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$B \to \pi\nu\bar{\nu}$, $\rho\nu\bar{\nu}$ and determination of $|V_{td}/V_{ub}|$

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The rare decays $B \to \pi\nu\bar{\nu}$ and $B \to \rho\nu\bar{\nu}$ are investigated. The ratio of branching fractions $B(B \to M\nu\bar{\nu})/B(B \to Ml\bar{\nu})$ is shown to depend on the ratio $|V_{td}/V_{ub}|$ of Kobayashi-Maskawa matrix elements with little hadronic uncertainty. The measurement of these fractions thus gives a robust constraint on the Kobayashi-Maskawa matrix. The branching fractions for $B \to \pi\nu\bar{\nu}$, $\rho\nu\bar{\nu}$ are estimated to range from 10^{-8} to 10^{-6}; an experimental method is presented which yields a single-event sensitivity below 10^{-4}.

The Kobayashi-Maskawa (KM) matrix [1] parametrizes the degree to which electroweak eigenstates mix to form mass eigenstates. The assumption of three and only three generations imposes a unitarity constraint upon the matrix which reduces the number of independent parameters to four. Thus, independently measuring the various elements serves as a powerful test of the validity of the three-generation standard model. This Brief Report discusses the KM element $|V_{td}|$, about which little is known at present. Information about $|V_{td}|$ can be extracted in four different ways: (1) Measuring KM-suppressed semileptonic branching ratios of the heavy top quark; (2) measuring the rate of rare K decays which proceed through a quark loop, the loop being sensitive to a heavy top; (3) measuring the parameter $x = \Delta m/\Gamma$ which governs the rate of B-$\bar{B}$ mixing; (4) measuring the rate of rare B decays which proceed through a top-dominated quark loop.

The first method may prove unfeasible as the large mass of top [2] ($m_t > 89$ GeV/c^2) is expected to cause it to decay to a jet rather than a simple M-l-ν final state. The second method is pursued in BNL experiment 787 [3] which measures $K^+ \to \pi^+\nu\bar{\nu}$, and BNL experiment 791 [4] and KEK experiment 137 [5] which measure $K_L \to \mu^+\mu^-$. The former decay proceeds via the short-distance diagrams of Fig. 1 while the latter decay proceeds through essentially identical diagrams plus the addition of a long-range 2-γ contribution. Both processes are sensitive to the product $|V_{ts}^* V_{td}|$ and in the absence of information about V_{ts} must invoke KM unitarity to extract information about $|V_{td}|$. In addition, these processes undergo significant QCD correction [6] and the dispersive 2-γ contribution to $K_L \to \mu^+\mu^-$ has not yet been reliably calculated.

The third method listed above is promising, as the mixing parameter x is proportional to $|V_{tb}^* V_{td}|^2 \approx |V_{td}|^2$. However, x also depends on the top mass and the combination $f_B^2 B_B$ of B-meson decay constant and B parameter . While m_t will presumably be measured one day, f_B and B_B have relatively large uncertainties [7] arising from hadronic effects which may not be so easily resolved.

The fourth method is analogous to $K \to \pi\nu\bar{\nu}$ and $K \to \mu^+\mu^-$ except the heavier B meson is investigated: $B \to \pi\nu\bar{\nu}$, $\rho\nu\bar{\nu}$, πl^+l^-, ρl^+l^-, $\mu^+\mu^-$, and $B \to \tau^+\tau^-$. As in $K^+ \to \pi^+\nu\bar{\nu}$, $B \to \pi\nu\bar{\nu}$ and $B \to \rho\nu\bar{\nu}$ are essentially free of long-distance contributions (these are suppressed by one or more factors of G_F in the amplitude). The b decay has an advantage over its s counterpart in that the rate is proportional to $|V_{tb}^* V_{td}| \approx |V_{td}|$ as in the case of B-$\bar{B}$ mixing. The theoretical uncertainty arises from form factors, though, rather than a decay constant. The form factors

