Baryogenesis, sphalerons, and the cogeneration of dark matter

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Processes involving the electroweak anomaly can erase completely a primordial baryon and lepton asymmetry if $B-L=0$. This has led to the search for plausible mechanisms for weak-scale baryogenesis, or for the generation of a primordial $B-L$ asymmetry. Here it is emphasized that if another quantum number conserved up to anomalies is present electroweak anomaly processes would not necessarily erase a primordial baryon asymmetry even if $B-L=0$. Moreover, an asymmetry in the new quantum number that is comparable to the baryon asymmetry is generated concomitantly due to the electroweak anomaly. This asymmetry could be the origin of dark matter.

It is generally thought that when the temperature in the early Universe was greater than the Fermi scale processes involving the electroweak anomaly ("sphaleron processes") violated baryon number (B) and lepton number (L) at a rate that exceeded the expansion rate of the Universe [1]. A consequence of this is that any primordial B and L asymmetry that had been generated at the grand-unified-theory (GUT) scale (or any other scale large compared to the Fermi scale) would have been erased *unless* there had been a primordial $(B-L)$ asymmetry also [2]. This has led to efforts to find plausible mechanisms of low-temperature baryogenesis [3] or to generate a $B-L$ asymmetry [4]. The point of this paper is to emphasize that if there is another global quantum number in the low-energy theory that is, like B and L, conserved (up to the weak anomaly) this conclusion is vitiated, and that not only will a baryon asymmetry arise even if $B-L=0$ but that at the same time an asymmetry in the new quantum number will be generated that can play the role of dark matter. Moreover, in simple realizations of this idea one can compute $\Omega_B/\Omega_{\text{dark matter}}$ in terms of only the mass of the dark-matter particle (which is the lightest particle carrying the new quantum number) and the number of species that exist of certain types of particle. If one specifies the number of species in the model and assumes that $1=\Omega\cong\Omega_{\text{dark matter}}$, as suggested by inflation, then one can relate Ω_B to the mass of the dark-matter particle alone. What one finds, typically, as will be shown later, is that the asymmetry in the new quantum number is comparable to the asymmetry in B and L. Therefore to have $\Omega_B\sim10^{-2}$ requires that the dark-matter particle have a mass in the 100-GeV range.

What seems attractive about this idea is that three problems are solved simultaneously. (a) $B\neq0$ is generated without having to violate $B-L$. (b) The dark matter (DM) is generated along with B and L. (c) It is explained why Ω_{DM} and Ω_B are comparable (in an order-of-magnitude sense), which in some scenarios is somewhat coincidental in that the physics behind the generation of the two densities is very different. The central ideas of this paper is contained in an earlier paper of the present author with Chivukula and Farhi [5]. These ideas were there presented in the context of technicolor theories. The point that we wish to make much clearer here is that these ideas apply much more generally, and not only do not require the formidable apparatus of technicolor, but require no gauge interactions beyond those of the standard model.

The reason why if $B-L$ were absolutely conserved (and there were no new quantum number besides B and L) anomaly processes would have wiped out both B and L is easily understood. Since anomaly processes violate both B and L (but not $B-L$) when these processes are in equilibrium there will be a condition of the form $\alpha B+\beta L=0$ expressing this equilibrium. There is another condition $B-L=0$ expressing the symmetric initial condition of the Universe and the fact that $B-L$ is absolutely conserved. These two conditions on the two quantities B and L can only be satisfied if $B=L=0$. (Note that, because the $B-L$ current is not anomalous, $\alpha\neq-\beta$.)

