Low-energy neutrino–two-photon interactions

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We present a systematic study of low-energy neutrino–two-photon interactions to lowest order of $O(\alpha_{\rm em}G_F)$. General structures of such interactions are analyzed. Differences due to whether neutrinos are Dirac particles or Majorana particles are discussed. Simple formulas are derived from which one can easily obtain the corresponding effective neutrino–two-photon interaction in all theoretical models. The compatibility of a small neutrino mass with a large neutrino–two-photon interaction is explored. Some physical implications of such interactions are also examined.

I. INTRODUCTION

Low-energy neutrino–two-photon interactions may potentially be of interest for a number of processes in astrophysics and cosmology. A well-known example would be the neutrino pair production via $\gamma\gamma\to\nu\bar{\nu}$. As this is a way for stars to loss energy, it could therefore be important in the study of stellar evolution [1]. Other closely related processes, such as the neutrino Compton scattering [2] $\nu\gamma\to\nu\gamma$, the two-photon production [3] $\bar{\nu}\nu\to\gamma\gamma$, and neutrino double-radiative decays [4] $\nu'\to\nu\gamma\gamma$, may also have interesting astrophysical and cosmological implications.

Also, some interests [5] have recently been aroused for a similar process: $\chi\bar{\chi}\to\gamma\gamma$, where χ, like a massive neutrino, is a neutral fermion that serves as a dark-matter candidate. It was suggested [6] that the detection of the two photons would be an interesting way of detecting dark matter in the galactic halo. Some of these ideas may soon be tested experimentally by the GRO (gamma-ray observatory) launched very recently.

In this article, we wish to study the general structures of the interactions that give rise to the aforementioned physical processes. While some efforts have already been made in this subject, discussions have so far largely been scattered over places where the results of analysis often turn out to be purely academic. From the point of view of particle physics, it is desirable to have a systematic study of neutrino–two-photon interactions, just as what we customarily do to the neutrino mass and mixing and to the neutrino magnetic moment [7]. Our purpose here is clear. We want to know if and where a relatively large result can be obtained, and how it will depend on the introduction of new physics.

In this paper, we will therefore mainly concentrate on issues related to particle-physics theory, while some of their physical consequences will also be explored. We will derive some simple formulas for the effective neutrino–two-photon interactions. As shown below, these formulas can be very easily employed in all theoretical models. We will provide some examples to demonstrate how this can be done. Our results are summarized at the end.

II. EFFECTIVE INTERACTIONS FOR DIRAC NEUTRINOS

How neutrinos interact with two real (on-shell) photons? If a neutrino ν is a Dirac particle, then the simplest answer which one would think of is

$$L_{\rm eff}=a\bar{\nu}\nu F^{\alpha\beta}F_{\alpha\beta}+ia'\bar{\nu}\gamma_5\nu\tilde{F}^{\alpha\beta}F_{\alpha\beta}\ , \tag{2.1a}$$

where $F^{\alpha\beta}$ is the electromagnetic field tensor with its dual give by $\tilde{F}^{\alpha\beta}=\frac{1}{2}\epsilon^{\alpha\beta\rho\lambda}F_{\rho\lambda}$, and ν is a neutrino field operator:

$$\nu(\mathbf{x},t)=\sum_{\alpha=\pm1/2}\int\frac{d^3\mathbf{k}}{(2\pi)^3}\frac{m_\nu}{k_0}[b_\alpha(k)u(k,\alpha)e^{-ik\cdot x}+d_\alpha^\dagger v(k,\alpha)e^{ik\cdot x}]\ . \tag{2.1b}$$

The coefficients a and a' are form factors which are invariant functions of kinematical variables, and they are real from Hermiticity [8]. Here, and throughout this paper, CP invariance is assumed for simplicity.

If there are more than one neutrino, then Eq. (2.1a) can be generalized as

$$L_{\rm eff}=\bar{\nu}_r(a_{rs}+b_{rs}\gamma_5)\nu_sF^{\alpha\beta}F_{\alpha\beta}+i\bar{\nu}_r(a'_{rs}\gamma_5+b'_{rs})\nu_s\tilde{F}^{\alpha\beta}F_{\alpha\beta}\ , \tag{2.2}$$

where r and s are flavor indices. CP invariance and Hermiticity now imply that

$$a_{rs}=a_{sr},\quad a'_{rs}=a'_{sr},\quad b_{rs}=-b_{sr},\quad b'_{rs}=-b'_{sr} \tag{2.3}$$

and all matrices are real. Evidently, terms proportional to b and b' violate parity P and charge conjugation C but remain CP invariant. Others are P, C, and hence CP invariant.

Suppose Eq. (2.2) is the only means by which neutrinos interact with the two photons; in a theory where neutrinos are massless due to a chiral symmetry such local interactions must vanish: $a_{rs},a'_{rs},b_{rs},b'_{rs}=0$. It should be pointed out, however, that one may in principle introduce massless neutrinos by invoking symmetries other than the chiral symmetry. In that case, massless neutrinos may still be able to interact locally with the two photons. The only possibilities would be, of course, like $a'_{rs}\bar{\nu}_r i\gamma_5\nu_s\tilde{F}^{\alpha\beta}F_{\alpha\beta}$, which can be either diagonal or off diagonal, or $ib'_{rs}\bar{\nu}_r\nu_s\tilde{F}^{\alpha\beta}F_{\alpha\beta}$, which can only be off diagonal because b'_{rs} is antisymmetric.

In this article, we are only interested in low-energy neutrino–two-photon interactions. By low energy we mean that energies associated with a specific interaction are much smaller than the W-boson mass M_W. For practical purposes, we are often interested in processes where the center-of-mass energy of a scattering, or the momentum transfer of a decay, is smaller than the e^+e^- threshold. In any case, one can show that, to the lowest order of $O(\alpha_{\rm em}G_F)$, where $\alpha_{\rm em}=e^2/4\pi$ and G_F is the Fermi coupling constant, our innocent guess of Eq. (2.2) actually exhausts all possibilities.

To show that to lowest-order weak interactions Eq. (2.2) indeed covers all possibilities, we consider low-energy neutrino interactions given by an effective four-fermion interaction Lagrangian, where the Lorentz-invariant term of interest is

$$[\bar{\nu}_r\Gamma_j(C^j_{rs}+iC'^j_{rs}\gamma_5)\nu_s][\bar{l}_{r'}\Gamma_j(D^j_{r's'}+iD'^j_{r's'}\gamma_5)l_{s'}]\ , \tag{2.4}$$

where

$$\Gamma_j=1,\gamma_\alpha,\sigma_{\alpha\beta},\gamma_\alpha\gamma_5,\gamma_5\ . \tag{2.5}$$

The coupling matrices C, C', D, and D' are specified by theoretical models. As these operators have a dimension of 6, their couplings are of the order $O(1/M_W^2)$, assuming weak interaction scales are of the order of or higher than M_W. In Eq. (2.4), l is a charged fermion (a charged lepton or a light quark) with its mass $m_l \ll M_W$. Others can always be arranged in such forms by a Fierz transformation. Interactions involving a heavy quark is negligible. This is because the scalar loop integral (see below) turns out to be inversely proportional to the squared mass of such a heavy quark.

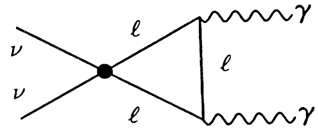

FIG. 1. An effective one-loop graph for a low-energy neutrino–two-phonon interaction. Depending on the theoretical model, the four-fermion vertex could either be a scalar, a pseudoscalar, or an axial-vector type.

Now, we must calculate the matrix element of $\bar{l}\Gamma_j l$ between the vacuum and a state of two photons. Matrices involve $\Gamma_j=\gamma_\alpha$ and $\Gamma_j=\sigma_{\alpha\beta}$, $\sigma_{\alpha\beta}\gamma_5$ vanish by charge-conjugation invariance analogous to the theorems of Furry [9] and Yang [10]. The remaining terms are not zero, and they can be calculated from a triangle Feynman diagram (Fig. 1) which contains a scalar, a pseudoscalar, and an axial-vector vertex, respectively.