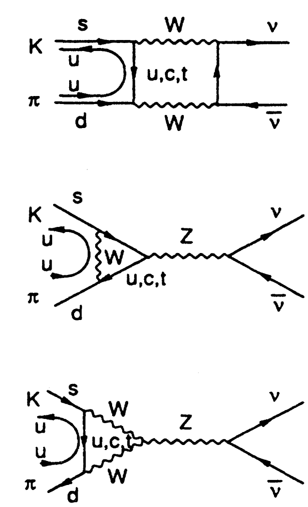

FIG. 1. Short-distance contributions to $K^+ \to \pi^+\nu\bar{\nu}$.

in this case are the same as those entering the semileptonic decays $B\to\pi l\bar{\nu}$ and $B\to\rho l\bar{\nu}$ and all dependence on these factors drops out in the ratio $B(B\to M\nu\bar{\nu})/B(B\to Ml\bar{\nu})$. The ratio then depends only on $|V_{td}/V_{ub}|$ and well-known parameters. The relationship is unique to transitions from b to the first quark generation: factoring out form factors for $B\to K\nu\bar{\nu}$ [8] requires invoking SU(3) symmetry, which is only approximate.

The effective Lagrangian for $b\to d\nu\bar{\nu}$ for each neutrino type is similar to that for $s\to d\nu\bar{\nu}$ given in Ref. [9]:

$$\mathcal{L}(b\to d\nu\bar{\nu})=\frac{G_F}{\sqrt{2}}\left[\frac{\alpha}{4\pi\sin^2\theta_W}\right]\times\sum_j(V_{jb}^*V_{jd})A_j\,\bar{d}L_\mu b\,\bar{\nu}L^\mu\nu\ , \tag{1}$$

where $L_\mu=\gamma_\mu(1-\gamma_5)$ and

$$A_j=C_j^{\text{box}}+C_j^Z\ ,$$

$$C_j^{\text{box}}=\frac{2x_j}{x_j-1}\left[1-\frac{\ln x_j}{x_j-1}\right],$$

$$C_j^Z=\frac{x_j}{4(x_j-1)}\left[x_j-6+\frac{3x_j+2}{x_j-1}\ln x_j\right],$$

$$x_j=\left[\frac{m_j}{m_W}\right]^2 .$$

The hadronic matrix element which enters the decay width for $B\to\pi\nu\bar{\nu}$ is

$$\langle\pi(k)|\bar{d}L_\mu b|B(p)\rangle=f^+(q^2)(p+k)_\mu$$

since the orthogonal term $f^-(q^2)q_\mu$ vanishes when contracted with $J^\mu_{\bar{\nu}\nu}$ due to current conservation ($m_\nu\approx0$). The diagrams are identical to those of Fig. 1 with s and K replaced by b and B, respectively.

The Lagrangian for the spectator decay $b\to ul\bar{\nu}$ is

$$\mathcal{L}(b\to ul\bar{\nu})=\frac{G_F}{\sqrt{2}}V_{ub}\,\bar{u}L_\mu b\,\bar{\nu}L^\mu l\ . \tag{2}$$

In this case $J^\mu_{l\bar{\nu}}$ is not conserved and $\langle\pi(k)|\bar{d}L_\mu b|B(p)\rangle$ has both $f^+(q^2)(p+k)_\mu$ and $f^-(q^2)q_\mu$ terms. However, the contraction of the second term with $J^\mu_{l\bar{\nu}}$ is proportional to m_l/m_b which for $l=e,\mu$ can be neglected. The hadronic matrix elements for $b\to d\nu\bar{\nu}$ and $b\to ul\bar{\nu}$ then depend on the same form factor, where isospin symmetry is assumed. The branching ratio for $B\to\pi\nu\bar{\nu}$ is then expressed in terms of $B(B\to\pi l\bar{\nu})$:

