ARTICLES

Solar neutrinos and the Mikheyev-Smirnov-Wolfenstein theory

Hans A. Bethe
Newman Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14853

John N. Bahcall
Institute for Advanced Study, Princeton, New Jersey 08540
(Received 9 July 1991)

The observation of solar neutrinos by Kamiokande shows that the solar-neutrino problem cannot be solved by changing the solar model. In combination with the observations with a chlorine detector, it makes the nonadiabatic form of the Mikheyev-Smirnov-Wolfenstein theory most likely, and determines $\Delta m^2 \sin^2\theta = 1.0\times10^{-8}$ eV^2. Probably all neutrinos go through the resonance in the Sun, those from ^{8}B nonadiabatically, all others adiabatically. The latter emerge from the Sun in the higher-mass eigenstate ν_2 and have a probability $\sin^2\theta$ to be detected as ν_e. The gallium experiments, when done with sufficient accuracy, will be able to determine $\Delta m^2 = m^2(\nu_\mu) - m^2(\nu_e)$ within fairly close limits. If the day-night effect can be measured, it will further constrain these limits. The small value of $\Delta m^2 \sin^2\theta$ explains why the oscillation from ν_e to ν_μ has not been observed in the laboratory. From existing experiments, the temperature at the center of the Sun can be determined to be within about 6% of that derived from the standard solar model; future neutrino experiments may determine it to within 1%.

I. INTRODUCTION

In a recent paper [1], we argued that the solar-neutrino problem can be explained by the nonadiabatic form of the Mikheyev-Smirnov-Wolfenstein (MSW) theory [2]. We also predicted that the Ga experiment which is now proceeding at SAGE [3] and GALLEX [4] would find an effect of only 5 solar-neutrino units (SNU), as compared with a prediction of 132 SNU from [5] the standard solar model (SSM).

Baltz and Weneser [6] (BW) have criticized our prediction for the Ga experiment, and calculate up to 90 SNU for Ga. We agree with their prediction. In fact, a similar prediction was made by Bahcall and Haxton [7], but we decided to leave the discussion of the effect involved for this present paper; see Secs. IV and VI. On our main result, they agree with us, namely that the combination of the Kamiokande II [8] and the chlorine [9] experiment can be explained by the MSW theory with a relation

$$\Delta m^2 \sin^2\theta = 10^{-8}\ \text{eV}^2 \tag{1}$$

between $\Delta m^2 = m_2^2 - m_1^2$ and θ, the mixing angle between the two neutrino types. This conclusion was already reached by Rosen and Gelb [10], and was recently used by Pakvasa and Pantaleone [11].

In this paper, we wish to discuss the physical aspects of the MSW theory and the relation of the MSW parameters to experiment. As much as possible, we shall use analytical calculations, but for accurate numerical answers, we shall use the results of BW.

II. THE EXPERIMENTS

The chlorine experiment [9] has yielded an average detection rate

$$\langle \phi\sigma \rangle_{\text{Cl,expt}} = 2.1 \pm 0.3\ \text{SNU}\quad (1\sigma\ \text{error}) \tag{2}$$

for neutrinos above the 0.81-MeV detector threshold energy. The prediction of the SSM is [5]

$$\langle \phi\sigma \rangle_{\text{Cl}} = 7.9 \pm 2.6\ \text{SNU}\ , \tag{3}$$

where the large error arises primarily from the uncertainty of the flux of ^{8}B neutrinos in the Sun. Disregarding this uncertainty,

$$\frac{\langle \phi\sigma \rangle_{\text{Cl,expt}}}{\langle \phi\sigma \rangle_{\text{Cl,SSM}}} = 0.27 \pm 0.04\ . \tag{4}$$

The Kamiokande II (KII) neutrino-electron scattering experiment [8] has given

$$\langle \phi\sigma \rangle_{\nu\text{-}e} = [0.46 \pm 0.05(\text{stat}) \pm 0.06(\text{syst})]\langle \phi\sigma \rangle_{\text{SSM}} \tag{5}$$

for recoil electrons of energy greater than 7.5 MeV (the uncertainty in the solar model prediction has again been disregarded).

The only solar neutrinos which can give recoil electrons of such high energy are those from [5] the decay of ^{8}B. This shows that ^{8}B is indeed produced in the Sun. Before the KII experiment, it was frequently suggested that the neutrino deficiency in the Cl experiment might be explained by reduced ^{8}B production in the Sun, and this might happen if the central temperature were

13.5×10^6 degrees, instead of 15.5 as derived from the SSM. One of us (JNB) has spent the last 20 years showing that such a reduction of T_c is incompatible with other properties of the Sun. The KII finding shows that indeed the ^{8}B production in the Sun cannot be strongly reduced, and that this is not the explanation of the neutrino deficiency (see Sec. VIII).

It is important that the Cl experiment records a smaller fraction of the theoretically predicted neutrinos than the KII. The Cl detector has a threshold of 0.81 MeV, with a rapidly increasing cross section as a function of energy, while KII responds only to neutrinos well above 7.5 MeV. The comparison of the two experimental results indicates that lower-energy neutrinos are more effectively "lost" than high-energy ones. Only by stretching the limits of error to the utmost could the results (4) and (5) be made equal. Adding the squares of the two errors in (5) we get ±0.08. Assuming then 1.6σ error in opposite directions in (4) and (5), which has a probability of e^{-5}, we would get

$$\frac{\langle\phi\sigma\rangle_{\rm expt}}{\langle\phi\sigma\rangle_{\rm SSM}}=0.33\ . \tag{6}$$

It should be noted that the comparison of (4) and (5) is approximately independent of the poorly known flux of ^{8}B neutrinos in the Sun, since both experiments respond primarily to ^{8}B neutrinos.

