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Relativistic constituent-quark model of electroweak properties of light mesons

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We investigate the predictive power of a relativistic constituent-quark model of electroweak properties of pseudoscalar and vector mesons in the *u*-, *d*-, *s*-quark sector. The rates for radiative and weak decay processes and electromagnetic form factors for charged and neutral pseudoscalar mesons are calculated and compared with experiment. This analysis determines the values of the angles for η-η' and ω-ϕ mixing, and also the Kobayashi-Maskawa matrix element V_{us}. A consistent picture of the $q\bar{q}$ system emerges which should allow a reliable treatment of electroweak phenomena in the heavy-meson sector also.

I. INTRODUCTION

Radiative decays of pseudoscalar and vector mesons have always been used as a testing ground for various theoretical models of hadron structure. We present in this work an analysis of a relativistic constituent-quark model which was formulated on the light cone in Ref. [1]. Nonrelativistic constituent-quark models, supplemented with dynamics suggested by QCD, treat mesons as bound quark-antiquark states and are quite successful in describing the mass spectrum of mesons [2]. The dominant effects of gluonic degrees of freedom are absorbed into an effective confining potential and constituent masses for the quarks. In Ref. [3] a one-gluon-exchange potential and various relativistic effects were taken into account. Godfrey and Isgur [3] analyzed also strong and electroweak couplings of mesons based upon nonrelativistic approximation methods for the relevant matrix elements, a treatment which is certainly inconsistent in the *u*-, *d*-, *s*-quark sector. The light-front formalism of Ref. [1] permits a fully relativistic treatment of the electroweak properties of mesons. The application of this method is more transparent if the diagrams of the perturbation theory on the light cone [4] are considered. The hadronic structure for small momentum transfer is nonperturbative and can be represented by one-loop diagrams. It is an attractive feature of the light-front formalism [1] that the time ordering of the loop diagram, involving quarks created out of or annihilating into the vacuum can be eliminated, thus leading to a relativistic quark model which retains the usual $q\bar{q}$ structure for mesons and is as simple to apply as its nonrelativistic counterparts.

The equation of motion of the bound $q\bar{q}$ state in the light-front formalism is a relativistic Schrödinger equation with an appropriate effective potential. However, wave functions that are consistent solutions of the model are not readily available. Therefore, we start with a simple ansatz for the meson wave functions which depend upon one parameter $1/\beta$ which essentially determines the confinement scale. The parameters of this model, the constituent masses m_q and the wave function parameters $\beta_{q\bar{q}}$, can be fixed by using the relevant experimental data. Once the values of the parameters are known, we can calculate the hadronic structure of mesons and compare with the available experimental information.

We have already used this relativistic quark model to investigate semileptonic [5] and rare [6] decays of *B* and *D* mesons. However, it is difficult to reliably fix the parameters of the model in the heavy-quark sector. It is the main purpose of the present work to explore the quality and the power of the model in the *u*-, *d*-, *s*-quark sector for which a large body of precise data exists.

The light-front formalism has been used also in Refs. [7–10] to determine selected electroweak properties of light mesons. The work of Aznauryan and Oganesyan [10] is close to ours; however, a different ansatz for the wave function has been employed in Ref. [10]. The choice of a particular trial wave function is somewhat arbitrary and can be justified only by the success of the corresponding predictions. A brief discussion of this problem can be found in Ref. [7].

In Sec. II we present a brief summary of the essential aspects of the light-front formalism for $q\bar{q}$ bound states. In Sec. III we fix the free parameters of the relativistic quark model (masses and wave-function parameters) for the *u*-, *d*-, *s*-quark sector by a comparison with the data for the decay constants f_π, f_K and leptonic decay rates of vector mesons. This analysis leads to a vector-mixing angle which is consistent with the current-mixing model, but disagrees with the presently advocated value based upon the quadratic mass-mixing model. In Sec. IV we discuss radiative transitions between pseudoscalar and vector mesons in the *u*-, *d*-, *s*-quark sector and compare the predicted rates with the experimental data. We find a value for the pseudoscalar mixing angle in excellent agreement with experiment. In Sec. V we calculate electroweak form factors of π and K. The predicted electromagnetic structure of the pion is in perfect agreement with a recent reanalysis of the data. Semileptonic kaon decays are dominated by the form factor F_+ and we emphasize that the usual linear approximation is inadequate to extract a precise value for the mean-square radius. We compare our result for F_+ with that of a chiral perturbation theory approach and give a value for the element V_{us} of the Kobayashi-Maskawa (KM) matrix, which has been calculated in the framework of the quark model. In Sec. VI the two-photon decays of π, η, and η' are analyzed. We first determine the slopes of the three transition form factors and find good agreement with the latest data. However, the calculated decay rates disagree with the measurements since two-photon processes are not entirely dominated by the one-loop diagram. An alternative approach is provided by the anomaly of the axial-vector current. The decay rates can be expressed in terms of three pseudoscalar decay constants, which can be calculated reliably in the quark model. Again, we can compare our numerical values with the respective results of chiral perturbation theory. The resulting decay rates are consistent with the data. Finally, we summarize our investigation in a concluding Sec. VII.

II. QUARK STRUCTURE OF THE MESON VERTEX

The composite meson state can be represented by the vertex of Fig. 1, and we briefly summarize the essential aspects of its treatment in the light-front formalism.

The four-momentum of the meson in terms of light-front components is $P=(P^-,P^+,\mathbf{P}_\perp)$, where $P^\pm=P^0\pm P^3$. We shall use also light-front vectors (denoted by an arrow over the latter throughout), which in this case is defined by $\vec{\mathbf{P}}=(P^+,\mathbf{P}_\perp)$. Light-front vectors have the unique property to be covariant under kinematic Lorentz transformations [11]. It is crucial to establish the appropriate variables for the internal motion of the constituents, whose momenta we shall denote by p_1 and p_2:

$$p_1^+=\xi P^+,\quad p_2^+=(1-\xi)P^+ ,$$
$$\mathbf{p}_{1\perp}=\xi\mathbf{P}_\perp+\mathbf{p}_\perp,\quad \mathbf{p}_{2\perp}=(1-\xi)\mathbf{P}_\perp-\mathbf{p}_\perp . \tag{2.1}$$

