Top-quark mass and a symmetric Cabibbo-Kobayashi-Maskawa matrix

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Phenomenological constraints due to CP violation in the K-$\bar{K}$ system and the extent of B-$\bar{B}$ mixing are used to determine the constraints on the various parameters of the Cabibbo-Kobayashi-Maskawa (CKM) matrix given the *Ansatz* is symmetric. An existing parametrization for the CKM matrix is shown to be the most general parametrization for the symmetric *Ansatz*. This *Ansatz* implies that the mass of the t quark is at least 180 GeV.

The standard electroweak model, despite its immense success, is somewhat arbitrary in that the Yukawa couplings of the fermions with the Higgs field are not constrained by any symmetry. The only way to determine the mixing angles and fermion masses is by empirical methods, although various constraints on the form of the mass matrices or the mixing angles have been advocated towards a partial solution of the problem. However, the phenomenological validity of such *Ansätze* are not guaranteed *a priori,* and comparisons with experimental results have to be explicitly made in each case.

In this paper we pursue the investigation of symmetric quark mixing [i.e., a symmetric (CKM) matrix [1,2]] in conjunction with CP-violation in the neutral-kaon system and the extent of the $B_d^0-\bar{B}_d^0$ mixings. Such a study is of particular interest for the following reasons. By defining the asymmetry parameter

$$A \equiv |V_{12}|^2-|V_{21}|^2=|V_{23}|^2-|V_{32}|^2 =|V_{31}|^2-|V_{13}|^2 \quad (1)$$

(where the latter equalities follow from unitarity in the three-generation case) it may be shown that the condition for the symmetric CKM parametrization implies that $A=0$ [2]. This actually implies that the CKM matrix is of symmetric modulus; by use of the rephasing freedom one can make it symmetric. Within (the quite small) limits of error the values of $|V_{12}|$ and that of $|V_{21}|$ are identical [3] and range between 0.217 and 0.223. This strongly suggests the possibility that the CKM matrix is symmetric.

By comparing the constraint of the symmetric CKM matrix and the results of B-$\bar{B}$ mixing and measurement of ϵ_K (the parameter associated with CP-violation in the K^0-$\bar{K}^0$ system) we show that a symmetric CKM matrix is consistent with experiment only if the top-quark mass m_t is higher than 180 GeV. For any given m_t, the value of $q=|V_{13}|/|V_{23}|$ and the CP-violating phase δ is strongly constrained.

For the three-generation CKM matrix, we adopt the parametrization [3,4]

$$V=\begin{bmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{i\delta} \\ -s_{12}c_{23}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{bmatrix}, \quad (2)$$

where $c_{ij}=\cos\theta_{ij}$; $s_{ij}=\sin\theta_{ij}$. The ansatz of a symmetric CKM mixing matrix can be studied by either constraining a nonsymmetric parametrization (such as the one given above) or by using the most general manifestly symmetric parametrization possible. This latter parametrization has only recently been explored [1,2]. Although in this paper we shall make use of the parametrization (2) above in deriving the limits on m_t, the parametrization of Ref. [1] is more useful in comparing our results to those of Refs. [5,6] where a more restricted symmetry on the CKM matrix was imposed. We shall employ it at the end of our paper for this purpose.

While θ_{12} is very accurately determined from K_{e3} and hyperon decays,

$$s_{12}=0.221\pm0.002\ , \quad (3)$$

θ_{23} and θ_{13} are rather poorly determined. The value of s_{23} may be extracted from a determination of V_{cb} (since $s_{23}\approx|V_{cb}|$ to a very good approximation) from the semileptonic B-meson partial width, under the assumption that it is given by the W-mediated process to be

$$\Gamma(b \to c l \bar{\nu}_l) = \left[\frac{G_F^2 m_b^5}{192\pi^3} \right] F(m_c^2/m_b^2) |V_{cb}|^2 , \quad (4)$$

where $F(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln(x)$ is a phase-space factor. Thus

$$s_{23}^2 = \left[\frac{192\pi^3}{G_F^2} \right] \frac{\mathcal{B}(b \to c l \bar{\nu}_l)}{\tau_b m_b^5 F(m_c^2/m_b^2)} . \quad (5)$$

Using the experimental results for the branching ratio and the B-meson lifetime,

$$\mathcal{B}(b \to c l \bar{\nu}_l) = 0.121 \pm 0.008 ,$$
$$\tau_b = (1.16 \pm 0.16) \times 10^{-12} \text{ sec} , \quad (6)$$

and the estimation for the quark masses

$$m_c = 1.5 \pm 0.2 \text{ GeV} , \quad m_b = 5.0 \pm 0.3 \text{ GeV} , \quad (7)$$

we get

$$s_{23} = 0.044 \pm 0.009 . \quad (8)$$

The charmless B-meson decay width imposes the limit

$$0.05 \le s_{13}/s_{23} \le 0.13 . \quad (9)$$

The CP violating phase δ is allowed to adopt any value in the range $[0,\pi]$ by these current experimental results.