Now let us examine what happens when there are three conserved (up to anomalies) quantum numbers B, L, and X. The condition expressing the equilibrium of the anomaly processes is of the form $\alpha B+\beta L+\gamma X=0$. If the X current has an electroweak anomaly then γ will be nonzero. [This means that there must be fermions carrying both X and $SU(2)_L$ quantum numbers. The implications of this will be seen below.] This relation and the relation $B-L=0$ coming from the absolute $B-L$ conservation give only two conditions on three quantities, so that a nontrivial solution exists:

$$B=L=-\left(\frac{\gamma}{\alpha+\beta}\right)X\neq0\ . \tag{1}$$

There are three interesting features of this equation. First, without $B-L$ violation one can still produce a nonvanishing baryon asymmetry. Second, one produces concomitantly an asymmetry of X that is comparable to the baryon asymmetry. Since X is conserved up to anomalies, the lightest X-bearing particle will be stable and will play the role of dark matter [6]. If this particle has a mass, m_{DM}, roughly of the order of the Fermi scale,

M_W, one then expects that

$$\Omega_B/\Omega_{\rm DM}=\left[\frac{\alpha+\beta}{\gamma}\frac{m_p}{m_{\rm DM}}\right]\sim m_p/M_W\sim 10^{-2}$$

which is a reasonable result, And, third, if one knows the parameters α, β, and γ one can compute $\Omega_B/\Omega_{\rm DM}$ in terms only of the mass of the dark-matter particle. But the parameters α, β, and γ are computable from elementary thermodynamic arguments, and in simple cases will only depend on the number of species of certain types of particles in the low-energy theory as will be seen in the example that will be described below.

There is one sticky point involved in this idea. It is necessary, as noted above, that there be fermions that carry both X and $\mathrm{SU}(2)_L$ quantum numbers. However, the lightest X-bearing particle, in order to play successfully the role of dark matter, must be electrically neutral and, perhaps, sterile under the weak interactions as well. In technicolor (where X=technibaryon number), as discussed in Ref. [7], it may happen that, while the techniquarks carry $\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ quantum numbers, the lightest stable technibaryon is neutral under $\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$. However, in this paper we wish to investigate what seems to us the simpler possibility, which does not involve extending the gauge group of the standard model, that some X-bearing particles are $\mathrm{SU}(2)_L$ nonsinglet, while others are $\mathrm{SU}(2)_L$ singlets. The former type are generated in the early Universe by the anomaly processes but ultimately decay into the latter, which are lighter and persist to make up the dark matter. We will now discuss a model that shows how this can happen in a realistic way.

To the standard model will be added a vectorlike set of leptons consisting of an ordinary family of leptons

$$L=\begin{bmatrix}\nu\\ l^-\end{bmatrix}_L,\quad l_L^+\quad \bar\nu_L\ ,$$

and a mirror family of leptons

$$\bar L=\begin{bmatrix}l^+\\ \nu\end{bmatrix}_L,\quad l_L^-,\quad \nu_L\ .$$

These will obtain mass, as do the known leptons

$$L^a=\begin{bmatrix}a\\ l^{-a}\end{bmatrix}_L,\quad l_L^{+a},\quad \text{and perhaps } \bar\nu_L^a\ ,$$

from the standard-model Higgs doublet

$$\Phi=\begin{bmatrix}\phi^0\\ \phi^0\end{bmatrix}.$$

(a here is a family label and is not carried by the new leptons.) The new leptons do not mix with the known leptons and carry the conserved (up to anomalies) quantum number X, as shown in Table I. Note that while L is a lepton and $\bar L$ is an antilepton, *both* doublets L and $\bar L$ carry the same value of X. If they carried the opposite values then X would have no electroweak anomaly. In addition to these leptons there are some $\mathrm{SU}(3)\times\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$-singlet fermions that will be denoted S_{+1}, S_{-1}, S_0. These are left-handed and the subscript refers to their X charge. These have lepton number zero. Finally, there are two new scalar multiplets,

$$\Phi'=\begin{bmatrix}\phi'^0\\ \phi^{-\prime}\end{bmatrix},$$

which has $X=0$, $L=+1$ and has a vanishing vacuum expectation value, and h, which is a singlet under everything and has $X=L=0$. h may or may not have an expectation value; it does not matter. The value of X and L for these various particles are listed in the table. It is easy to see that there are the following terms possible among these particles.

(a) $\tilde\Phi^\dagger(L^aCl_L^{+b})$, which is the lepton Yukawa coupling of the standard model. If there are right-handed neutrinos there are also the terms $\Phi^\dagger(L^aC\bar\nu_L^b)$ and $(\bar\nu_L^aC\bar\nu_L^b)$.