The minus components can be defined covariantly such that $p_i^2=m_i^2$, where m_1,m_2 are the constituent masses of the quarks

$$p_1^-=\frac{\mathbf{p}_{1\perp}^2+m_1^2}{\xi P^+},\quad p_2^-=\frac{\mathbf{p}_{2\perp}^2+m_2^2}{(1-\xi)P^+} . \tag{2.2}$$

Note that only light-front vectors are conserved, i.e., $\vec{\mathbf{p}}_1+\vec{\mathbf{p}}_2=\vec{\mathbf{P}}$, but $p_1+p_2\neq P$; instead we find

$$(p_1+p_2)^2=M_0^2 ,$$
$$M_0^2=\frac{\mathbf{p}_\perp^2+m_1^2}{\xi}+\frac{\mathbf{p}_\perp^2+m_2^2}{1-\xi} . \tag{2.3}$$

The meaning of M_0 becomes more transparent if the momentum fraction ξ is replaced by another variable p_z defined by

$$\xi=\frac{E_1+p_z}{E_1+E_2},\quad 1-\xi=\frac{E_2-p_z}{E_1+E_2} , \tag{2.4}$$

where $E_i=(m_i^2+\mathbf{p}^2)^{1/2}$ and $\mathbf{p}^2=\mathbf{p}_\perp^2+p_z^2$. In terms of the new variable $M_0=E_1+E_2$.

The wave function $\psi(\mathbf{p},\lambda\bar{\lambda},JJ_3)$ of the bound state depends only upon the inner momentum $\mathbf{p}$, the spin variables $\lambda,\bar{\lambda}=\pm\frac{1}{2}$ of the quarks, and the spin J of the meson. Its spin structure can be chosen to be that of a noninteracting system where the spins of the constituents are rotated by a Melosh-type rotation [11]. The wave function for S- and P-state mesons can be represented in the appropriate basis of Dirac spinors [5] in terms of the vertex operator $\Gamma(\mathbf{p})$ for Fig. 1:

$$(P^2-M_0^2)\psi(\mathbf{p},\lambda\bar{\lambda},JJ_3)=\left[\frac{M_0}{2E_1E_2}\right]^{1/2}\bar{u}(\vec{\mathbf{p}}_1,\lambda)\Gamma(\mathbf{p})v(\vec{\mathbf{p}}_2,\bar{\lambda}) , \tag{2.5a}$$

for $^1S_0$ mesons

$$\Gamma(\mathbf{p})=h_0(p)\gamma_5 , \tag{2.5b}$$

for $^3S_1$ mesons with $J_3=\pm1$, 0,

$$\Gamma(\mathbf{p})=-h_0(p)\hat{\epsilon}_\mu(J_3)\left[\gamma_\mu-\frac{(p_1-p_2)_\mu}{M_0+m_1+m_2}\right] , \tag{2.5c}$$

for $^3P_1$ mesons,

$$\Gamma(\mathbf{p})=h_1(p)\hat{\epsilon}_\mu(J_3)\times\left[\gamma_\mu\gamma_5+\frac{m_1-m_2}{M_0^2-(m_1-m_2)^2}(p_1-p_2)_\mu\gamma_5\right] . \tag{2.5d}$$

The polarization vector depends upon the total momentum

$$\hat{\epsilon}(\pm1)=\epsilon(\pm1)=\left[\frac{2}{P^+}\boldsymbol{\epsilon}_\perp\mathbf{P}_\perp,\ 0,\ \boldsymbol{\epsilon}_\perp\right] ,$$
$$\boldsymbol{\epsilon}_\perp(\pm1)=\mp(1,\pm i)/\sqrt{2} , \tag{2.6}$$
$$\hat{\epsilon}(0)=-\frac{1}{M_0}\left[\frac{-M_0^2+\mathbf{P}_\perp^2}{P^+},P^+,\mathbf{P}_\perp\right] .$$

Note that $(p_1+p_2)\hat{\epsilon}(J_3)=0$. The function $h_0(p)$ is defined by

$$h_0(p)=\left[\frac{2E_1E_2}{M_0}\right]^{1/2}\frac{P^2-M_0^2}{\sqrt{2}[M_0^2-(m_1-m_2)^2]^{1/2}}\phi(p) , \tag{2.7}$$

and the S-state orbital wave function $\phi(p)$ is approximated by a harmonic-oscillator function

$$\phi(p)=\pi^{-3/4}\beta^{-3/2}((2\pi)^3/3)^{1/2}\exp(-\mathbf{p}^2/2\beta^2) , \tag{2.8}$$

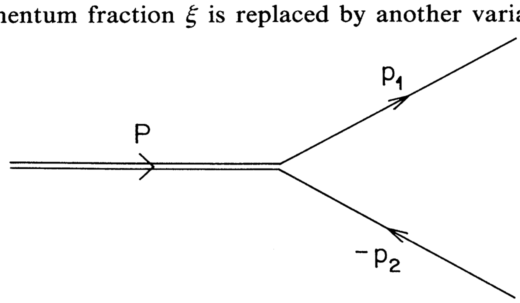

FIG. 1. The $q\bar{q}$ vertex of the composite meson.

which is normalized according to

$$\frac{N_c}{(2\pi)^3}\int d^3p\,|\phi(p)|^2=1\ . \tag{2.9}$$

The function $h_1(p)$ is then given by

$$h_1(p)=\frac{M_0^2-(m_1-m_2)^2}{2M_0\beta}h_0(p)\ . \tag{2.10}$$

III. LEPTONIC DECAY CONSTANTS

The pseudoscalar decay constant f_p for $\pi^-=d\bar{u}$ and $K^-=s\bar{u}$ is given by the matrix element of the axial-vector current $\bar{u}\gamma_\mu\gamma_5 d$ and $\bar{u}\gamma_\mu\gamma_5 s$, respectively, which we express in the general form

$$\langle 0|\bar{q}''\gamma_\mu\gamma_5 q'|\vec{P},\ 00\rangle\sqrt{2p^+}=iP_\mu\sqrt{2}f_p\ . \tag{3.1}$$

The matrix element can be represented in the one-loop approximation by the diagram of Fig. 2, which is given in the light-front formalism by

$$P^+\sqrt{2}f_p=\frac{N_c}{(2\pi)^3}\int d^3p\left[\frac{M_0}{2E_1E_2}\right]^{1/2}\times\sum_{\lambda\bar{\lambda}}\psi(\mathbf{p},\lambda\bar{\lambda},\ 00)\bar{v}(\vec{P}_2,\bar{\lambda})\gamma^+\gamma_5 u(\vec{P}_1,\lambda)\ . \tag{3.2}$$