III. MSW THEORY: THE NONADIABATIC TRANSITION

Since the solar-neutrino problem cannot be explained by anomalous temperature of the Sun, the clue must lie in properties of the neutrino. The MSW theory is the natural explanation. In this theory, the electron neutrinos produced near the center of the Sun are converted on their way out of the Sun into μ or τ neutrinos which cannot be detected by a radioactive detector like Cl, and are detected with reduced efficiency (about $\frac{1}{6}$) by ν-e collisions, as in KII. Many authors have published plots of Δm^2 vs $\sin^2\theta$ which give contour lines of parameters which are compatible with the result of the Cl experiment, Eq. (2). These show three legs: a horizontal leg at $\Delta m^2\simeq10^{-4}$, a slant leg corresponding approximately to Eq. (1), and a vertical leg at $\sin^2 2\theta\simeq1$. If the solution is on the horizontal leg, then ^{8}B neutrinos will almost completely be converted into ν_x ($x=\mu$ or τ). This is contradicted by the KII experiment; hence, the horizontal leg is disproved.

The slant leg was discovered by Rosen and Gelb [12] and by Kolb *et al.* [13] and interpreted by Kolb *et al.* [13], by Haxton [14], and by Parke [14] as representing a nonadiabatic transition. The best analytical treatment was given by Pizzochero [15], who assumed an exponential distribution of density (actually, electron density) in the Sun:

$$\rho=\rho_0\exp(-r/R_s)\ . \tag{7}$$

By a simple analytical argument he shows that the survival probability of a ν_e when going through the resonance is

$$p=\exp(-C/E)\ , \tag{8}$$

where E is the neutrino energy and

$$C=\pi R_s\Delta m^2\sin^2\theta\ , \tag{9}$$

$$R_s=6.6\times10^9\ \text{cm}\ . \tag{10}$$

Baltz and Weneser [16] have shown by computer calculation that (8),(9) is very accurate.

Equation (8) is in qualitative agreement with the experimental result that the Cl detection (4) is more strongly diminished relative to theory than the KII detection (5). Determining C from the Cl results gives

$$C=10.5\pm1.5\ \text{MeV}\quad(1\sigma) \tag{11}$$

disregarding the uncertainty of the ^{8}B neutrino flux. Using this value, the KII result is predicted to be

$$\frac{\langle\phi\sigma\rangle_{\rm expt}}{\langle\phi\sigma\rangle_{\rm SSM}}=0.455\pm0.045\ , \tag{12}$$

in excellent agreement with the experimental result (5).

This result makes it very probable that the nonadiabatic form of the MSW theory is indeed the correct explanation of the solar-neutrino results and that the flux of ^{8}B neutrinos that is produced in the Sun is not very different from the best theoretical estimate.

If the uncertainty of the ^{8}B neutrino flux is taken into account, (11) must be replaced by

$$C=10.5^{+5}_{-3.5}\ \text{MeV}\ . \tag{13}$$

Equation (8) shows that only high-energy neutrinos have a good chance of surviving the resonance as electron neutrinos. ^{7}Be neutrinos ($E=0.87$ MeV) for example, have a probability

$$p(\text{Be})\sim10^{-5}\ . \tag{13a}$$

They, and other low-energy neutrinos, are totally converted into ν_x. This is what is expected for an *adiabatic* transition: as the density decreases, the neutrino "slides down" the mass-vs-density curve and ends up as a ν_x. Thus the slant line gives a nonadiabatic transition only for ^{8}B neutrinos, for all others the transition is adiabatic.

From the value of C, and using (9), we find

$$\Delta m^2\sin^2\theta=(1.0\pm0.5)\times10^{-8}\ \text{eV}^2\ . \tag{14}$$

This is the *only* quantity we can derive from a combination of the Cl and the KII experiment.

Baltz and Weneser [6] have done an accurate computer calculation which is reproduced in Fig. 1. The narrow hatched band is compatible with both the Cl and the KII experiment. The center of this band lies at

$$\Delta m^2\sin^2\theta=1.1\times10^{-8}\ \text{eV}^2 \tag{15}$$

for $\Delta m^2=2\times10^{-7}$ to 10^{-5}, in good agreement with (14). So far, they and we have only used the KII and Cl experiments.

Once we accept the MSW theory, we have to realize

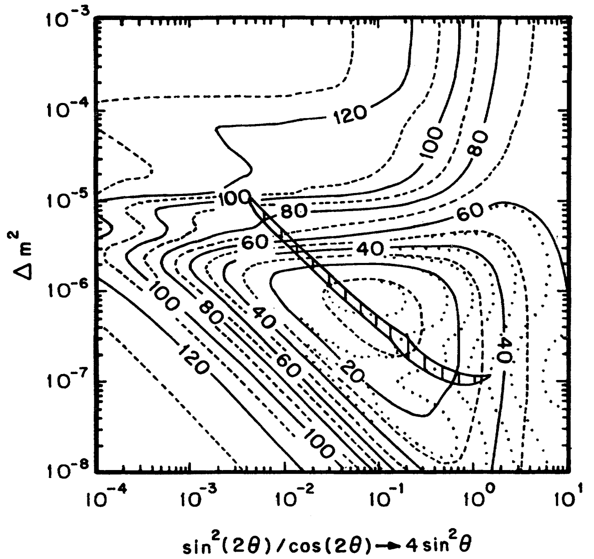

FIG. 1. Relation between Δm^2 and $\sin^2\theta$, according to Baltz and Weneser [6]. (Their $\sin^2 2\theta/\cos 2\theta$ should be replaced by $4\sin^2\theta$; see end of Sec. VI.) The hatched band is the region permitted by both the Cl and the KII experiment. The solid contours give the Ga counts, in SNU, to be expected if the neutrinos arrive directly at the Earth. The dotted lines take into account also the neutrinos which get to the counter at night, after having gone through the Earth.