The relation $|V_{12}| = |V_{21}|$ obviously restricts one to a three-dimensional hypersurface in the parameter space spanned by s_{12}, s_{23}, $q = |V_{13}|/|V_{23}|$ and δ. While $J = \mathrm{Im}(V_{11} V_{22} V_{12}^* V_{21}^*)$, the rephasing-invariant measure of CP violation does vary with s_{23}, q and δ do not show any such variations, as their dependence on θ_{23} is very weak. Taking s_{12} and s_{23} as phenomenological inputs from (3) and (8) leaves us with a curve in the q-δ plane for fixed values of s_{12} and s_{23}. For the situation described in Ref. [5] the curve shrinks to a point. By determining whether this curve lies within the region in the q-δ plane allowed by the ϵ_K and B-$\bar{B}$ mixing data, we are therefore able to obtain limits on the mass m_t of the t quark as a consequence of the symmetric CKM *ansatz* since these latter quantities depend upon m_t.

The K^0-$\bar{K}^0$ system indirect CP violating measure ϵ_K in the CKM picture is expressed as [7]

$$|\epsilon_K| = C B_K s_{23}^2 q \sin(\delta) \{ [\eta_3 f_3(y_t) - \eta_1] y_c s_{12} + \eta_2 y_t f_2(y_t) s_{23}^2 [s_{12} - q\cos(\delta)] \} , \quad (10)$$

where

$$C \equiv \frac{(G_F f_K M_W)^2 M_K}{6\pi^2 \sqrt{2} (\Delta M_K)} ,$$

$$f_2(y_t) = 1 - \frac{3}{4} \frac{y_t(1+y_t)}{(1-y_t)^2} \left[1 + \frac{2y_t}{1-y_t^2} \ln(y_t) \right] ,$$
$$f_3(y_t) = \ln\left[\frac{y_t}{y_c} \right] - \frac{3}{4} \frac{y_t}{1-y_t} \left[1 + \frac{y_t}{1-y_t} \ln(y_t) \right] , \quad (11)$$

and $y_i \equiv m_i^2/M_W^2 (i = c,t)$. The parameters η_i are QCD corrections [8]

$$\eta_1 = 0.7 , \quad \eta_2 = 0.6 , \quad \eta_3 = 0.4 . \quad (12)$$

The experimental result $|\epsilon_K| = 2.3 \times 10^{-3}$ gives a parabola in the q-δ plane for given B_K, s_{23} and m_t. The bag factor B_K is very poorly determined and various theoretical estimates only find the bounds $\frac{1}{3} \le B_K \le 1$. The expression for the $B_d^0 - \bar{B}_d^0$ mixing parameters $x_d = \Delta M/\Gamma$ is, on the other hand,

$$x_d = \tau_b \frac{G_F^2}{6\pi^2} \eta M_B (B_B f_B^2) M_W^2 y_t f_2(y_t) |V_{tb} V_{td}^*|^2 , \quad (13)$$

where $M_B = 5.28$ GeV, $B_B f_b^2 = (0.15 \pm 0.05 \text{ GeV})^2$ and the QCD correction $\eta = 0.85$. Experimentally $|V_{tb}| \approx 1$ to a high degree of accuracy and

$$|V_{td}|^2 = s_{23}^2 (s_{12}^2 + q^2 - 2 s_{12} q \cos\delta) . \quad (14)$$

The ARGUS result

$$x_d = 0.73 \pm 0.18 \quad (15)$$

thus gives another curve in the q-δ plane for given s_{23} and m_t.

It is straightforward to see that the symmetric *Ansatz* implies a strong lower bound on m_t. Equation (13) shows that $x_d \approx m_t^2 |V_{31}|^2$ which by the symmetric *Ansatz* is $m_t^2 |V_{13}|^2$. However Eqs. (8 and 9) impose a severe upper limit on $|V_{13}|$, in turn yielding a strong lower bound on m_t.

