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Energetic neutrinos from heavy-neutralino annihilation in the Sun

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Neutralinos may be captured in the Sun and annihilate therein producing high-energy neutrinos. Present limits on the flux of such neutrinos from underground detectors such as Irvine-Michigan-Brookhaven (IMB) and Kamiokande II may be used to rule out certain supersymmetric dark-matter candidates, while in many other supersymmetric models the rates are large enough that if neutralinos *do* reside in the galactic halo, observation of a neutrino signal may be possible in the near future. Neutralinos that are either nearly pure Higgsino or a Higgsino-gaugino combination are generally captured in the Sun by a scalar interaction with nuclei in which a virtual lightest Higgs boson is exchanged. If the squark mass is not much greater than the neutralino mass then capture of neutralinos that are primarily gaugino occurs predominantly by spin-dependent scattering off hydrogen in the Sun. Although only neutrinos from annihilation in the Sun are considered here, the neutrino signal from weakly interacting massive particle annihilation in the Earth should be of comparable strength. Detection rates for mixed-state neutralinos are generally higher than those for Higgsinos or gauginos.

I. INTRODUCTION

The idea that stable weakly interacting massive particles (WIMP's) make up the bulk of the dark matter in the Universe and galactic halo has been the focus of much theoretical and experimental research recently [1]. Now that the original WIMP, the Dirac neutrino, has been ruled out [2], the neutralino [3], a linear combination of the supersymmetric partners of the photon, Z^0, and Higgs bosons, has become the preferred thermal relic. Although the original treatises considered only neutralinos lighter than the $W^{\pm}$ [4,5], heavy neutralinos, those more massive than the W, may also be suitable dark-matter candidates [6,7]. Although "extremely" massive neutralinos are not favored theoretically [8], neutralinos in the 100-GeV range may still solve the naturalness problem and become increasingly attractive as unsuccessful accelerator searches push the mass scale for supersymmetry upward.

Since many neutralinos are not yet accessible in accelerators and are such compelling dark-matter candidates, a variety of complementary experiments to detect neutralinos in our galactic halo are currently being pursued. Some seek to observe neutralinos by detecting the energy deposited in an ultralow background detector when a neutralino elastically scatters off of a nucleus therein [9]. Alternatively, neutralino dark matter in the galactic halo may be indirectly detected by its annihilation products. A continuum spectrum of cosmic-ray antiprotons [10], γ rays [11], and positrons [12] are produced in the cascade resulting from the annihilation products of the neutralinos; however, astrophysical uncertainties involving the propagation of cosmic rays from conventional sources are so great that it seems unlikely that WIMP-induced continuum cosmic rays could ever be distinguished from those from standard sources. Some authors have boldly suggested that annihilation of WIMP's in the galactic halo could produce either γ-ray [13] or positron [14,15] line radiation which could be readily distinguished from the background. While such a signal would provide unambiguous evidence for particle dark matter, because of astrophysical uncertainties an observable signal of this kind is not guaranteed even if suitable WIMP's *do* reside in the galactic halo.

In this paper we address the possibility of indirect detection of heavy neutralinos by observation of yet another annihilation product: high-energy neutrinos [16]. WIMP's in the galactic halo will be captured in the body of the Sun or Earth [17–20] and annihilate therein, producing high-energy neutrinos that may be observable in underground neutrino detectors. This method of detection has several advantages over cosmic-ray signatures: First of all, whereas cosmic rays are expected to be isotropically distributed, the neutrino signal comes from a fixed direction and is therefore much more easily distinguished from background. The number density $n_{\tilde{\chi}}$ of neutralinos in the halo is inversely proportional to the neutralino mass, and as we shall see, the annihilation rate in the Sun is $\propto n_{\tilde{\chi}}$, while the annihilation rate in the halo is $\propto n_{\tilde{\chi}}^2$, making the neutrino signal favored for higher neutralino masses. In addition, the uncertainties in the predicted rates for neutrino events are smaller than those in the predicted cosmic-ray fluxes (roughly factors of about 2 for neutrino events and orders of magnitude for cosmic-ray fluxes). Basically, this is because the local halo density is known better than the dark-matter distribution throughout the Galaxy and propagation of neutrinos through the Sun is more easily modeled than cosmic-ray propagation through the Galaxy. It should also be noted that neutrino and cosmic-ray searches are mutually complementary: For example, the neutralinos that may be discovered through distinctive cosmic-ray positron signatures are primarily Higgsinos [15], whereas neutrino signals are strongest for neutralinos that are a mixed Higgsino-gaugino state.

Unlike Dirac neutrinos, which annihilate directly into light (i.e., ν_e, ν_μ, and ν_τ) neutrinos, neutralinos are Majorana particles and therefore do *not* produce prompt neutrinos; the neutrinos from neutralino-neutralino annihilations come from the decays of the annihilation products, and so the neutrino spectrum is considerably softer. Detailed neutrino spectra from energetic quarks and leptons injected into the core of the Sun were calculated by Ritz and Seckel (RS) [21]. The analysis for light neutralinos was originally carried out by Giudice and Roulet [22], who considered only annihilation into fermion-antifermion pairs, and more completely by Gelmini, Gondolo, and Roulet [23], who considered annihilation into pairs of Higgs bosons as well. Here we extend this work to heavy neutralinos by considering the effect of the gauge-boson, Higgs-boson, and top-quark annihilation channels which open up for heavy neutralinos. We also consider the effect of the interactions of the annihilation products and resulting high-energy neutrinos in the Sun which become important at higher energies.

First, let us briefly review the minimal supersymmetric extension of the standard model (MSSM) and the properties of the neutralino. For more details we refer the reader to Ref. [3] and Griest, Kamionkowski, and Turner (GKT) [5], whose notation we use throughout. There are actually four neutralinos, and the lightest (the nth of the four neutralinos) is assumed to be the lightest supersymmetric particle (LSP) and stable and is denoted as *the* neutralino,

$$\tilde{\chi}=Z_{n1}\tilde{B}+Z_{n2}\tilde{W}^3+Z_{n3}\tilde{H}_1+Z_{n4}\tilde{H}_2 \ , \qquad (1)$$

where $(Z)_{ij}$ is a real orthogonal matrix that diagonalizes the neutralino mass matrix (Eq. (C38) of Ref. [3]) and depends only on the gaugino mass parameter M, Higgsino mass parameter μ, and the ratio of Higgs vacuum expectation values $\tan\beta$. In Fig. 1 we plot neutralino mass contours (dashed curves) and contours of $Z_{n1}^2+Z_{n2}^2$ (solid curves), the gaugino fraction, for $\tan\beta=2$ (plots for other values of $\tan\beta$ are similar). As noted originally by Olive and Srednicki [6], in much of parameter space where the neutralino is heavier than the W, the gaugino fraction is greater than 0.99 and the neutralino is almost pure B-ino. In much of parameter space, the gaugino fraction is less than 0.01 and the neutralino is almost pure Higgsino. Near the 0.5 gaugino fraction curve, a curve that asymptotes to $\mu=\frac{5}{3}M\tan^2\theta_W$ at high neutralino mass, the neutralino is a mixed state, half gaugino and half Higgsino.

In the MSSM there are three neutral Higgs bosons [24]: The mass of the lightest H_2^0, which must be less than $m_Z\cos2\beta$ (provided the top quark is not unusually heavy; see Ref. [25]), and $\tan\beta$ determine the masses of the other two, H_1^0, which must be heavier than the Z, and H_3^0, whose mass falls between $m_{H_0^2}$ and $m_{H_1^0}$. There are also charged Higgs bosons $H^\pm$ which are always heavier than the W and two charginos, linear combinations of the supersymmetric partners of the W and the charged Higgs bosons. The masses of the superpartners of the quarks and leptons, which we will collectively refer to as squarks, are all undetermined, but for simplicity we give them all the same mass $M_{\tilde{q}}$, which, assuming the neutralino is the LSP, is greater than $m_{\tilde{\chi}}$.

Although the MSSM has many undetermined parameters [3] ($\tan\beta$, M, μ, $m_{H_2^0}$, $M_{\tilde{q}}$, and the top-quark mass m_t), the parameters are not entirely unconstrained, and by studying several "corners" of parameter space, we can get an understanding of the dependence of detection rates on the different parameters of the model. Although m_t is constrained only to be greater than 80 GeV [26] (from unsuccessful accelerator searches) and less than about 200 GeV [27] (from limits on radiative corrections to $\sin^2\theta_W$), we will assume $m_t=120$ GeV throughout; as we will discuss later, varying the top-quark mass should have little effect on our results. Recent searches for neutral Higgs bosons at the CERN e^+e^- collider LEP have constrained regions of $m_{H_2^0}$-$\tan\beta$ space [28]. In addition, we will only consider $\tan\beta>1$, since radiative corrections drive $\tan\beta$ to values greater than one when $m_t \gg m_b$, and

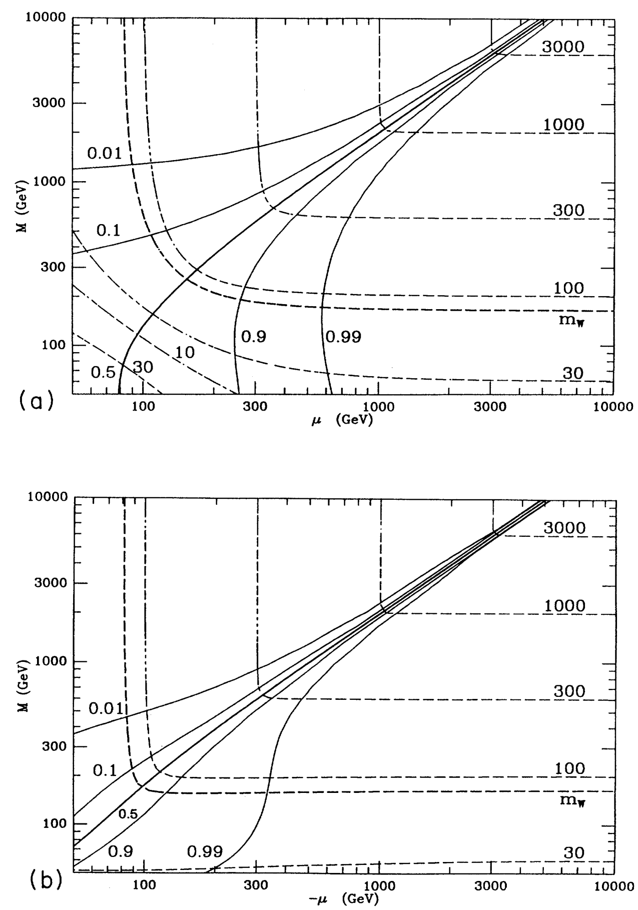

FIG. 1. Lightest neutralino composition and mass for $\tan\beta=2$. The dashed curves are contours of constant neutralino mass $m_{\tilde{\chi}}$, and the solid curves are contours of constant gaugino fraction ($Z_{n1}^2+Z_{n2}^2$); in (a) $\mu>0$ and in (b) $\mu<0$.

$\tan\beta < m_t/m_b \simeq 25$, required for electroweak symmetry breaking in many supergravity models [29]. To see the range of possibile capture and detection rates due to the range of all possible values for the squark mass, we will present results assuming the squark mass is infinite and then show results assuming the squark mass is slightly heavier than the neutralino mass.

Although determination of the event rate is relatively straightforward, it is quite lengthy and depends on a variety of input physics such as solar physics, neutrino physics, hadronization of quarks, underground detectors, and, of course, the interactions of neutralinos with ordinary matter. The flux of high-energy neutrinos of type i (e.g., $i = \nu_\mu$, $\bar{\nu}_\mu$, etc.) from neutralino annihilation in the Sun is simply

$$\left[\frac{d\phi}{dE}\right]_i = \frac{\Gamma_A}{4\pi R^2} \sum_F B_F \left[\frac{dN}{dE}\right]_{Fi} . \tag{2}$$

The quantity Γ_A is the rate of neutralino-neutralino annihilations in the Sun, and R is simply the distance of the Earth from the Sun. Neutralinos from the galactic halo are accreted onto the Sun and their number in the Sun is depleted by annihilation. In most cases of interest these two processes come to equilibrium on a time scale much shorter than the solar age, in which case $\Gamma_A = C/2$, where C is the rate for capture of neutralinos from the halo. As one might imagine, the capture rate is basically determined by the flux of neutralinos incident on the Sun and a probability for capture, which in turn depends on kinematic factors and the cross sections for elastic scattering of the neutralino off of the elements in the Sun. The sum is over all annihilation channels F (e.g., pairs of gauge or Higgs bosons or fermion-antifermion pairs), B_F is the annihilation branch for channel F, and $(dN/dE)_{Fi}$ is the differential energy spectrum of neutralino type i at the *surface* of the Sun expected from injection of the particles in channel F in the *core* of the Sun. The spectrum $(dN/dE)_{Fi}$ is a function of the energy of the neutrino and energy of the injected particles. Determination of these spectra is quite complicated as it involves hadronization of the annihilation products, interaction of the particles in the resulting cascade with the solar medium, and the subsequent interaction of high-energy neutrinos with the solar medium as they propagate from the core to the surface of the Sun [21].

The experimental signature on which we will eventually focus will be the number of upward-moving muons induced by high-energy neutrinos from the Sun that are observed in underground detectors. Given the fluxes $(d\phi/dE)_i$, the final result for the rate (per unit detector area) for neutrino-induced upward-moving muons may be written simply as

$$\Gamma_{\text{detector}} = \sum_i D_i \int \left[\frac{d\phi}{dE}\right]_i E^2 dE , \tag{3}$$

where the sum is over ν_μ, which produce muons, and $\bar{\nu}_\mu$, which produce antimuons. Since the cross section for the neutrino to produce a muon in the rock below the detector is proportional to the neutrino energy E and the range of the muon is roughly proportional to its energy, the probability that a neutrino of energy E produces a muon which traverses the detector is E^2 times a constant D_i, hence the integral in Eq. (3). Neutrinos may also be detected by contained events in which a charged lepton is produced within the detector, but because this process is proportional only to the neutrino energy E (as opposed to E^2 for throughgoing events), the throughgoing muons should provide a more promising signature for heavy neutralinos.