A. Axial-vector-current contributions

For the axial-vector triangle diagram, the four-fermion interaction can be written as

$$[\bar{\nu}_r(p_r)\gamma_\alpha\gamma_5(A_{l,rs}+A'_{l,rs}\gamma_5)\nu_s(p_s)(\bar{l}\gamma^\alpha\gamma_5 l)\ , \tag{2.6}$$

where CP invariance and Hermiticity require that the matrices $A_{l,rs}$, $A'_{l,rs}$ are real and symmetric:

$$A_{l,rs}=A_{l,sr},\quad A'_{l,rs}=A'_{l,sr}\ , \tag{2.7}$$

where, and henceforth, l runs over all the participating charged particles. One finds [11]

$$\begin{aligned}J_{5,\alpha}&\equiv\langle 0|\bar{l}\gamma_\alpha\gamma_5 l|\gamma(k_1)\gamma(k_2)\rangle\\&=-\frac{2i}{\pi}\alpha_{\rm em}Q_l^2 f_l\epsilon_1^{\mu_1}\epsilon_2^{\mu_2}[(k_{1,\mu_2}\epsilon_{\mu_1\nu_1\nu_2\alpha}-k_{2,\nu_1}\epsilon_{\mu_2\nu_1\nu_2\alpha})k_1^{\nu_1}k_2^{\nu_2}+(k_1\cdot k_2)\epsilon_{\mu_1\mu_2\nu\alpha}(k_1-k_2)^\nu]\ ,\end{aligned} \tag{2.8}$$

where $\epsilon_1^{\mu_1}$ and $\epsilon_2^{\mu_2}$ are the photon polarizations, and eQ_l is the charge carried by l. k_1 and k_2 are the momenta carried by the two photons. The result should be multiplied by a color factor 3 if l is a quark. The scalar integral f_l is given by

$$f_l=\int_0^1 dx\int_0^{1-x}dy\frac{xy}{m_l^2-2xyk_1\cdot k_2-i\epsilon}=-\frac{1}{4k_1\cdot k_2}+\frac{m_l^2}{4(k_1\cdot k_2)^2}I_l\ , \tag{2.9}$$

with

$$\begin{aligned}I_l&=2k_1\cdot k_2\int_0^1 dx\int_0^{1-x}dy\frac{1}{m_l^2-2xyk_1\cdot k_2-i\epsilon}\\&=\begin{cases}2(\arcsin\sqrt{k_1\cdot k_2/2m_l^2})^2 & \text{if } k_1\cdot k_2<2m_l^2,\\ \dfrac{\pi^2}{2}-\dfrac{1}{2}\ln^2\dfrac{1+\sqrt{1-2m_l^2/k_1\cdot k_2}}{1-\sqrt{1-2m_l^2/k_1\cdot k_2}}+i\pi\ln\dfrac{1+\sqrt{1-2m_l^2/k_1\cdot k_2}}{1-\sqrt{1-2m_l^2/k_1\cdot k_2}} & \text{if } k_1\cdot k_2\ge 2m_l^2\ .\end{cases}\end{aligned} \tag{2.10}$$

Equation (2.8) can be simply obtained by extracting out the gauge-invariant part of the result of the effective triangle diagram by imposing on-shell Ward identities (Ref. [11]). Alternatively, it can be obtained from an explicit model calculation [12,13].

The scalar integrals f_l and I_l have a notable feature that their integrands become most singular and hence their contributions become significant when $m_l^2 \sim 2k_1 \cdot k_2$. For low-energy neutrino–two-photon interactions, i.e., $2k_1 \cdot k_2 \ll M_W^2$, this implies that only those diagrams which have an internal light particle ($m_l \ll M_W$) can be relatively important. This is the reason why we have discarded operators involving a heavy quark in Eq. (2.4).

One can simplify the result by rewriting

$$J_{5,\alpha} = -\frac{2i}{\pi}\alpha_{\rm em}Q_l^2 f_l (k_1+k_2)_\sigma (\tilde{F}_{1,\alpha\lambda}F_2^{\sigma\lambda} + \tilde{F}_{2,\alpha\lambda}F_1^{\sigma\lambda})$$
$$= \frac{i}{2\pi}\alpha_{\rm em}Q_l^2 f_l (k_1+k_2)_\alpha (\tilde{F}_{1,\rho\lambda}F_2^{\rho\lambda} + \tilde{F}_{2,\rho\lambda}F_2^{\rho\lambda}) , \tag{2.11}$$

where the last step follows from the identity

$$\tilde{F}_{\alpha\beta}F^{\sigma\beta} = -\tfrac{1}{4}\delta_\alpha^\sigma \tilde{F}_{\rho\lambda}F^{\rho\lambda} , \tag{2.12}$$

where $\delta_\alpha^\sigma = g^{\sigma\beta}g_{\beta\alpha}$. After contracting with the γ^α matrix, the momentum transfer $(k_1+k_2)_\alpha = (p_r+p_s)_\alpha$ acts on the neutrino field generating a neutrino mass m_ν via the equation of motion. Its sign is determined by the direction of momentum flow with respect to the effective local interaction vertex. The final result is of the form given by the second term of Eq. (2.2). Thus, in a theory, such as the standard model where the interaction currents are chiral, a massless neutrino cannot interact with two photons in lowest-order weak interactions. This is basically the result of Gell-Mann's theorem [14].

Now, the effective neutrino–two-photon interaction due to axial-vector currents is reduced to the standard form given by the second term of Eq. (2.2) with

$$\tilde{F}^{\alpha\beta}F_{\alpha\beta} \equiv \tfrac{1}{2}(\tilde{F}_1^{\alpha\beta}F_{2,\alpha\beta} + \tilde{F}_2^{\alpha\beta}F_{1,\alpha\beta}) , \tag{2.13}$$

and

$$a'_{rs} = -\frac{1}{\pi}\alpha_{\rm em}Q_l^2 N_l f_l (m_{\nu_s} + m_{\nu_r}) A_{l,rs} ,$$
$$b'_{rs} = \frac{1}{\pi}\alpha_{\rm em}Q_l^2 N_l f_l (m_{\nu_s} - m_{\nu_r}) A'_{l,rs} . \tag{2.14}$$

$N_l = 1$ (3) if l is a lepton (quark). Others are zero.

B. Pseudoscalar current contributions

The effective four-fermion interaction can be written as

$$[\bar{\nu}_r \gamma_5 (P_{l,rs} + P'_{l,rs}\gamma_5)\nu_s](\bar{l}\gamma_5 l) . \tag{2.15}$$

Now, CP invariance and Hermiticity require that

$$P_{l,rs} = P_{l,sr}, \quad P'_{l,rs} = -P'_{l,sr} \tag{2.16}$$

and P, P' real. For the triangle diagram with a pseudoscalar vertex, the calculation is straightforward. The result of our calculation is again of the form given by the second term of Eq. (2.2), where the two photons are in a CP-odd eigenstate $F\cdot\tilde{F}$, with

$$a'_{rs} = \frac{1}{\pi k_1\cdot k_2}\alpha_{\rm em}Q_l^2 N_l I_l m_l P_{l,rs} ,$$
$$b'_{rs} = \frac{1}{\pi k_1\cdot k_2}\alpha_{\rm em}Q_l^2 N_l I_l m_l P'_{l,rs} . \tag{2.17}$$

In contrast with the axial-vector contribution, the pseudoscalar contribution is directly proportional to an internal charged fermion mass rather than a neutrino mass. As a consequence, models with physical pseudoscalar interactions may generate a much enhanced neutrino–two-photon interaction because $m_l \gg m_\nu$. Equation (2.17) also indicates that even a massless neutrino may still possibly interact with two photons in lowest-order weak interactions.

C. Scalar current contributions

The effective four-fermion interaction now takes the form

$$[\bar{\nu}_r (S_{l,rs} + S'_{l,rs}\gamma_5)\nu_s](\bar{l}l) . \tag{2.18}$$

CP invariance and Hermiticity imply that

$$S_{l,rs} = S_{l,sr}, \quad S'_{l,rs} = -S'_{l,sr} \tag{2.19}$$

and both matrices are real. Now, in contrast with the previous two cases, the result is of the form given by the first term of Eq. (2.2), where the two photons are in a CP-even eigenstate $F\cdot F$. Explicitly,

$$a_{rs} = -\frac{1}{\pi k_1\cdot k_2}\alpha_{\rm em}Q_l^2 N_l \left[1+\frac{1}{2}\left[1-\frac{4m_l^2}{2k_1\cdot k_2}\right]I_l\right] \times m_l S_{l,rs} ,$$
$$b_{rs} = -\frac{1}{\pi k_1\cdot k_2}\alpha_{\rm em}Q_l^2 N_l \left[1+\frac{1}{2}\left[1-\frac{4m_l^2}{2k_1\cdot k_2}\right]I_l\right] \times m_l S'_{l,rs} , \tag{2.20}$$

where we have defined

$$F^{\alpha\beta}F_{\alpha\beta} = \tfrac{1}{2}(F_1^{\alpha\beta}F_{2,\alpha\beta} + F_2^{\alpha\beta}F_{1,\alpha\beta}) . \tag{2.21}$$

Once again, the result of our calculation is directly proportional to an internal charged fermion mass rather than a neutrino mass.

This concludes that, in lowest-order weak interactions, Eq. (2.2) is the most general effective low-energy neutrino–two-photon interaction Lagrangian. The couplings are subject to constraints given by Eq. (2.3) arising from CP invariance and Hermiticity.

The above discussion also illustrates how one can simply obtain results without actually doing model calculations. Basically, for a given theoretical model, one needs only to find out the effective four-fermion interactions and then substitute the results into the corresponding master equations (2.14), (2.17), and (2.20). The calculation for the four-fermion interaction is straightforward.

It only involves evaluating some trivial tree graphs followed by a Fierz transformation.

This conclusion implies that photons cannot be polarized from neutrino decay or scattering if the spin degrees of freedom of the neutrino are summed. This follows because the two terms in Eq. (2.2) cannot interfere. This conclusion then has an important astrophysical implication. Namely, to leading order, photons from (neutral) dark-matter annihilation or scattering or decay are not polarized. In order to have them polarized, one has to either violate *CP* invariance or go beyond lowest-order weak interactions. As a consequence, the effect (if any) is unlikely to be significant. This statement also holds for Majorana neutrinos.