In our numerical analysis we hold B_K and m_t fixed and consider the total variation of all other parameters, taken in quadrature. Thus we get two interesting bands in the q-δ plane coming from ϵ_K and x_d. If this zone does contain the curve obtained from the symmetrical *Ansatz,* then the assumptions are obviously valid for the given choice of m_t and B_K.

A plot of the curve in the q-δ plane for the symmetric *Ansatz* (henceforth called the symmetric curve) is given in Fig. 1. We find a very narrow curve considering all the variations of s_{12} and s_{23}. We next superimpose on the symmetric curve in the q-δ plane curves parametrizing the regions allowed by the experiments with B-$\bar{B}$ mixing and the measurement of ϵ_K [9].

We find that when the top-quark mass is lighter than 180 GeV, the symmetric curve does not intersect with the ARGUS measurement of x_d, implying that the top quark must be at least this heavy if the symmetric *Ansatz* is correct. Imposing the K-$\bar{K}$ mixing result we find that for $B_K = \frac{1}{3}$, the symmetric *Ansatz* implies $m_t > 275$ GeV (although for $B_K = \frac{2}{3}$ and 1, the lower limit of 180 GeV is unaltered). Alternatively, for given values of m_t, when the symmetric curve overlaps with the measurements of x_d and ϵ_K we find that the symmetric *Ansatz* allows only a restricted range of values for q and δ, i.e., the CP violating phase is not completely arbitrary. The value of δ lies between 8° and 31°, while q is restricted to lie between 0.113 and 0.13. We have shown the allowed regions of q

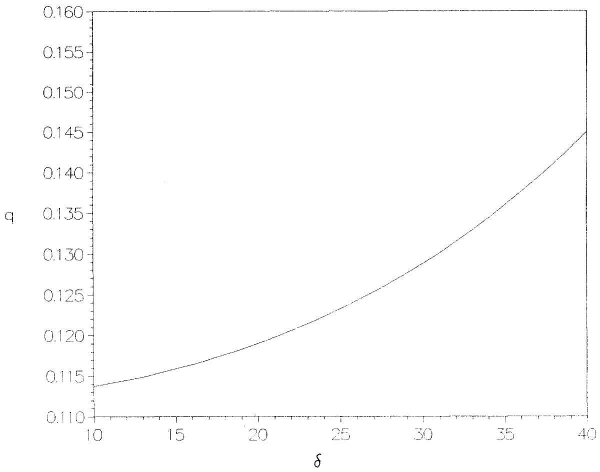

FIG. 1. The symmetric curve for q vs δ. Note that existing data implies that $8° \leq \delta \leq 31.1°$.

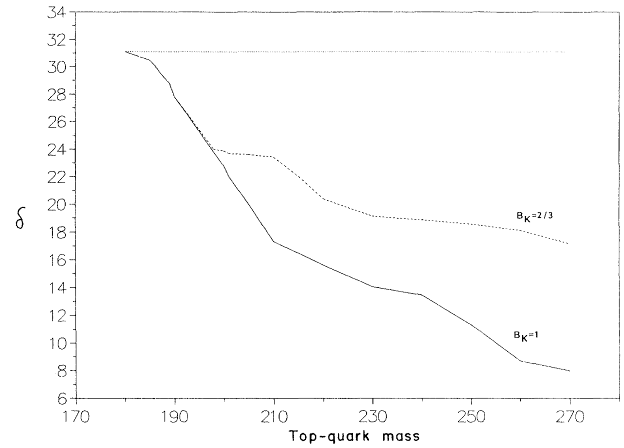

FIG. 3. Allowed region of δ as a function of m_t for $B_K=1$ (solid line) and $B_K=\frac{2}{3}$ (dashed line). The dotted line is the upper limit on δ, valid for all B_K.

and δ for different m_t values in Figs. 2 and 3 for two different values of B_K, namely, $B_K=\frac{2}{3}$ and 1.

We have also checked the above results using the parametrization of Ref. [1] which is related to (2) by a rephasing $\tilde{V}=P_1VP_2$ where P_1 and P_2 are phase matrices. This parametrization is given in terms of the eigenvalues and eigenvectors of the CKM matrix itself. If λ_i are the eigenvalues and ω_i the normalized eigenfunctions, then

$$\tilde{V}=\sum_{i=1}^{3}\lambda_i\omega_i\otimes\omega_i^\dagger , \tag{16}$$

where

$$w_1=\begin{bmatrix} c_1 \\ s_1c_2 \\ s_1s_2 \end{bmatrix}, \quad w_2=\begin{bmatrix} -s_1c_3 \\ c_1c_2c_3-s_2s_3e^{i\alpha} \\ c_1s_2c_3+c_2s_3e^{i\alpha} \end{bmatrix}, \quad w_3=\begin{bmatrix} s_1s_3 \\ -c_1c_2s_3-s_2c_3e^{i\alpha} \\ -c_1s_2s_3+c_2c_3e^{i\alpha} \end{bmatrix}, \tag{17}$$

where $c_i \equiv \cos(\beta_i)$ and $s_i \equiv \sin(\beta_i)$. By an appropriate redefinition of the quark phases the angles β_i can be constrained to be in the first quadrant.