In the next section we discuss the rate Γ_A of neutralino-neutralino annihilation in the Sun and Earth. The annihilation rate is proportional to the square of the number of neutralinos in the Sun or Earth, and this number is increased by capture of neutralinos from the halo while neutralinos are depleted by annihilation. Capture occurs by elastic scattering of neutralinos in the galactic halo off of nuclei in the Sun. We show the regions of parameter space in which capture occurs predominantly by scattering off of heavy nuclei via a scalar ("spin-independent") interaction involving exchange of the lightest Higgs boson and the regions where capture occurs primarily by scattering via an axial ("spin-dependent") interaction involving squark exchange off of hydrogen. We also show the regions of parameter space where the capture and annihilation rates are large enough that the annihilation rate is half the capture rate and the neutrino flux is at "full signal."

In Sec. III we discuss the neutrino spectra $(dN/dE)_{Fi}$ from products of neutralino-neutralino annihilation in the Sun and Earth. We describe the hadronization and decays of the annihilation products and the interaction of the annihilation products and high-energy neutrinos with the Sun. In Sec. IV we discuss detection of high-energy neutrinos from the Sun (and Earth). We then point out that the most promising method of detection is via observation of upward-moving throughgoing muons induced by high-energy neutrinos in the rock below the detector and discuss the calculation of the event rate.

In Sec. V we present our results and discuss which supersymmetric candidates for the primary component of the galactic halo are already ruled out by current neutrino-flux limits and which may be observable in the near future. Most of the models that are inconsistent with current limits from Irvine-Michigan-Brookhaven (IMB) [30] and Kamiokande [31] on high-energy neutrino fluxes are those where the neutralino is a mixed gaugino-Higgsino state and the mass of the lightest Higgs boson is near the current lower limits imposed by LEP [28]. We find that if observational neutrino-flux limits are improved by a factor of 10, say, many more supersymmetric models will become detectable by these methods. The neutrino signal from neutralinos that are primarily gaugino is greater for models where the squark mass is smaller, while the neutrino rates from neutralinos that are Higgsino or mixed gaugino-Higgsino states are relatively insensitive to the squark mass. In the final section we discuss our results, briefly discuss backgrounds and detection strategies, and make some concluding remarks. In Appendix A we display the cross section for elastic scattering of a neutralino off of nuclei, and Appendix B contains new results for cross sections for annihilation of neutralinos into mixed Higgs/gauge-boson final states.

II. RATE OF ANNIHILATION IN THE SUN

The first step in calculating the rate for WIMP-induced neutrino events from the Sun is the determination of the rate at which neutralinos annihilate in the Sun. As mentioned previously, neutralinos accumulate in the Sun or Earth by capture from the galactic halo and are depleted by annihilation. If N is the number of neutralinos in the Sun, then the differential equation governing the time evolution of N is

$$\dot{N}=C-C_A N^2 , \tag{4}$$

where the dot denotes differentiation with respect to time. Here C is the rate of accretion of neutralinos onto the Sun (or Earth). The determination of C is straightforward and will be discussed in detail below, and if the halo density of neutralinos remains constant in time, C is, of course, time independent.

The second term on the right-hand side is twice the annihilation rate in the Sun (or Earth), $\Gamma_A=C_A N^2/2$, and accounts for depletion of neutralinos. The quantity C_A depends on the cross section for neutralino-neutralino annihilation and the distribution of neutralinos in the Sun (or Earth) [32]:

$$C_A=\frac{\langle \sigma v \rangle_A V_2}{V_1^2} , \tag{5}$$

where $\langle \sigma v \rangle_A$ is the spin-averaged total annihilation cross section times relative velocity in the limit of zero relative velocity (since captured neutralinos move *very* slowly) and can be evaluated using the formulas of GKT and Appendix B, and the quantities V_j are effective volumes for the Sun or Earth [32,18]:

$$V_j=\left[\frac{3m_{\mathrm{Pl}}^2 T}{2jm_{\tilde{\chi}}\rho}\right]^{3/2} , \tag{6}$$

where T is the temperature of the Sun or Earth, m_{Pl} is the Planck mass, and ρ is the core density of the Sun or Earth. In Ref. [32] it is found that $V_j=6.5\times 10^{28}(jm_{\tilde{\chi}}^{10})^{-3/2}$ cm^3, where $m_{\tilde{\chi}}^{10}$ is the neutralino mass in units of 10 GeV, for the Sun, and in Ref. [18] it is found that $V_j=2.0\times 10^{25}(jm_{\tilde{\chi}}^{10})^{-3/2}$ cm^3 for the Earth.

Solving Eq. (4) for N, we find that the annihilation rate at any given time is

$$\Gamma_A=\frac{C}{2}\tanh^2(t/\tau_A) , \tag{7}$$

where $\tau_A=(CC_A)^{-1/2}$ is the time scale for capture and annihilation to equilibrate. Therefore, if the age of the Sun is much greater than the equilibration time scale ($t_\odot=1.5\times 10^{17}$ sec $\gg\tau_A$), then the neutrino flux is at "full signal" ($\Gamma_A=C/2$), but if $\tau_A\gg t_\odot$, then the annihilation rate is smaller and the neutrino signal is diluted accordingly. As we shall see, the capture rate in the Earth is generally $\lesssim 10^{-9}$ that in the Sun, while the value of V_j in the Earth is only about 3×10^{-4} that in the Sun, and so the value of τ_A is always larger in the Earth than in the Sun; consequently, the fraction of full signal in the Earth can never be greater than that in the Sun.

Since the calculation of the capture rate that we use has been completed by Gould [18–20], we will only review the ingredients and refer the reader to the original papers for details. The basic idea is simple: When passing through the Sun, if a WIMP of mass $m_{\tilde{\chi}}$ scatters off of a nucleus of mass m_i to a velocity less than the escape velocity v_{esc} at that point, it will be captured. Since the typical halo velocity of a WIMP is about 300 km sec^{-1} and the escape velocity in the Sun is 618 km sec^{-1} (just at the surface), WIMP's are captured in the Sun quite efficiently. On the other hand, the escape velocity in the Earth is 11 km s^{-1}, and so unless the WIMP mass closely matches the mass of an element abundant in the Earth, the conditional probability that the WIMP will be captured in a collision is quite small. This kinematic suppression can be quantified by the parameter

$$A=\frac{3}{2}\frac{m_{\tilde{\chi}}m_i}{(m_{\tilde{\chi}}-m_i)^2}\left[\frac{v_{\mathrm{esc}}^2}{\bar{v}^2}\right]\phi_i , \tag{8}$$

where v_{esc} is the escape velocity at the surface of the Sun or Earth, $\bar{v}$ is the velocity dispersion of the WIMP's in the halo, and ϕ_i is about 3.3 in the Sun and 1.4 in the Earth (the exact values for element i are listed in Table I in Ref. [23]). When $A\gg 1$ the kinematic suppression factor $S_i(m_{\tilde{\chi}})\rightarrow 1$ (i.e., there is no suppression and WIMP's are efficiently captured) and when $A\ll 1$, $S_i(m_{\tilde{\chi}})\rightarrow A$ [18].

The neutralino scatters off of nuclei with spin (which for the purpose of capture in the Sun or Earth includes only the hydrogen in the Sun) via an axial or "spin-dependent" interaction characteristic of Majorana particles. In addition, the neutralino may scatter off of any nucleus via a scalar interaction in which the neutralino couples to the mass of the entire nucleus; for heavy neutralinos the scalar cross section σ_{SC} is proportional to the fourth power of the nuclear mass. For the elastic scattering cross section, we use the results of Griest [5,33], which include both a spin-dependent and a scalar term due to the exchange of a squark and the Z boson, and of Barbieri, Frigeni, and Giudice [34], which includes a scattering term due to the exchange of the lightest Higgs boson. We also include the effect of the exchange of H_1^0, the heavier scalar Higgs boson (which increases the elastic scattering cross section only slightly). As recently pointed out by Gelmini, Gondolo, and Roulet [23], the cross section for scalar interactions of neutralinos with nuclei is larger than that given in Refs. [5] and [34] when one takes into account the substantial strange-quark content in the nucleus as implied by the pion-nucleon σ term [35]. For the convenience of the reader, the complete formulas for the elastic scattering cross section are listed in Appendix A.

If the neutralino has scalar interactions with the nucleus and the momentum transfer q is not small compared to the inverse of the nuclear radius R, the neutralino does not "see" the entire nucleus and the cross section for scattering of neutralinos off of nuclei is form-factor suppressed (like that for electromagnetic elastic scattering of electrons from nuclei). In terms of the energy loss ΔE, the form-factor suppression may be written as [36]

$$|F(q^2)|^2=\exp(-\Delta E/E_0)\ , \tag{9}$$

where $E_0=3/(2m_iR^2)$. As shown by Gould [18], form-factor suppression is negligible for capture of heavy WIMP's in the Earth, but a proper calculation of accretion of heavy WIMP's in the Sun must include form-factor suppression. One finds that the form-factor suppression of capture from hydrogen and helium is negligible and capture from scattering off of elements with atomic masses 12–32 is moderately suppressed, while capture from scattering off of iron is suppressed by several orders of magnitude for WIMP's in the several hundred GeV range. If there were no form-factor suppression, owing to the factor of M_i^4 [see Eqs. (10) and (11)] in the scalar cross section, one would expect scattering from iron nuclei to dominate the capture of WIMP's in the Sun; however, because of the form-factor suppression, capture of heavy WIMP's in the Sun occurs primarily by scattering off of oxygen [18]. Even so, capture from scattering off of iron nuclei is still significant. When considering the complete capture rate due to scalar interaction of WIMP's off of nuclei in the Sun, one finds that the form-factor suppression of the scalar elastic-scattering cross section decreases the capture rate by a factor of about 0.3 for WIMP's of mass 80 GeV and about 0.07 for TeV-mass WIMP's. Incidentally, as the neutralino mass is increased past 1 TeV, the form-factor suppression ceases to decrease with increasing WIMP mass; the reason is that if the nuclear mass is negligible compared to the WIMP mass, the momentum transfer does not depend on the WIMP mass.

The full capture-rate calculation assumes that the astrophysical object moves through a homogeneous Maxwell-Boltzmann distribution of WIMP's and requires information about the elemental composition of the object and distribution of elements in the object. One must integrate over the trajectories of the WIMP through the Sun and over the velocity distribution of the WIMP's. The final result for the capture rate, adapted from Gould [18], is

$$C=c\frac{\rho_{0.4}^{\chi}}{m_{\tilde{\chi}}\bar{v}_{300}}\sum_i\,[\sigma_{\rm SD}^{i(40)}+F_i(m_{\tilde{\chi}})\sigma_{\rm SC}^{i(40)}]f_i\phi_iS_i(m_{\tilde{\chi}})/m_i\ , \tag{10}$$

where $c=5.8\times10^{24}\ \mathrm{sec}^{-1}$ for the Sun and $c=5.7\times10^{15}\ \mathrm{sec}^{-1}$ for the Earth, $\rho_{\chi}^{0.4}$ is the mass density of neutralinos in the galactic halo in units of 0.4 GeV cm^{-3}, $m_{\tilde{\chi}}$ is the neutralino mass in units of GeV, and $\bar{v}_{300}$ is the velocity dispersion of the neutralinos in the galactic halo in units of 300 km sec^{-1}. The sum is over all species of nuclei in the astrophysical object (here the Earth or Sun), m_i is the mass of the ith nuclear species in GeV, f_i is the mass fraction of element i, $\sigma_{\rm SD}^{i(40)}$ is the cross section for elastic scattering off of nucleus i via an axial interaction (given in Appendix A) in units of 10^{-40} cm^2, and $\sigma_{\rm SC}^{i(40)}$ is the cross section for elastic scattering of the neutralino off of nucleus i via a scalar interaction (given in Appendix A) in units of 10^{-40} cm^2. The quantities ϕ_i describe the velocity distribution of element i in the Sun or Earth and are given in the Appendix of Ref. [23] as are the quantities f_i. In principle, the axial interaction may also be form-factor suppressed [37], but since the only element with a spin that is significant for capture is hydrogen, we do not consider it here.

Instead of listing accurate expressions for $S_i(m_{\tilde{\chi}})$ and $F_i(m_{\tilde{\chi}})$ for each element i, it is simpler to give the results obtained by summing the capture rate from Ref. [20] over all nuclear species using the matrix elements for elastic scattering given in Appendix A. We find that the capture rate in the Sun (from scalar interactions) can be approximated by

$$C^{\rm SC}(m_{\tilde{\chi}})=\left|\frac{\langle f|\mathcal{L}_{\rm eff}|i\rangle}{m_i}\right|^2 f_s(m_{\tilde{\chi}})\ , \tag{11}$$

where $\langle f|\mathcal{L}_{\rm eff}|i\rangle$ is given by Eq. (A10) (and is independent of m_i) and

$$f_s(m_{\tilde{\chi}})=\begin{cases}2.04\times10^{38}\exp[-0.0172(m_{\tilde{\chi}}-10)] & \text{if } m_{\tilde{\chi}}\le 80\ \text{GeV},\\ 6.10\times10^{37}(m_{\tilde{\chi}}/80)^{-1.06-0.38[(m_{\tilde{\chi}}-80)/920]^{1/2}} & \text{if } 80\ \text{GeV}\le m_{\tilde{\chi}}\le 1000\ \text{GeV}\ ,\\ 1.72\times10^{36}(m_{\tilde{\chi}}/1000)^{-1.88} & \text{for } m_{\tilde{\chi}}\ge 1000\ \text{GeV}\ .\end{cases} \tag{12}$$

Similarly, the capture rate in the Sun due to axial interactions with hydrogen may be approximated by

$$C^{\rm SD}(m_{\tilde{\chi}})=2.13\times10^{26}\left[\sum A_q'\Delta q\right]^2(m_{\tilde{\chi}}/80)^{-1.937}\ . \tag{13}$$

These functional forms are accurate to about 5% for neutralino masses greater than a few GeV and less than a few TeV. Note that $C^{\rm SC}\propto m_{\tilde{\chi}}^{-\alpha}$, with $1<\alpha<2$ [20]. One power of $m_{\tilde{\chi}}^{-1}$ comes from the $m_{\tilde{\chi}}$ dependence of the number density of neutralinos in the galactic halo, and the additional $m_{\tilde{\chi}}$ dependence comes from the kinematic and form-factor suppression. Note that the cross section due to scalar interactions does *not* decrease significantly as the neutralino mass is increased. The $m_{\tilde{\chi}}$ dependence of C^{SD} is similar, but contains additional $m_{\tilde{\chi}}$ dependence since $\sum A\Delta q$ is a function of $(m_{\tilde{\chi}}^2 - M_{\tilde{q}}^2)$ for heavy neutralinos [see Eq. (A3)].