III. EFFECTIVE INTERACTIONS FOR MAJORANA NEUTRINOS

We now turn to Majorana neutrinos constrained by the Majorana condition

$$\xi^c=\eta\xi \; . \tag{3.1a}$$

Here $\xi^c \equiv C\bar{\xi}^T$ and η is a phase factor. With the charge conjugation defined above one finds that η is real, i.e., $\eta=\pm 1$. It also can be shown that the *CP* property of the Majorana field ξ is given by $i\eta$ [15]. The plane wave expansion of a Majorana field is

$$\xi(\mathbf{x},t)=\sum_{\alpha=\pm 1/2}\int \frac{d^3\mathbf{k}}{(2\pi)^3}\frac{m_\xi}{k^0}[b_\alpha u(k,\alpha)e^{-ik\cdot x} + \eta b_\alpha^\dagger u^c(k,\alpha)e^{ik\cdot x}]. \tag{3.1b}$$

By complying with the Majorana condition (3.1a) new constraints arise. Suppose that the effective interaction Lagrangian of Majorana neutrinos are analogous to that of Dirac neutrinos:

$$L_{\text{eff}}=\bar{\xi}_r(a_{rs}^M+b_{rs}^M\gamma_5)\xi_s F^{\alpha\beta}F_{\alpha\beta} + i\bar{\xi}_r(a_{rs}^{M\prime}\gamma_5+b_{rs}^{M\prime})\xi_s \tilde{F}^{\alpha\beta}F_{\alpha\beta} \; , \tag{3.2}$$

where the superscript M refers to Majorana neutrinos. In the phase convention chosen above, we find that all the coupling matrices must again be real by *CP* invariance and

$$a_{rs}^M=a_{sr}^M, \quad a_{rs}^{M\prime}=a_{sr}^{M\prime}, \quad b_{rs}^M=-b_{sr}^M, \quad b_{rs}^{M\prime}=-b_{sr}^{M\prime} \tag{3.3}$$

by Hermiticity. In addition, we can rewrite Eq. (3.2) as

$$\begin{aligned} L_{\text{eff}} &= \bar{\xi}_s^c(a_{rs}^M+b_{rs}^M\gamma_5)\xi_r^c F^{\alpha\beta}F_{\alpha\beta} \\ &\quad + i\bar{\xi}_s^c(a_{rs}^{M\prime}\gamma_5+b_{rs}^{M\prime})\xi_r^c \tilde{F}^{\alpha\beta}F_{\alpha\beta} \\ &= [\bar{\xi}_r(a_{rs}^M-b_{rs}^M\gamma_5)\xi_s F^{\alpha\beta}F_{\alpha\beta} \\ &\quad + i\bar{\xi}_r(a_{rs}^{M\prime}\gamma_5-b_{rs}^{M\prime})\xi_s \tilde{F}^{\alpha\beta}F_{\alpha\beta}]\eta_r\eta_s \; , \end{aligned} \tag{3.4}$$

where the last step follows from the Majorana condition (3.1a) and a rearranging of flavor indices. Equating Eq. (3.2) to (3.4), one finds two constraints:

$$1\pm\eta_r\eta_s=0 \; . \tag{3.5}$$

Similar to neutrino–one-photon interactions [16,17], the physical distinction between these two constraints is as follows: In one case, $\eta_r\eta_s=1$, the two Majorana neutrinos have the same *CP* property. As a consequence, $b_{rs}^M=b_{rs}^{M\prime}=0$ and

$$L_{\text{eff}}(\eta_r\eta_s=1)=a_{rs}^M\bar{\xi}_r\xi_s F^{\alpha\beta}F_{\alpha\beta}+ia_{rs}^{M\prime}\bar{\xi}_r\gamma_5\xi_s\tilde{F}^{\alpha\beta}F_{\alpha\beta} \; , \tag{3.6}$$

where a_{rs}^M and $a_{rs}^{M\prime}$ are real and symmetric. In another, $\eta_r\eta_s=-1$, the two Majorana neutrinos have the opposite *CP* property. Now $a_{rs}^M=a_{rs}^{M\prime}=0$ and the interaction is

$$L_{\text{eff}}(\eta_r\eta_s=-1)=b_{rs}^M\bar{\xi}_r\gamma_5\xi_s F^{\alpha\beta}F_{\alpha\beta}+ib_{rs}^{M\prime}\bar{\xi}_r\xi_s\tilde{F}^{\alpha\beta}F_{\alpha\beta} \; , \tag{3.7}$$

where b_{rs}^M and $b_{rs}^{M\prime}$ are real and antisymmetric. In other words, the interaction must be either Eq. (3.6) or Eq. (3.7), but not a mixture of these two. In particular, for interactions involving two identical neutrinos, where $n_r=\eta_s$, the effective interaction must be of the form given by Eq. (3.6), which is very similar to the case of a Dirac neutrino [see Eq. (2.1a)]. This is in contrast with the neutrino–one-photon interaction, where *CPT* invariance implies that a Majorana neutrino cannot have a magnetic moment.

Again, to show that to lowest order weak interactions Eqs. (3.6) and (3.7) cover all possibilities. Let us reexamine the different situations.

A. Axial-vector-current contributions

We start by replacing the Dirac neutrinos in Eq. (2.6) by Majorana neutrinos. The coupling matrices $A_{l,rs}$ and $A_{l,rs}^\prime$ are changed to $A_{l,rs}^M$ and $A_{l,rs}^{M\prime}$. The superscript is to remind us that now we are dealing with Majorana neutrinos. Again, one can show that from *CP* invariance and Hermiticity both $A_{l,rs}^M$ and $A_{l,rs}^{M\prime}$ are real and symmetric with respect to the neutrino flavor indices r and s. In addition, the Majorana neutrino condition implies that

$$\begin{aligned} &[\bar{\xi}_r\gamma_\alpha\gamma_5(A_{l,rs}^M+A_{l,rs}^{M\prime}\gamma_5)\xi_s](\bar{l}\gamma^\alpha\gamma_5 l) \\ &\quad =\eta_r\eta_c[\bar{\xi}_r\gamma_\alpha\gamma_5(A_{l,rs}^M-A_{l,rs}^{M\prime}\gamma_5)\xi_s](\bar{l}\gamma^\alpha\gamma_5 l) \; . \end{aligned} \tag{3.8}$$

The value of $\eta_r\eta_s$ is determined by the Majorana mass matrix. Evidently, the effective interaction Lagrangian must be a form given by either Eq. (3.6) or (3.7), with the by now standard procedure, they are given by

$$\begin{aligned} a_{rs}^{M\prime} &= -\frac{1}{2\pi}\alpha_{\text{em}}Q_l^2 N_l f_l(m_{\xi_s}+m_{\xi_r})A_{l,rs}^M(1+\eta_r\eta_s) \; , \\ b_{rs}^{M\prime} &= \frac{1}{2\pi}\alpha_{\text{em}}Q_l^2 N_l f_l(m_{\xi_s}-m_{\xi_r})A_{l,rs}^{M\prime}(1-\eta_r\eta_s) \; , \end{aligned} \tag{3.9}$$

where the phase factor $(1\pm\eta_r\eta_s)/2$ is introduced in accordance with the Majorana condition.

B. Pseudoscalar current contributions

The effective four-fermion interaction still takes the form of Eq. (2.15). From *CP* invariance and Hermiticity, the analogous coupling matrices are again real and

$$P^M_{l,rs}=P^M_{l,sr},\quad P^{M\prime}_{l,rs}=-P^{M\prime}_{l,sr}\ . \tag{3.10}$$

With the Majorana condition, the interaction must satisfy additionally

$$[\bar{\xi}_r\gamma_5(P^M_{l,rs}+P^{M\prime}_{l,rs}\gamma_5)\xi_s](\bar{l}\gamma_5 l)$$
$$=\eta_r\eta_s[\bar{\xi}_r\gamma_5(P^M_{l,rs}-P^{M\prime}_{l,rs}\gamma_5)\xi_s](\bar{l}\gamma_5 l)\ . \tag{3.11}$$

The operator $\bar{l}\gamma_5 l$ is *CP* odd. Since the *CP* properties of the matrix elements of $\bar{\xi}_r\gamma_5\xi_s$ and $\bar{\xi}_r\xi_s$ are opposite, there are again two possibilities to make the four-fermion interaction *CP* invariant. In one case, $\eta_r\eta_s=1$, *CP* invariance and Hermiticity require that $P^{M\prime}_{l,rs}=0$. In another, $\eta_r\eta_s=-1$, it is $P^M_{l,rs}$ that must be zero. In any case, the effective Lagrangian must be of the form given by the second term of Eq. (3.2), where, depending on the relative *CP* properties of the two neutrinos, the only nonzero form factor could be either $a^{M\prime}_{rs}$ or $b^{M\prime}_{rs}$, but not a mixing of both:

$$a^{M\prime}_{rs}=\frac{1}{2\pi k_1\cdot k_2}\alpha_{\rm em}Q_l^2N_lI_lm_lP^M_{l,rs}(1+\eta_r\eta_s)\ ,$$
$$b^{M\prime}_{rs}=\frac{1}{2\pi k_1\cdot k_2}\alpha_{\rm em}Q_l^2N_lI_lm_lP^{M\prime}_{l,rs}(1-\eta_r\eta_s)\ . \tag{3.12}$$

C. Scalar current contributions

The analysis is exactly the same. Given an effective four-fermion interaction

$$[\bar{\xi}_r(S^M_{l,rs}+S^{M\prime}_{l,rs}\gamma_5)]\xi_s(\bar{l}l)\ , \tag{3.13}$$

where $S^M_{l,rs}$ and $S^{M\prime}_{l,rs}$ are symmetric and antisymmetric respectively. The effective Lagrangian must take the form given by the first term of Eq. (3.2) with

$$a^M_{rs}=-\frac{1}{2\pi k_1\cdot k_2}\alpha_{\rm em}Q_l^2N_l\left[1+\frac{1}{2}\left[1-\frac{4m_l^2}{2k_1\cdot k_2}\right]I_l\right]\times m_lS^M_{l,rs}(1+\eta_r\eta_s)\ ,$$
$$b^M_{rs}=-\frac{1}{2\pi k_1\cdot k_2}\alpha_{\rm em}Q_l^2N_l\left[1+\frac{1}{2}\left[1-\frac{4m_l^2}{2k_1\cdot k_2}\right]I_l\right]\times m_lS^{M\prime}_{l,rs}(1+\xi_r\eta_s)\ . \tag{3.14}$$

Again, depending on the relative *CP* properties of the two participating Majorana neutrinos, the only nonzero form factor is either a^M or b^M.