(b) $\tilde\Phi^\dagger(LCl_L^+)$, $\Phi^\dagger(LC\bar\nu_L)$, $\Phi^\dagger(\bar LCl_L^-)$, and $\tilde\Phi^\dagger(\bar LC\nu_L)$ (but not $\bar\nu_LC\bar\nu_L$ or $\nu_LC\nu_L$ because of X conservation), which are the analogous Yukawa terms for the new leptons and give them all mass of order M_W.

(c) $\Phi'^\dagger(L^aCS_0)$, $\Phi'^\dagger(LCS_{-1})$, and $\tilde\Phi'^\dagger(\bar LCS_{-1})$. These couple the doublet leptons to the singlet fermions and thus allow the former (which are produced by anomaly processes) to decay into the latter (which are the dark matter).

(d) $h(S_{+1},CS_{-1})$, $(S_{+1}CS_{-1})$, $h(S_0CS_0)$, (S_0CS_0),

TABLE I. The assignments of lepton number (L) and the new quantum number X to the fields of the illustrative model. The first set of fields with the family index a ($a=1,2,3$), are the known families of leptons; the second set is the exotic lepton family; and the third set is the exotic mirror lepton family. The S_i ($i=\pm1,0$) are standard-model singlet fermions whose subscripts refer to the value of X. Φ is the standard-model Higgs doublet. Φ' and h are additional scalars.

Field	X	L
$L^a=\begin{bmatrix}\nu^a\\ l^{-a}\end{bmatrix}_L$	0	+1
l_L^{+a}	0	−1
$\bar\nu_L^a$(?)	0	−1
$L=\begin{bmatrix}\nu\\ l^-\end{bmatrix}_L$	+1	+1
l_L^+	−1	−1
$\bar\nu_L$	−1	−1
$\bar L=\begin{bmatrix}l^+\\ \bar\nu\end{bmatrix}_L$	+1	−1
l_L	−1	+1
ν_L	−1	+1
S_{+1}	+1	0
S_{-1}	−1	0
S_0	0	0
$\Phi=\begin{bmatrix}\phi^0\\ \phi^-\end{bmatrix}$	0	0
$\Phi'=\begin{bmatrix}\phi'^0\\ \phi'^-\end{bmatrix}$	0	+1
h	0	0

and h^2, h^3, h^4. These all involve only singlet particles. These couplings allow the S particles to annihilate with their antiparticles into h particles [see step (6) below].

The cosmological evolution was as follows. (1) At very early times when the temperature was far above M_W, some processes—perhaps grand unification processes—generated $B=L\neq 0$. Whether $X\neq 0$ was also generated at this point does not matter, but let us say not. (2) When anomaly processes were in equilibrium (down to about $T=200$–300 GeV, see [8]) they violated B, L, and X (but not $B-L$) and converted some of the asymmetry in B and L into an asymmetry in X, as described in Eq. (1). The X-bearing particles that are generated by the anomaly processes are the doublets, L and $\bar{L}$. (3) Anomaly processes went out of equilibrium, i.e., "froze-out." (4) When the temperature fell below the masses of L and $\bar{L}$, annihilations occurred between L and $\bar{L}$ and their antiparticles leaving only the excess of L and $\bar{L}$ (due to the X asymmetry). (5) These excess L and $\bar{L}$ decayed through Φ'-mediated processes into ordinary leptons and the X-bearing singlets S_{-1}, as follows, $L\to L^a+S_0+S_{-1}^{(c)}$ and $\bar{L}\to L^{a(c)}+S_0^{(c)}+S_{-1}^{(c)}$. Here (c) means antiparticle. Thus since $S_{-1}^{(c)}$ has $X=+1$, the X asymmetry existed (and still exists) as an asymmetry in S_{-1}. This is the dark matter. (6) When the temperature further dropped below the masses of the $S_\pm$ and S_0, annihilations occurred between them and their antiparticles to give h particles. Only the asymmetry in S_{-1} now remains of the singlet fermions, S_i. (7) When the temperature dropped below the mass of h, they decay away into ordinary particles. For example, the loop diagram shown in Fig. 1 can give $h\to 2$ leptons. One is left, finally, with the B, L, and X asymmetries resulting from step (2). The B asymmetry is today in the form of protons and neutrons. The X asymmetry is in the form of S_{-1}, which being the lightest particles having X are stable. To compute the relative abundances of B, L, and X it is thus only necessary to compute what they were when the anomaly processes froze out. This is an elementary thermodynamic exercise. Given the processes that are allowed by the couplings (a) through (d) above one can write the chemical potentials of all the particles in terms of four quantities: μ_q, μ_l, μ_W, μ_X. One finds for the particles of the standard model