The relative importance of the capture rates due to spin-dependent scattering opposed to spin-independent scattering due to squark and Higgs-boson exchange depends on the supersymmetry model. Spin-independent scattering vanishes as the neutralino becomes a pure B-ino or Higgsino as does spin-dependent scattering due to Z exchange. To study the effect of Higgs-boson-exchange scattering on the capture rate, we set the squark mass to infinity. Doing so, we find that the capture rate due to Higgs-boson exchange is generally more important than that due to Z exchange when the neutralino is heavier than the W. In Fig. 2 we show contour plots in the M-μ plane of the rate of capture of neutralinos in the Sun for (a) $\tan\beta=2$, $m_{H_2^0}=35$ GeV, and $\mu>0$; (b) $\tan\beta=2$, $m_{H_2^0}=35$ GeV, and $\mu<0$; (c) $\tan\beta=2$, $m_{H_2^0}=50$ GeV, and $\mu>0$; and (d) $\tan\beta=25$, $m_{H_2^0}=45$, and $\mu>0$, assum-

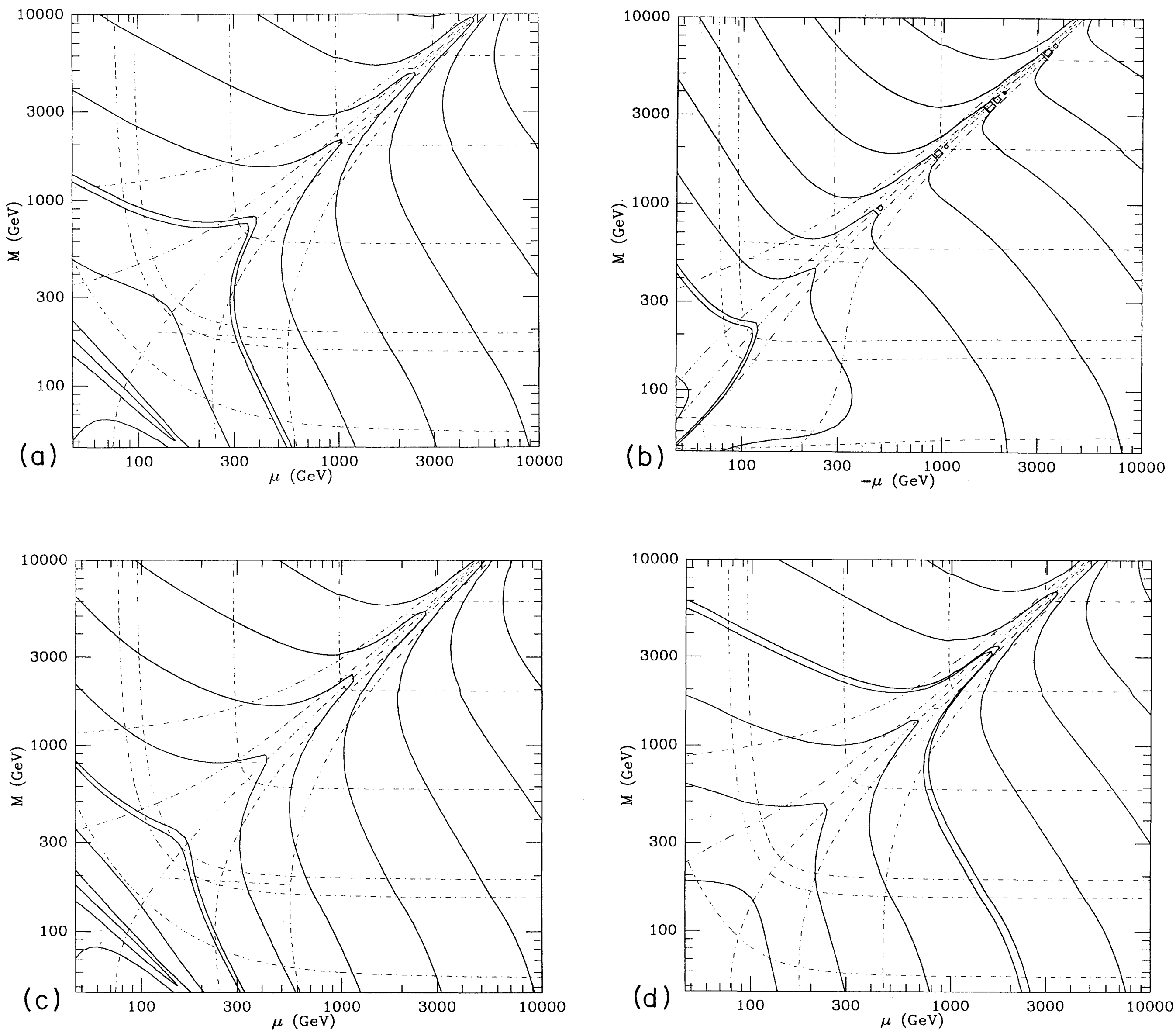

FIG. 2. Contour plots of the capture rate of neutralinos in the Sun assuming neutralinos make up the primary component of the halo dark matter and that the squark mass is infinite. The double curve indicates a capture rate of 10^{24} sec^{-1}; the spacing between other curves are decades, the capture rate decreasing toward higher masses. In (a) $\tan\beta=2$, $m_{H_2^0}=35$ GeV, and $\mu>0$, and (b) is the same except $\mu<0$. In (c) $\tan\beta=2$ and $m_{H_2^0}=50$, and in (d) $\tan\beta=25$ and $m_{H_2^0}=45$. In (c) and (d) only regions of positive μ are shown; the plots for negative μ are similar. For convenience, the mass and composition contours are also shown.

ing the squark mass is infinite. As expected, when squark exchange is negligible, mixed-state neutralinos are captured far more readily than pure B-inos or pure Higgsinos, and for fixed masses the capture rate decreases with increasing purity.

From Fig. 2 we also find that if $\tan\beta$ is held fixed, the capture rate generally decreases with increasing $m_{H_2^0}$ as a result of the propagator suppression, and if we hold $m_{H_2^0}$ fixed, the capture rate generally increases with increasing $\tan\beta$; this is simply because the Higgs couplings contain terms inversely proportional to $\cos\beta$.

To see the effect of the squark mass on the capture rate, we show in Fig. 3 the rate of capture of neutralinos in the Sun when we take the squark mass to be 20 GeV heavier than the neutralino mass. Doing so, we find that the capture rate for Higgsinos and mixed-state neutralinos is similar to that when the squark mass is infinite; this implies that capture of Higgsinos and mixed-state neutralinos occurs primarily by Higgs-boson-exchange scattering and that the capture rate is insensitive to the squark mass. On the other hand, for models where the neutralino is mostly B-ino and the squark is taken to be 20 GeV heavier than the neutralino, capture occurs primarily by spin-dependent scattering of the neutralino off of the hydrogen in the Sun. This is illustrated in Fig. 4 where we show contours of the fraction of the capture rate that occurs as a result of spin-dependent scattering. Scattering that occurs via spin-dependent exchange of the squark depends only very weakly on $\tan\beta$ and does not depend on $m_{H_2^0}$ at all; therefore, if the squark mass is small enough so that capture of the neutralino occurs primarily by squark-exchange scattering, the capture rate depends primarily on the squark mass. We should also mention that in computing the spin-dependent cross section we used the (still controversial) European Muon Collaboration (EMC) [38] results for the spin content of the proton. As discussed in Appendix A, if instead we used the naive flavor-SU(3) quark model for the proton, the spin-dependent cross section due to squark exchange would be roughly 3 times larger.

Now that we have results for the capture rate, we can see where the annihilation rate is at full signal, $\Gamma_A = C/2$, and where the time scale for equilibration of the number of WIMP's N, is so large that $\Gamma_A \ll C$. In Fig. 5 we show the regions of parameter space where energetic neutrinos are not at full signal because neutralinos have not had sufficient time to collect in the Sun. In the dark shaded regions the signal is less than 10% of the full signal ($t_\odot/\tau_A < 0.33$), and in the light shaded region the signal is less than 90% of the full signal ($t_\odot/\tau_A < 1.82$); elsewhere, capture and annihilation of neutralinos occurs rapidly enough so that the neutrino rates are at full signal ($t_\odot/\tau_A > 1.82$). In Fig. 5(a), $\tan\beta = 2$, $m_{H_2^0} = 35$, the squark mass is taken to be infinite, and $\mu > 0$; Fig. 5(b) is similar, but $\mu < 0$ is shown; and Fig. 5(c) is similar to Fig. 5(a), but the squark mass is taken to be 20 GeV heavier than the neutralino mass. Note that in most models where the neutralino is lighter than 1 TeV the neutrino flux is at full signal. Later, we will find that in regions of parameter space, where the neutrino flux is large enough to be near current observational limits, the flux is at full signal. We will also see that τ_A generally stays small enough so that the rates remain at full signal even for

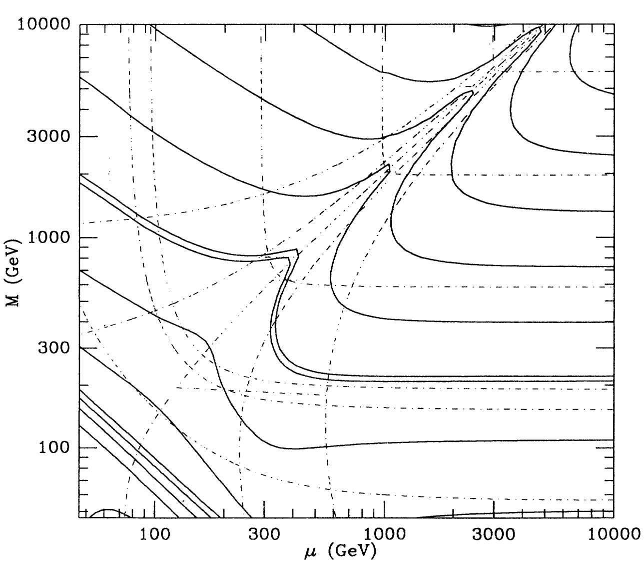

FIG. 3. Same as Fig. 2(a), but here the squark mass is assumed to be 20 GeV heavier than the neutralino mass.

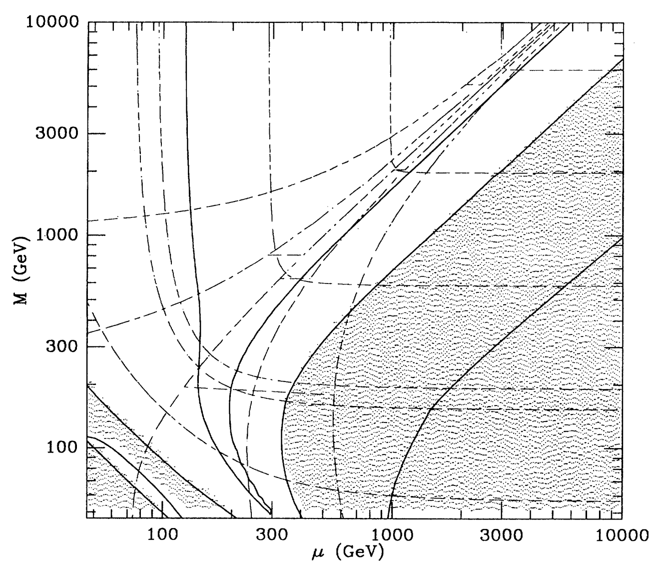

FIG. 4. Contours of the fraction of the capture rate due to spin-dependent scattering when the squark is assumed to be 20 GeV heavier than the neutralino and $\tan\beta = 2$ and $m_{H_2^0} = 35$. In the shaded regions the fraction is greater than 0.5, and the contours indicate where the fraction is 0.01, 0.5, and 0.99. Again, mass and composition contours are also shown, and plots for other values of $\tan\beta$ and $m_{H_2^0}$ are qualitatively similar.

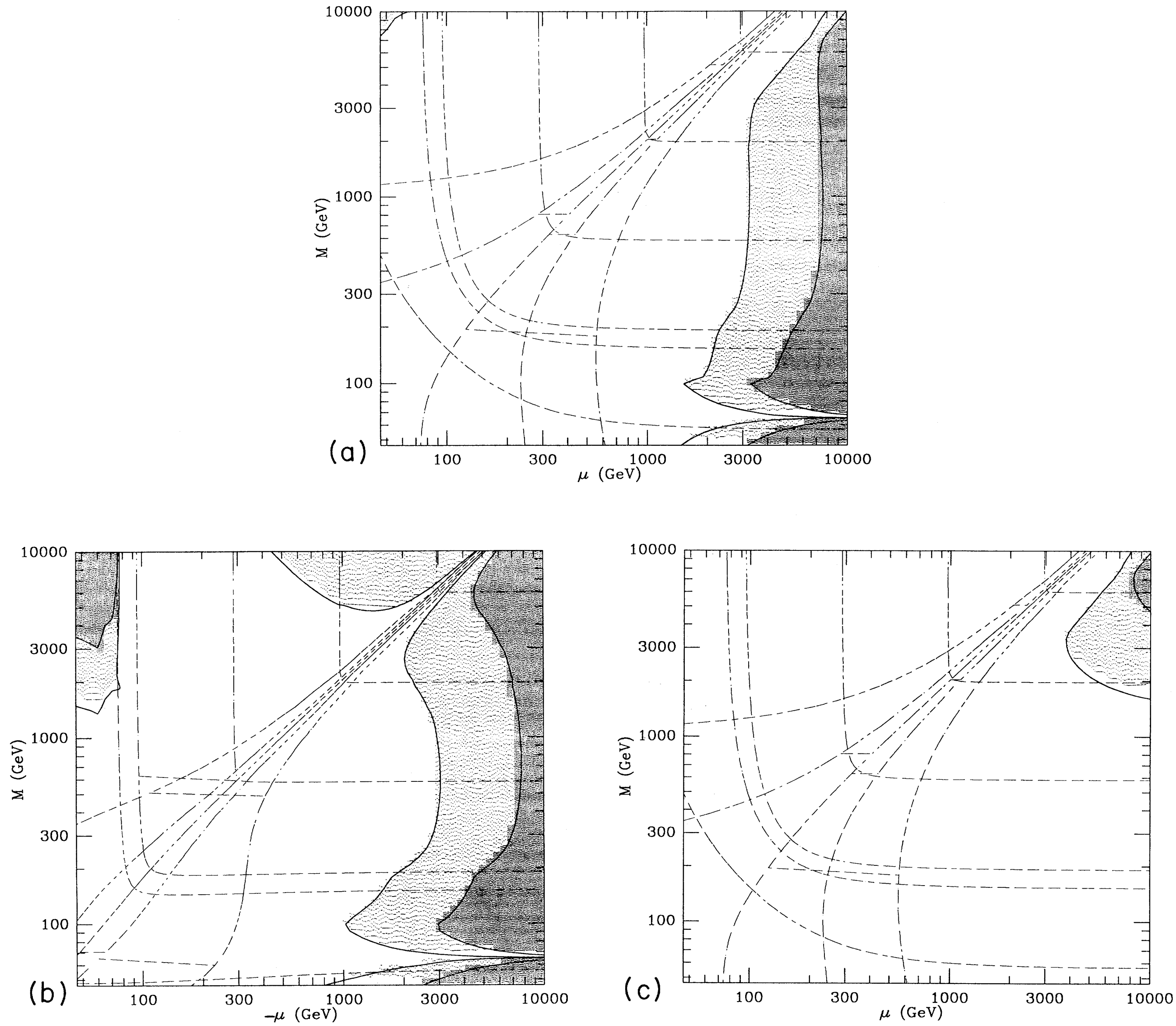

FIG. 5. Contours of $t_\odot/\tau_A$. In the dark shaded regions, $t_\odot/\tau_A < 0.33$, and in the light shaded region, $t_\odot/\tau_A < 1.82$; elsewhere, $t_\odot/\tau_A > 1.82$. In (a) $\tan\beta = 2$, $m_{H_2^0} = 35$, the squark mass is taken to be infinite, and $\mu > 0$; (b) is similar, but $\mu < 0$ is shown; and (c) is similar to (a), but the squark mass is taken to be 20 GeV heavier than the neutralino mass. Plots for other values of $\tan\beta$ and $m_{H_2^0}$ are similar.

most models with a neutrino flux several orders of magnitude weaker than the current observational limits.