Except a phase factor $(1\pm n_rn_s)/2$, formulas for Majorana neutrinos are very similar to those for Dirac neutrinos. It is not even necessary to introduce these phase factors explicitly in the formula. As discussed below, they will arise automatically when we calculate the matrix element, due to the self-conjugate property of Majorana neutrinos [see Eq. (3.1b)].

IV. WHERE COULD NEUTRINO–TWO-PHOTON INTERACTIONS BE RELATIVELY LARGE?

Except the τ neutrino, empirically the effective low-energy four-fermion interactions [Eq. (2.4)] are known. To a good approximation, they are given by the standard weak interactions. In particular, in the standard model the couplings of Eq. (2.6) are

$$A_{l,rs}=-A'_{l,rs}=-\frac{G_F}{2\sqrt{2}}\delta_{rs}\theta_l\ , \tag{4.1}$$

where

$$\theta_l=\begin{cases}1 & \text{if } l \text{ is an up quark or a lepton of the } r\text{th generation}\ ,\\ -1 & \text{if } l \text{ is a down quark or other leptons}\ .\end{cases} \tag{4.2}$$

Contributions to neutrino–two-photon interactions are obtained by substituting Eq. (4.1) into Eq. (2.14). The final outcome is zero, however, because neutrinos are massless in the standard model. Because of this, neutrino–two-photon interactions could be very sensitive to new physics, even if the effective low-energy four-fermion interaction is dominantly given by the standard physics. Now, the question of interest is what kind of new physics may potentially make such interactions relatively large.

Let us consider some examples. For simplicity, we will ignore quark loop contributions in the following discussion (they can always be included in a straightforward way).

A. The standard model with massive neutrinos

Let us start from the standard model by introducing neutrino Dirac masses. The mixing matrix of the lepton sector is given by V_{rl}. Some of the calculations presented below have been done by others [18]. We present them here just for completeness.

The effective four-fermion interaction is the axial-vector type. In terms of Eq. (2.6), the couplings are

$$A_{l,rs}=-A'_{l,rs}=-\frac{G_F}{2\sqrt{2}}(2V_{rl}V_{sl}-\delta_{rs}), \tag{4.3}$$

where the first term comes from a virtual W exchange. The second term arises from a virtual Z, it is diagonal because of the absence of flavor-changing neutral currents. Substituting Eq. (4.3) into Eq. (2.14), we obtain lepton-contributions

$$a'_{rs}=\frac{G_F\alpha_{\rm em}}{2\pi\sqrt{2}}(m_{\nu_s}+m_{\nu_r})f_l(2V_{rl}V_{sl}-\delta_{rs})\ ,$$
$$b'_{rs}=\frac{G_F\alpha_{\rm em}}{\pi\sqrt{2}}(m_{\nu_s}+m_{\nu_r})f_lV_{rl}V_{sl}\ . \tag{4.4}$$

As we mentioned before, the results are directly proportional to a Dirac neutrino mass.

Next, we consider the situation where left-handed neutrinos form massive Majorana particles [19]. The Majorana neutrino fields satisfying the standard equation of motion with a positive mass are now given by

$$\xi_r = \nu_{L,r} + \eta_r \nu_{L,r}^c \ , \tag{4.5}$$

where $\nu_{L,r}^c \equiv (\nu_{L,r})^c$. Here, we will only consider contributions arising from gauge interactions. In terms of the Majorana field ξ, the charged current can be written as

$$L^{\rm CC} = \frac{g}{\sqrt{2}} V_{rl} (\bar{l}_L \gamma^\lambda \xi_r W_\lambda^- + \bar{\xi}_r \gamma^\lambda l_L W_\lambda^+) \ . \tag{4.6}$$

The effective four-fermion interaction due to a W exchange is therefore

$$L_{\rm eff}(W) = -\frac{G_F}{\sqrt{2}} V_{rl} V_{sl} [\bar{\xi}_r \gamma^\lambda (1-\gamma_5) \xi_s][\bar{l}\gamma_\lambda \gamma_5 l] \ . \tag{4.7}$$

Terms not contributing to the two-photon interaction have been discarded.

The neutral current due to a Z exchange can be written as

$$\begin{aligned} L^{\rm NC} &= -\frac{g}{4\cos\theta_W} \delta_{rs} (\bar{\nu}_{L,r} \gamma^\lambda \nu_{L,s} - \bar{\nu}_{L,r}^c \gamma^\lambda \nu_{L,s}^c) Z_\lambda \\ &= \frac{g}{4\cos\theta_W} \delta_{rs} \bar{\xi}_r \gamma^\lambda \gamma_5 \xi_s Z_\lambda \ , \end{aligned} \tag{4.8}$$

where we have used $\gamma_5(1\pm\gamma_5) = \pm(1\pm\gamma_5)$. Together with the lepton-Z interaction $-(g/4\cos\theta_W)\bar{l}_L \gamma^\lambda Z_\lambda + \cdots$, one finds that the effective four-fermion interaction due to a Z exchange is

$$L_{\rm eff}(Z) = \frac{G_F}{2\sqrt{2}} \delta_{rs} (\bar{\xi}_r \gamma^\lambda \gamma_5 \xi_s)(\bar{l}\gamma_\lambda \gamma_5 l) \ . \tag{4.9}$$

In terms of axial-vector-current four-fermion interactions (3.8), it then follows that the couplings are basically the same as for the Dirac neutrinos:

$$\begin{aligned} A_{l,rs}^M &= -\frac{G_F}{2\sqrt{2}} (2V_{rl} V_{sl} - \delta_{rs}) \ , \\ A_{l,rs}^{M\prime} &= \frac{G_F}{\sqrt{2}} V_{rl} V_{sl} \ . \end{aligned} \tag{4.10}$$

The form factor of the neutrino–two-photon interaction Lagrangian is therefore either

$$a_{rs}^{M\prime} = \frac{G_F \alpha_{\rm em}}{4\pi\sqrt{2}} (2V_{rl} V_{sl} - \delta_{rs}) f_l(m_{\xi_s} + m_{\xi_r})(1+\eta_r \eta_s) \tag{4.11}$$

or

$$b_{rs}^{M\prime} = \frac{G_F \alpha_{\rm em}}{2\pi\sqrt{2}} V_{rl} V_{sl} f_l(m_{\xi_s} - m_{\xi_r})(1-\eta_r \eta_s) \ , \tag{4.12}$$

but not a mixture of both.

Again, the results are proportional to neutrino masses. Furthermore, except for a phase factor $(1\pm\eta_r\eta_s)/2$, the effective interaction for Majorana neutrinos look very much the same as for Dirac neutrinos. As mentioned before, it is not even necessary to introduce the phase factor explicitly in the formula. They arise automatically when we calculate the matrix element. Indeed, one can show from Eq. (3.1b) that, for a diagonal interaction $r=s$,

$$\begin{aligned} \int d^3x \langle \xi_r(k) | a_{rr}^{M\prime} \bar{\xi}_r(\mathbf{x},t) i\gamma_5(\mathbf{x},t) \xi_r(\mathbf{x},t) | \xi_r(k') \rangle &= a_{rr}^{M\prime} [\bar{u}_r(k) i\gamma_5 u_r(k') - \eta_r^2 \bar{u}_r^c(k') i\gamma_5 u_r^c(k)] \\ &= a_{rr}^{M\prime}(1+\eta_r^2) \bar{u}_r(k) i\gamma_5 u_r(k') \ , \end{aligned} \tag{4.13}$$

which is two times bigger than the corresponding Dirac neutrino matrix element. As a consequence, the cross section of the scattering $\gamma\gamma \to \bar{\xi}\xi$ will be enhanced with respect to that of $\gamma\gamma \to \bar{\nu}\nu$, where the neutrinos are Dirac particles, by a factor of $4/2=2$, where the factor of 2 in the denominator arises from the identical particle statistics.