$$\mu(u^a)=\mu_q+\tfrac{1}{2}\mu_W\ , \tag{2a}$$

$$\mu(d^a)=\mu_q-\tfrac{1}{2}\mu_W\ , \tag{2b}$$

$$\mu(\nu^a)=\mu_l+\tfrac{1}{2}\mu_W\ , \tag{2c}$$

$$\mu(l^{-a})=\mu_l-\tfrac{1}{2}\mu_W\ , \tag{2d}$$

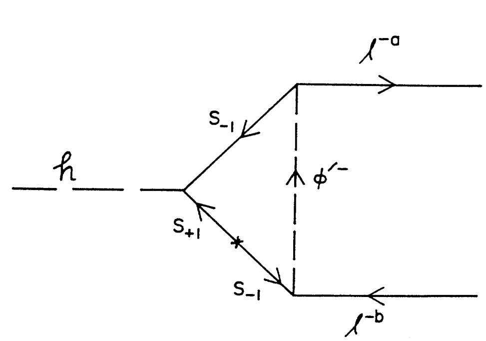

FIG. 1. Diagram showing how the h particle can decay into ordinary matter in the illustrative model described in the text.

$$\mu(W^+)=\mu_W\ , \tag{2e}$$

$$\mu(\Phi)=0\ . \tag{2f}$$

For the new particles

$$\mu(\nu)=\mu_l+\mu_X+\tfrac{1}{2}\mu_W\ , \tag{2g}$$

$$\mu(l^-)=\mu_l+\mu_X-\tfrac{1}{2}\mu_W, (\nu, l^- \epsilon L)\ , \tag{2h}$$

$$\mu(\bar{\nu})=-\mu_l+\mu_X-\tfrac{1}{2}\mu_W, (\bar{\nu}, l^+ \epsilon \bar{L})\ , \tag{2i}$$

$$\mu(l^+)=-\mu_l+\mu_X+\tfrac{1}{2}\mu_W\ , \tag{2j}$$

$$\mu(S_{+1})=-\mu(S_{-1})=\mu_X\ , \tag{2k}$$

$$\mu(S_0)=0\ , \tag{2l}$$

$$\mu(\phi'^0)=\mu_l+\tfrac{1}{2}\mu_W\ , \tag{2m}$$

$$\mu(\phi'^-)=\mu_l-\tfrac{1}{2}\mu_W\ , \tag{2n}$$

$$\mu(h)=0\ . \tag{2o}$$

One can now eliminate μ_W using the condition that the Universe has net electric charge zero [9]. From the condition that the anomaly processes be in equilibrium, and from the condition $B-L=0$, one can solve for μ_X and μ_l to get everything in terms of μ_q. The ratio of B and X can then be determined.

For simplicity, we will make the following plausible assumptions, $m_{\phi^{-\prime}}=m_{\phi^{0\prime}}$, $m_\nu \cong m_{l^-}$, and $m_{\bar{\nu}}\cong m_{l^+}$. These are plausible because to assume otherwise would give significant contributions to the ρ parameter. (These masses are all assumed to be in the 100–300-GeV range.) Also it is assumed that $m_{l^-}\cong m_{l^+}$. If T^* is the temperature at which the anomaly processes froze out, we assume that $m_{S_{\pm 1}} \ll T^*$.