$$\begin{aligned}B(B\to\pi\nu\bar{\nu})&=3B(B\to\pi l\bar{\nu})\frac{\alpha^2}{8\pi^2\sin^4\theta_W}\times\frac{\left|\sum_j V_{jb}^*V_{jd}C_j\right|^2}{|V_{ub}|^2}\\&\simeq3B(B\to\pi l\bar{\nu})(1.276\times10^{-5})\times\frac{|V_{cb}^*V_{cd}C_c+V_{td}C_t|^2}{|V_{ub}|^2}\ ,\end{aligned} \tag{3}$$

where

$$C_q=C_q^{\text{box}}+C_q^Z=\frac{x_q}{4}\left[\frac{3(x_q-2)}{(x_q-1)^2}\ln x_q+\frac{x_q+2}{x_q-1}\right],$$

and a factor of 3 is included on the right-hand side to account for all known neutrino types. The kinematic factor C_q is numerically negligible for m_u, approximately 0.004 for $m_c=1.5$ GeV/c^2, and in the range 1.5–3.3 for m_t between 90 and 180 GeV/c^2. The relative contribution of charm to top is less than 0.8% and QCD corrections to the charm contribution can be neglected. QCD corrections to the top contribution can also be neglected due to large top mass. Equation (3) thus has essentially no theoretical uncertainty.

To estimate the size of $B(B\to\pi\nu\bar{\nu})$ we use the independently determined quantities $|V_{cb}|$ (from the B semileptonic decay rate), $|V_{ub}/V_{cb}|^2$ (from the end point of the semileptonic momentum spectrum), and $|V_{cd}|$ (from the charm production rate in deep-inelastic neutrino scattering). Substituting in the measured values $|V_{cb}|=0.040\pm0.006\pm0.006$ [10], $|V_{ub}/V_{cb}|^2\simeq0.015\pm0.006$ [11], $|V_{cd}|=0.204\pm0.017$ [12], and taking $B(B\to\pi l\bar{\nu})\approx12\times|V_{ub}|^2$ from a recent theoretical estimate [13], one finds, for $m_t=180$ GeV/c^2,

$$B(B\to\pi\nu\bar{\nu})\simeq(5.0\times10^{-3})|V_{td}|^2\ . \tag{4}$$

For $|V_{td}|$ ranging over the Particle Data Group values 0.003–0.019 [12], $B(B\to\pi\nu\bar{\nu})$ ranges from 4.5×10^{-8} to 1.8×10^{-6}.

The decay $B\to\rho\nu\bar{\nu}$ is favored with respect to $B\to\pi\nu\bar{\nu}$ because of the three possible spin states of the final-state meson. The hadronic matrix element is expressed in terms of four different form factors:

$$\begin{aligned}\langle\rho(k)|\bar{d}L_\mu b|B(p)\rangle=&\ A_1(q^2)(m_B^2-m_\rho^2)\epsilon_\mu(k)\\&-A_2(q^2)(\epsilon\cdot q)(p+k)_\mu\\&-A_3(q^2)(\epsilon\cdot q)q_\mu\\&+V(q^2)i\epsilon_{\mu\nu\lambda\sigma}\epsilon^\nu(k)(p+k)^\lambda(p-k)^\sigma\end{aligned}$$

where $q=p-k$. For $B\to\rho\nu\bar{\nu}$ these terms contract with $\bar{\nu}L^\mu\nu$ while for $B\to\rho l\bar{\nu}$ they contract with $\bar{\nu}L^\mu l$. Since both lepton currents have the same chirality and m_l can be neglected, the two different contractions result in the same linear combination of form factors. As before, A_3 does not contribute due to current conservation. The branching ratio for $B\to\rho\nu\bar{\nu}$ is then expressed in terms of $B(B\to\rho l\bar{\nu})$ with little uncertainty as was done before for $B\to\pi\nu\bar{\nu}$. The momentum spectrum of the ρ in $\rho\nu\bar{\nu}$ and $\rho l\bar{\nu}$ decays are in fact identical; the processes differ only in their overall normalization. In analogy with expression (3) one obtains, for $m_t=180$ GeV/c^2,