The situation for an off-diagonal interaction $\xi_s \to \xi_r (r \neq s)$ is slightly more complicated. It should be noticed that, for Majorana neutrinos, both $\bar{\xi}_r i\gamma_5 \xi_s$ and $\bar{\xi}_s i\gamma_5 \xi_r$, and similarly $\bar{\xi}_r \xi_s$ and $\bar{\xi}_s \xi_r$, contribute to the matrix element of the same physical process. This is evidently not the case for Dirac neutrinos. In any case, one finds

$$\begin{aligned} \int d^3x \langle \xi_r(k) | a_{rs}^{M\prime} [\bar{\xi}_r(\mathbf{x},t) i\gamma_5 \xi_s(\mathbf{x},t) + \bar{\xi}_s(\mathbf{x},t) i\gamma_5 \xi_r(\mathbf{x},t)] | \xi_s(k') \rangle &= a_{rs}^{M\prime} [\bar{u}_r(k) i\gamma_5 u_s(k') - \eta_r \eta_s \bar{u}_s^c(k') i\gamma_5 u_r^c(k)] \\ &= a_{rs}^{M\prime}(1+\eta_r\eta_s) \bar{u}_r(k) i\gamma_5 u_s(k') \ . \end{aligned} \tag{4.14}$$

Similarly,

$$\int d^3x \langle \xi_r(k) | b_{rs}^{M\prime} [\bar{\xi}_r(\mathbf{x},t) \xi_s(\mathbf{x},t) - \bar{\xi}_s(\mathbf{x},t) \xi_r(\mathbf{x},t)] | \xi_s(k') \rangle = b_{rs}^{M\prime}(1-\eta_r\eta_s) \bar{u}_r(k) u_s(k') \ . \tag{4.15}$$

The matrix element for Majorana neutrinos are again twice as large when compared with Dirac neutrinos. Nevertheless, depending on their relative *CP* properties, they can have only one term, i.e., either $a^{M\prime}$ or $b^{M\prime}$, but not both. By contrast, both terms are allowed for Dirac neutrinos, and their form factors are about the same when $m_{\nu_s} \gg m_{\nu_r}$. Since these two terms do not interfere, the decay rate or scattering cross section for Majorana neutrinos is still about a factor of 2 bigger than that of Dirac neutrinos.

More generally, the mass matrix of Majorana neutrinos can have Dirac mass terms if one also introduces right-handed neutrinos. Some interesting features of such mass matrix will be examined explicitly when we turn to consider some other nonstandard models.

B. Two-Higgs-doublet model

Including one more Higgs doublet, there will be a physical charged scalar $\phi^\pm$ in the $SU(2)_L \times U(1)_Y$ model. The interaction of ϕ^+ and the leptons is

$$L_{\phi^+} = \frac{g}{\sqrt{2}M_W}\phi^+ V_{rl}\bar{\nu}_r\left[\frac{v_1}{v_2}m_l R + \frac{v_2}{v_1}m_{\nu_r}L\right]l + \text{H.c.} \ , \tag{4.16}$$

where v_1 and v_2 are the vacuum expectation values of the two Higgs doublets, and L and R are the helicity-projection operators. Here, we assume for simplicity that neutrinos are Dirac particles. It then follows that the effective low-energy four-fermion interaction is given by

$$L_{\text{eff}} = -\frac{G_F}{2\sqrt{2}}V_{rl}V_{sl}\left\{\frac{m_l m_{\nu_r}}{m^2_{\phi^+}}[\bar{\nu}_r(1-\gamma_5)\nu_s][\bar{l}(1-\gamma_5)l] + \frac{m_l m_{\nu_s}}{m^2_{\phi^+}}[\bar{\nu}_r(1+\gamma_5)\nu_s][\bar{l}(1+\gamma_5)l] + \left[\frac{v_1}{v_2}\right]^2\frac{m_l^2}{m^2_{\phi^+}}[\bar{\nu}_r\gamma_\alpha(1-\gamma_5)\nu_s][(\bar{l}\gamma^\alpha\gamma_5 l) - \left[\frac{v_2}{v_1}\right]^2\frac{m_{\nu_r}m_{\nu_s}}{m^2_{\phi^+}}[\bar{\nu}_r\gamma_\alpha(1+\gamma_5)\nu_s](\bar{l}\gamma^\alpha\gamma_5 l)\right\} + \cdots \ , \tag{4.17}$$

where the ellipses represent terms which do not contribute.

Here, all terms in Eq. (4.17) are essentially proportional to a neutrino mass. Phenomenologically, we have $m_l^2/m^2_{\phi^+} \ll 1$. Thus, unless one stretches the theory to the breaking point where the ratio of v_1/v_2 becomes extremely large, contributions to the neutrino–two-photon interaction from the charged scalar will be much smaller than that from the standard physics (4.4). An enhancement could be obtained provided

$$\frac{v_1}{v_2} > \frac{m_{\phi^+}}{m_l} \ . \tag{4.18}$$

In that case, the dominant term would come from the third term of Eq. (4.17) and it would take the similar form as that from the standard-model physics:

$$a'_{rs} = \frac{G_F\alpha_{\text{em}}}{2\pi\sqrt{2}}(m_{\nu_s}+m_{\nu_r})V_{rl}V_{sl}f_l\left[\frac{v_1}{v_2}\right]^2\left[\frac{m_l}{m_{\phi^+}}\right]^2 ,$$
$$b'_{rs} = \frac{G_F\alpha_{\text{em}}}{2\pi\sqrt{2}}(m_{\nu_s}-m_{\nu_r})V_{rl}V_{sl}f_l\left[\frac{v_1}{v_2}\right]^2\left[\frac{m_l}{m_{\phi^+}}\right]^2 . \tag{4.19}$$

The possibility of having an extremely large ratio in a three-Higgs-doublet model [20] has recently been explored in connection with the study of *CP* violation in the K_{l3} decay [21].

For a reasonable choice of the parameters, contributions from neutral scalars are negligible. This is again due to the fact $m_l^2/m^2_{\phi^0} \ll 1$, here m_{ϕ^0} is the mass of a neutral physical scalar.

C. Left-right model

All models discussed so far have a common feature that the required helicity flip of the effective Lagrangian Eq. (2.2) or (3.2) is achieved by an explicit neutrino mass insertion. We now consider the $SU(2)_L \times SU(2)_R \times U(1)$ left-right model [22], where the helicity flip can be achieved via a charged lepton mass insertion.

Once again, we start from Dirac neutrinos. The relevant interaction Lagrangian of the model is given by

$$L = \frac{g}{\sqrt{2}}W_a^\alpha V_{rl}^a \bar{l}\gamma_\alpha(L+\theta R)\nu_r + \text{H.c.}, \tag{4.20}$$

where $a=1,2$ refers to the two W's of the model: W_1 is the one observed in the laboratory and W_2 is the so called right-handed W. $V^1(V^2)$ is the mixing matrix associated with the left- (right-) handed charged current. θ is the left-right mixing. Phenomenologically, it is generally required that

$$\theta \lesssim \tfrac{1}{400} \ . \tag{4.21}$$

Because of the left-right mixing, $\theta \neq 0$, new terms are generated in addition to the standard ones discussed before. In terms of the effective four-fermion interaction, terms which could potentially be important are those from the left-right mixing ($V \equiv V^1$)

$$L_{\text{eff}} = 2\sqrt{2}G_F\theta V_{rl}V_{sl}[(\bar{\nu}_s\nu_r)(\bar{l}\,l) - (\bar{\nu}_s\gamma_5\nu_r)(\bar{l}\gamma_5 l)] \ . \tag{4.22}$$

Comparing Eq. (4.22) with Eqs. (2.15) and (2.18), one sees that $P'_{l,rs} = S'_{l,rs} = 0$ and

$$P_{l,rs} = -S_{l,rs} = -2\sqrt{2}G_F\theta V_{rl}V_{sl} \ . \tag{4.23}$$

It then follows from Eqs. (2.17) and (2.20) that the only nonzero terms arising from the left-right mixing are those which are symmetric in r and s:

$$a_{rs}=-\frac{2\sqrt{2}G_F\alpha_{\rm em}}{\pi k_1\cdot k_2}\theta V_{rl}V_{sl}m_l\left[1+\frac{1}{2}\left[1-\frac{4m_l^2}{2k_1\cdot k_2}\right]I_l\right],$$
$$a'_{rs}=-\frac{2\sqrt{2}G_F\alpha_{\rm em}}{\pi k_1\cdot k_2}\theta V_{rl}V_{sl}m_lI_l\ . \tag{4.24}$$

An interesting feature of Eq. (4.24) is that the results are directly proportional to a charged-lepton mass rather than a neutrino mass. Now, if one takes the limit of θ from phenomenology shown in Eq. (4.21) literally, one would see that results given by Eq. (4.24) would be much larger than those from the standard model physics (4.4) because, for a reasonable choice of m_{ν_e}, $m_e/m_{\nu_e}=5\times10^5$ eV$/m_{\nu_e}\gg\theta$. In the small family mixing limit, the enhancements for the diagonal neutrino–two-photon interactions are $\theta(m_e/m_{\nu_e})$, $\theta(m_\mu/m_{\nu_\mu})$, and $\theta(m_\tau/m_{\nu_\tau})$, respectively. In addition, in the standard model (with massive neutrinos), the two photons can only be in a CP-odd eigenstate $\tilde{F}\cdot F$, whereas here both CP-odd and CP-even eigenstates are allowed.

However, we will argue in the next section that, in the simple version of left-right models, it is technically not natural to have a small neutrino mass and a large left-right mixing. Some fine-tunings have to be introduced.

