2}g^{\sigma\nu}\frac{\partial m}{\partial x^\nu}\frac{1}{m} . \tag{B4}$$

In the solar reference system, the Newtonian limit for x' of this equation is

$$\frac{d^2x'}{dt'^2}-\nabla'\phi=\frac{1}{2}\nabla'm\frac{1}{m} . \tag{B5}$$

We obtain the expression for ∇m from Eq. (B2), and then Eq. (B5) becomes

$$\frac{d^2x'}{dt'^2}=\nabla\phi+\frac{1}{2}\frac{\dot{m}}{m}w . \tag{B6}$$

The anomalous acceleration is proportional to the comoving time-logarithmic derivative $\dot{m}/m$. If two bodies differ in chemical composition, this latter quantity may differ between them, and a violation of the WEP will appear. If not, any difference between the relative contributions of the gravitational binding energy to the total mass of each body will produce a Nórdtvedt-type effect, and so a violation of the general weak equivalence principle (GWEP).

Braginsky and Panov [29] have set upper bounds for the difference between the gravitational accelerations induced by the Sun on aluminum and platinum bodies. The time variation of their masses will be mainly determined by the time variation of the aluminum and platinum nuclear masses, respectively. Using the results obtained in Sec. II and in SV1 for the time variation of nuclear masses we obtain Eq. (20) of Table III.

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