Using the representation (2.5) for the wave function, the integrand of Eq. (3.2) is seen to be the trace of Dirac matrices and f_p is found to be

$$f_p=\frac{N_c}{(2\pi)^3}\int d^3p\ \phi(p)\left[\frac{2M_0}{E_1E_2}\right]^{1/2}\times\frac{(1-\xi)m_1+\xi m_2}{[M_0^2-(m_1-m_2)^2]^{1/2}}\ . \tag{3.3}$$

The width for the leptonic decay of vector mesons is given, for leptons of zero mass, by

$$\Gamma(V\to e^+e^-)=\frac{4\pi\alpha^2 f_V^2}{3m_V^3}\ . \tag{3.4}$$

The decay constant f_V is defined by the matrix element of the electromagnetic current whose quark structure in lowest order is formally described again by Fig. 2:

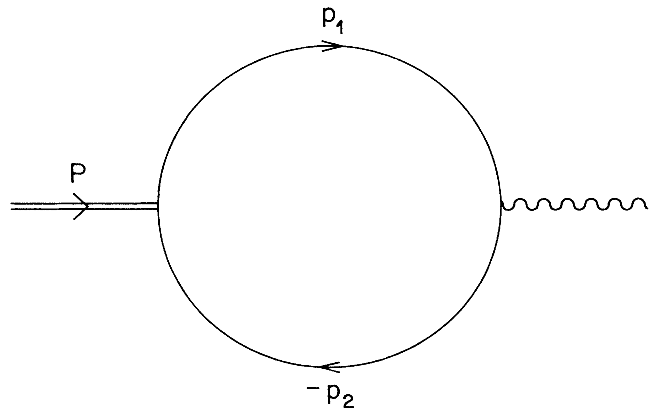

FIG. 2. One-loop Feynman diagram representing the hadronic structure of the meson decay amplitude.

$$j_\mu=Q_u\bar{u}\gamma_\mu u+Q_d\bar{d}\gamma_\mu d+Q_s\bar{s}\gamma_\mu s\ , \tag{3.5a}$$

$$\langle 0|j_\mu|\vec{P},1J_3\rangle\sqrt{2P^+}=\epsilon_\mu(J_3)f_V\ . \tag{3.5b}$$

For the decay of $\rho^0=(u\bar{u}-d\bar{d})/\sqrt{2}$ we obtain

$$f_\rho/m_\rho=g(m)(Q_u-Q_d)/\sqrt{2}\ , \tag{3.6}$$

where $g(m)\equiv I_2(m,m)$ with $m=m_u=m_d$ is the loop integral corresponding to Fig. 2 and in analogy to Eq. (3.2) we have

$$\epsilon^+(J_3)I_2(m_1,m_2)=\frac{N_c}{(2\pi)^3}\int d^3p\left[\frac{M_0}{2E_1E_2}\right]^{1/2}\times\sum_{\lambda\bar{\lambda}}\psi(\mathbf{p},\lambda\bar{\lambda},1J_3)\bar{v}(\vec{p}_2,\bar{\lambda})\gamma^+u(\vec{p}_1,\lambda). \tag{3.7}$$

This expression has to be evaluated for a longitudinally polarized state as given in Eqs. (2.5) and 92.6), with the result

$$I_2(m_1,m_2)=\frac{N_c}{(2\pi)^3}\int d^3p\ \phi(p)\left[\frac{M_0}{E_1E_2}\right]^{1/2}\frac{2}{[M_0^2-(m_1-m_2)^2]^{1/2}}\left[(1-\xi)m_1+\xi m_2+\frac{2p_\perp^2}{M_0+m_1+m_2}\right]\ . \tag{3.8}$$

We have given the result for unequal constituent-quark masses since we shall use it to calculate, e.g., the decay constant f_{K^*}.

We shall use the conventions of Ref. [12] to define the simplest possible mixing scheme for ω and ϕ. The starting point is the set of SU(3) basis states:

$$\omega_8=(u\bar{u}+d\bar{d}-2s\bar{s})/\sqrt{6}\ ,\qquad \omega_0=(u\bar{u}+d\bar{d}+s\bar{s})/\sqrt{3}\ . \tag{3.9}$$

In terms of these states the flavor mixing of ω and ϕ is represented by

$$\phi=\cos\theta_V\omega_8-\sin\theta_V\omega_0\ ,\qquad \omega=\sin\theta_V\omega_8+\cos\theta_V\omega_0\ . \tag{3.10}$$

For our purpose it is more convenient to use a quark basis

$$\phi=-\sin\delta_V(u\bar{u}+d\bar{d})/\sqrt{2}-\cos\delta_V s\bar{s} \, ,$$
$$\omega=\cos\delta_V(u\bar{u}+d\bar{d})/\sqrt{2}-\sin\delta_V s\bar{s} \, . \quad (3.11)$$

The decay constants for ω and ϕ are now obviously given by

$$f_\phi/m_\phi=-\sin\delta_V g(m)(Q_u+Q_d)/\sqrt{2}-\cos\delta_V g(m_s)Q_s \, ,$$
$$f_\omega/m_\omega=\cos\delta_V g(m)(Q_u+Q_d)/\sqrt{2}-\sin\delta_V g(m_s)Q_s \, . \quad (3.12)$$

where $g(m_s)\equiv I_2(m_s,m_s)$ depends upon the s-quark mass and the wave-function parameter $\beta_{s\bar{s}}$. For the loop integral $g(m)$ the parameter β has to be used and we assume that $\beta=\beta_{u\bar{u}}=\beta_{d\bar{d}}$.