The *Ansatz* $s_3=0$ yields a manifestly symmetric CKM

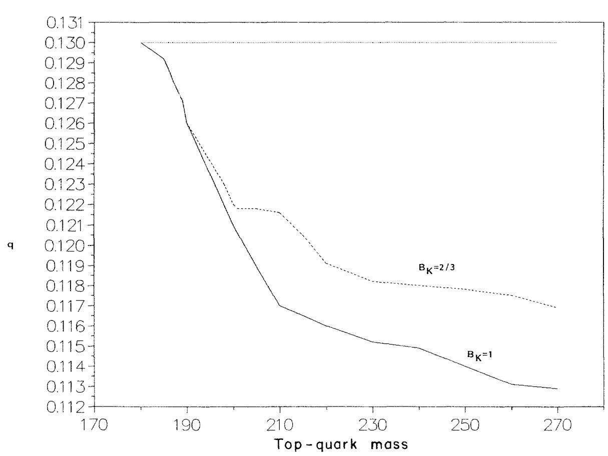

FIG. 2. Allowed region of q as a function of m_t for $B_K=1$ (solid line) and $B_K=2/3$ (dashed line). The dotted line is the upper limit on q, valid for all B_K.

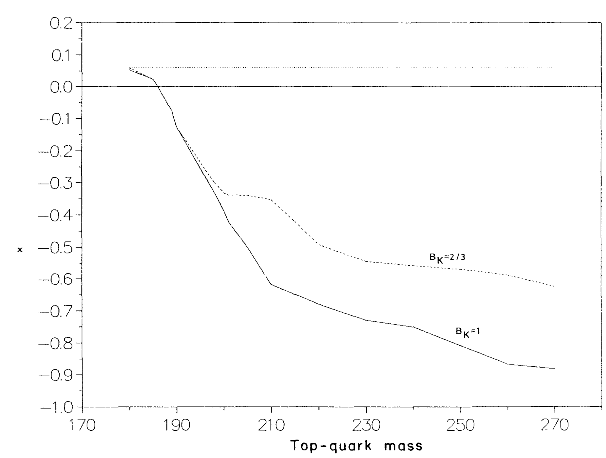

FIG. 4. Allowed region of x as a function of m_t for $B_K=1$ (solid line) and $B_K=\frac{2}{3}$ (dashed line). The dotted line is the upper limit on x, valid for all B_K.

matrix whose elements may be expressed in terms of β_1, β_2, and $\mathrm{tr}\,\tilde{V} \equiv x$ where the trace of $\tilde{V}$ may be chosen to be real [1]. A special case was considered by Kielanowski [5], who made the further assumption that V was traceless to have a two angle parametrization. It is straightforward to show that parameters of the Wolfenstein-type parametrization for a general symmetric CKM matrix as given in [2] are in 1-1 correspondence with x, $\sin\beta_1$, and $\sin\beta_2$, showing that the choice $s_3=0$ yields the most general symmetric CKM matrix possible for the parametrization (16).

Plotting q in terms of β_1, β_2, and x we again obtain the symmetric curve in Fig. 1, providing a numerical check on our results. The experimental constraints imply that x must lie between -0.882 and 0.02. We also show the allowed regions of the parameter x for different values of m_t in Fig 4. From the allowed region of x for different m_t, we can immediately conclude that $x=0$ is allowed for m_t about 185 GeV, in accord with an earlier result of Rosner and Kielanowski [6].

In conclusion, we find that if the CKM matrix is symmetric then the top-quark mass has to be heavier than 180 GeV, to be consistent with the experiments on B-$\bar{B}$ mixing and the measurement of ϵ_K; if the bag constant $B_K=\frac{1}{3}$ then $m_t>275$ GeV. The parameters q and δ are constrained to be in the range

$$0.130 \le q \le 0.113\ , \quad 8.0^\circ \le \delta \le 31.1^\circ \tag{18}$$

for the symmetric CKM matrix over the allowed range of the top-quark mass.

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