$$B(B\to\rho\nu\bar{\nu})=(4.16\times10^{-4})B(B\to\rho l\bar{\nu})|V_{td}/V_{ub}|^2\ , \tag{5}$$

where the charm contribution is neglected. Taking $|V_{ub}|=|V_{cb}|\times|V_{ub}/V_{cb}|$ and $B(B\to\rho l\bar{\nu})=(0.56\pm0.29)\times10^{-3}$ from a preliminary measurement at CLEO [14], one finds $B(B\to\rho\nu\bar{\nu})$ in the range 8.8×10^{-8} to 3.5×10^{-6}.

The presence of a fourth generation is expected to influence $B(B\to\pi\nu\bar{\nu})$ and $B(B\to\rho\nu\bar{\nu})$ via the additional heavy quark in the internal loop. Such a generation may have a heavy neutrino (e.g., $m_\nu > m_Z/2$) and thus not be constrained by recent measurements of Γ_Z [15]. The contribution is proportional to $|V^*_{t'b}V_{t'd}|C_{t'}$, where $V_{t'b}$ may be on the order of $\sin\theta_C$ in analogy with V_{cd}, $V_{t'd}$ may be very small since it is three generations away from the diagonal, and $C_{t'}$ may be very large if the t' quark is heavy. Figure 2 shows $B(B\to\rho\nu\bar{\nu})$ plotted vs $|V_{t'}|\equiv|V^*_{t'b}V_{t'd}|$ for different values of $m_{t'}$. Unitarity demands that $|V_{t'}|<0.3$ and that $V_{t'}+V^*_{tb}V_{td}+V^*_{cb}V_{cd}+V^*_{ub}V_{ud}=0$. The KM matrix element V_{td} is thus taken to be $0.01-V_{t'}$ such that unitarity is preserved and the average Particle Data Group value of $|V_{td}|$ is recovered in the limit of no fourth-generation mixing. The top mass is taken to be 120 GeV/c^2.

The figure shows that for $m_{t'}=500$ GeV/c^2, $B(B\to\rho\nu\bar{\nu})$ can rise by three orders of magnitude to the level of 10^{-4}. If one includes the constraint on $V_{t'}$ resulting from B-$\bar{B}$ mixing which we find to be $|V_{t'}|<0.01$, the branching ratio can still rise by more than an order of magnitude for heavy t'. The sharp drops shown represent a type of Glashow-Iliopoulos-Maiani (GIM) suppression where the contributions from t and t' identically cancel each other. If $m_{t'}=m_t$ a similar mechanism causes the branching ratio to become independent of $V_{t'}$ and equal to the three-generation value. One concludes that with $\sim 10^4$ fully reconstructed B decays an *upper* limit on the mass of a fourth-generation quark can be found as a function of mixing angle. Such a constraint cannot be found from charged semileptonic decays such as $B\to\rho l\bar{\nu}$, $Dl\bar{\nu}$ because there is no quark loop in the lowest-order diagrams.

The measurement of rare B decay modes is currently at the level of 10^{-4} [16]. To push this down to 10^{-6}–10^{-7} requires a large increase in luminosity such as that provided by a B factory, and good background suppression. The decays $B\to\rho\nu\bar{\nu}$ and $B\to\pi\nu\bar{\nu}$ are especially difficult to detect because the B of interest cannot be reconstructed. Instead one must rely on the opposite-side B being reconstructed and then looking on the side of interest for a ρ+missing energy. This method is not particularly efficient as only about 1% of all B decays are fully reconstructed due to the large track multiplicities and combinatorics. Nonetheless it is estimated that CLEO-II will collect on the order of $(3\text{–}9)\times10^4$ fully reconstructed B decays over the next few years [17], and a B factory can be expected to yield an order of magnitude more than this. Thus, using such a sample should yield a sensitivity to $\rho\nu\bar{\nu}$ in the range 10^{-5} to 10^{-6}.