We shall investigate first how the parameters of the quark model are constrained by selected data. For the u-, d-quark sector we use the new value for the pion decay constant $f_\pi=92.4\pm0.2$ MeV of Ref. [13] and the ρ decay constant $f_\rho/m_\rho=152.9\pm3.6$ MeV, since the corresponding model values depend sensitively upon the u-, d-quark mass m and the wave-function parameter β, where we assume also $\beta=\beta_{u\bar{d}}$. We find for the quark mass $m=250\pm5$ MeV. It might be interesting to note that already the analysis of isospin violations in mesons [14] of Godfrey and Isgur led to the conclusion that the masses of u and d quarks lie in a narrow range around 250 MeV. A similar calculation for the kaon based upon the decay constant $f_K=113.4\pm1.1$ MeV of Ref. [13] and the decay rate for $K^{*+}\to K^+\gamma$, which we shall treat in Sec. IV, leads to the s-quark mass $m_s=370\pm20$ MeV. The same masses have been found in Ref. [10]. We shall use the values for masses and wave-function parameters given in Table I for all states in the u-, d-, s-quark sector.

All free parameters of the quark model are now fixed and the loop integral defined in Eq. (3.8) can be calculated for the u-, d-, and s-quark loops:

$$g(m)=214.79 \text{ MeV}, \quad g(m_s)=230.78 \text{ MeV} \, . \quad (3.13)$$

The resulting leptonic decay rates together with the experimental data have been collected in Table II. Of special interest is the value of the mixing angle $\delta_V=-3.30°$ or $\theta_V=31.96°$ which follows from our analysis. We find that δ_V is negative, which disagrees with the usual convention (e.g., Ref. [12]) that is based upon the quadratic mass mixing model. Our result is independent of the particular choice of parameters as the following relation, which is a consequence of Eqs. (3.6) and (3.12), shows

$$\tfrac{1}{3}f_\rho/m_\rho=\cos\delta_V f_\omega/m_\omega-\sin\delta_V f_\phi/m_\phi \, ,$$
$$51.0\pm1.2=\cos\delta_V(45.7\pm0.8)-\sin\delta_V(79.1\pm1.3) \text{ MeV} \, . \quad (3.14)$$

If the experimental values for decay constants are used, Eq. (3.14) requires $\delta_V<0$.

Other well-known relations can be reproduced if the small SU(3) breaking, which manifests as the difference between $g(m)$ and $g(m_s)$ in Eq. (3.13), is neglected. If we put $g(m)=g(m_s)=g$, Eq. (3.12) can be simplified:

TABLE I. Table of the parameters of the relativistic quark model in the u-, d-, s-quark sector. With these parameters the central values of the decay constants f_π and f_K, quoted in the text, are reproduced.

q	m_q (GeV)	$\beta_{q\bar{q}}$ (GeV)	$\beta_{u\bar{q}}$ (GeV)
u,d	0.25	0.3194	
s	0.37	0.3478	0.3949

$$f_\rho/m_\rho=g/\sqrt{2} \, ,$$
$$f_\phi/m_\phi=\cos\theta_V g/\sqrt{6} \, , \quad (3.15)$$
$$f_\omega/m_\omega=\sin\theta_V g/\sqrt{6} \, ,$$

which leads to relations between decay rates

$$\tfrac{1}{3}m_\rho\Gamma_\rho=m_\omega\Gamma_\omega+m_\phi\Gamma_\phi \, , \quad (3.16)$$

$$\Gamma_\omega/\Gamma_\phi=\tan^2\theta_V p m_\phi/m_\omega \, . \quad (3.17)$$

Equation (3.16) has been derived long ago in Ref. [15] by saturating the first spectral sum rule of Weinberg [16] by ρ, ω, and ϕ in the narrow-width approximation. Equation (3.17) has been derived, e.g., by Oakes and Sakurai (Ref. [17]), who have also shown that the current-mixing model [18] is the only theory of $\omega-\phi$ mixing compatible with the first sum rule of Weinberg. These arguments in favor of the current-mixing model suggested a negative mixing angle δ_V already more than two decades ago.

We can also evaluate the decay constant f_{K^*}, which for $K^{*+}=u\bar{s}$, is defined by the matrix element

$$\langle 0|\bar{s}\gamma_\mu u|\vec{P},1J_3\rangle\sqrt{2P^+}=\epsilon_\mu(J_3)\sqrt{2}f_{K^*} \quad (3.18)$$

and

$$f_{K^*}/m_{K^*}=g(m,m_s)/\sqrt{2}=186.73 \text{ MeV} \, , \quad (3.19)$$

where Eq. (3.8) with the wave-function parameter $\beta_{u\bar{s}}$ has been used. The difference between f_{K^*}/m_{K^*} and $f_\rho/m_\rho=151.88$ MeV is due to SU(3) breaking, which is larger here than in Eq. (3.13). It is common practice to estimate f_{K^*} using the Das-Mathur-Okubo sum rules [19]. If the relevant spectral functions are dominated by the vector mesons ρ and K^*, one obtains the broken SU(3) relation $f_\rho/m_\rho=f_{K^*}/m_{K^*}$. This relation has been derived in the narrow-width approximation, which might be the reason for its disagreement with the quark-model results.

TABLE II. Rates and decay constants for the leptonic decays $V\to e^+e^-$, where the mixing angle $\delta_V=-3.3°$ has been used. The experimental data have been taken from the PDG (Ref. [12]).

V	f_V/m_V^2	Γ_{th} (keV)	Γ_{expt} (keV)
ρ	0.198	6.73	6.77±0.32
ω	0.0591	0.609	0.60±0.02
ϕ	0.0782	1.391	1.37±0.05

IV. RADIATIVE TRANSITIONS BETWEEN PSEUDOSCALAR AND VECTOR MESONS

The rate for the decay $V \to P\gamma$ is given by

$$\Gamma=\frac{1}{3}\alpha g_{VP\gamma}^2\left[\frac{m_V^2-m_P^2}{2m_V}\right]^3 , \qquad (4.1)$$

where the coupling constant $g_{VP\gamma}$ is defined by the matrix element of the electromagnetic current (3.5a) whose quark structure in the one-loop approximation is formally described by Fig. 3:

$$\langle \vec{P}'',00|j_\mu|\vec{P}', 1J_3\rangle\sqrt{4P'^+P''^+} = ig_{VP\gamma}\epsilon_{\mu\nu\alpha\beta}\epsilon_\nu(J_3)P'_\alpha P''_\beta . \qquad (4.2)$$