III. NEUTRINO SPECTRA FROM NEUTRALINO ANNIHILATION

Give the annihilation rate $\Gamma_A = C \tanh^2(t_\odot/\tau_A)/2$, the diferential flux of neutrino type i (e.g., ν_e, ν_μ, $\bar{\nu}_\mu$, etc.) produced by the annihilation of neutralinos in the Sun or Earth at a distance R from the source is

$$\left[\frac{d\phi}{dE}\right]_i = \frac{\Gamma_A}{4\pi R^2} \sum_F B_F \left[\frac{dN}{dE}\right]_{Fi} , \tag{14}$$

where the sum is over all annihilation channels.

The quantities B_F are the branching ratios for annihilation into final state F. Since the neutralinos are moving nonrelativistically in the Sun or Earth, B_F may be determined by the relative magnitude of the cross sections for annihilation into channel F at zero velocity given in Ref. [6] [Eqs. (A10), (B7), (C11), and (D6)] and in Appendix B.

The final states F into which neutralinos may annihilate at zero relative velocity are $f\bar{f}$, where f is a quark or charged lepton, W^+W^-, Z^0Z^0, $H_1^0H_3^0$, $Z^0H_1^0$, $Z^0H_2^0$, W^+H^-, and W^-H^+ [39]. The cross sections for annihilation into other combinations of gauge and Higgs bosons vanish as the relative velocity approaches zero. The calculation of the cross sections for annihilation into the mixed gauge- and Higgs-boson final states $Z^0H_1^0$, $Z^2H_2^0$, W^+H^-, and W^-H^+ at zero relative velocity are new and the results are presented in Appendix B. As noted by Olive and Srednicki [7], annihilation into the mixed gauge-boson–Higgs-boson final states is generally small for pure B-inos and Higgsinos, but may be important for mixed-state neutralinos. For models where the neutralino is a pure B-ino and the squark masses are much larger than all other masses involved, annihilation into the mixed gauge-boson–Higgs-boson states may be comparable to annihilation into Higgs-boson states; in this case neutralinos annihilate predominantly into these states, but the total rate for annihilation is very small and the neutralinos are generally very weakly interacting.

The $(dN/dE)_{Fi}$ are the differential energy spectra of neutrino type i at the surface of the Sun (or Earth) that result from the injection of particles in final state F at the center of the Sun (or Earth). These spectra are functions of the neutrino energy E and energy E_i of the injected particles. Calculation of the spectra requires information about the cascade following the decay of the annihilation products, the hadronization of heavy quarks in the cascade, and the interactions of particles in the cascade with the medium at the core of the Sun or Earth. Since the $(dN/dE)_{Fi}$ are the neutrino spectra at the surface of the astrophysical object, while neutralino annihilation occurs at the center and the Sun is not transparent to neutrinos with energies in the 100-GeV range, absorption and energy loss of neutrinos by the solar medium must also be included in the calculation. Since the density and thickness of the Earth are different from those in the Sun, the $(dN/dE)_{Fi}$ from particles injected in the Earth will be different than those from the Sun.

A detailed calculation of the neutrino spectrum from injected quarks and leptons was performed by the authors of Ref. [21] using the Lund Monte Carlo program. Their calculation includes hadronization of quarks and interactions of the fermions and neutrinos with the solar medium. Electrons, muons, and light (u,d,s) quarks are stopped in the Sun before they decay and therefore do not produce high-energy neutrinos. The top quark is expected to hadronize and then decay far before it can lose a substantial amount of energy, and the τ will also decay immediately. Bottom and charm quarks hadronize, and because of the high density of the core of the Sun, the heavy hadron may subsequently lose a significant fraction of its energy before decaying. RS estimate that if E_0 is the initial heavy-hadron energy in the Sun, the mean energy of the hadron when it decays will be

$$\langle E \rangle = E_c e^{E_c/E_0} \int_{E_c/E_0}^{\infty} \frac{e^{-x}}{x} dx \ , \tag{15}$$

and they estimate that $E_c \approx 250$ GeV for charmed hadrons and $E_c \approx 470$ GeV for bottom hadrons. Evaluating the integral, one finds that $\langle E \rangle \simeq E_0(1-E_0/E_c)$ for $E_0 \ll E_c$ and $\langle E \rangle \simeq E_c[\ln(E_0/E_c)-\gamma_E]$ for $E_0 \gg E_c$, where $\gamma_E = 0.577\ldots$ is Euler's constant, and so the mean energy of the decaying hadron never grows much larger than E_c.

At high energies the Sun is no longer transparent to neutrinos and interactions of neutrinos with the solar medium may significantly alter the energy spectrum. For τ's injected at energies above several hundred GeV, the flux of muon neutrinos may be significantly enhanced by the decay of additional τ's produced by charged-current interactions of τ neutrinos with the solar medium. Electron and muon neutrinos are absorbed by charged-current interactions: The probability that a neutrino of initial energy E_i will escape from the Sun is $\exp(-E_i/E_{\text{abs}})$, where $E_{\text{abs}} = 198$ GeV for neutrinos and $E_{\text{abs}} = 296$ GeV for antineutrinos. Furthermore, at high energies neutral-current interactions degrade the neutrino energy.

Since the density of the core of the Earth is about $\frac{1}{12}$ that of the core of the Sun, muons and light quarks are still stopped before they decay, while stopping of heavy hadrons may be ignored until several TeV. Moreover, the optical depth of the Earth is much smaller than that of the Sun, and so interactions of neutrinos with the Earth may be ignored for neutrino energies less than several TeV. As a result, Ritz and Seckel's noninteracting results may be used for the neutrino spectra from the Earth.

The results presented by RS are for neutrino spectra from fermions injected into the core of the Sun at 60 and 1000 GeV; however, we need to obtain information about the spectra for fermions injected at any energy up to 1 TeV. For reasons to be discussed below, we will eventually focus on detection of neutrinos via neutrino-induced upward-moving muons. Since the cross section for a neutrino to produce a muon in the rock below the detector is proportional to the neutrino energy and the range of the muon is roughly proportional to the energy, the probability for a neutrino to produce a throughgoing muon is proportional to the energy squared. Therefore, to obtain event rates we need only the second moments $\langle Nz^2 \rangle_{Fi} m_{\tilde{\chi}}^2$, where

$$\langle Nz^2 \rangle_{Fi} \equiv \frac{1}{m_{\tilde{\chi}}^2} \int \left(\frac{dN}{dE} \right)_{Fi} E^2 dE \ . \tag{16}$$

The functional forms of the spectra are not required.

For fermions injected into the core of the Earth, interactions are negligible and the moments of the neutrino spectra are easily obtained from the noninteracting results of Ritz and Seckel. In this case [21],

$$\langle Nz^2 \rangle = \frac{1}{3} \langle N \rangle \langle y^2 \rangle \left[\langle z_f^2 \rangle - \frac{m_f^2}{4E_i^2} \right] , \tag{17}$$

where E_i is the fermion injection energy, $\langle N \rangle$, and $\langle y^2 \rangle$ are the rest-frame yield and second moments listed in Table 2 of RS, $\langle z_f^2 \rangle$ is the second moment of the fragmentation function listed in Table 3 of RS, and m_f is the mass of the injected fermion.

For fermions injected into the core of the Sun, the calculation is much more difficult since one must take interactions into account. RS outline a procedure for analytically estimating the effect of interactions which reproduces the Monte Carlo results reliably for injection energies $\lesssim 200$ GeV. An effort to modify and apply the corrections to describe interactions at higher energies resulted in moments of the neutrino spectra that reproduced those obtained from the Monte Carlo program only to within $\sim 50\%$; however, in doing so one finds that for injected b and c quarks the most important effect is the stopping of heavy hadrons. Therefore, for the scaled second moment of the neutrino spectra for b and c quarks, we assumed that

$$\langle Nz^2 \rangle = \left[a x_0 e^{x_0} \int_{x_0}^{\infty} \frac{e^{-x}}{x} dx \right]^2 , \tag{18}$$

where $x_0 \equiv E_c / E_i$, and fitted a and E_c to match the interacting results of RS at 60 and 1000 GeV. (Actually, since RS did not present interacting results at 60 GeV for antineutrinos or, in the case of the b quark, for neutrinos, we obtained these numbers using the corrections for interactions described in their paper.) We found that for neutrinos from c quarks, $a = 0.056$ and $E_c = 155$ GeV, for antineutrinos from c quarks, $a = 0.052$ and $E_c = 275$ GeV; for neutrinos from b quarks, $a = 0.086$ and $E_c = 185$ GeV, and for antineutrinos from b quarks, $a = 0.082$ and $E_c = 275$ GeV.

Since τ leptons are not stopped and do not hadronize, absorption of muon neutrinos is the most important interaction effect for the spectra from τ leptons; production of muon neutrinos from interactions of τ neutrinos is also significant at high energies, but these neutrinos are predominantly low energy and do not contribute significantly to the second moment. Thus we take the second moment of the neutrino spectrum from injected τ leptons to be

$$\langle Nz^2 \rangle = a e^{-E_i / E_{\text{abs}}} , \tag{19}$$

and fitted a and E_{abs} to reproduce the RS result of 60 and 1000 GeV. For neutrinos, $a = 0.0204$ and $E_{\text{abs}} = 476$ GeV, and for antineutrinos, $a = 0.0223$ and $E_{\text{abs}} = 599$ GeV.

Our estimates for the spectra from the top quark are far more uncertain. RS used a top-quark mass of 40 GeV, and here we have assumed that it is 120 GeV. Since even 40 GeV is so much heavier than all other lighter particle masses, we assumed that the scaled rest-frame neutrino spectra would be the same for a top quark of 120 GeV as it would for a top quark of 40 GeV. We then estimated the effect of interactions for a top quark injected into the solar core at 120 GeV and assumed that the RS interacting results at 1000 GeV would also be valid for a 120-GeV top quark. At injection energies just above threshold, the moments of the neutrino distribution have a strong dependence on the fragmentation function, and at higher energies, absorption of neutrinos determines the behavior of the spectral moments. Therefore, neither of the expressions in Eq. (18) or (19) really describe the injection-energy dependence of $\langle Nz^2 \rangle$. Nevertheless, the effect of interactions, which we can reliably estimate at low energies, is better described by Eq. (18) than by Eq. (19), and so we use the form of Eq. (18) with $a = 0.18$ and $E_c = 110$ GeV for neutrinos and $a = 0.14$ and $E_c = 380$ GeV for antineutrinos.

Although these estimates of $\langle Nz^2 \rangle$ are somewhat *ad hoc* and admittedly crude for arbitrary injection energies between 60 and 1000 GeV, they should be relatively accurate for neutrino spectra from annihilation of neutralinos not much heavier than the W or Z; at higher energies our approximations are far from pinpoint accuracy, but they should still be good enough to indicate the effect of interactions of the decay products and neutrinos with the solar medium.

Since Higgs and vector bosons decay into pairs of quarks and leptons immediately, it is easy to obtain $\langle Nz^2 \rangle$ for injected bosons from our previous results for the neutrino spectra from injected fermions [40]. Suppose boson B undergoes two-body decays into fermions f, and N_f^B is the number of fermions of type f produced on average per B decay (i.e., $\sum_f N_f^B = 2$) and which can be obtained from the branching ratios for decay of B into the various final states and contents of those channels. If E_i is the injected boson energy, then the energy of the fermion in the rest frame of the B is $m_B/2$, where m_B is the B mass, and in the moving frame it is $E_f = E_i(1+\beta\cos\theta)/2$, where β is the velocity of B in units of the speed of light and θ is the angle between the direction of motion of the decay product and direction of motion of B. For Higgs bosons and unpolarized vector bosons (which are produced by the annihilation of neutralinos, provided the interactions of the neutralinos are CP conserving, which is assumed throughout here), the decay is isotropic, which means that the laboratory-frame energies of the fermions from the decay of B are evenly distributed from $E_i(1-\beta)/2$ to $E_i(1+\beta)/2$. Therefore,

$$\langle Nz^2 \rangle_{Bi} = \sum_f \frac{N_f^B}{\beta E_i} \int_{E_i(1-\beta)/2}^{E_i(1+\beta)/2} \langle Nz^2 \rangle_{fi}(E) dE , \tag{20}$$

where $\langle Nz^2 \rangle_{fi}(E)$ are the second moments of the neutrino spectra presented above as a function of the injected fermion energy E.

The three neutral Higgs bosons of the minimal extension of the supersymmetric standard model decay into fermion-antifermion pairs. The branching ratios for the decays of H_2^0 and H_3^0, from which the N_f^B are obtained, are given in the Appendix of Ref. [23] and are proportional to the fermion mass squared (so the Higgs bosons do not decay directly into energetic neutrinos), and the branching ratios for the decay of H_1^0 may be obtained from those for H_2^0 decay by switching $\cos\alpha$ and $\sin\alpha$.