that the KII experiment measures both ν_e and ν_x, albeit the latter with a cross section of only about $\frac{1}{6}$ of the former. If p is the fraction of neutrinos surviving as ν_e, the total fraction measured by KII is

$$(\mathrm{KII}) = p + (1-p)/6 \ . \tag{16}$$

In the experimental ratio (5), we combine the statistical and the systematic errors by adding the squares; thus,

$$(\mathrm{KII})_{\mathrm{expt}} = 0.46 \pm 0.08 \ , \tag{5a}$$

then

$$p = 0.35 \pm 0.10 \ . \tag{17}$$

This is still larger than the Cl result, (4), but the two are now within the probable error. A compromise value would be

$$p' = 0.29 \tag{18}$$

which is within 60% of the standard error of (4) and (17).

BW, and Bahcall and Haxton [7] also predict the result of the ongoing gallium experiment [3,4], using contour lines. They stress that values up to 90 SNU are compatible with the narrow allowed band. In the next three sections, we shall discuss how this comes about.

IV. THE RESONANCE CONDITION

The condition for the MSW resonance is [17]

$$\frac{n_e(\text{resonance})}{n_e(\text{center of Sun})} = 0.67 \cos 2\theta \left[\frac{\Delta m^2}{10^{-4}\,\mathrm{eV}^2} \right] \left[\frac{10\ \mathrm{MeV}}{E} \right] , \tag{19}$$

where n_e is the electron number density. The production of ^{8}B and ^{7}Be neutrinos in the Sun peaks at about $n_e(\text{resonance})/n_e(\text{center of Sun})$ equal to 0.9 and 0.85, respectively, whereas the pp neutrinos originate in a broader region further out, at $n_e(\text{resonance})/n_e(\text{center of Sun}) \approx 0.65$. For the interesting cases, $\cos 2\theta$ is very near 1. In order for an average neutrino of given energy to encounter an MSW resonance on its way out, it is necessary that

$$\Delta m^2 < 1.1 \times 10^{-5} E \ . \tag{20}$$

Thus a ^{8}B neutrino, E around 10, will have a resonance of $\Delta m^2 < 10^{-4}$ eV^2, the well-known horizontal leg. A ^{7}Be neutrino, $E = 0.87$ MeV, will only go through a resonance if $\Delta m^2 < 10^{-5}$. And a pp neutrino detectable by Ga, $E = 0.2$ to 0.4 will only experience a resonance if $\Delta m^2 < (2 \text{ to } 4) \times 10^{-6}$ eV^2.

From the Cl-KII comparison, Sec. II, it is very likely that ^{7}Be does go through a resonance. If it did not, all the ^{7}Be neutrinos would arrive on Earth as ν_e, which would greatly aggravate the discrepancy between (4) and (5). Therefore the shaded band of BW begins only at $\Delta m^2 = 10^{-5}$. At this Δm^2, however, pp neutrinos still do not go through a resonance, so they arrive on Earth as ν_e; this explains the large Ga count which is possible according to BW. As Δm^2 decreases, the fraction of pp neutrinos having a resonance increases, and the expected Ga count goes down to 20 SNU at $\Delta m^2 = 2 \times 10^{-6}$.

The result of the gallium experiment by the SAGE group[3], after 5 months of actual counting, is

$$\mathrm{SAGE} = 0 \pm 55\ \mathrm{SNU} \quad (680002\ \mathrm{C.L.}) \ , \tag{21}$$

$$\mathrm{SAGE} = 0 \pm 79\ \mathrm{SNU} \quad (90\%\ \mathrm{C.L.}) \ . \tag{21a}$$

Using the result for 68% C.L. with Fig. 1,

$$\Delta m^2 < 3 \times 10^{-6} \ ; \tag{22}$$

hence,

$$\sin^2\theta > 0.003 \ . \tag{23}$$

We can now draw a purely theoretical conclusion. In the standard model of particle physics, the Cabibbo-Kobayashi-Maskawa theory [18], the coupling between first and third generation is very small. Since neutrinos have nonzero mass and are mixed, it is reasonable to assume that they also follow a similar theory as CKM. Then the coupling between ν_e and ν_τ should also be very small, much smaller than our lower limit (23). We suggest that the conversion in the Sun is not to ν_τ, but instead to ν_μ. If we were to use the 90% C.L. SAGE limit of 79 SNU, we would find $\sin^2\theta > 0.0015$.

The predictions of grand unified theories have recently been thoroughly reviewed by Bludman, Kennedy, and Langacker [19]. They give reasons why the CKM mixing may be applicable to the neutrino sector, and state that in this case

$$10^{-6} < \sin^2\theta_{e\tau} < 4\times 10^{-4} . \tag{24}$$

On this basis, we may conclude that indeed ν_x is not ν_τ but that $\nu_x = \nu_\mu$.