Charge neutrality implies that

$$0=18(\tfrac{2}{3})\mu(u^a)+18(-\tfrac{1}{3})\mu(d^a)+6(-1)\mu(l^{a-})+2n_L(+1)[f_F(m_{l^+}/T^*)/f_F(0)]\mu(l^+)$$
$$+2n_{\phi'}(-1)[f_B(m_{\phi'}/T^*)/f_F(0)]\mu(\phi^{-\prime})+3(+1)[f_B(M_W/T^*)/f_F(0)]\mu(W^+)\ .$$

Here f_F and f_B represent the fermion and boson distribution functions

$$f_F(x)\equiv \frac{1}{4\pi^2}\int_0^\infty \frac{y^2dy}{\cosh^2(\frac{1}{2}\sqrt{y^2+x^2})}\ ,$$

$$f_B(x)\equiv \frac{1}{4\pi^2}\int_0^\infty \frac{y^2dy}{\sinh^2(\frac{1}{2}\sqrt{y^2+x^2})}\ .$$

We have assumed that the chemical potentials are the same for the three known generations and have used that the masses of the known quarks and leptons are small compared to T^*. Using also that $M_W/T^* \ll 1$ and that $f_B(0)/f_F(0)=2$, the last term in the equation becomes $6\mu(W^+)$. $n_{\phi'}$ and n_L are the number of flavors of Φ' and L, respectively. We will define

$$2\tilde{n}_{\phi'} \equiv n_{\phi'}[f_B(m_{\phi'-}/T^*)/f_F(0)] \; ,$$
$$\tilde{n}_L = n_L[f_F(m_l/T^*)/f_F(0)] \; . \quad (3)$$

Then using Eqs. (2) one has

$$0=6\mu_q-(6+4\tilde{n}_L-2\tilde{n}_{\phi'})\mu_l + \mu_W(18+2\tilde{n}_L+\tilde{n}_{\phi'})$$

or

$$\mu_W = \frac{-6\mu_q+(6+4\tilde{n}_L+2\tilde{n}_{\phi'})\mu_l}{(18+2\tilde{n}_L+\tilde{n}_{\phi'})} \; . \quad (4)$$

Moreover lepton and baryon number are given by

$$B(T^*) \propto 12\mu_q \; , \quad (5a)$$
$$L(T^*) \propto 6(\mu_l - \tfrac{1}{2}\mu_W) + 3(\mu_l + \tfrac{1}{2}\mu_W) + 8\tilde{n}_L\mu_l + 4\tilde{n}_{\phi'}\mu_l$$
$$= (9+8\tilde{n}_L+4\tilde{n}_{\phi'})\mu_l - \tfrac{3}{2}\mu_W \; , \quad (5b)$$

(Here we have used the fact that there is only one polarization of the neutrinos, ν^a, and two polarizations of the charged leptons, $l^{\bar{a}}$.) Since the combination $(2\tilde{n}_L+\tilde{n}_{\phi'})$ appears in several places we will call this n. Thus the condition $B-L=0$ gives

$$12\mu_q = (9+4n)\mu_l - \tfrac{3}{2}\mu_W \; . \quad (6)$$

Equation (4), which can be rewritten as

$$\mu_W = [-6\mu_q + (6+2n)\mu_l]/(18+n) \; ,$$

allows Eq. (6) to be expressed as

$$(207+12n)\mu_q = (153+78n+4n^2)\mu_l \; . \quad (7)$$

Finally, there is the condition that anomaly processes are in equilibrium. This gives

$$0 = 9\mu_q + 3\mu_l + 2n_L\mu_X \; , \quad (8)$$

This comes from the fact that a weak instanton emits 9 quark lepton doublets (3 families), and n_L each of the doublets L and $\bar{L}$. Equations (7) and (8) imply that

$$2n_L\mu_X = -\mu_q\left[9+3\frac{\mu_l}{\mu_q}\right]$$
$$= -3\mu_q\left[\frac{666+246n+12n^2}{153+78n+4n^2}\right] \; . \quad (9)$$