If the neutralinos annihilate into τ leptons or b, c, or t quarks and an energetic neutrino is produced in the decay of these fermions, then the typical neutrino energy is $\frac{1}{3}$ the mass of the neutralino. If the neutralinos annihilate into Higgs bosons, there is another step in the decay chain before energetic neutrinos are produced, and so their energies would typically be $\frac{1}{6}$ the mass of the neutralino. This is partially compensated by the fact that each Higgs boson produces two fermions, but since the detection rates are proportional to the energy squared, the net effect is that, if the neutralinos annihilate into Higgs bosons, the detection rate is roughly half the rate had they annihilated into fermions (assuming, of course, that the branching ratio for the various fermions from Higgs-boson decays is nearly the same as the branching ratios for the various fermions from neutralino annihilation if only fermion final states are considered). Although H_2^0 must be lighter than $m_Z \cos 2\beta$ and most certainly decays only into quarks and leptons, the other Higgs bosons may be much heavier and may include other exotic decay channels as well, which may also produce energetic neutrinos which would most likely have a much softer spectrum. If this is the case, then by assuming that they decay only into quarks and leptons, we are overestimating the neutrino yields.

It turns out that the most favorable annihilation channel for observing high-energy neutrinos is the gauge-boson final state. The reason is that W and Z bosons decay directly into neutrinos with appreciable branching ratios. Compared with the event rate from these "semiprompt" neutrinos, the even rate for neutrinos which come from the quark and charged-lepton decay products of the gauge bosons is negligible. A W decays to a muon and a muon neutrino about 11% of the time [41], and so, neglecting interactions, $\langle Nz^2 \rangle$ is roughly 0.025 for slow W's and 0.033 for relativistic W's. This is larger than all the values expected from fermion-antifermion pairs (see Table I of RS), although $\tau^\pm$ final states come close. Furthermore, at higher energies no energy is lost from hadronization or stopping of the vector bosons. (At higher energies the value of $\langle Nz^2 \rangle$ for gauge-boson final states becomes smaller than that from $\tau^\pm$ final states; this is because the energies of neutrinos from gauge-boson decays are generally larger than those from τ decays so that absorption of neutrinos in the Sun from gauge-boson decays is stronger than absorption of neutrinos from τ decays. Even so, if the neutralino annihilates to $\tau^\pm$ pairs, it will also have a significant and usually larger annihilation branch to $b\bar{b}$, $c\bar{c}$, and if kinematically accessible, $t\bar{t}$ pairs, and so the total neutrino yield from gauge-boson final states will be greater than the total yield from fermion-antifermion states.) The branching ratio for $Z^0 \to \nu\bar{\nu}$ is slightly smaller than the branching ratio for $W \to \mu\bar{\nu}_\mu$, but two neutrinos are produced so that $\langle Nz^2 \rangle$ is a little larger.

For W bosons injected in the core of the Earth with velocity β, we can ignore interactions of the neutrinos with the Earth, and

$$\langle Nz^2 \rangle_{Wi} = \Gamma_{W \to \mu\bar{\nu}_\mu} (3+\beta^2)/12 \ , \tag{21}$$

where i is a neutrino or antineutrino; $\langle Nz^2 \rangle_{Zi}$ may be obtained by multiplying by 2 and replacing $\Gamma_{W \to \mu\bar{\nu}_\mu}$ by $\Gamma_{Z \to \nu\bar{\nu}}$. To account for interactions of the neutrinos with the solar medium for vector bosons injected into the core of the Sun, we use the estimate of RS that a neutrino injected with an energy E leaves the Sun with energy

$$E_f = \frac{E}{1+E\tau_i} \ , \tag{22}$$

where $\tau_\nu = 1.01 \times 10^{-3}$ GeV^{-1} and $\tau_{\bar{\nu}} = 3.8 \times 10^{-4}$ GeV^{-1}, and the probability

$$P_f = \left[\frac{1}{1+E\tau_i} \right]^{\alpha_i} \ , \tag{23}$$

where $\alpha_\nu = 5.1$ and $\alpha_{\bar{\nu}} = 9.0$ for antineutrinos. Doing so, we find that

$$\langle Nz^2 \rangle_{Wi} = \frac{\Gamma_{W \to \mu\bar{\nu}_\mu}}{\beta E_i^3} \left. \frac{2+2E\tau_i(1+\alpha_i)+E^2\tau_i^2\alpha_i(1+\alpha_i)}{\alpha_i \tau_i^3 (\alpha_i^2-1)(1+E\tau_i)^{\alpha_i+1}} \right|_{E=E_i(1+\beta)/2}^{E=E_i(1-\beta)/2} \ , \tag{24}$$

for W's injected into the core of the Sun with energy E_i.

In Fig. 6 we show the second moments $m_{\tilde{\chi}}^2 \langle Nz^2 \rangle$ of the neutrino yield from the Sun for the $c\bar{c}$, $b\bar{b}$, $t\bar{t}$, $\tau^\pm$, $W^\pm$, and $H_2^0 H_3^0$ (using $\tan\beta = 2$ and $m_{H_2^0} = 35$) final states as a function of the neutralino mass. The neutrino yields from Z^0 pairs (not shown) is similar to, but slightly smaller, than the yields from $W^\pm$ pairs, and the yield from the $H_1^0 H_3^0$ (when it is kinematically accessible) final state is similar to that from the $H_2^0 H_3^0$ final state. We remind the reader that although the yield from $\tau^\pm$ pairs surpasses that from gauge-boson pairs for neutralinos heavier than about 200 GeV, if the neutralino annihilates to lepton pairs, then it also has a significantly annihilation branch into quark-antiquark pairs and the yield from gauge-boson pairs is still larger than the *total* yield from fermion-antifermion final states.

IV. RATES FOR DETECTION IN UNDERGROUND DETECTORS

Generally, neutrinos are detected either by contained events where the neutrino undergoes a charged-current interaction and produces a lepton in the detector or by upward-moving throughgoing muons in which a muon neutrino undergoes a charged-current interaction in the rock below the detector and produces a muon which then passes through the detector. Since the cross section for a charged-current interaction is proportional to the neutrino energy and the effective range of a muon is proportional to the muon energy, the rate for contained events is roughly proportional to the neutrino energy and the rate for neutrino-induced throughgoing muons is proportional to the square of the neutrino energy. Therefore, at sufficiently high energies the rate for throughgoing

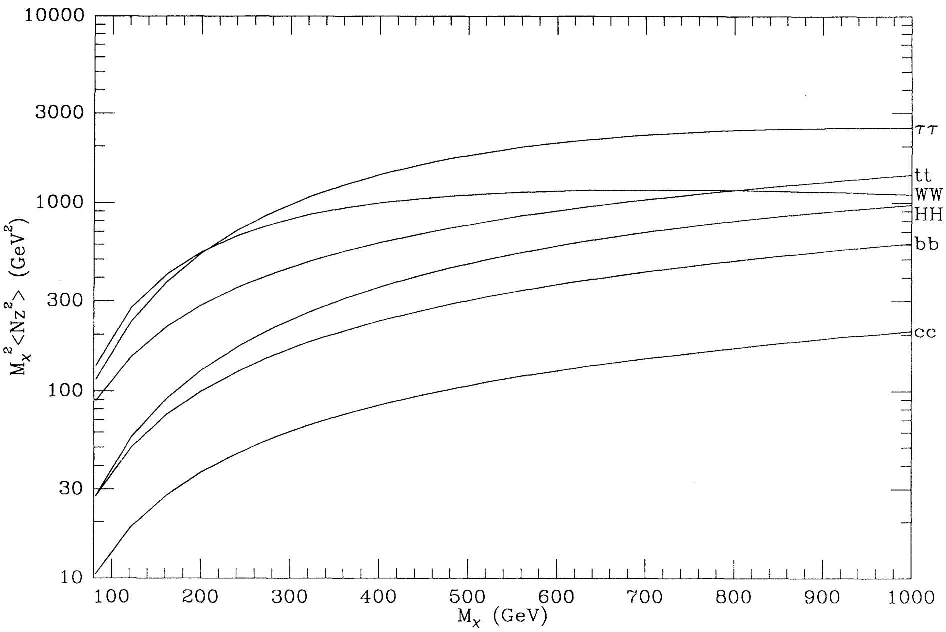

FIG. 6. Second moments of the neutrino yields from the Sun from the $c\bar{c}$, $b\bar{b}$, $t\bar{t}$, $\tau^{\pm}$, $W^{\pm}$, and $H_2^0 H_3^0$ (where $\tan\beta=2$ and $m_{H_2^0}=35$ GeV) annihilation channels as a function of the neutralino mass.

muons should be greater than that for contained events. In Ref. [23] the regions of parameter space ruled out by searches for contained events from Frejus [42] very nearly matches those regions ruled out by searches for throughgoing muons for IMB [30] for neutralinos less massive than the W. [In addition, the Nuclear Stability Experiment (NUSEX) [43] reports that limits on muons produced by neutrino interactions in the rock below the detector that stop inside the fiducial volume of the detector are in agreement with those from contained or throughgoing events from IMB, Frejus, and Kamiokande.] Furthermore, Ref. [22] indicates that the rate for detecting high-energy neutrinos from the Sun via throughgoing muons per 100 m^2 becomes larger than that for contained events per kiloton for neutralinos heavier than roughly 60 GeV, while the rate for observing throughgoing muons is greater than that for contained events from neutrinos from the Earth for neutralinos heavier than roughly 20 GeV. Therefore, since neutralinos heavier than the W are considered here, we will concentrate on detection of neutrinos via throughgoing muon events.

After taking the cross section for muon production in the rock and the effective range of the muons into account, but ignoring detector thresholds (which are near 2 GeV, far lower than the average neutrino energies considered here), the rate (per unit detector area) for neutrino-induced throughgoing muon events is [21]

$$\Gamma_{\text{detect}}=1.27\times10^{-29}Cm_{\tilde{\chi}}^2 \times\sum_i a_i b_i \sum_F B_F \langle Nz^2\rangle_{Fi}\ \text{m}^{-2}\,\text{yr}^{-1}, \tag{25}$$

for neutrinos from the Sun; the same expression multiplied by 5.6×10^8 (the square of the ratio of the Earth-Sun distance to the Earth's radius) gives the rate for neutrino events from the Earth. Here C is the capture rate in units of s^{-1}, the sum on i is over muon neutrinos and antineutrinos, the a_i are neutrino-scattering coefficients, $a_\nu=6.8$ and $a_{\bar{\nu}}=3.1$, the b_i are muon-range coefficients, $b_\nu=0.51$ and $b_{\bar{\nu}}=0.67$, and $\langle Nz^2\rangle_{Fi}$ is the second moment of the spectrum of neutrino type i from final state F scaled by the neutralino mass squared. [Note that neutrino-induced muons from the Sun can only be observed when the Sun is below the horizon; Eq. (25) does *not* consider this.]

Given the expressions for $\langle Nz^2\rangle$ for neutrino spectra from the Sun and Earth, we estimate that, if the neutralino has only scalar interactions with nuclei, the strength of the neutrino signal from the Earth should be comparable to that from the Sun. (If the neutralino has only axial interactions, then *no* signal is expected from the Earth since no nuclei with spin are abundant in the Earth.) To see this, first note that the difference in the prefactors c for the Sun and Earth in the capture-rate equation (10) is roughly compensated by the geometric factor 5.6×10^8 accounting for the difference in the distances between us and the Sun or the center of the Earth (5.8×10^{24} sec^{-1} for the Sun opposed to 3.2×10^{24} sec^{-1} for the Earth). Therefore, for dark-matter candidates with masses in the Earth's resonance range 10 GeV $\lesssim m_{\tilde{\chi}} \lesssim$ 75 GeV [that is, where $A \gg 1$; see Eq. (8)], the kinematic suppression factor S_i is nearly unity, and since the fraction of the Earth's mass due to heavy elements is higher than that in the Sun, the neutrino flux from the Earth will be greater than

that from the Sun (again, only if the WIMP in question has scalar interactions with nuclei) [18].
tions with nuclei) [18].

In contrast, the heavy neutralinos considered here have masses outside the Earth's resonance range, and so capture by the Earth is suppressed as a result of the factor of $(v_{\rm esc}/\bar{v})^2 \approx 1.4\times10^{-3}$ [see Eq. (8)], which is always much smaller than the analogous quantity in the Sun. On the other hand, form-factor suppression of capture by heavy elements in the Sun tends to suppress capture in the Sun relative to that in the Earth. A careful calculation [20] shows that the ratio of the capture rates (scaled by the squares of the relative distance) due to scalar interactions with nuclei in the Sun and Earth is about unity for WIMP's of mass $m_{\tilde{\chi}} \simeq 80$ GeV, rises to about 5.5 for a WIMP mass of 1 TeV, and asymptotes to 6.5 for heavier WIMP's. One should also note that, in addition, if the capture and annihilation rates for the neutralino in question are small, then the neutralino signal from the Earth may be further weakened relative to that from the Sun as the time τ_A for the number of neutralinos to reach equilibrium in the Earth is generally smaller than that in the Sun. Since only very slow heavy WIMP's may be captured in the Earth, there is an additional uncertainty in the rate of accretion of heavy WIMP's onto the Earth because of the (roughly a factor of 2) uncertainty in the zero-velocity phase-space density of WIMP's [19]. On the other hand, as discussed in Sec. III, interactions of decay products and neutrinos with the solar medium weaken the neutrino signal from the Sun relative to that in the Earth by a few percent at $m_{\tilde{\chi}} \simeq 80$ GeV and by a factor of order 10^{-1} for WIMP's with masses near 1 TeV. In addition, one must also consider that neutrino-induced muons from the Earth can be continuously observed, while the Sun can only be observed when it is below the horizon. We again remind the reader that if the WIMP in question has only axial interactions with nuclei (such as a B-ino in models with a relatively light squark), it may be captured in the Sun by scattering off of hydrogen, but it will *not* be captured in the Earth.