If Δm^2 is between 10^{-7} and 10^{-6}, as seems likely from Fig. 1—see also Sec. VII—then the resonance for a 10-MeV ^{8}B neutrino is at a density (in g/cm^3)

$$0.1 < \rho < 1 , \tag{25}$$

which occurs between $0.55R_\odot$ and $0.8R_\odot$. The resonance for ^{7}Be neutrinos (0.87 Mev) is at about 10 times this density, corresponding to between $0.3R_\odot$ and $0.55R_\odot$.

Reduction of the error of the SAGE experiment will permit a more accurate determination of Δm^2; see Fig. 1. For this purpose, it would also be very useful to measure the day-night effect. This affects chiefly the ^{7}Be neutrinos. It is likely that these neutrinos could be measured electronically, and thus in real time, in the borexino experiment [23] which may operate in the Gran Sasso laboratory, perhaps in 1995. Referring to Fig. 1, the dotted curves (which include neutrinos coming through the Earth at night) differ greatly from the solid ones (which only include neutrinos coming directly from the Sun) for $\Delta m^2 = (1-3)\times 10^{-7}$, indicating a strong day-night effect.

V. ADIABATIC TRANSITION

Neutrinos generally behave adiabatically as they escape from the Sun, with the sole exception of ^{8}B neutrinos when conditions (14) is satisfied. If we had instead

$$\Delta m^2 \sin^2\theta \gg 10^{-8}$$

then also ^{8}B neutrinos would behave adiabatically.

The wave function of a neutrino, and hence its properties when it leaves the Sun, depends on the relation of its place of emission to the resonance (19). The wave function at the place of emission can be derived from the mass matrix which is [20], in the ν_e, ν_μ representation,

$$\mathcal{M} = \begin{bmatrix} B - \cot 2\theta & 1 \\ 1 & -B + \cot 2\theta \end{bmatrix} , \tag{26}$$

where, using the notation of Ref. [20],

$$B = A/\Delta m^2 \sin 2\theta \tag{27}$$

$$= \frac{1.5\times 10^{-7} \rho Y_e E}{\Delta m^2 \sin 2\theta} . \tag{28}$$

Using for ρY_e the value near the ^{8}B-production peak, 75 g cm^{-3} [see below (19)],

$$B = \frac{10^{-5} E}{\Delta m^2 \sin 2\theta} . \tag{29}$$

Writing the eigenfunctions of $\mathcal{M}$ in the form

$$\psi_a = \begin{bmatrix} \cos\chi \\ -\sin\chi \end{bmatrix} , \quad \psi_b = \begin{bmatrix} \sin\chi \\ \cos\chi \end{bmatrix} , \tag{30}$$

we find

$$\cot 2\chi = -B + \cot 2\theta \tag{31}$$

and the eigenvalues

$$\varepsilon = \pm[(B - \cot 2\theta)^2 + 1]^{1/2} . \tag{32}$$

At the place of emission, the neutrino is an electron neutrino, so its wave function is

$$\psi = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \cos\chi\psi_a + \sin\chi\psi_b . \tag{33}$$

If, at the place of emission, the density is high above the resonance,

$$B \gg (\cot 2\theta, 1) \tag{34}$$

then, according to (31),

$$\chi = \frac{\pi}{2} - \frac{1}{2B} . \tag{35}$$

Then, according to (30), the neutrino will be essentially in state ψ_b. It will remain in this state throughout its travel through the Sun. When it exits from the Sun, $B = 0$, and ψ_b becomes the eigenfunction of the higher-mass free-space neutrino which is close to ν_μ.

If the neutrino is emitted at the resonance, then $\chi = \pi/4$ and ψ contains equal amounts of ψ_a and ψ_b. We shall be mostly interested in the neighborhood of the vertical leg so $\cot 2\theta < 1$; cf. Sec. VI.

Returning to the case of large B, the coefficient of ψ_a is

$$\cos\chi \approx \frac{1}{2B} = \frac{\Delta m^2 \sin 2\theta}{2\times 10^{-5} E} . \tag{36}$$

It is small, but for given Δm^2, it is larger for lower neutrino energy E.

VI. REGENERATION OF ν_e

On the way from the Sun to the Earth, the neutrinos remain in state $|\nu_1\rangle$ or $|\nu_2\rangle$ if they were in ψ_a or ψ_b at the exit from the Sun. The interference between these two states is probably not important. As they enter the detector, $\nu_1 = \nu_e$ with a probability $\cos^2\theta$, while $\nu_2 = \nu_e$ with probability $\sin^2\theta$. Therefore, with the wave function (33) and neglecting the interference term, the probability of detecting a ν_e is

$$p = \cos^2\chi \cos^2\theta + \sin^2\chi \sin^2\theta \tag{37}$$

$$= \sin^2\theta + \cos^2\chi \cos 2\theta . \tag{38}$$

If Δm^2 is small, then according to (36), $\cos\chi$ is small, and p is given essentially by $\sin^2\theta$.

Thus the neutrinos which have been converted into ν_2 by the resonance, still have a considerable probability to be detected as ν_e on Earth provided $\sin\theta$ is large enough. This is the explanation of the vertical leg in the plot of Δm^2 versus $\sin^2\theta$. The mixing angle θ can be determined from the fraction of SSM recorded in the detector. For the Cl detector, this fraction is 0.27 ± 0.04, so this is the value of $\sin^2\theta$ provided $\cos\chi$ in (36) is small. For larger Δm^2, $\sin^2\theta$ will be somewhat smaller. This determines the location of the vertical leg if only the Cl detector results are used.