Except for charges and mixing amplitudes the coupling constant $g_{VP\gamma}$ can be expressed in terms of the loop integral $I_3(m_1,m_2)$ which has been calculated already in Ref. [5]. We quote the result for the case that the transition takes place between S-state mesons, whose vertex structure is given in Eq. (2.5) and which have identical orbital wave functions ϕ (2.8):

$$I_3(m_1,m_2)=\frac{N_c}{(2\pi)^3}\int d^3p\,|\phi(p)|^2\frac{2}{M_0^2-(m_1-m_2)^2}\times\frac{1}{\xi}\left[(1-\xi)m_1+\xi m_2+\frac{p_\perp^2}{M_0+m_1+m_2}\right] . \qquad (4.3)$$

If the quark basis (3.11) is used, we find for the coupling constants $g_{V\pi\gamma}$:

$$\begin{aligned} g_{\rho\pi\gamma}&=(Q_u+Q_d)I(m) ,\\ g_{\omega\pi\gamma}&=\cos\delta_V(Q_u-Q_d)I(m) , \qquad (4.4)\\ g_{\phi\pi\gamma}&=-\sin\delta_V(Q_u-Q_d)I(m) , \end{aligned}$$

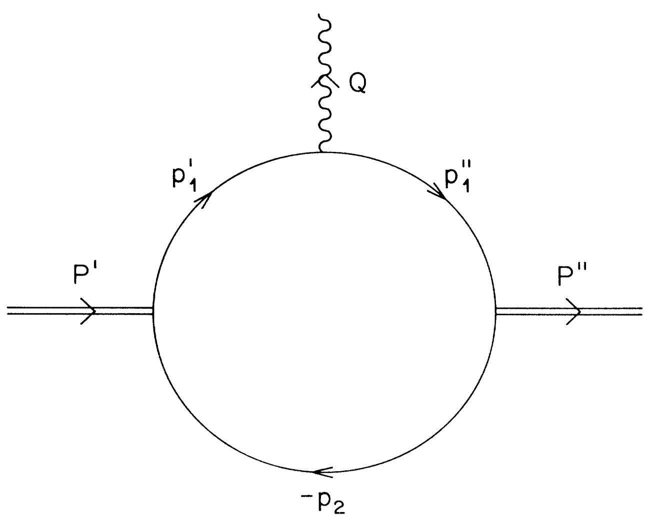

FIG. 3. One-loop Feynman diagram representing the hadronic structure of the amplitude for the electroweak transition between two mesons, with $P=P'+P''$ and $Q=P'-P''$.

where $I(m)\equiv I_3(m,m)=2.3496$ GeV^{-1} with the parameters of Table I. The resulting rates together with the corresponding data are shown in Table III. While our results for the three transitions are consistent with the averaged experimental rates of the Particle Data Group [12] (PDG), the quark model favors the somewhat higher rates for $\rho^+\to\pi^+\gamma$ reported in Ref. [20]: $\Gamma=81\pm4\pm4$ keV and $\Gamma=71\pm7$ keV. From the uncertainty of the experimental rate for $\phi\to\pi\gamma$ the range of admissible values for the mixing angle δ_V is found to be $\delta_V=-3.37°\pm0.17°$.

The mixing scheme for η and η' can be defined in analogy to the $\omega-\phi$ mixing, if the possibility of heavy flavor mixing is ignored. For this purpose, we replace ϕ, ω, θ_V, δ_V in Eqs. (3.9)–(3.11) by η, η', θ_P, δ_P, respectively. In this manner we find, for the coupling constants $g_{V\eta\gamma}$,

$$\begin{aligned} g_{\rho\eta\gamma}&=-\sin\delta_P(Q_u-Q_d)I(m) ,\\ g_{\omega\eta\gamma}&=-\cos\delta_V\sin\delta_P(Q_u+Q_d)I(m)\\ &\quad+\sin\delta_V\cos\delta_P 2Q_s I(m_s) , \qquad (4.5)\\ g_{\phi\eta\gamma}&=\sin\delta_V\sin\delta_P(Q_u+Q_d)I(m)\\ &\quad+\cos\delta_V\cos\delta_P 2Q_s I(m_s) , \end{aligned}$$

where $I(m_s)\equiv I_3(m_s,m_s)=1.8740$ GeV^{-1} with the parameters of Table I. The results are given in Table III. Of particular interest is the value for the mixing angle θ_P which we determine by a comparison with the experimental rate for $\phi\to\pi\gamma$. We give our value together with the experimental results:

$$\theta_P=\begin{cases}-18.5°\pm1.0° \text{ (this work)} ,\\ -19.8°\pm2.2° \text{ (Ref. [21])} ,\\ -19.1°\pm1.4° \text{ (Ref. [22])} .\end{cases} \qquad (4.6)$$

Finally the coupling constants $g_{V\eta'\gamma}$ are given by

TABLE III. Rates and decay constants for the radiative decays $M'\to M''\gamma$, where the mixing angles $\delta_V=-3.3°$ and $\theta_P=-19°$ have been used. The experimental data have been taken from the PDG (Ref. [12]).

$M'\to M''\gamma$	$g_{M'M''\gamma}$ (GeV^{-1})	$\Gamma_{\rm th}$ (MeV)	$\Gamma_{\rm expt}$ (MeV)
$\rho^+\to\pi^+\gamma$	0.783	0.076	0.068±0.007
$\omega\to\pi\gamma$	2.35	0.73	0.72±0.04
$\phi\to\pi\gamma$	0.135	0.0056	0.0058±0.0006
$\rho\to\eta\gamma$	1.91	0.059	0.058±0.011
$\omega\to\eta\gamma$	0.677	0.0087	0.0061±0.0025
$\phi\to\eta\gamma$	−0.692	0.0553	0.0567±0.0028
$\eta'\to\rho\gamma$	1.37	0.0675	0.062±0.007
$\eta'\to\omega\gamma$	0.40	0.0048	0.0062±0.0009
$\phi\to\eta'\gamma$	1.04	5.7×10^{-4}	
$K^{*+}\to K^+\gamma$	0.831	0.050	0.050±0.005
$K^{*0}\to K^0\gamma$	−1.274	0.117	0.117±0.010

$$g_{\rho\eta'\gamma}=\cos\delta_P(Q_u-Q_d)I(m)\ ,$$
$$g_{\omega\eta'\gamma}=\cos\delta_V\cos\delta_P(Q_u+Q_d)I(m)+\sin\delta_V\sin\delta_P 2Q_s I(m_s)\ , \quad (4.7)$$
$$g_{\phi\eta'\gamma}=-\sin\delta_V\cos\delta_P(Q_u+Q_d)I(m)+\cos\delta_V\sin\delta_P 2Q_s I(m_s)\ .$$

The corresponding results are shown in Table III. The comparison of theory and experiment for η' decays is not as favorable as before and additional experimental data would be helpful in order to decide if η' contains admixtures of heavy flavors and gluons.