So the neutrino signal from the Earth should be comparable to that from the Sun for neutralinos heavier than the W and less than about 1 TeV. For even heavier WIMP's, if capture and annihilation in the Earth are large enough that they remain in equilibrium, then the signal from the Earth should be stronger than that from the Sun since interactions with the solar medium deplete the solar signal; if, however, the equilibration time scale in the Earth is larger than the age of the Earth, the signal from the Earth will be suppressed relative to that in the Sun. In the following we will focus our attention on the neutrino signal from WIMP annihilations in the Sun only, but we stress that, for most of the models we are considering here, a comparable signal should be observed from the Earth. Furthermore, Gould [20] has recently discussed the intriguing possibility that a measurement of the relative strength of the neutrino signals from the Sun and Earth could provide information about the cosmological abundance of dark matter. We also point out that observation of a neutrino signal from the Sun and the absence of one from the Earth would be a signature of particle dark matter with predominantly axial interactions.

V. RESULTS

Since the MSSM has many undetermined parameters, we will show results in the M-μ plane for several values of $\tan\beta$ and $m_{H_2^0}$ allowed by null results from searches for neutral Higgs bosons at LEP [28]. Again, we will first

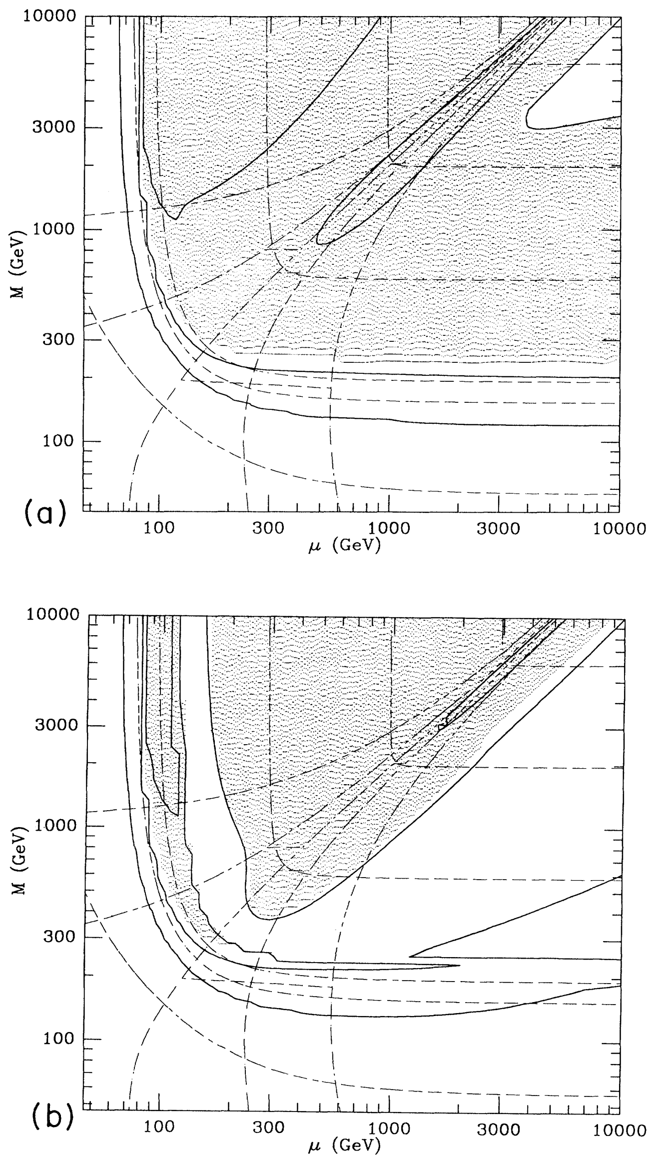

FIG. 7. Contours of the fraction of the neutrino signal that comes from gauge-boson final states. In the shaded regions the fraction is greater than 0.5, and the contours indicate where the fraction is 0.01, 0.5, and 0.99. In (a) the squark mass is taken to be infinite, and in (b) the squark mass is assumed to be 20 GeV heavier than the neutralino mass. In both, $\tan\beta=2$ and $m_{H_2^0}=35$ GeV and $\mu>0$. Plots for other values of $\tan\beta$ and $m_{H_2^0}$ and for negative μ are qualitatively similar.

take the squark masses to be infinite; this minimizes the capture rate and emphasizes gauge- and Higgs-boson final states. Then we will consider squark masses 20 GeV higher than the neutralino mass; this will emphasize capture by spin-dependent scattering and fermion final states for neutralinos where such effects are important.

When the neutralino is mostly Higgsino, it annihilates primarily into gauge bosons, and the effects of the squark, Higgs-boson, and top-quark masses are relatively unimportant [6]. When the neutralino is mostly B-ino, it annihilates primarily into fermions (provided the squark mass is not too large), and when the top-quark channel is open, it annihilates predominantly into the top quark. Mixed-state neutralinos generally annihilate into gauge bosons, fermions, and Higgs bosons as well with comparable magnitudes.

In Fig. 7 we plot contours of the fraction of the neutrino signal that comes from gauge bosons. When the squark mass is taken to be infinite [Fig. 7(a)], the neutralino does not annihilate into fermions, and since gauge bosons yield a much harder spectrum of neutrinos than Higgs bosons, virtually all of the neutrino signal from heavy neutralinos comes from gauge-boson final states. When the squark mass is 20 GeV heavier than the neutralino mass [Fig. 7(b)], fermions are the dominant annihilation products from B-inos, and so the neutrino signal is not always dominated by neutrinos from gauge bosons. Still, neutrinos from gauge-boson final states dominate the signal for Higgsino and contribute a signal comparable to that from fermions in many regions of parameter space with mixed-state neutralinos and B-inos.

The IMB Collaboration has found an upper limit on the flux of upward-moving muons induced by neutrinos from the Sun with energy larger than 2 GeV of 2.65×10^{-2} m^{-2} yr^{-1} [30] (and similar, though slightly weaker, limits have been found by Kamiokande II [31]). Therefore, supersymmetric models in which the capture and annihilation of the neutralino yields larger neutrino fluxes are inconsistent candidates for the primary component of the galactic halo. (To be precise we do not implement the 2-GeV cutoff in our calculation, but since we are primarily interested in heavy neutralinos here, the fraction of our signal from lower-energy neutrinos should be insignificant.) In Fig. 8 the dark shading denotes the regions of parameter space excluded by this constraint. The light shaded regions are those that would be excluded or observed if the observational flux limits were to be improved by a factor of 100. The curve inside the light shaded areas encloses regions of parameter space that would be excluded or observed if current observational limits were improved by a factor of 10. To indicate the sensitivity of these results to uncertainties in the calculation, the dashed curve inside the excluded region indicates the region excluded if the true neutrino rate is only $\frac{1}{5}$ as large as our calculations indicate. In Fig. 8(a), $\tan\beta=2$, $m_{H_2^0}=35$ GeV, the squark mass is taken to be infinite, and $\mu>0$, and Fig. 8(b) is similar except that $\mu<0$. In Fig. 8(c), $\tan\beta=2$ and $m_{H_2^0}=35$ GeV, in Fig.

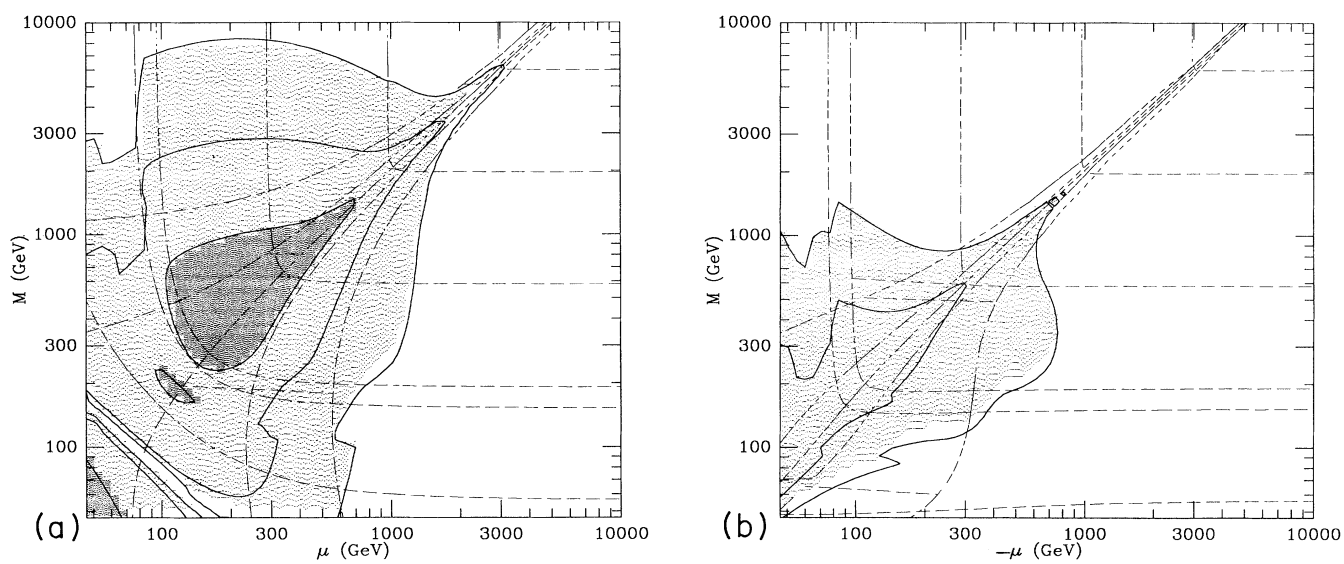

FIG. 8. Regions where the neutralino is excluded as the primary component of the galactic halo by limits on the flux of upward-moving neutrino-induced muons from the Sun. The dark shaded regions are those excluded by current IMB limits. The light shaded regions are those that would be excluded if current observational limits were improved by a factor of 100. The curve inside the excluded region encloses the region that would be excluded if the true neutrino flux was $\frac{1}{5}$ of the results of the calculation here, and the curve inside the light shaded region encloses regions that would be excluded if the current observational limits were improved by a factor of 10. In (a) $\tan\beta=2$, $m_{H_2^0}=35$ GeV, $\mu>0$, and the squark mass is taken to be infinite, and (b) is the same except $\mu<0$. In (c) $\tan\beta=2$ and $m_{H_2^0}=35$ GeV, in (d) $\tan\beta=2$ and $m_{H_2^0}=50$ GeV, and in (e) $\tan\beta=25$ and $m_{H_2^0}=45$ GeV. In (c)–(e) the squark mass is assumed to be 20 GeV greater than the neutralino mass, and only regions of positive μ are shown. Plots for negative μ are similar, but excluded regions are smaller.

8(d), $\tan\beta=2$ and $m_{H_2^0}=50$ GeV, and in Fig. 8(e), $\tan\beta=25$ and $m_{H_2^0}=45$ GeV. In Figs. 8(c)–8(e), the squark mass is assumed to be 20 GeV greater than the neutralino mass and only regions of positive μ are shown.

From Fig. 8 we see that the limits on energetic neutrino fluxes from the Sun already exclude many supersymmetric models with heavy mixed-state neutralinos lighter than about 1 TeV when the lightest Higgs boson is light and $\tan\beta$ is small [Figs. 8(a)–8(c)] or when $\tan\beta$ is large [Fig. 8(e)], independent of the squark mass. Unfortunately, the region of $m_{H_2^0}$-$\tan\beta$ parameter space in which current neutrino limits exclude neutralinos as dark-matter candidates is similar to that excluded by current LEP results [28]; the rates for neutrino events from models with larger values of $m_{H_2^0}$ [Fig. 8(d)] are much smaller. Also, current neutrino-flux bounds are ineffective in ruling out neutralinos that are almost pure Higgsino or B-ino; however, if the observational bounds are improved by a factor of 10, far more supersymmetric dark-matter candidates would be observable. Also, note that the event rates are much smaller from supersymmetric models with negative μ. This is because the elastic-scattering cross sections are generally smaller [44], which leads to a smaller capture rate.

For values of $\tan\beta$ and $m_{H_2^0}$ near the current observational limits [Figs. 8(a), 8(b), and 8(e)], most heavy

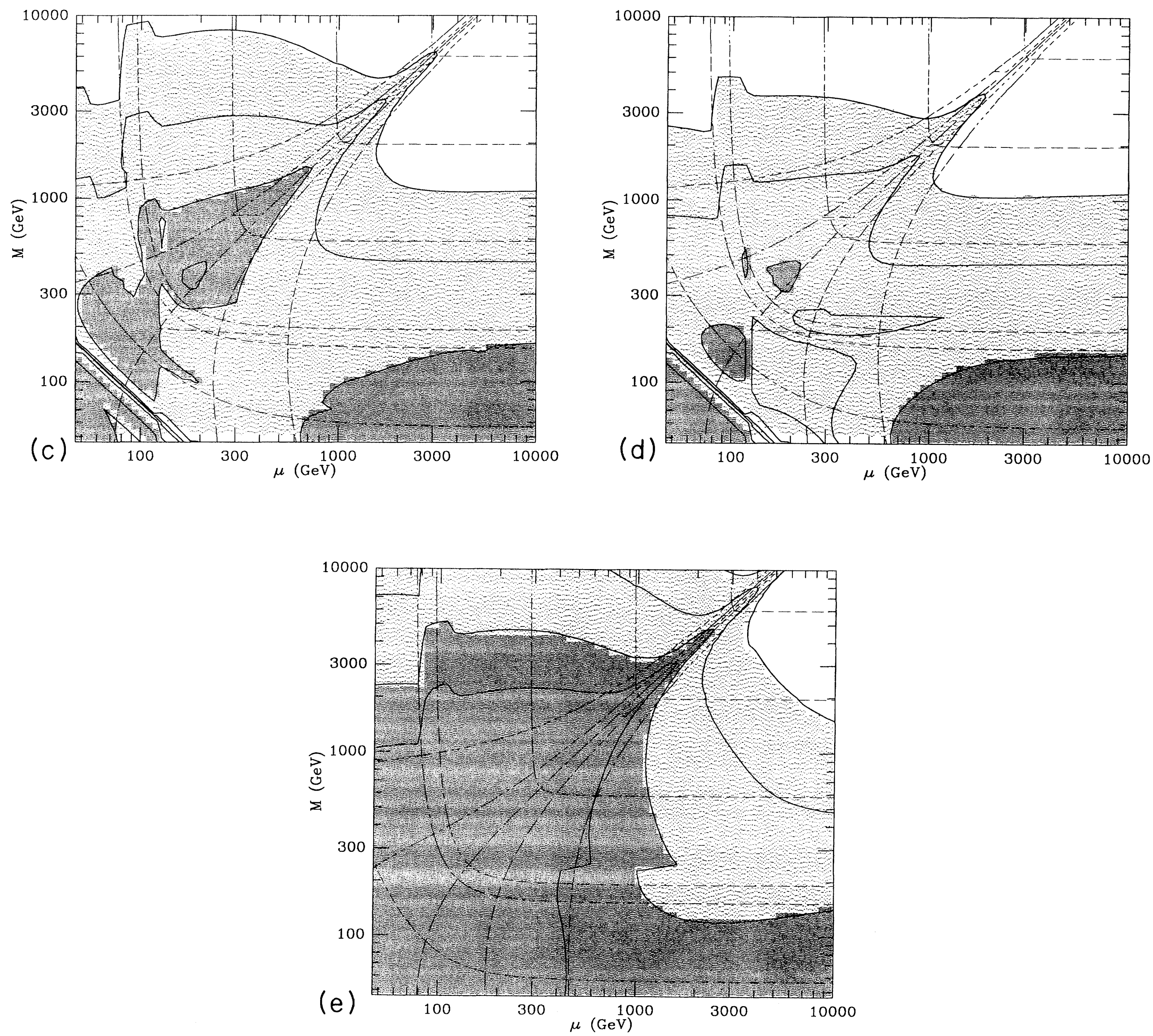

FIG. 8. (*Continued*).