We shall now investigate to what extent the vertical leg is compatible with both KII and Cl. For the KII detector, we take the result (17), viz.,

$$p=0.35\pm0.10 \ . \tag{17}$$

Taking the central value, this gives a $\sin^2\theta$ incompatible with that derived from the Cl experiment. But going toward the lower limit, the two measurements are compatible with $p'=0.29$, Eq. (18). So we may choose

$$\sin^2\theta\simeq0.29 \ . \tag{39}$$

This result was derived neglecting the second term in (38). If we postulate that this second term is <0.02 for neutrinos of the Be group with $E\approx1$ MeV, and use (39), then (36) requires

$$\Delta m^2<4\times10^{-6}\ \mathrm{eV}^2 \ . \tag{40}$$

Thus the lower part of the vertical leg is still compatible with both experiments.

The Baltz-Weneser band, Fig. 1, terminates at a similar point:

$$\sin^2 2\theta=0.75, \quad \sin^2\theta=0.25, \quad \Delta m^2=1.1\times10^{-7} \tag{41}$$

and has no vertical leg. The evaluation by Hirata *et al.* [21] gives a band centered on relation (1), and similar to the band of Fig. 1, plus a vertical leg extending to $\Delta m^2>10^{-4}$. The latter is permitted because these authors considered *only* the KII experiment. The requirement of compatibility with the Cl experiment leads to the limitation (40).

It is rather easy to exclude the solution near $\Delta m^2=10^{-4}$ found by Hirata *et al.* [21] For this purpose we take the minimum KII value from (17), $p=0.25$. Since in the SSM, ^{8}B contributes 6.1 SNU to the Cl result [5], the minimum permitted contribution by ^{8}B is 1.5 SNU. (Note that this is essentially independent of the solar model, it is just based on the Kamiokande measurement). Be and other neutrinos of energy around 1-MeV energy contribute (in the SSM [5]) 1.7 SNU (^{7}Be formation is quite insensitive to the solar temperature); so the total predicted count in Cl is 3.2 SNU. The maximum Cl count from experiment is 2.4 SNU, Eq. (2). Thus there is a deficit of 0.8 SNU which must be due to the Be group since the ^{8}B contribution comes directly from the KII experiment. Therefore at least about a half of the Be-type neutrinos must go through the resonance. But, using (19), this is only possible if $\Delta m^2<10^{-5}$ eV^2.

[There is 68% confidence level that the Kamiokande p lies between the limits given by (17), so the probability of $p<0.25$ is 16%. Similarly, the probability of the correct observed level being above the upper limit of (2), 2.4 SNU, is 16%. The combined probability is 2.6%; hence the statements in the preceding paragraph have a confidence limit over 97%.]

On theoretical grounds, we do not believe the vertical leg. As we previously mentioned, we think the standard model of particle theory should apply to the neutrino sector, and in this model $\sin^2\theta$ is unlikely to be greater than 0.1.

To prove this experimentally, the limits of both the systematic and the statistical errors on the KII experiment will have to be reduced. Perhaps this can be done by the SNO experiment [22] where the ν_e and ν_μ from ^{8}B can in principle be separately measured.

Incidentally, our discussion shows that it is $\sin^2\theta$ which is relevant for the regeneration, Eq. (38), not $\sin^2 2\theta/\cos 2\theta$. Likewise, in the nonadiabatic model, Eq. (9), $\sin^2\theta$ occurs, not the other expression. Therefore one should plot $\sin^2\theta$ versus Δm^2.

VII. THE GALLIUM EXPERIMENT

While KII and the Cl experiment have determined the product $\Delta m^2\sin^2\theta$ fairly accurately, determination of Δm^2 itself has to wait for a more accurate result from the Ga experiment. The preliminary result from that experiment, (21), was most welcome because it confirmed (a) that the problem is not in the solar model, and (b) that the nonadiabatic form of the MSW theory is likely to be correct.

The relation between Ga count and Δm^2, as calculated by Baltz and Weneser (BW), is shown in Fig. 1. With the present SAGE result of 68% confidence limits (21), we find

$$10^{-7}<\Delta m^2<3.3\times10^{-6} \ . \tag{42}$$

Since the ν_μ is likely to be much heavier than ν_e, this means

$$0.3<m(\nu_\mu)<1.8\ \mathrm{meV} \ . \tag{43}$$

Looking at the solid lines and their intersection with the hatched band, it would be nice if the experiments could prove that

$$\mathrm{Ga}<20\ \mathrm{SNU} \ . \tag{44}$$

This would still permit a wide range of Δm^2:

$$10^{-7}<\Delta m^2<2\times10^{-6} \ . \tag{44a}$$

The dotted curves give the result when the conversion of neutrinos in the Earth [6] is included. In this case, an experimental result like (44) would show that

$$3\times10^{-7}<\Delta m^2<2\times10^{-6} \ . \tag{45}$$

To show Ga <20 SNU will be a major experimental effort. Because of the limited amount of Ga available [3,4], the counting rate is very low; at present, 1 SNU corresponds to $\frac{1}{3}$ count a year. There is also substantial background. We hope that the limit (44) can be achieved after a few years.