Let us now turn to consider Majorana neutrinos, where the mass Lagrangian

$$\begin{aligned}-L_{\rm mass}=&m_{rs}(\bar{\nu}_{L,r}\nu^c_{L,s}+\bar{\nu}^c_{L,s}\nu_{L,r})\\&+m_{D,rs}(\bar{\nu}_{L,r}\nu_{R,s}+\bar{\nu}_{R,s}\nu_{L,r})\\&+M_{rs}(\bar{\nu}_{R,r}\nu^c_{R,s}+\bar{\nu}^c_{R,s}\nu_{R,r})\end{aligned} \tag{4.25}$$

contains both Dirac- and Majorana-mass terms. We will assume that the neutrino mass matrix can be diagonalized perturbatively. Then in the basis where the charged-lepton mass matrix and the submatrices of m_{rs} and M_{rs} are diagonal, the charged-current interaction Lagrangian is still given by Eq. (4.20). Here, we choose the matrices diagonalizing m_{rs} and M_{rs} to be the same for simplicity.

The mass eigenstate fields ξ_r and ξ'_r are obtained by diagonalizing the remaining mass matrix. Perturbatively, one finds

$$\begin{aligned}\xi_r&\approx(\nu_{L,r}+U'_{rs}\nu^c_{R,s})+\eta_r(\nu^c_{L,r}+U'_{rs}\nu_{R,s})\ ,\\ \xi'_r&\approx(\nu_{R,r}+U_{rs}\nu^c_{L,s})+\eta'_r(\nu^c_{R,r}+U_{rs}\nu_{L,s})\ ,\end{aligned} \tag{4.26}$$

where $\eta_r,\eta'_r=\pm1$ and

$$U_{rs},U'_{rs}\approx\tfrac{1}{2}O(m_DM^{-1})_{rs} \tag{4.27}$$

are the mixing matrices which connect a heavy neutrino to a light one. Evidently, the self-conjugate fields given above satisfy the Majorana condition (3.1a). Now, the interaction Lagrangian involving light Majorana neutrinos is

$$L\approx\frac{g}{\sqrt{2}}V_{rl}W^-_\alpha(\bar{l}_L\gamma^\alpha\xi_r-\theta U_{rs}\eta_s\bar{l}_R\gamma^\alpha\xi_s)+\text{H.c.} \tag{4.28}$$

It then follows that the effective four-fermion interaction due to the left-right mixing is

$$L_{\rm eff}=-\sqrt{2}G_F\theta\{(\eta_rk_{l,rs}+\eta_sK_{l,sr})[(\bar{\xi}_s\xi_r)(\bar{l}l)-(\bar{\xi}_s\gamma_5\xi_r)(\bar{l}\gamma_5l)]-(\eta_rK_{l,rs}-\eta_sK_{l,sr})[(\bar{\xi}_s\gamma_5\xi_r)(\bar{l}l)-(\bar{\xi}_s\xi_r)(\bar{l}\gamma_5l)]\}\ , \tag{4.29}$$

where

$$K_{l,rs}\equiv V_{sl}V_{ql}U_{qr}\ . \tag{4.30}$$

The overall sign of the effective interaction Lagrangian is determined by the sign of the phase factor. This is in contrast with a chiral-type interaction, i.e., left$\times$left or right$\times$right currents, where phase factors always enter as a combination of $(1\pm\eta_r\eta_s)/2$, which is either zero or 1. Hence, the overall sign of the interaction Lagrangian remains the same irrespective of the sign of η_r and η_s. Now, using our master equations (3.12) and (3.14), one finds that the results due to the left-right mixing are

$$\begin{aligned}\begin{bmatrix}a^M_{sr}\\b^M_{sr}\end{bmatrix}&=\frac{\sqrt{2}G_F\alpha_{\rm em}}{\pi k_1\cdot k_2}\theta m_l(\eta_sK_{l,sr}\pm\eta_rk_{l,sr})\\&\quad\times\left[1+\frac{1}{2}\left[1-\frac{4m_l^2}{2k_1\cdot k_2}\right]I_l\right],\\ \begin{bmatrix}a^{M\prime}_{sr}\\b^{M\prime}_{sr}\end{bmatrix}&=\frac{\sqrt{2}G_F\alpha_{\rm em}}{\pi k_1\cdot k_2}\theta m_l(\eta_sK_{l,sr}\pm\eta_rk_{l,rs})I_l\ .\end{aligned} \tag{4.31}$$

They are again directly proportional to a charged lepton mass. If the mixing angles in U_{rs} were not very small, contributions arising from the left-right mixing could dominate.

Again, when we calculate the matrix element, for $\eta_r\eta_s=-1\ (+1)$ only $b^M_{rs}(a^M_{rs})$ and $b^{M\prime}_{rs}\ (a^{M\prime}_{rs})$ terms are not zero. Unlike in the $SU(2)_L\times U(1)_Y$ model, there are no simple relations between the matrix elements of Dirac neutrinos and Majorana neutrinos.

D. The Zee model

The Zee model [23] is a simple extension of the standard model, where one introduces a charged scalar singlet in the Higgs sector. This charged singlet h^+ is assumed to carry a lepton number and therefore couples only to leptons. Such a simple extension has a number of interesting physical implications. One example would be the neutrino magnetic (transition) moment. A large portion of theoretical models [24–28] constructed so far, which could provide an extremely large neutrino magnetic moment, bare some resemblance to the Zee model.

Let us first consider the case where neutrinos are Dirac particles. To do so, we also introduce right-handed neutrino into the model [29].

The new physics arising from interactions with h^+ is

$$L=[F_{rl}(\bar{\nu}^c_{L,r}l_L h^+ + \bar{l}_L \nu^c_{L,r} h^-) + F'_{rl}(\bar{\nu}^c_{R,r} l_R h^+ + \bar{l}_R \nu^c_{R,r} h^-)] , \tag{4.32}$$

where

$$F_{rl}=-F_{lr} \tag{4.33}$$

due to Fermi statistics. To maximize the effect of interest, here we assume that there is no mixing in the lepton sector at the tree level. Introducing small mixing angles will not alter our conclusion.

One can easily calculate the effective four-fermion interaction Lagrangian from Eq. (4.32) and subsequently obtain

$$\begin{Bmatrix} a'_{rs} \\ b'_{rs} \end{Bmatrix} = -\frac{G_F\alpha_{\text{em}}}{\sqrt{2}\pi}\left[\frac{M_W^2}{g^2m_h^2}\right]\left[(F_{rl}F_{sl}\pm F'_{rl}F'_{sl})f_l(m_{\nu_s}\pm m_{\nu_r}) - (F_{rl}F'_{sl}\pm F_{sl}F'_{rl})m_l\frac{I_l}{k_1\cdot k_2}\right], \tag{4.34}$$

$$\begin{Bmatrix} a_{rs} \\ b_{rs} \end{Bmatrix} = \frac{G_F\alpha_{\text{em}}}{\sqrt{2}\pi k_1\cdot k_2}\left[\frac{M_W^2}{g^2m_h^2}\right](F_{rl}F'_{sl}\pm F_{sl}F'_{rl}) \times m_l\left[1+\frac{1}{2}\left[1-\frac{4m_l^2}{2k_1\cdot k_2}\right]I_l\right], \tag{4.35}$$

where m_h is the mass of h.

It is evident that the final results have both neutrino-mass and charged-lepton-mass-dependent terms. Literally, the charged-lepton-mass-dependent term could dominate, just like in the left-right model, provided the corresponding couplings in F and F' are not very small. Phenomenologically, the most stringent constraint arises from the absence of $\mu\to e\gamma$. There, one finds (Ref. [29]) $F_{12}^2/m_h^2<10^{-8}$ GeV^{-2}. Constraints on other combinations of the parameters are much weaker.

In addition, the constraint that F must be antisymmetric introduces a special feature to the model. Consider, for instance, a low-energy photon annihilation $\gamma\gamma\to\bar{\nu}(k_1)\nu(k_2)$ with $2k_1\cdot k_2\lesssim 1$ MeV. In the absence of h, the product of the annihilation would dominantly be ν_e. This follows because the loop integral [see Eq. (2.9)] f_l or I_l is inversely proportional to the mass square of the charged lepton of the same generation:

$$\frac{f_e}{f_{\mu,\tau}}\sim\frac{m_{\mu,\tau}^2}{m_e^2}\gg 1 . \tag{4.36}$$

As a consequence, $m_{\nu_e}f_e/m_{\nu_{\mu,\tau}}\gg 1$ unless $m_{\nu_{\mu,\tau}} \gtrsim m_{\nu_e}m_{\mu,\tau}^2/m_e^2$. However, in the new physics, ν_e is forced to couple to the heavier charged leptons but $\nu_{\mu,\tau}$ can interact directly with e. Consequently, by switching on the new physics, one could have, depending on the explicit values of F and F', a sizable (and even an equal) amount of $\nu_{\mu,\tau}$ generated entirely from the new physics. This is the major difference between the Zee model and the conventional left-right model.

Combining these two aspects, even if we apply the most stringent phenomenological constraint $F_{rl}F'_{rl}\sim 10^{-9}$ GeV^{-2} uniformly to all the possible nonzero combinations, enhancements to some of the diagonal neutrino–two-photon interactions are still very large. When compared with the standard physics result, the ratios of the enhancement are approximately 10^{-1} eV/m_{ν_e} for ν_e, 10^6 eV/m_{ν_μ} for ν_μ, and 10^8 eV/m_{ν_τ} for ν_τ, respectively. Also, the ratio of $\nu_{\mu,\tau}$ and ν_e in the photon annihilation can be 10 eV/m_{ν_e}.