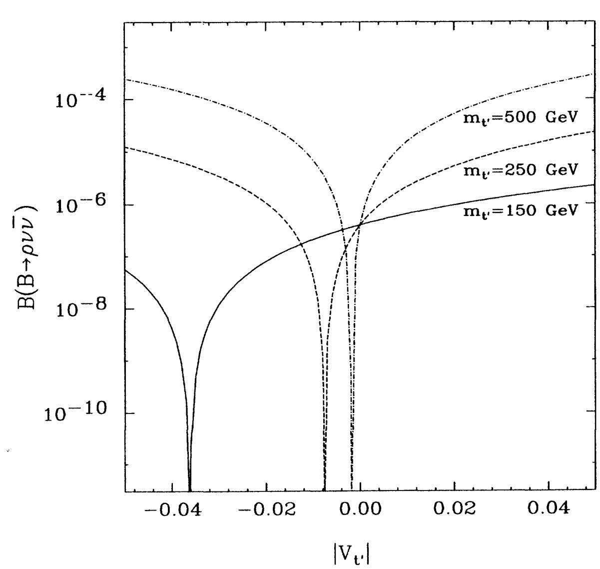

FIG. 2. $B(B\to\rho\nu\bar{\nu})$ plotted vs $|V_{t'}|\equiv|V^*_{t'b}V_{t'd}|$ for different values of $m_{t'}$. The top mass is taken to be 120 GeV/c^2.

The detection efficiency for $\rho\nu\bar{\nu}$ or $\pi\nu\bar{\nu}$ is improved by relaxing the requirement of full reconstruction of the opposite-side B and requiring only momentum and energy conservation. At an e^+e^- machine running at the $\Upsilon(4S)$ resonance which decays to $B\bar{B}$, one requires that (1) the scalar sum of all energy in an event except for that of the ρ or π adds up to the beam energy, and (2) the vector sum of all momenta in an event except for that of the ρ or π adds up to $\simeq 0$. While more efficient, this selection yields more background. We have investigated what backgrounds are likely to be encountered and what background rejection is possible by doing a Monte Carlo study using the standard CLEO $\Upsilon(4S)\to B\bar{B}$ event-generator program. The B's are decays into about two dozen channels with appropriate (where possible, measured) branching fractions. To find the efficiency of various cuts we also generate a $B\bar{B}$ sample where one B is decayed to $\rho\nu\bar{\nu}$ using appropriate form factors, in this case those of Isgur *et al.* [18]. Details of the study can be found in Ref. [19]. For simplicity we require only one ρ in an event and we do not simulate detector response: neutrons, neutrinos, and K_L's are considered invisible while everything else is considered well measured. Requiring that the amount of energy in an event excluding that of the ρ adds up to within ±50 MeV of the beam energy and that the total momentum excluding that of the ρ vectorially adds up to less than 500 MeV/c preserves 12% of the signal but only 0.007% of the background (21 out of 300 000 events). The ρ momentum spectrum of signal and background samples both before and after the cuts are shown in Fig. 3. The energy and momentum windows were chosen to retain reasonable efficiency for $\rho\nu\bar{\nu}$ events and are consistent with the measured energy and momentum resolutions of the CLEO detector [16].