Some help in narrowing the limits (45) could come from the day-night effect [6]. The neutrinos passing through the Earth at night go mostly through the mantle where the density varies from 3.4 to 5.5. Using Eq. (16) for the position of the resonance, setting $Y_e=0.5$ and

$\cos 2\theta = 1$, we find for Be neutrinos, $E = 0.87$, that we must have

$$\Delta m^2 = (2.2 \text{ to } 3.6) \times 10^{-7}\ \text{eV}^2 . \tag{45a}$$

This is one of the most interesting ranges in Fig. 1. Thus Be neutrinos are the prime candidate for a day-night effect. This has also been pointed out by Pakvasa and Pantaleone [11]. (For B neutrinos, the accessible range of Δm^2 is about 10 times higher, which is not very interesting; besides, the effect would be small.)

Fortunately, there is a reasonable chance for an electronic detector of Be neutrinos. This is the Borexino experiment proposed by Raghavan [23] which may operate in the Gran Sasso laboratory around 1995. If this is successful and shows a day-night effect, it would show that Δm^2 lies in the interval (45); if it shows zero effect, it would show that Δm^2 lies outside this interval.

The minimum for Ga found by BW is 7, as compared to our previous prediction [1] of 5. In Ref. [1] we took into account only the ^{8}B neutrinos detected in Ga. The larger numbers obtained by BW for $\sin^2 2\theta > 0.1$ are due to the regeneration of ν_e, Sec. VI. Those for $\sin^2 2\theta < 0.03$ are due to residual *pp* neutrinos which did not go through the resonance at all; Sec. IV.

VIII. INFORMATION ABOUT THE SOLAR INTERIOR

We can conclude from the previous discussion, especially the result given in Eq. (12) of Sec. III, that the flux of ^{8}B neutrinos which is produced in the Sun is not very different from the flux given by the best theoretical estimates. If the value of the theoretical flux were very different from what the Sun is producing, then it would be a somewhat surprising accident that we can come so close to accounting for the absolute rates of two different kinds of solar-neutrino experiments, electron-scattering (Kamiokande II) and radiochemical (chlorine), with a single, attractive MSW solution.

The flux that is actually produced in the Sun cannot be much less than the best estimate of the calculated flux since MSW does not produce new electron-type neutrinos, and the best-estimate rate, assuming no MSW effect, is only about twice the observed rate. Surely the Sun is producing at least as many ^{8}B neutrinos as the minimum counted by Kamiokande II, which can be inferred from Eq. (5) by including the experimental errors. The upper limit results from the requirement of consistency between the comparison of observed and calculated rates for the chlorine and the electron-scattering experiments. Because the chlorine rate is more sensitive to large values of the constant C, defined by Eqs. (8)–(10), than is the Kamiokande II rate, one cannot *increase* the produced flux relative to the calculated flux by very much and still fit the results of both experiments.

Consider the ratio of experimental results

$$R = \frac{\langle \phi\sigma \rangle_{\text{Cl}}}{\langle \phi\sigma \rangle_{\nu\text{-}e}} , \tag{46}$$

which is independent of the absolute value of the ^{8}B flux for the solutions we discuss. Experimentally,

$$R = 0.75^{+0.38}_{-0.23} , \tag{47}$$

where we have used 1σ uncertainties in *opposite* directions for both experiments. Since Cl can record lower-energy neutrinos than KII, it is more sensitive to C, so C can be determined from the ratio R; the lower R, the larger C. Thus the lowest value for R, 0.52, corresponds to the largest allowed value for the flux of ^{8}B neutrinos.

Numerically, we find that

$$0.35 < \frac{\phi_{\text{produced}}(^8\text{B})}{\phi_{\text{calculated}}(^8\text{B})} < 2.0 . \tag{48}$$

The upper limit is slightly larger here than quoted in Ref. [1] because we have used here the 7.5-MeV data of KII for consistency with the discussion in the other parts of this paper.

Although the current limits on the flux of ^{8}B neutrinos span nearly a factor of 6, the limits are significant because they demonstrate that astronomically useful information can be obtained from just two solar-neutrino experiments—even if the MSW effect is operative. Because of the strong dependence of the temperature of calculated ^{8}B neutrino flux on the central temperature of the Sun [5], $\approx T_{\text{central}}^{18}$, the existing limits correspond to about a 6% uncertainty in the central temperature, i.e.,

$$\frac{\Delta T_{\text{central}}}{T_{\text{central}}} = \pm 0.06 . \tag{49}$$

It seems likely that future measurements with the SNO deuterium detector [22] and with the recently funded Super-Kamiokande detector [24] will lead to a determination of the flux of ^{8}B neutrinos that is accurate to 10%, corresponding to a determination of the central temperature of the Sun to an accuracy of better than 1%.

IX. SUPERNOVA NEUTRINOS

In the case of SN 1987A, we observed (mainly) electron antineutrinos. In the MSW theory, these have no resonances [20] with ν_μ or ν_τ, so we can observe just as many $\bar{\nu}_e$ as are produced in the supernova.

If, in the future, from a supernova in our own Galaxy, neutrinos ν_e can also be observed, we must expect similar transformations as in the Sun since typical energies of supernova neutrinos are 10–25 MeV, similar to the neutrinos from ^{8}B. We showed in (25) that the density of the MSW transition is about $\rho = 0.1-1$: So there will not be a transition in that region of the supernova where the neutrinos are produced and where typically [25] $\rho > 10^9$; therefore, the dynamics of the supernova will not be affected. But the transition will occur as the neutrinos travel out of the supernova, just as in the case of the Sun.