We turn to the case where neutrinos are Majorana particles. In this case, we do not introduce ν_R. Now, in order to have neutrinos be massive, one needs to have two Higgs doublets $\phi_{1,2}$, which mix with each other via a term such as $\epsilon_{ij}\phi_1^i\phi_2^j h^- + \text{H.c.}$ in the Higgs potential. The Yukawa coupling of the model is given by

$$L_y=F_{rl}\bar{\nu}^c_{L,r}l_L h^+ - \frac{g}{\sqrt{2}}\frac{m_L}{M_W}\frac{\delta_{rl}}{\cos\beta}\bar{\nu}_{L,r}l_R\phi_1^+ + \text{H.c.} , \tag{4.37}$$

where $\tan\beta$ measures the ratio of the vacuum expectation values of the two Higgs doublets. For simplicity, we only allow ϕ_1 couple to the leptons so that the model has a natural flavor conservation.

In terms of the Majorana fields defined in Eq. (4.5), the effective four-fermion interaction Lagrangian of the model can be calculated easily from Eq. (4.37). Omitting terms which are directly proportional to a neutrino mass, we find

$$\begin{Bmatrix} a_{rl}^{M'} \\ b_{rl}^{M'} \end{Bmatrix} = \begin{Bmatrix} +a_{lr}^{M'} \\ -b_{lr}^{M'} \end{Bmatrix} = -\frac{g\alpha_{\text{em}}}{8\sqrt{2}\pi\cos\beta k_1\cdot k_2}\frac{m_l^2}{M_W\langle h\phi_1\rangle}\eta_r F_{rl} I_l ,$$

$$\begin{Bmatrix} a_{rl}^{M} \\ b_{rl}^{M} \end{Bmatrix} = \begin{Bmatrix} +a_{lr}^{M} \\ -b_{lr}^{M} \end{Bmatrix} = -\frac{g\alpha_{\text{em}}}{8\sqrt{2}\pi\cos\beta k_1\cdot k_2}\frac{m_l^2}{M_W\langle h\phi_1\rangle}\eta_r F_{rl}\left[1+\frac{1}{2}\left[1-\frac{4m_l^2}{2k_1\cdot k_2}\right]I_l\right] . \tag{4.38}$$

Here, $\langle h\phi_1\rangle$, arising from the h-ϕ_1 mixing, has a dimension of $mass^2$, and $\langle h\phi_1\rangle^{-1}\sim M_1^{-2}-M_2^{-2}$ where M_1 and M_2 are the masses of the two charged Higgs bosons of the model.

One can show that this model also generates off-diagonal calculable neutrino masses. Thus, ξ_r is not a mass-eigenstate field. The mass-eigenstate fields can be obtained by diagonalizing the neutrino mass matrix. It can be shown that the same parameter which enters into Eq. (4.38) also appears in the radiatively generated neutrino mass matrix. Therefore, in the absence of fine-tunings, Eq. (4.38) is, in fact, effectively proportional to a neutrino mass.

On the other hand, the fact that the neutrino mass matrix is off-diagonal implies large mixings in the lepton sector. Thus, similar to the case of Dirac neutrinos, large enhancements for $\nu_{\mu,\tau}$ can be expected from the new physics and, particularly, from the off-diagonal standard charged-current interaction, where the enhancement can reach the maximum of the order of $m_{\mu,\tau}^2/m_e^2$ for $\nu_{\mu,\tau}$.

E. Supersymmetric model

In a supersymmetric model [30], additional contributions arise from supersymmetric particle interactions. All these contributions are basically generated from axial-vector-current interactions and hence the results are directly proportional to a neutrino mass.

There is a class of models [31] which contain R-parity-violating interactions of the form

$$\lambda_{ijk}L_iL_jE_k^c+\lambda'_{ijk}L_iQ_jD_k^c, \tag{4.39}$$

where L, Q, E^c, D^c are the usual lepton and quark $SU(2)_L$ doublets and singlets respectively and i, j, k are generation indices. Because of Fermi statistics, the coupling λ_{ijk} is antisymmetric under the exchange of i and j. These interactions can provide a contribution (through effective scalar or pseudoscalar interactions) to the neutrino–two-photon interaction which appears not directly suppressed by a small neutrino mass. If the left- and the right-handed slepton (squark) mixing is not zero, it will be directly proportional to a charged-lepton (quark) mass. However, in that case, a nonzero and calculable neutrino mass will also be generated, and the same parameter that makes the neutrino–two-photon interaction large also enters into the neutrino mass. Thus, this contribution is again effectively proportional to a neutrino mass.

On the other hand, interactions given by Eq. (4.39) can induce significant flavor-changing neutral currents. Particularly, a large off-diagonal ν_τ-e coupling would allow the scalar-integral of a tau-neutrino–two-photon interaction be f_e or I_e rather than f_τ or I_τ. The consequence is very similar to that discussed in the Zee model.

From the above model discussions, one sees that to have a relatively large neutrino two-photon interaction it is necessary to introduce new physics such that (1) its interaction strength with neutrinos are not very small, (2) the new interaction will not generate a calculable neutrino mass such that the same parameter, which determines the neutrino mass, also enters into the effective neutrino–two-photon interaction, (3) the required helicity flip in Eqs. (2.2) and (3.2) is realized by a charged-fermion mass insertion, and finally (4) the resulting scalar loop integrals [f_l or I_l in Eqs. (2.9) and (2.10)] are not suppressed by a heavy-mass scale. Theoretical models which contain nonchiral weak interactions may potentially be able to satisfy these conditions. Our model discussions also show that, numerically, it appears possible to obtain a large enhancement in some nonstandard theoretical models.

V. THE COMPATIBILITY OF A SMALL NEUTRINO MASS AND A LARGE NEUTRINO–TWO-PHOTON INTERACTION

Although it appears that a much larger neutrino–two-photon interaction can be obtained in some theoretical models, fine-tunings have to be introduced in order to keep neutrino mass small. It would be of interest, of course, if such fine-tunings could be avoided. This is a technical issue related to the naturalness of having a small neutrino mass and a large "helicity-flip" interaction. The problem is that, for a given Feynman graph contributing to the neutrino-two-photon interaction, it will also generate a neutrino mass after removing the two external photon lines. As far as neutrino–two-photon interaction is concerned, this problem has not yet been fully addressed.

To illustrate where the fine-tuning enters, let us consider the left-right model with Majorana neutrinos as an example. The results from the new physics due to the left-right mixing are given in Eq. (4.31). When compared with those arising from standard-model physics, they are directly proportional to a charged-lepton mass rather than a neutrino mass.

However, the same parameter which enters into the neutrino–two-photon interaction Lagrangian also appears in the neutrino mass matrix. Indeed, the mixing matrix $K_{l,rs}$ defined in Eq. (4.30) contains a matrix called U, which connects a light left-handed neutrino to a heavy right-handed neutrino. Its order of magnitude is given by Eq. (4.27), where m_D is typically of the order of the charged lepton mass m_l. Now, combining the light-heavy mixing and the charged lepton mass in Eq. (4.31) one sees that

$$m_lU\sim m_D^2M^{-1}\ , \tag{5.1}$$

which is basically a "seesaw" formula [32,33] for a light-neutrino mass. As a consequence, contributions from the new physics and from the standard physics are not that different [34].

One could make the new physics result much bigger than the standard result by introducing fine-tunings. This can be done because the light neutrino mass is actually given by

$$m_\nu\approx m-m_D^2M^{-1}/4\ . \tag{5.2}$$

Thus, one could fine-tune the difference of these two terms such that the outcome, but not the individual term, is small. In that case, one would still have $m_lU\gg m_\nu$ and the new physics result would be much bigger than the standard one. Such a fine-tuning is unnatural because it has to be done in all order.

Similarly, one can show that it is also necessary to introduce fine-tunings if neutrinos are Dirac particles [35], where the fine-tuning enters into the determination of the left-right mixing θ. Similar situations happen also in other simple theoretical models [36].

For neutrino–one-photon interactions, it is known that the naturalness problem could be solved by a "charge" symmetry introduced by Voloshin [37]. Unfortunately, this symmetry will not help us to solve the naturalness problem in neutrino–two-photon interactions. This is because at one-loop level only those intermediate states invariant under charge conjugation contribute.

Moreover, the effective neutrino–two-photon interaction operator [see Eq. (2.2)] has a dimension of 7, which is a factor of 2 higher than a neutrino–one-photon interaction operator, $\bar{\nu}\sigma_{\alpha\beta}\nu F^{\alpha\beta}$. Comparing the form factors of these two, the form factor of a neutrino–two-photon interaction will have an additional mass square in the denominator. Therefore, when introducing new physics, we must make sure that the heavy mass of the new physics will not go to the denominator. Otherwise, we would still end up with a much suppressed neutrino–two-photon interaction, even though the suppression itself does not directly result from the smallness of a neutrino mass.