The coupling constants for neutral and charged K^* decays are given by

$$g_{K^{*0}K^0\gamma}=Q_d I_3(m,m_s)+Q_s I_3(m_s,m)\ ,$$
$$g_{K^{*+}K^+\gamma}=Q_u I_3(m,m_s)+Q_s I_3(m_s,m)\ . \quad (4.8)$$

The charged decay rate depends sensitively upon the value of the s-quark mass and has been used to determine its range: $m_s=370\pm20$ MeV. The predicted rate for the neutral decay agrees with experiment as shown in Table III.

V. ELECTROWEAK FORM FACTORS OF π and K

The most general form of the hadronic matrix element of the vector current must be represented in terms of two form factors:

$$\langle \vec{P}'',00|\bar{q}''\gamma_\mu q'|\vec{P}',\ 00\rangle\sqrt{4P'^+P''^+}=F_+P_\mu+F_-Q_\mu\ . \quad (5.1)$$

The underlying hadronic structure is generated in the one-loop approximation by the graph of Fig. 3. We shall consider only the form factor F_+; using symbolic notation, it can be written as

$$F_+(Q^2)=H(m_1'm_1'',\ m_2;\ \beta',\ \beta'') \quad (5.2)$$

and the loop integral H has been calculated for $Q^2\le 0$ in Ref. [5] [Eq. (4.1)]. For low-momentum transfer we can parametrize the form factors derived from (5.2) in the following way:

$$F_M(Q^2)\simeq\frac{F_M(0)}{1-Q^2/\Lambda_1^2-Q^4/\Lambda_2^4},\quad M=+,\pi,K\ . \quad (5.3)$$

The parameters Λ_1, Λ_2 are determined from Eq. (5.2) by calculating the first and second derivative at zero-momentum transfer of the loop integral H. The mean-square radius is defined as $r_M^2=6/\Lambda_1^2$.

Let us first discuss the charge form factor of the pion, which is given by

$$F_\pi(Q^2)=(Q_u-Q_d)H(m,m,m\,;\ \beta,\beta)\ , \quad (5.4)$$

with $F_\pi(0)=1$. Using the parameters of Table I, Eq. (5.4) agrees with the available experimental data for spacelike momentum transfer, and for $\Lambda_{1,2}$ we find

$$\Lambda_1=721\ \text{MeV}(r_\pi^2=0.449\ \text{fm}^2),\quad \Lambda_2=1280\ \text{MeV}\ . \quad (5.5)$$

The minus sign of the Q^4 term in Eq. (5.3) is remarkable (for constituent-quark masses $m>275$ MeV the sign is positive). Our result is in excellent agreement with the result of a reanalysis [23] of the CERN, Orsay, and Novosibirsk data [24]. A fit on the basis of the parametrization (5.3) gave [23]

$$\Lambda_1=720\pm4\ \text{MeV}\quad (r_\pi^2=0.451\pm0.005\ \text{fm}^2)\ ,$$
$$\Lambda_2=1420\pm560\ \text{MeV}\ . \quad (5.6)$$

A similar analysis of the electromagnetic form factors of charged and neutral kaons based on the expression

$$F_{K^+,K^0}(Q^2)=Q_{u,d}H(m,m,m_s;\ \beta_{u\bar{s}},\beta_{u\bar{s}})-Q_sH(m_s,m_s,m;\beta_{u\bar{s}},\beta_{u\bar{s}})\ , \quad (5.7)$$

with $F_{K^+}(0)=1$ and $F_{K^0}(0)=0$, leads to the following result for the charged form factor:

$$\Lambda_1=845\ \text{MeV}\ (r_{K^+}^2=0.327\ \text{fm}^2),\quad \Lambda_2=1270\ \text{MeV}\ , \quad (5.8)$$

while $r_{K^0}^2=-0.045$ fm^2. The CERN data [25] led to

$$\Lambda_1=830\pm61\ \text{MeV}\ (r_{K^+}^2=0.34\pm0.05\ \text{fm}^2)\ , \quad (5.9)$$

and $r_{K^0}^2=-0.054\pm0.026$ fm^2. The result (5.9) was obtained after taking into account the estimated systematic error of 1% and happens to be insensitive to the functional form assumed for the kaon form factor.

For K_{e3} decay, one may neglect the electron mass in the calculation of the rate, which then depends only on the form factor F_+ defined in Eqs. (3.1), (3.2) and for which we obtain

$$F_+(0)=0.965\ ,$$
$$\Lambda_1=858\ \text{MeV}\ (r_+^2=0.318\ \text{fm}^2),\quad \Lambda_2=1850\ \text{MeV}\ . \quad (5.10)$$

The experimental data are usually analyzed in terms of a linear parametrization

$$F_+(Q^2)\simeq F_+(0)(1+\lambda_+Q^2/m_\pi^2) \quad (5.11)$$

and the averaged slope of the K_{e3}^0 data as given by the PDG [12] is $\lambda_+=0.0300\pm0.0016$, which would correspond to a mean-square radius $r_+^2=0.36\pm0.02$ fm^2. However, in view of the precise data, it is not correct to derive the radius from the linear approximation (5.11). This point is illustrated by the analysis of semileptonic decays of neutral kaons of Ref. [26]: If the parametrization (5.11) is used, the slope $\lambda_+=0.0306\pm0.0034$ is obtained from the K_{e3}^0 experiment of Ref. [26]. If the K_{e3}^0 data are fitted by a monopole approximation

$$F_+(Q^2)=\frac{F_+(0)}{1-Q^2/\Lambda_1^2}\ , \quad (5.12)$$

the result is [26]

$$\Lambda_1=835\pm40\ \text{MeV}\ (r_+^2=0.335\pm0.032\ \text{fm}^2)\ . \quad (5.13)$$

The quark model prediction (5.10) is consistent only with the experimental result (5.13).