Higgsinos would be observable, independent of the squark mass, should they be the primary component of the galactic halo; for larger $m_{H_2^0}$ the rates are smaller [Fig. 8(d)]. The rates from heavy B-inos are sensitive to the squark mass as may be seen by comparing Figs. 8(a) and 8(c). If the squark mass is much greater than the neutralino mass [Fig. 8(a)], the B-inos that are extremely pure will not be observable, but if the squark mass is near the neutralino mass [Fig. 8(c)], the event rates are much greater. Since relic-abundance calculations show that B-inos heavier than about 500 GeV are cosmologically inconsistent [6], Figs. 8(c), 8(d), and 8(e) suggest that forseeable improvements in the observational data from the Sun (but not the Earth) will probe almost the entire range of promising (i.e., models where the squark is not unusually heavy) supersymmetric models in which the LSP is a B-ino. Although improvements in the observational data will not be quite as conclusive for neutralinos that are primarily Higgsino or mixed states, it is still interesting to note that the region of parameter space probed by energetic neutrino searches (i.e., models with neutralino masses up to a few TeV or so) will nearly overlap with those regions which are cosmologically inconsistent (and unsuitable for preserving the mass hierarchy).

Throughout, we have taken the top-quark mass to be 120 GeV; however, our results are generally insensitive to this assumption. This is because the event rates are determined primarily by the capture rates in the Sun which do not depend on the top-quark mass. Increasing the top-quark mass would increase the fraction of annihilation products that are top quarks relative to the fraction that are gauge or Higgs bosons, and the neutrino spectrum from top quarks is generally softer than that from gauge bosons. Therefore, an increase in the top-quark mass would result in a slightly lower event rate for models where the number of top-quark final states is comparable to the number of gauge-boson final states.

By comparing Fig. 8 with Fig. 5, we find that in the excluded regions the capture and annihilation rates are large enough that the number of neutralinos in the Sun has reached equilibrium ($t_\odot > \tau_A$). Generally, we find that current observational limits on energetic neutrino fluxes would have to be increased by about two orders of magnitude until neutralinos that have not yet reached their equilibrium in the Sun are detected.

VI. CONCLUDING REMARKS

One of the most important questions facing particle physics and cosmology is the nature of the dark matter known to exist throughout the Universe and in our galactic halo. A well-motivated extension of the $SU(3)_C \times SU(2)_L \times U(1)_Y$ model of particle interactions is the minimal supersymmetric standard model. If low-energy supersymmetry exists in nature, then it is likely that the neutralino is the lightest supersymmetric particle. Although the neutralino was originally taken to be light, its mass could also lie in the 100-GeV range, and as unsuccessful accelerator searches push the mass scale for supersymmetry upward, this possibility becomes more attractive. Calculations [6] show that in much of parameter space the neutralino has a relic abundance suitable for solving the dark-matter problem. Given this result, it remains to be seen experimentally whether neutralinos do indeed populate our halo.

In this paper we have proposed that the presence of heavy neutralino dark matter be inferred through the observation of energetic neutrinos produced by neutralino annihilation in the Sun and Earth. Neutralinos that are primarily Higgsino or a mixed Higgsino-gaugino state are captured in the Sun and Earth by elastic scattering due to light-Higgs-boson exchange off of nuclei, and for mixed-state neutralinos the capture is quite efficient. If the squark is not much heavier than the neutralino, gauginos are captured via spin-dependent squark-exchange scattering off of hydrogen in the Sun but not the Earth.

Neutralinos that have been captured will annihilate and high-energy neutrinos will be produced by the decays of the annihilation products. Calculation of the energy spectrum of neutrinos from such a source as they emerge from the Sun is quite involved as the cascade from the annihilation products must be modeled considering, among other things, the effect of the solar medium on the shower. In addition, since the neutrinos have very high energies, absorption and energy loss of the neutrinos as they pass through the Sun must be included in the calculation.

The most promising method of detection of these neutrinos is through observation in underground detectors of upward-moving muons produced by the neutrinos in the rock below the detector. Current limits from IMB on the number of such throughgoing muons may already be used to constrain regions of heavy-neutralino parameter space where the neutralino is a mixed Higgsino-gaugino state and with a mass less than about 300 GeV. Furthermore, in other regions of parameter space, where the neutralino is either slightly heavier (though still in the sub-TeV range) or closer to being a pure Higgsino or gaugino state, the predicted event rates are large enough that energetic neutrino signals may be observable in the near future with increased observing time or larger detectors. Given the enormous importance of such a discovery and the promise of observation of such a signal from many supersymmetric dark-matter candidates, the search for energetic neutrinos from the Sun should be pursued.

The final result of our calculation that was compared with experiment was the flux of neutrino-induced upward-moving muons; therefore, the strongest limits should eventually come from detectors with the largest surface area or longest exposure time. The current IMB [30] limits come from a detector of area roughly 400 m^2 and an exposure time of about a year, and the limits from Kamiokande II [31] come from a slightly smaller exposure. The next improvement should come from the Monopole, Astrophysics, and Cosmic-Ray Observatory (MACRO) [45], which will have an area more than twice as large as IMB, and in the more distant future there may be a factor of 10 improvement in the collection area with a deep-sea detector [46]. There is also the intriguing possibility of an increase in detector area of several orders of magnitude by looking for Cherenkov radiation from energetic muons in deep Antarctic ice [47].

To see the prospects for discovery of dark-matter candidates via observation of muons induced by neutrinos from WIMP annihilation in the Sun, let us consider the background of throughgoing muons induced by atmospheric neutrinos. The flux of such muons (with energies larger than 2 GeV) is [30]

$$\Phi_\mu(E > 2\ \text{GeV}) = 0.075\ \text{m}^{-2}\,\text{yr}^{-1}\,\text{sr}^{-1}\ . \quad (26)$$

Now, although the angular size of the Sun in the sky is quite small and the detector resolution may be quite good, the angle between the muon direction and the direction of the parent neutrino has an intrinsic distribution with average of roughly $\bar{\theta}_{\mu\nu} \simeq 15°/[E_\mu/(2\ \text{GeV})]^{1/2}$, and so muon tracks from within 15° of the Sun need to be accepted. We see that the background from an angular window of this size is comparable to the IMB limit of $0.0265\ \text{m}^{-2}\,\text{yr}^{-1}$. So additional exposure will improve this flux limit by providing the statistics needed to distinguish excess signal from the background.

Another strategy for improving the signal-to-noise ratio is to raise the muon-energy cutoff E_μ^{cut}. Since the atmospheric neutrino flux decreases roughly as E_ν^{-3} (to be conservative) and the probability for detection of a neutrino of energy E_ν is proportional to E_ν^2, the background event rate decreases only logarithmically with increasing cutoff energy; of course, this is not the whole story. Since the mean muon-production angle $\bar{\theta}_{\mu\nu} \propto E_\mu^{-1/2}$, the size of the angular window around the Sun from which muon tracks must be accepted is accordingly smaller; consequently, the background event rate is proportional to $(E_\mu^{\text{cut}})^{-1}$. On the other hand, most of the neutrinos from WIMP's with masses of 100–1000 GeV should have energies well above 10 GeV; furthermore, the detectability of energetic neutrinos is proportional to the neutrino energy. So, by accepting muons with energies greater than 10 GeV, for example, the background is decreased by a factor of 5, while the dark-matter signal should be reduced only slightly. Of course, if such a cutoff is to be implemented, the neutrino spectra from heavy-WIMP annihilation in the Sun should be more carefully determined, either through Monte Carlo or more detailed analytic modeling of interactions of decay products and neutrinos with the solar medium to determine exactly how much of the signal is lost by rejecting muon events with energies lower than the cutoff.

We should mention that throughout we have assumed that neutralinos are the primary component of the galactic halo. Of course, if neutralinos constitute only a fraction of the dark matter, then the rates for detection will be lowered accordingly. There is also the question of whether the relic abundance of the LSP associated with a given supersymmetric model can account for the dark matter in galactic halos. Generally, it is assumed that if the fraction of critical density contributed by neutralinos today is $0.025 \lesssim \Omega_{\tilde{\chi}} h^2 \lesssim 1$, where h is the present Hubble parameter in units of $100\ \text{km}\,\text{sec}^{-1}\,\text{Mpc}^{-1}$, then the neutralino is a good dark-matter candidate. If $\Omega_{\tilde{\chi}} h^2 \gtrsim 1$, the relic density is too large to be consistent with the observed age of the Universe, and if $\Omega_{\tilde{\chi}} h^2 \lesssim 0.025$, the relic abundance is too small to make up the primary component of the galactic halo.

Here we assume that all of the heavy neutralinos we consider *are* candidates for the primary component of the galactic halo. The relic abundance of a WIMP depends on its abundance in the early Universe at "freeze-out," when the annihilation rate of the WIMP falls below the expansion rate. The annihilation rate at any given time depends on the temperature of the Universe and the cross section for annihilation of the WIMP, which is determined by the particle-physics model. On the other hand, since we have little familiarity with the conditions in the Universe before big-bang nucleosynthesis, the expansion rate at freeze-out cannot be reliably predicted. If one makes the simplest, and standard, assumption, that the early Universe was radiation dominated, then it is found that the relic abundance of heavy neutralinos is generally greater than 0.001 [48]. However, many nonstandard scenarios accommodate an expansion rate at freeze-out larger than that in the radiation-dominated Universe [49], and so, if the standard calculations find a relic abundance greater than 0.001, nonstandard scenarios allow for a relic abundance greater than 0.025. Conversely, if standard calculations yield $\Omega_{\tilde{\chi}} h^2 > 1$, a value of $\Omega_{\tilde{\chi}} h^2 < 1$ is possible if the abundance was diluted by some entropy-producing process such as inflation, a quark-hadron or electroweak phase transition, or out-of-equilibrium decay of a massive particle. Therefore, since the standard calculations yield relic abundances for LSP's within a few orders of magnitude of the dark-matter window, $0.025 \lesssim \Omega_{\tilde{\chi}} h^2 \lesssim 1$, and the abundance of a thermal relic in nonstandard cosmological models may differ from that in the standard radiation-dominated Universe by a few orders of magnitude, almost all heavy neutralinos should be considered dark-matter candidates.

Given that energetic neutrinos from heavy-neutralino annihilation in the Sun may be observable, we speculate that neutrinos from annihilation of other heavy dark-matter candidates (such as Majorana neutrinos) may also be observable. Such a heavy WIMP would have to be captured readily in the Sun, either by a scalar interaction with heavy nuclei or by a sizable spin-dependent elastic-scattering cross section that could result from the exchange of another particle not much heavier than the WIMP (e.g., a heavy lepton in the case of a Majorana neutrino) or maybe by a strong coupling to the Z. Even if the dark matter consists of some heavy WIMP other than the MSSM neutralino, the MSSM provides a good example of a particle-physics model with a well-determined phenomenology that is consistent with current laboratory results and contains an excellent dark-matter candidate. This example shows that the idea that galactic halos are populated by (possibly detectable) WIMP's is alive and well and that the quest for their discovery should be pursued vigorously.

To conclude, we note that the properties of the heavy neutralino in many models are such that their capture and annihilation in the Sun yields an observable flux of energetic neutrinos. We also point out that in many models a heavy neutralino may easily make up the primary component of the galactic halo while remaining invisible to neutrino detectors, and so null results from energetic neutrino searches are not likely to rule out supersymmetric dark matter. Nevertheless, given the present uncertainty as to the nature of the dark matter, the popularity of supersymmetry in particle physics, and the interesting "coincidence" that the relic abundance of the LSP in most supersymmetric models falls near the dark-matter window, it is clear that the search for energetic neutrinos from the Sun holds considerable promise for discovery, should neutralinos reside in the galactic halo.

ACKNOWLEDGMENTS

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APPENDIX A: ELASTIC-SCATTERING CROSS SECTION

The neutralino may elastically scatter off of a nucleus via a scalar interaction where the WIMP interacts with the mass of the nucleus, and if the nucleus has spin, the neutralino may also scatter via an axial interaction. The cross section for scattering of a neutralino off of nucleus i via an axial interaction (the "spin-dependent cross section") is [5]

$$\sigma_{\rm SD}=\frac{24m_{\tilde\chi}^2 m_i^2 G_F^2}{\pi(m_{\tilde\chi}+m_i)^2}\,\frac{4}{3}\lambda^2 J(J+1)\left[\sum_{u,d,s} A_q'\Delta q\right]^2 , \tag{A1}$$

where

$$A_q'=\tfrac{1}{2}T_{3L}^q(Z_{n3}^2-Z_{n4}^2)-x_q^2\left[\frac{2m_q^2 d_q^2}{4m_W^2}+[T_{3L}^q Z_{n2}-\tan\theta_W(T_{3L}^q-e_q)Z_{n1}]^2+\tan^2\theta_W e_q^2 Z_{n1}^2\right], \tag{A2}$$

$$x_q^2=\frac{m_W^2}{(m_{\tilde\chi}+m_i)^2-(M_{\tilde q}+m_i)^2} \tag{A3}$$

is the squark-exchange suppression factor [33], and

$$\lambda=\tfrac{1}{2}\{1+[s_p(s_p+1)-l(l+1)]/[J(J+1)]\} , \tag{A4}$$

is the Landé factor from the one-particle nuclear shell model for a nucleus with spin J, an unpaired nucleon with spin s_p, and orbital angular momentum l. Here m_q is the (current) quark mass, $d_q=-Z_{n3}/\cos\beta$ for down-type quarks, $d_q=Z_{n4}/\sin\beta$ for up-type quarks, T_{3L}^q is the weak isospin of the quark, e_q is its charge, and θ_W is the Weinberg angle. The quantity Δq measures the fraction of the nucleon spin carried by the quark. In the naive flavor-SU(3) quark model, $\Delta u=0.97$, $\Delta d=-0.28$, and $\Delta s=0$; however, the EMC reports $\Delta u=0.746$, $\Delta d=-0.508$, and $\Delta s=-0.226$ [38].