A typical example of this kind is the following: one could imagine an interaction such as

$$L = iF_{rs}\bar{\nu}^c_{R,r}\nu_{R,s}\phi^0 + iF'\bar{E}\gamma_5 E\phi^0 + \text{H.c.} , \tag{5.3}$$

where

$$F_{rs} = F_{sr} . \tag{5.4}$$

Here, ϕ^0 is a neutral pseudoscalar and E is a heavy charged fermion. All the particles in Eq. (5.3) are $SU(2)_L$ singlets and the whole theory can be easily made gauge invariant. One can make the neutrinos massless by, for instance, imposing a discrete symmetry $\nu_{R,i} \to -\nu_{R,i}$, so that the right-handed singlets cannot couple to the left-handed $SU(2)_L$ doublets. Also, one can choose F_{rs} diagonal by an orthogonal rotation of $\nu_{R,r}$. Now, we are in a situation where neutrinos are massless but the neutrino–two-photon interaction is not zero

$$L_{\text{eff}} = ia'_{rr}\bar{\xi}_r\gamma_5\xi_r\tilde{F}^{\alpha\beta}F_{\alpha\beta} , \tag{5.5}$$

with $\xi_r \equiv \nu_{R,r} + \eta_r\nu^c_{R,r}$ and

$$a'_{rs} = -\frac{\alpha_{\text{em}}F_{rr}F'}{\pi k_1\cdot k_2 m^2_{\phi^0}} m_E I_E .$$

I_E is given in Eq. (2.10) where one replaces m_l by m_E, the mass of the heavy fermion. For a low-energy process, $m_E I_E / k_1\cdot k_2 = 1/m_E$.

Now, suppose we introduce small masses for the neutrinos. Then the standard physics contribution to the neutrino–two-photon interaction is no longer zero. Comparing these two, one finds

$$\frac{\text{new result}}{\text{standard result}} \sim \frac{F'FM^2_W}{g^2 m^2_{\phi^0}} \frac{m_E I_E}{m_{\nu_e} I_e} \approx \frac{F'FM^2_W}{g^2 m^2_{\phi^0}} \frac{m^2_e}{m_{\nu_e} m_E} . \tag{5.6}$$

Here, the point is that even if one assumes the maximal allowed value of $F'F/m^2_{\phi^0} \sim g^2/M^2_W$, the new physics contribution can hardly be much bigger than the standard result obtained from a much simpler way. In fact, this could happen only if $m_E \lesssim 25(10\ \text{eV}/m_{\nu_e})$ GeV, but one knows that m_E cannot be much smaller than M_W. Thus, the new physics result given by Eq. (5.3) is in fact equally suppressed.

Such a heavy-mass suppression could have been avoided if the last term of Eq. (5.3) were $i\bar{e}\gamma_5 e\phi^0$. But then, by gauge invariance, one finds that ϕ^0 has to be a member of an SU(2) multiplet. In that case, the other members of this multiplet will necessarily generate a neutrino mass, and there is no guarantee in general that the result will be sufficiently small once one makes the neutrino–two-photon interaction large.

While at present we do not yet have a solution to this naturalness problem, a toy model outlined below appears to work in the desired direction. The idea is to find a mechanism in which one-loop contributions to the neutrino mass, but not to the neutrino–two-photon interaction, cancel exactly.

One could imagine an interaction like

$$L = \frac{g'_{rl}}{\sqrt{2}} \left[V^-_\mu \bar{l}\gamma^\mu(L + \theta R)\nu_r + \sum_{\alpha=1,2,3,4} \phi^-_a(\bar{l}_L\nu^c_{L,r} + \theta\bar{l}_R\nu^c_{R,r}) \right] + \text{H.c.} , \tag{5.7}$$

where one has a massive charged vector boson V_μ and four charged scalars ϕ_a(ϕ^-_a is a gauge singlet, $a = 1,2,3,4$). Their interaction strength with the normal leptons and their "left-right mixing," θ, are equal. If their masses are also the same, $m_V = m_{\phi_a} \equiv M$, then one-loop radiative corrections to the neutrino mass from V^- and from ϕ^-_a would be the same but with the opposite sign, and thus cancel (see Fig. 2). Unlike a scalar multiplet, the neutral partner of $V^\pm_\mu$ will not generate a neutrino mass, if neutrino masses are assumed to be zero at the tree level.

One thing special about this toy model is that there are no complete cancellations for the neutrino–two-photon interaction. In fact, the effective pseudoscalar current contributions from V^-_μ and ϕ^-_a actually add up, although the scalar part cancels. This follows because a scalar charged-current contributes, after a Fierz transformation, equally to a scalar- and a pseudoscalar-four-fermion interaction, whereas those from a vector charged current, although equal in size, have the opposite sign.

In any case, the effective-four fermion interaction of Eq. (5.7) is

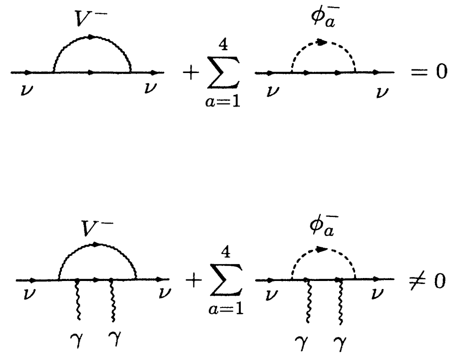

FIG. 2. Depending on the relative interaction strength and the participating particle masses, contributions to the neutrino mass, but not to the neutrino–two-photon interaction, could cancel in these two sets of graphs.

$$L_{\text{eff}} = -\theta \frac{g'_{rl} g'_{sl}}{2M^2} [(\bar{\nu}_r \gamma_5 \nu_s)(\bar{l} \gamma_5 l) - \tfrac{1}{4}(\bar{\nu}_r \sigma_{\alpha\beta} \nu_s)(\bar{l} \sigma^{\alpha\beta} l) - \tfrac{1}{4}(\bar{\nu}_r \sigma_{\alpha\beta} \gamma_5 \nu_s)(\bar{l} \sigma^{\alpha\beta} \gamma_5 l)] \ . \tag{5.8}$$

Clearly, up to one-loop level, no term in Eq. (5.8) would disturb the requirement that neutrinos are massless, because

$$\mathrm{tr}(\overline{\bar{l} \gamma_5 l}), \mathrm{tr}(\overline{\bar{l} \sigma_{\alpha\beta} l}), \mathrm{tr}(\overline{\bar{l} \sigma_{\alpha\beta} \gamma_5 l}) = 0 \ . \tag{5.9}$$

This would have not been the case, if there were terms such as $(\bar{\nu}_r \nu_s)(\bar{l} l)$. In any case, the one loop neutrino–two-photon interaction is

$$i a'_{rs} \bar{\nu}_r \gamma_5 \nu_s \tilde{F}^{\alpha\beta} F_{\alpha\beta} \ , \tag{5.10}$$

with [see Eq. (2.17)]

$$a'_{rs} = -\frac{\theta g'_{rl} g'_{sl} \alpha_{\text{el}}}{2\pi M^2} \frac{I_l}{k_1 \cdot k_2} m_l \ . \tag{5.11}$$

While the first term in Eq. (5.8) leads to a neutrino–two-photon interaction, the presence of the last two terms implies that such a mechanism can also generate a neutrino magnetic moment which is not suppressed by a neutrino mass. Our explicit loop calculation shows, indeed, that is the case. Potentially, this might be yet another viable mechanism for producing a large neutrino magnetic moment.

It is of course not obvious if this idea can be materialized in a realistic model. The trouble is that there does not seem to have a viable symmetry reason to relate a vector boson to a scalar. On the other hand, introducing small asymmetries (explicitly in all possible ways) is not crucial for our results. Although, in that case, one would have at the one-loop level a nonzero neutrino mass, it would be suppressed by the small asymmetry parameters chosen explicitly at the tree level. The point is that these small parameters will not enter into the leading terms of neutrino–two-photon interactions. Thus, one could have a theoretical model where the neutrino–two-photon interaction is large and the neutrino mass is arbitrarily small.

The question of compatibility becomes irrelevant if one considers nonlocal interaction arising from two-loop graphs. It is known [38] that a vanishingly small nonlocal neutrino–two-photon interaction can be generated at two-loop level, even if neutrinos are massless.

VI. CONCLUSION

We have presented a systematic study of neutrino–two-photon interactions. At one-loop level, these effective interactions turn out to be local and they can at most have two independent terms for a diagonal process, such as a neutrino–two-photon scattering. The number of independent terms becomes 4 for an off-diagonal interaction, such as a flavor-changing two-photon decay. We have derived some simple formulas for the corresponding form factors. By using these formulas, one can easily obtain results in all theoretical models. We have also provided some examples to illustrate how this can be done. By doing so, we also show that a relatively larger neutrino–two-photon interaction can be obtained in some nonstandard theoretical models. Because of the possibility of having a large flavor-changing neutral interaction, one can have a much enhanced neutrino–two-photon interaction invoking ν_μ and, particularly, ν_τ.

Differences arising from whether a neutrino is a Dirac particle or a Majorana particle are examined in some detail. In the absence of *CP* violation, we find that the structure of a Majorana neutrino–two-photon interaction is determined by the relative *CP* properties of the participating Majorana neutrinos.

We have also examined the question of compatibility of having small neutrino masses and a large neutrino–two-photon interaction. A new illustrative mechanism is discussed. It shows potentially how this problem might be solved.

Even with a much enhanced neutrino–one-photon interaction suggested by some nonstandard theoretical models (often via some arbitrary fine tunings), it appears difficult to have its effects be significant. Only in some very special circumstances might such effects be able to compete with those from, for instance, neutrino-electron(nucleon) scattering or neutrino–one-photon interaction. On the other hand, to have a large neutrino–two-photon interaction, we have to introduce new physics any way. It is therefore conceivable that, in some special situation, those seemingly dominant effects might be canceled once we switch on the new physics, leaving the neutrino–two-photon interaction as the only dominant interaction. Detailed phenomenological studies will be presented in a separate article.

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