We will define a function

$$c(n) = \frac{2}{3}\left[\frac{153+78n+4n^2}{111+41n+2n^2}\right]$$

which can be seen to be very slowly varying: $c(0)=\frac{34}{37}\cong 0.92$; $c(1)\cong 1.02$; $c(3)\cong 1.12$; $c(10)\cong 1.23$; and $c(\infty)=1.33$. For realistic values of $n\equiv 2\tilde{n}_L+\tilde{n}_{\phi'}$, $c(n)=1.2\pm 0.1$. In terms of this Eq. (9) becomes

$$\mu_X/\mu_q = \frac{6}{c(n)n_L} \; . \quad (10)$$

Now if we call the number of species of S_{-1} by n_S the value of $X(T)/B(T)$ is given by

$$X(T)/B(T) = (8\tilde{n}_L + 2n_S)\mu_X/12\mu_q \; .$$

Or,

$$\frac{X}{B} = 4\left[\frac{\tilde{n}_L+\frac{1}{4}n_S}{n_L}\right]c(n)^{-1} \; , \quad (11)$$

$$\Omega_B \cong \frac{\Omega_B}{\Omega_{\rm DM}} = \frac{1}{4}\frac{m_p}{m_{\rm DM}}\left[\frac{n_L}{\tilde{n}_L+\frac{1}{4}n_S}\right]c(n) \; . \quad (12)$$

The factor $c(n)$ is indistinguishable from unity in practice since Ω_B will probably never be known to 10%. All of the practical uncertainty is in the factor $n_L/(\tilde{n}_L+\frac{1}{4}n_S)$. If m_L is significantly less than T^*, then this factor is just $(1+\frac{1}{4}n_s/n_L)^{-1}$.

This is a simple rational number characterizing the model and is for small values of n_S and n_L (and therefore for the simplest models) also indistinguishable from unity. Thus it is fair to say that if $n_S/n_L \lesssim 1$ that within about 20%

$$\Omega_B \cong \frac{1}{4}\frac{m_p}{m_{\rm DM}} \; . \quad (13)$$

For m_L approximately equal to or larger than T^* the factor $n_L/(\tilde{n}_L+\frac{1}{4}n_S)$ becomes larger, but is still of order unity.

The above model is certainly not unique. There are several simple sets of fermions that could be added to the standard model and given an anomalous but otherwise conserved global quantum number. Nevertheless, the qualitative features should remain unaffected, and one expects to find

$$\Omega_B = km_p/m_{\rm DM} \; , \quad (14)$$

where k is a number roughly of order unity determined by simple thermodynamic considerations, and involving the details of the model. The dark-matter particle is then a standard model singlet with mass in the 10–100-GeV range. It would be hard to see this particle directly. But as we have noted there must be some X-bearing fermions that have $\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y$ quantum numbers (in this model L and $\bar{L}$). These can be produced in the laboratory, *in pairs* because of the X conservation. Then they will decay, as in the early Universe, into ordinary leptons (or quarks) and the dark-matter particles. In that way the properties of the dark-matter particle and the other particles in the 'X sector' could be indirectly inferred.

A general consideration in any model of dark matter where the dark matter abundance is supposed to be entirely due to an asymmetry, as here, is whether the symmetric component has sufficient annihilation cross section to disappear [6]. A problem can arise if the cross section required for the symmetric component to annihilate makes the dark matter so interactive that it would have been seen in laboratory searches. This problem does not arise here because the annihilations of the S_i and their antiparticles happen through the coupling to the singlet scalar h. This interaction can be supposed to be fairly strong without leading to any problems because it involves only exotic, gauge-singlet particles. The h does not mediate any interaction of the dark-matter particle, S_{-1}, with ordinary quarks and leptons.

A final remark is in order about how the primordial B and L asymmetries might have arisen. One possibility is through conventional grand unified theories in the familiar "drift and decay" scenario. For example, one can embed the illustrative model outlined above in the most straightforward way in an SU(5) model, assigning the new lepton family and mirror family to $(\mathbf{10}+\overline{\mathbf{5}}+\mathbf{1})$ and $(\overline{\mathbf{10}}+\mathbf{5}+\mathbf{1})$, respectively, and the standard model singlet fields S_i and h to SU(5) singlets. One finds that the SU(5) gauge interactions (or the exchange of color-triplet scalars) violate B and L as in minimal SU(5), but both $B-L$ and X (which is assigned to whole SU(5) multiplets in the way consistent with Table I) are exactly conserved—except that X is anomalous, of course. Thus the primordial asymmetry would be $B=L\neq 0$, $X=0$.

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