The value for $F_+(0)$ given in Eq. (5.10) has been calculated for $m_s=370$ MeV. The uncertainty of the s-quark mass ($m_s=370\pm20$ MeV) leads to a range of values $0.959\le F_+(0)\le 0.967$. The quark model result agrees with the result of Leutwyler and Roos [37] who used chiral perturbation theory to estimate

$$F_+(0)=0.961\pm0.008\ . \tag{5.14}$$

We emphasize this independent confirmation of the value of $F_+(0)$ since the reliability of (5.14) has been questioned in Ref. [28] due to the uncertainties inherent in the chiral perturbation theory approach.

We have also performed an analysis of K_{e3} decays on the basis of the quark model form factor (5.10) in order to determine the value V_{us} of the KM quark-mixing matrix. We give our value together with the result of Leutwyler and Roos:

$$V_{us}=\begin{cases}0.2199\pm0.0017 \text{ (this work and Ref. [29])}\ ,\\ 0.2196\pm0.0023 \text{ (Ref. [27])}\ .\end{cases} \tag{5.15}$$

Though the methods used to obtain the results of (5.15) are quite different in detail, the agreement is excellent.

VI. ANALYSIS OF THE DECAYS $P\to\gamma\gamma$

It seems obvious to treat the two-photon decays of pseudoscalar mesons with the method used in Sec. IV to investigate radiative transitions between pseudoscalar and vector mesons. This approach introduces in a natural way the $P-\gamma$ transition form factor $G_P(Q^2)$. The rate for the decay $P\to\gamma\gamma$ is

$$\Gamma=\frac{\pi}{4}\alpha^2 g_{P\gamma\gamma}^2 m_P^3,\quad g_{P\gamma\gamma}=G_P(0)\ . \tag{6.1}$$

The coupling constant is defined in terms of the transition form factor $G_P(Q^2)$, which is given by the matrix element of the electromagnetic current (3.5a) between the meson state and the on-shell photon state $|\vec{P}'',M\rangle$:

$$\langle\vec{P}'',M|j_\mu|\vec{P}',00\rangle\sqrt{4P'^+P''^+}=iG_P\epsilon_{\mu\nu\alpha\beta}\epsilon_\nu^*P'_\alpha P''_\beta\ , \tag{6.2}$$

where $\epsilon(M)$ is the transverse polarization vector of the photon and $Q=P'-P''$. The matrix element (6.2) is represented by the one-loop graph of Fig. 4. The transition form factor G_P can be expressed in terms of the corresponding loop integral I_4 which is very similar to the loop integral I_3 of Sec. IV:

$$\begin{aligned}G_\pi(Q^2)&=(Q_u^2-Q_d^2)\sqrt{2}I_4(m,m)\ ,\\ G_\eta(Q^2)&=-\sin\delta_P(Q_u^2+Q_d^2)\sqrt{2}I_4(m,m)\\ &\quad-\cos\delta_P Q_s^2 2I_4(m_s,m_s)\ ,\\ G_{\eta'}(Q^2)&=\cos\delta_P(Q_u^2+Q_d^2)\sqrt{2}I_4(m,m)\\ &\quad-\sin\delta_P Q_s^2 2I_4(m_s,m_s)\ ,\end{aligned} \tag{6.3}$$

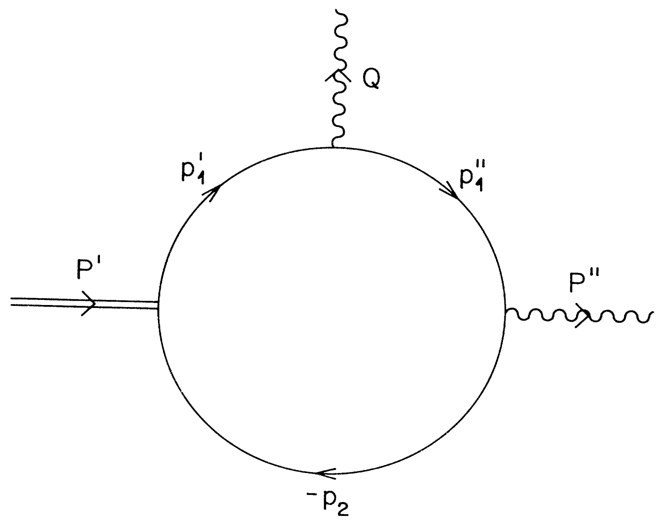

FIG. 4. One-loop Feynman diagram representing the hadronic structure of the amplitude for the two-photon decay of a meson, with $Q=P'-P''$ and $m_1''=m_2$.

$$\begin{aligned}I_4(m_1,m_2)=&\frac{N_c}{(2\pi)^3}\int d^3p\,\phi(p)\left[\frac{M_0}{E_1E_2}\right]^{1/2}\\ &\times\frac{2}{[M_0^2-(m_1-m_2)^2]^{1/2}}\frac{1-\xi}{p_\perp'^2+m_2^2}\\ &\times\left[(1-\xi)m_1+\xi m_2\right.\\ &\left.+\frac{m_1-m_2}{Q^2}\mathbf{p}_\perp\mathbf{Q}_\perp\right]\ ,\end{aligned} \tag{6.4}$$

where $\mathbf{p}_\perp'=\mathbf{p}_\perp-(1-\xi)\mathbf{Q}_\perp$ and $Q^2=-Q_\perp^2$ since we have to impose the condition $Q^+=0$. For low momentum transfer the form factors can be approximated by a monopole form

$$G_P(Q^2)=\frac{g_{P\gamma\gamma}}{1-Q^2/\Lambda_P^2}\ , \tag{6.5}$$

where the pole mass Λ_P is determined by the derivative of $G_P(Q^2)$ for $Q^2=0$. Our results for Λ_P are shown in Table IV and agree with the latest experimental results of Refs. [30,31]. Similar results have been obtained in the framework of the vector-dominance model [32]: $\Lambda_\pi=770$ MeV, $\Lambda_\eta=730$ MeV, $\Lambda_{\eta'}=820$ MeV.

TABLE IV. Transition form factor parameters Λ_P have been calculated for $\theta_P=-19°$. Experimental results of pole fits to the form factors from the CELLO (Ref. [30]) and TPC/2γ (Ref. [31]) Collaborations.

Λ_P (MeV)	Theory	CELLO	TPC/2γ
P: π^0	756	748±30	
η	730	839±63	700±80
η'	796	794±44	850±70

TABLE V. Radiative widths $\Gamma(P\rightarrow\gamma\gamma)$ have been calculated for $\theta_P=-19°$. The experimental data have been taken from Ref. [12] (PDG), Ref. [33] (A), and Ref. [30] (C).