But in contrast with the Sun, the supernova produces neutrinos of all flavors. Thus at the resonance, ν_e will be converted into ν_μ, but at the same place the ν_μ produced in the core of the supernova will be converted into ν_e. Theory predicts [25] that the individual ν_μ will have somewhat higher energy than the ν_e but probably somewhat lower total number. This difference from $\bar{\nu}_e$ may be observable in the ν_e which we would detect. (We have here assumed that $\nu_x = \nu_\mu$, which we deduced as being reasonable in Sec. IV.)

What about ν_τ? Presumably, ν_τ has a larger mass than ν_μ; hence, its resonance with ν_e will lie at a higher density. If $m(\nu_\tau)$ is of the order of 1 eV, which is possible in the see-saw model (Sec. XI), then the resonance for neutrinos of about 20 MeV will be at a density $\rho \approx 10^6$. Such densities do not exist in the Sun, but are available in the mantle of a supernova. There the ν_e from the core may change into ν_τ and vice versa. Whether this actually happens depends on the mixing angle $\theta_{e\tau}$. The crucial quantity is C, Eq. (9); it has to be about 20 MeV or greater for the transition to occur. A density of 10^6 is found in an exploding supernova at $R \simeq 10^8$ cm, and R_s, Eq. (10), is of the same order. Therefore

$$\frac{C_{\rm SN}}{C_\odot} = \frac{10^8}{6.6\times 10^9}\,\frac{1(10^{-6}-5\times 10^{-4})}{10^{-8}} = 6-300\ , \tag{50}$$

where we have assumed the mass of the ν_τ to be 1 eV, and have adopted the range of $\theta_{e\tau}$ deduced by Bludman, Kennedy, and Langacker [19]. Since $C_\odot = 10$ MeV, $C_{\rm SN}$ is enough to cause effectively complete transition of a 20-MeV neutrino to ν_τ. But we have no experimental means to tell ν_μ and ν_τ of 20 MeV apart.

Once the ν_e from the supernova have been converted into ν_τ, they will remain in that flavor, they will not be converted into ν_μ because neither the ν_μ nor the ν_τ change their masses by the MSW effect; their mass curves [20] are flat and do not cross.

However, the ν_e which have been made from ν_τ in the high-density resonance (at $\rho = 10^6$ or so) will, on coming to the low-density resonance (ρ about 1) convert into ν_μ. At that same resonance, ν_μ from the core are converted into ν_e.

X. LABORATORY OSCILLATIONS

Many experimenters have tried to observe the oscillation of ν_e into ν_μ, or vice versa, in the laboratory, but no definitive positive results have been found: In light of the results from the solar-neutrino experiments, this is not surprising. For a given energy E, the difference of momentum of two neutrino flavors is given by

$$\Delta p = \Delta m^2/2E\ . \tag{51}$$

As we have seen, the Δm^2 for ν_μ and ν_e is of order 10^{-6} eV^2, and typical experimental energies are of order 100 MeV, so

$$\Delta p \approx 10^{-8}\ \mu\text{eV}\ . \tag{52}$$

(μeV = micro eV). The oscillation length is given by

$$L = 2\pi/\Delta p = 2\pi 19.7\ \text{cm}/[\Delta p\ (\mu\text{eV})] \approx 10^5\ \text{km}\ . \tag{53}$$

Evidently, no oscillation can be observed in the laboratory.

The situation is different for $\nu_\mu - \nu_\tau$ oscillation. Here Δm^2 may be of order 1 eV^2, so even if $E > 2$ GeV so as to permit ν_τ be detected by conversion into τ mesons, we find

$$\Delta p \approx 10^{-4}\ \mu\text{eV} \tag{54}$$

giving an oscillation length

$$L \simeq 10\ \text{km}\ . \tag{55}$$

This is still large but not hopeless for an experiment.

Indeed, Ushida *et al.* [26] and Batusov *et al.* [27] have done two such experiments at Fermi National Accelerator Laboratory, putting the τ-neutrino detector at 1.5 km from the source of μ neutrinos. They got negative results from which they conclude that there is an upper limit on the mass of the τ neutrino of about 2 eV, if $\sin^2\theta_{\mu\tau} > 1\times 10^{-3}$. However, if $\sin^2\theta < 10^{-3}$, m_τ may be anything. Bludman, Kennedy, and Langacker [30] give the likely range as

$$1\times 10^{-3} < \sin^2\theta_{\mu\tau} < 4\times 10^{-3}\ . \tag{56}$$

Thus the Ushida experiment may not restrict the mass of the τ neutrino.

XI. THE SEESAW MECHANISM

Gell-Mann, Ramond, and Slansky [28] and independently Yamagita [29] have proposed that right-handed neutrinos have masses of the order of the mass of grand unified theory (GUT) and that the mass of a left-handed neutrino of flavor i is roughly

$$m(\nu_i) = m^2(q_i)/M(\text{GUT})\ , \tag{57}$$

where q_i is the quark of the same flavor. In the first place, this permits an estimate of the mass of the electron neutrino:

$$\frac{m(\nu_e)}{m(\nu_\mu)} = \frac{m^2(q_{\rm up})}{m^2(q_{\rm charm})} = \left[\frac{4}{1100}\right]^2 = 10^{-5}\ . \tag{58}$$

Similarly,

$$\frac{m(\nu_\tau)}{m(\nu_\mu)} = \frac{m^2(q_{\rm top})}{m^2(q_{\rm charm})} = \left[\frac{200}{1.1}\right]^2 \simeq 3\times 10^4\ . \tag{59}$$

Our estimate (43) from solar neutrinos is that the mass of ν_μ is between 0.3 and 1.8 meV. Then the mass of the electron neutrino should be

$$m(\nu_e) \approx 10^{-8}\ \text{eV}\ . \tag{60}$$

The mass of ν_τ on this basis would be

$$m(\nu_\tau) \simeq 10\ \text{to}\ 60\ \text{eV}\ . \tag{61}$$

The see-saw mechanism has been the subject of many papers. An illuminating recent one is by Bludman, Kennedy, and Langacker [30]. They show that many choices are possible for M(GUT), but that the quadratic relation between the masses of neutrinos and quarks is likely to be true. Then the mass of the τ neutrino would fall in the range (61). Bludman *et al.* [30] then show that a mass in this range may close the Universe. This opens the exciting possibility [30] that the τ neutrino may be the dark matter which closes the Universe.