Examining the surviving background events reveals that almost half of them have ρ's originating from two-body decays such as $B\to D^*\rho$ or $D^0\to K^-\rho^+$. The two-body B decays are eliminated by a cut on ρ momentum (excluding the window 2.0–2.4 GeV/c retains 90% of the signal) while most of the D decays are cut by pairing the ρ with a $K^\pm$ or K_S to reconstruct the D invariant mass. The background is reduced further by exploiting the fact that background events have larger multiplicities (tracks $+\gamma$'s) than $\rho\nu\bar{\nu}$ events; in the latter case only one of the B's produces any detectable energy after the ρ is excluded. We find that requiring that the multiplicity be less

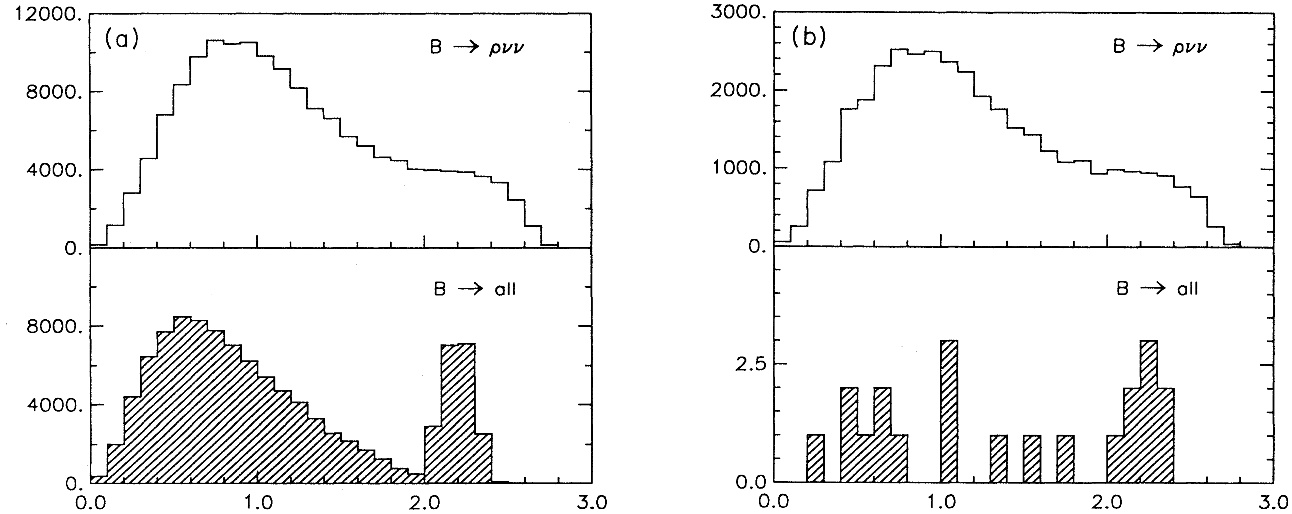

FIG. 3. The ρ momentum spectrum of signal and background samples (a) before energy and momentum conservation cuts, and (b) after cuts. The excess of background events between 2.0 and 2.4 GeV/c is due mostly to $B \to D^* \rho$ decays.

than 14 eliminates almost 75% of remaining background while retaining 65% of remaining signal. The overall background rejection is 1×10^{-5} with a $\rho\nu\bar{\nu}$ efficiency of 7.2%; the single-event sensitivity of the experiment is then $(10^{-5}/0.072)\times\frac{1}{2}=6.9\times10^{-5}$ where a factor of $\frac{1}{2}$ is included to account for the fact that *either* B produced from $\Upsilon(4S)$ decay can branch to $\rho\nu\bar{\nu}$. This sensitivity is equivalent to having 14 400 fully reconstructed events.

These results are to a large extent qualitative as the B branching fractions used in the Monte Carlo simulation are only approximately known. Within this approximation, however, using reasonable branching fractions, form factors, and selection cuts, it appears possible to reject background at the level of 1 part in 10^5 while preserving a detectable amount of signal. Hopefully more insight into whether this is realistic and how one can improve upon it will be gained over the next few years as large samples of reconstructed B's are collected. We have considered only the case of a *symmetric* e^+e^- collider; additional background rejection may be possible at an asymmetric machine by requiring that the two tracks comprising the ρ originate from an otherwise isolated vertex.

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