P	one loop	Anomaly	Experiment
π	5.30 eV	7.73 eV	7.25±0.23 eV (A)
η	485 eV	485 eV	510±27 eV (PDG)
η'	2.90 keV	4.45 keV	4.51±0.26 keV (PDG) 3.62±0.14±0.48 keV (C)

The loop integrals for $Q^2=0$ are given by

$$Q^2=0:\ I_4(m,m)=0.482\ \text{GeV}^{-1}\ ,\quad I_4(m_s,m_s)=0.329\ \text{GeV}^{-1} \tag{6.6}$$

and the coupling constant $g_{P\gamma\gamma}$ can be evaluated, using Eq. (6.3). The values for the radiative widths (6.1) resulting from the one-loop approximation are shown in Table V and, except for η decay, disagree with the data. It seems that the hadronic structure of the neutral pseudoscalar mesons is not well enough approximated by the one-loop diagram of Fig. 4. Gluon-exchange effects introduce additional structure which may manifest itself as the formation of intermediate vector mesons. We suspect in particular that a mechanism analogous to flavor mixing of isoscalar $q\bar{q}$ states plays a dominant role. For $\tan\delta_P\simeq-\sqrt{2}$, η decay is not affected by such a mixing process, and this picture would explain why the one-loop approximation works so well in this case.

An alternative approach to determine the coupling constants is provided by the anomaly of the axial-vector current [34], which predicts the magnitude of two-photon decays [35]:

$$\begin{aligned} g_{\pi\gamma\gamma}&=\frac{1}{4\pi^2 f_\pi}\ ,\\ g_{\eta\gamma\gamma}&=\frac{1}{4\pi^2\sqrt{3}}\left[\frac{1}{f_8}\cos\theta_P-\frac{2\sqrt{2}}{f_0}\sin\theta_P\right],\\ g_{\eta'\gamma\gamma}&=\frac{1}{4\pi^2\sqrt{3}}\left[\frac{1}{f_8}\sin\theta_P+\frac{2\sqrt{2}}{f_0}\cos\theta_P\right]. \end{aligned} \tag{6.7}$$

The three pseudoscalar decay constants f_P can be calculated reliably in the quark model. They are defined by the matrix elements of the axial-vector currents:

$$A_\mu^i=\bar{q}\frac{\lambda_i}{2}\gamma_\mu\gamma_5 q,\quad i=0,1,\ldots,8\ ,$$

with $\lambda_0=(\frac{2}{3})^{1/2}\mathbb{1}$. The relevant matrix elements are

$$\begin{aligned} \langle 0|A_\mu^3|\vec{P},\ \pi^0\rangle\sqrt{2P^+}&=iP_\mu f_\pi\ ,\\ \langle 0|A_\mu^n|\vec{P},\ \eta_\eta\rangle\sqrt{2P^+}&=iP_\mu f_n,\quad n=0,\ 8\ . \end{aligned} \tag{6.8}$$

The states η_8 and η_0 are defined in analogy to Eq. (3.9), which leads to

$$\begin{aligned} f_8&=(f_\pi+2f_{s\bar{s}})/3\ ,\\ f_0&=(2f_\pi+f_{s\bar{s}})/3\ , \end{aligned} \tag{6.9}$$

where $f_{s\bar{s}}$ is given by the loop integral (3.3), represented by Fig. 2, with $m_1=m_2=m_s$ and the wave-function parameter $\beta_{s\bar{s}}$. Our results are (with $f_\pi=92.4$ MeV)

$$\begin{aligned} f_{s\bar{s}}&=114.00\pm2.3\ \text{MeV}.\\ f_8/f_\pi&=1.156\pm0.017\ ,\\ f_0/f_\pi&=1.078\pm0.008\ , \end{aligned} \tag{6.10}$$

and the corresponding decay rates [calculated with the central values of Eq. (6.10)] are shown in Table V. The errors assigned to the numbers in Eq. (6.10) reflect the uncertainty of the s-quark mass ($m_s=370\pm20$ MeV). This approach gives results which are consistent (within the assumptions of current algebra) with the data, perhaps except for η' decay where more precise data would be welcome in order to draw conclusions about possible admixtures of heavy-flavor $q\bar{q}$ states or gluonium.

The decay constant f_8 has been determined before in chiral perturbation theory. Gasser and Leutwyler [35] have calculated the first nonleading term in the quark mass expansion of the decay constants in the pseudoscalar octet with the result

$$\begin{aligned} \langle 0|A_\mu^8|\vec{P},\ \eta\rangle\sqrt{2P^+}&=iP_\mu f_\eta\ ,\\ f_\eta/f_\pi&=1.3\pm0.05\ . \end{aligned} \tag{6.11}$$

At this level of approximation there is no η-η' mixing and consequently $f_\eta=f_8$. It is not known how the terms of second order, in particular the effect of η-η' mixing, would change the value of f_η given in (6.11). An independent calculation has been reported in Ref. [36] with the result $f_8/f_\pi=1.25$, which is different from (6.11) due to additional assumptions made in Ref. [36]. The disagreement between the quark-model results (6.10) and the first-order chiral perturbation theory result (6.11) cannot be reconciled by any admissible modification of the values of our parameters.

To our knowledge f_0 has not been calculated previously, but has been used as a free parameter together with the mixing angle θ_P in the analysis [36,37] of two photon decays of η and η'. The empirical result $f_0/f_\pi=1.04\pm0.04\pm0.05$ is in agreement with (6.10).

VII. CONCLUSIONS

The relativistic quark model provides a framework, wherein we have overall an excellent and consistent picture of electroweak transitions of pseudoscalar and vector mesons in the u-, d-, s-quark sector. The data can be fitted in terms of only five parameters, and though we have considered a great variety of processes, the agreement of the predictions with experiment (within the errors) apparently requires no additional structure of the constituent quarks. This simple picture differs from the present quark model description of baryons, where nonvanishing Pauli magnetic moments of the constituent quarks are needed in order to achieve a better fit to the data.

The extension of this approach to include c and b quarks should allow a reliable treatment of electroweak phenomena in the heavy-meson sector [5,6], once the respective constituent-quark masses and wave-function parameters can be determined by comparison with the data.

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