For the capture-rate calculation the spin-dependent cross section and sum that appears in Eq. (A1) may be simplified considerably. The only element with spin in the Sun found in abundance is hydrogen. For hydrogen $\frac{4}{3}\lambda^2 J(J+1)=1$ and in the EMC model,

$$\sum A_q'\Delta q=0.37(Z_{n3}^2-Z_{n4}^2)-x_q^2\left[-3.98\times10^{-7}\frac{Z_{n3}^2}{\cos^2\beta}+2.86\times10^{-9}\frac{Z_{n4}^2}{\sin^2\beta}+0.003Z_{n2}^2+0.133Z_{n2}Z_{n1}+0.073Z_{n1}^2\right], \tag{A5}$$

while in the flavor-SU(3) model,

$$\sum A_q'\Delta q=0.3125(Z_{n3}^2-Z_{n4}^2)-x_q^2\left[-3.5\times10^{-10}\frac{Z_{n3}^2}{\cos^2\beta}+3.72\times10^{-9}\frac{Z_{n4}^2}{\sin^2\beta}+0.173Z_{n2}^2+0.1125Z_{n2}Z_{n1}+0.122Z_{n1}^2\right]. \tag{A6}$$

The term proportional to $(Z_{n3}^2-Z_{n4}^2)$ arises from Z exchange, and the second term arises from squark exchange. For heavy B-inos, $Z_{n3}\simeq Z_{n4}\simeq0$, for heavy Higgsinos, $Z_{n3}^2\simeq Z_{n4}^2$, and as we will see below, for heavy mixed-state neutralinos, the axial interaction is much weaker than the scalar interaction; therefore, scattering of heavy neutralinos via Z exchange is essentially negligible. In addition, from Eqs. (A5) and (A6) one can see that if the neutralino is pure Higgsino, spin-dependent scattering due to squark exchange is also negligible, but if the neutralino is pure B-ino ($Z_{n1}\simeq1$ and $Z_{n2}\simeq Z_{n3}\simeq Z_{n4}\simeq0$) and the squark is not much heavier than the neutralino, then spin-dependent scattering due to squark exchange may be significant. By comparing Eqs. (A5) and (A6), we also see that had we used the flavor-SU(3) quark model the capture rates would be roughly 3 times as large as those obtained using the EMC results, which we used in this work.

The cross section for scattering via a scalar interaction is obtained from Refs. [5], [34], [35], and [23]. Griest [5] obtained the results for a scalar interaction via exchange of a virtual squark, and Barbieri, Frigeni, and Guidice [34] obtained results for a scalar interaction in which a Higgs boson is exchanged; however, in both of these papers it was assumed that the nucleon mass is due to gluons [50]. Recent measurements of this pion-nucleon σ term imply that a significant fraction of the nuclear mass is due to a sea of strange quarks [35]. When applied to neutralino-nucleus scattering via a scalar interaction, it is found that, although the component of squark- and Higgs-boson–nucleon coupling due to gluons is reduced, there is an additional component due to squark and Higgs coupling to the strange-quark sea, and the net effect is a significant increase in the squark- and Higgs-boson–nucleon coupling [23].

The scalar cross section may be derived from the effective Lagrangian [5,34]

$$\mathcal{L}_{\rm eff}=\sqrt{2}G_F(Z_{n2}-Z_{n1}\tan\theta_W)\times\sum_q\left[\frac{m_W}{m^2_{H^0_2}}g_{H_2}k^{(2)}_q+\frac{m_W}{m^2_{H^0_1}}g_{H_1}k^{(1)}_q+\frac{\epsilon d_q x_q^2}{m_W}\right]\times m_q\bar{\tilde{\chi}}\tilde{\chi}\bar{q}q\ , \tag{A7}$$

where $k^{(1)}_q=\sin\alpha/\sin\beta$ and $k^{(2)}_q=\cos\alpha/\sin\beta$ for up-type quarks, $k^{(1)}_q=\cos\alpha/\cos\beta$ and $k^{(2)}_q=-\sin\alpha/\cos\beta$ for down-type quarks, $g_{H_2}=(Z_{n3}\sin\alpha+Z_{n4}\cos\alpha)$, $g_{H_1}=(Z_{n3}\cos\alpha+Z_{n4}\sin\alpha)$, and ϵ is the sign of the lightest-neutralino mass, $|m_{\tilde{\chi}}|/m_{\tilde{\chi}}$ (see Griest, Kamionkowski, and Turner [6]). In addition to terms due to exchange of the lightest Higgs boson [34] and the squark [5], to be complete we have included a term due to exchange of the heaviest Higgs boson, although it should generally be smaller than that due to exchange of the lightest Higgs boson.

The scattering cross section is obtained from the square of the matrix element $\langle f|\mathcal{L}_{\rm eff}|i\rangle$ of this effective Lagrangian between the initial and final neutralino-nuclear states. In Ref. [50] (as modified by Ref. [35]) it is shown that the coupling of a scalar field to the gluons in

$$\langle N|m_h\bar{h}h|N\rangle=\tfrac{2}{27}m_i(0.56)\ , \tag{A8}$$

where h is a heavy-quark field, m_h is the heavy-quark mass, and $|n\rangle$ is the nuclear wave function. In addition, measurements of the pion-nucleon σ term imply that [35]

$$\langle N|m_s\bar{s}s|N\rangle=\tfrac{2}{27}m_i(5.94)\ , \tag{A9}$$

where s is the strange-quark field and m_s is its mass. The matrix elements of $m_q\bar{q}q$ for the u and d quarks are much smaller. With these results it is easy to find that the matrix element is

$$\langle f|\mathcal{L}_{\rm eff}|i\rangle=\sqrt{2}G_F\tfrac{2}{27}m_i(Z_{n2}-Z_{n1}\tan\theta_W)\times\left[\frac{m_W}{m^2_{H^0_2}}g_{H_2}\left[1.12\frac{\cos\alpha}{\sin\beta}-6.5\frac{\sin\alpha}{\cos\beta}\right]+\frac{m_W}{m^2_{H^0_1}}g_{H_1}\left[1.12\frac{\sin\alpha}{\sin\beta}+6.5\frac{\cos\alpha}{\cos\beta}\right]+\frac{\epsilon x_q^2}{m_W}\left[1.12\frac{Z_{n4}}{\sin\beta}-6.5\frac{Z_{n3}}{\cos\beta}\right]\right]\ , \tag{A10}$$

and the cross section for scattering off of nucleus i via a scalar interaction is

$$\sigma_{\rm SC}=\frac{4m^2_{\tilde{\chi}}m_i^2}{\pi(m_{\tilde{\chi}}+m_i)^2}|\langle f|\mathcal{L}_{\rm eff}|i\rangle|^2\ . \tag{A11}$$

We should clarify that this is the cross section that would be measured only *if* the neutralino interacted coherently with the entire nucleus. If the inverse of the momentum transfer $1/q$ in the scattering event is small compared with the nuclear radius R, then the neutralino does not interact coherently with the entire nucleus and the total cross section is momentum-transfer dependent (or, equivalently, scattering-angle dependent) and is given by Eq. (A11) times $|F(q^2)|$, the form-factor suppression. The effect of the form-factor suppression on capture in the Sun and Earth is discussed in Sec. II.

APPENDIX B: MIXED GAUGE-BOSON–HIGGS-BOSON FINAL STATES

In addition to the gauge- and Higgs-boson final states considered in Ref. [6], neutralinos may annihilate into mixed Higgs-boson–gauge-boson final states when the mass of the neutralino exceeds the average of the gauge- and Higgs-boson masses. At zero relative velocity the available channels are ZH^0_1, ZH^0_2, W^+H^-, and W^-H^+. Annihilation into ZH^0_3 is possible in general, but does not occur at zero relative velocity for CP-conserving theories. The reason is that at zero relative velocity neutralino-neutralino annihilation occurs via an s wave, and because of Fermi statistics, the initial state has $CP=-1$. Since the Z has spin 1 and the Higgs particle is a scalar, the orbital wave function of the outgoing state must have $l=1$, and since the Z is CP even and the H^0_3 is CP odd, the final state must have $CP=1$ and is therefore inaccessible from the initial state.

Since the ZH^0_2 final state is the first mixed channel to open up as the neutralino mass is increased, we will consider it first. [Incidentally, since $(m_{H^0_2}+m_Z)/2$ may be less than m_W, this channel may be open for neutralinos that are lighter than the W, a possibility that was not considered in previous work.] Throughout this appendix we will use the notation of Griest, Kamionkowski, and Turner (GKT) [6], and some of the couplings we will use here are defined there.

Annihilation of two neutralinos into ZH^0_2 occurs via s-channel exchange of a Z and a H^0_3 and by t- and u-channel exchange of all four neutralinos. The cross section $\sigma_{ZH^0_2}$ for this process as relative velocity $v_{\rm rel}\to 0$ is

$$\sigma_{ZH^0_2}v_{\rm rel}=\frac{kX_{ZH^0_2}}{32\pi m^3_{\tilde{\chi}}}\ , \tag{B1}$$

where

$$k=\left[m^2_{\tilde{\chi}}-\frac{1}{2}(m_Z^2+m^2_{H^0_2})+\frac{(m_Z^2-m^2_{H^0_2})^2}{16m^2_{\tilde{\chi}}}\right]^{1/2} \tag{B2}$$

is the momentum of the outgoing particles and

$$X_{ZH_2^0}=2k^2\frac{m_{\tilde\chi}^2}{m_Z^2}\left[\frac{zF_{nn}}{m_Z^2}+\frac{4M_{3nn}hm_{\tilde\chi}}{s-m_{H_3^0}^2}+\sum_k\frac{2gM_{2nk}F_{nk}(m_{\tilde\chi_k^0}-m_{\tilde\chi})}{t-m_{\tilde\chi_k^0}^2}\right]^2 . \tag{B3}$$

Here $z=gm_Z\sin(\beta-\alpha)/\cos\theta_W$ is the coupling at the H_2^0ZZ vertex [where g is the $SU(2)_L$ coupling constant], $F_{ij}=g(Z_{i3}Z_{j3}-Z_{i4}Z_{j4})/2\cos\theta_W$ is the coupling at the $Z\tilde\chi_i^0\tilde\chi_j^0$ vertex, M_{ijk} is the $H_i^0\chi_j^0\tilde\chi_k^0$ coupling and is given in Eq. (C9) of GKT, $h=g\cos(\alpha-\beta)/2\cos\theta_W$ is the $ZH_2^0H_3^0$ coupling, the sum is over all four neutralinos, and $t=[(m_Z^2+m_{H_2^0}^2)/2]-m_{\tilde\chi}^2$.

For larger neutralino masses the ZH_1^0 channel opens up. (Recall that the H_1^0 is always heavier than the Z.) The cross section for annihilation into ZH_1^0 may be obtained from annihilation into ZH_2^0 by simply replacing $m_{H_2^0}$ by $m_{H_1^0}$, M_{2ij} by M_{1ij}, and using $z=gm_Z\cos(\beta-\alpha)/2\cos\theta_W$ and $h=g\sin(\alpha-\beta)/2\cos\theta_W$.

Annihilation into $WH^\pm$ final states occurs thorough s-channel exchange of the H_3^0 and t- and u-channel exchange of the two charginos. The cross section for this process as relative velocity $v_{\rm rel}\to 0$ is

$$\sigma_{WH^\pm}v_{\rm rel}=\frac{kX_{WH^\pm}}{32\pi m_{\tilde\chi}^3} , \tag{B4}$$

where

$$X_{WH^\pm}=\left[\frac{gm_{\tilde\chi}k}{m_W}\right]^2\left[\frac{4M_{3nn}m_{\tilde\chi}}{s-m_{H_3^0}^2}+\sum_i\frac{[m_{\tilde\chi_i^+}(e_iQ_R^i-f_iQ_L^i)+m_{\tilde\chi}(f_iQ_R^i-e_iQ_L^i)]}{t-m_{\tilde\chi_i^+}^2}\right]^2 . \tag{B5}$$

The sum is over the two charginos, the quantities e_i and f_i are given in GKT [Eq. (A2) of GKT],

$$\begin{aligned}\begin{bmatrix}Q_L^1\\ Q_L^2\end{bmatrix}&=g\cos\beta\left[Z_{n4}\begin{bmatrix}\cos\phi_+\\ \sin\phi_+\end{bmatrix}+\frac{1}{\sqrt2}(Z_{n2}+Z_{n2}\tan\theta_W)\begin{bmatrix}-\epsilon\sin\phi_+\\ \epsilon\cos\phi_+\end{bmatrix}\right],\\ \begin{bmatrix}Q_R^1\\ R_L^2\end{bmatrix}&=g\sin\beta\left[Z_{n3}\begin{bmatrix}\cos\phi_-\\ \sin\phi_-\end{bmatrix}+\frac{1}{\sqrt2}(Z_{n2}+Z_{n2}\tan\theta_W)\begin{bmatrix}-\sin\phi_-\\ \cos\phi_-\end{bmatrix}\right],\end{aligned} \tag{B6}$$

where the angles ϕ_+ and ϕ_- are related to the diagonalization of the chargino mass matrix and are given in Ref. [51], $\epsilon=\det X/|\det X|$, and X is the matrix defined in Eq. (C9) of Ref. [3]. Here,

$$k=\left[m_{\tilde\chi}^2-\frac{1}{2}(m_W^2+m_{H^+}^2)+\frac{(m_W^2-m_{H^+}^2)^2}{16m_{\tilde\chi}^2}\right]^{1/2} , \tag{B7}$$

and $t=[(m_W^2+m_{H^+}^2)/2]-m_{\tilde\chi}^2$.

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the squark is not much heavier than the neutralino), then there may be a comparable annihilation branch into gluons and the neutrino signal may be diluted accordingly. In the majority of the models considered here, the neutralino is either a pure Higgsino or is heavier than the top quark and the dominant final states are gauge bosons or top quarks; so the gluon final states have little or no effect.

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