ACKNOWLEDGMENTS

This work was supported in part by the NSF via Grants No. PHY-86-20266 at the Institute for Advanced Study and No. PHY-87-15272 at Cornell. We are much indebted to Sidney Bludman for illuminating discussions, and for making Refs. [19] and [30] available to us.

[1] J. N. Bahcall and H. A. Bethe, Phys. Rev. Lett. **65**, 2233 (1990).

[2] S. P. Mikheyev and A. Yu Smirnov, Yad. Fiz. **42**, 1441 (1986) [Sov. J. Nucl. Phys. **42**, 913 (1986)]; L. Wolfenstein, Phys. Rev. D **17**, 2369 (1979).

[3] I. R. Barabanov *et al.*, in *Solar Neutrinos and Neutrino Astronomy,* Proceedings of the Conference, Lead, South Dakota, 1984, edited by M. L. Cherry, W. A. Fowler, and K. Lande, AIP Conf. Proc. No. 126 (AIP, New York, 1985), p. 175.

[4] T. Kirsten, in *1986 Massive Neutrinos in Astrophysics and Particle Physics,* Proceedings of the Sixth Moriond Workshop, Tignes, France, 1986, edited by O. Fackler and J. Tran Thanh Van (Editions Frontieres, Gif-sur-Yvette, 1986), p. 119.

[5] J. N. Bahcall and R. K. Ulrich, Rev. Mod. Phys. **60**, 297 (1988).

[6] A. J. Baltz and J. Weneser, Phys. Rev. Lett. **66**, 520 (1991).

[7] J. N. Bahcall and W. C. Haxton, Phys. Rev. D **40**, 931 (1989).

[8] K. S. Hirata *et al.*, Phys. Rev. Lett. **65**, 1297 (1990).

[9] R. Davis, Jr., in *Proceedings of the Seventh Workshop on Grand Unification,* ICOBAN '86, Toyama, Japan, 1986, edited by J. Arafune (World Scientific, Singapore, 1987), p. 237.

[10] S. P. Rosen and J. M. Gelb, Phys. Rev. D **39**, 3190 (1989).

[11] S. Pakvasa and J. Pantaleone, Phys. Rev. Lett. **65**, 2479 (1990); T. K. Kuo and J. Pantaleone, Rev. Mod. Phys. **61**, 937 (1989).

[12] S. P. Rosen and J. M. Gelb, Phys. Rev. D **34**, 969 (1986).

[13] E. W. Kolb, M. S. Turner, and T. P. Walker, Phys. Lett. B **175**, 478 (1986).

[14] W. C. Haxton, Phys. Rev. Lett. **57**, 1271 (1986); S. J. Parke, *ibid.* **57**, 1275 (1986).

[15] P. Pizzochero, Phys. Rev. D **36**, 2293 (1982).

[16] A. J. Baltz and J. Weneser, Phys. Rev. D **35**, 528 (1987); M. Gribier, W. Hampel, J. Rich, and D. Vignaud, Phys. Lett. B **182**, 89 (1986).

[17] J. N. Bahcall, *Neutrino Astrophysics* (Cambridge University Press, Cambridge, England, 1989), Eq. (9.53).

[18] M. Kobayashi and M. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).

[19] S. A. Bludman, D. C. Kennedy, and P. G. Langacker, Center for Particle Astrophysics, Berkeley Report No. CfPA-Th-91-003 Rev (unpublished).

[20] H. A. Bethe, Phys. Rev. Lett. **56**, 1305 (1986).

[21] Hirata *et al.* [8].

[22] G. Aardsma *et al.*, Phys. Lett. B **194**, 321 (1987).

[23] R. S. Raghavan *et al.*, AT&T Bell Laboratories Technical Memorandum 11121-910204-16 (unpublished).

[24] Y. Totsuko, in *Proceedings of the Seventh Workshop on Grand Unification* [9], p. 118.

[25] H. A. Bethe, Rev. Mod. Phys. **62**, 801 (1990).

[26] N. Ushida *et al.*, Phys. Rev. Lett. **57**, 2897 (1986).

[27] Yu. Batusov *et al.*, Z. Phys. C **48**, 209 (1990).

[28] M. Gell-Mann, P. Ramond, and R. Slansky, in *Proceedings of the Supergravity Workshop,* Stony Brook, New York, 1979, edited by P. van Nieuwenhuizen and D. Z. Freedman (North-Holland, Amsterdam, 1979), p. 315.

[29] T. Yanagida, Prog. Theor. Phys. **64**, 1103 (1980).

[30] S. A. Bludman, D. C. Kennedy, and P. G. Langacker, Center for Particle Astrophysics/Berkeley Report No. CfPA-TH-91-002 (unpublished).