Photoproduction signatures of hybrid baryons: An application of the quark model with gluonic degrees of freedom

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An application of the quark model with gluonic degrees of freedom to hybrid baryons is discussed. The photon-gluon transition operator for the process $\gamma q \rightarrow qG$ is added to the electromagnetic interaction, where it plays an explicit role in quark-gluon systems. The successes of the three-valence-quark model in describing the magnetic moments of baryons is preserved in the new model. It is shown that the Barnes-Close selection rule for hybrid baryons (q^3G states) is equivalent to the Moorhouse selection rule for conventional P-wave baryons (q^3 states). We predict a gluonic partner of the nucleon with the same ratio of magnetic moments. It is possible that the Roper resonance $P_{11}(1440)$ is such a state. The phenomenological implications of such an assignment and important tests accessible to forthcoming photoproduction experiments at CEBAF are discussed.

I. INTRODUCTION

Historically, most investigations of hybrid (quark plus gluon) resonances were carried out in the context of the bag model [1,2]. A general conclusion of these studies is that the hybrid states q^3G or $q\bar{q}G$ should exist in nature. Some more specific conclusions about the valence gluon in these studies are as follows: the confined gluon may be divided into TE (tranverse electric) modes with $J^{PC}=1^{+-}$, whose color-magnetic field is proportional to $\mathbf{k}_g \times \epsilon_g$ in momentum space, and TM (transverse magnetic) modes with $J^{PC}=1^{--}$, whose color-electric field is proportional to $[\epsilon_g-(\mathbf{n}_g\cdot\epsilon_g)\mathbf{n}_g]$. A TE mode is the lowest eigenmode. As a consequence of the spin-color correlation, the totally antisymmetric states with three quarks and a gluon span a **70** of SU(6). The lightest q^3G basis states have positive parity and are composed of three valence quarks and a TE gluon [3].

There are two aspects to an investigation of the role of gluonic degrees of freedom in spectroscopy. The first is simply a search for "extra" states in nature that cannot be explained by the conventional quark potential model. For example, in the meson sector a $q\bar{q}G$ state may have $J^{PC}=1^{-+}$, which cannot be constructed from $q\bar{q}$ alone. In the baryon sector, the spectrum expected for q^3G states is quite different from that of the conventional q^3 baryon resonances. The second aspect involves a study of transition properties, such as selection rules, which might provide us a signature of gluonic degrees of freedom. One such signature was found by Barnes and Close [4], who showed that for bag-model wave functions the photoexcitation of the lightest hybrid state is strongly suppressed by a proton target but allowed by a neutron target.

In this paper we investigate some implications and phenomenological consequences of the presence of gluonic degrees of freedom in baryon wave functions. As gluonic degrees of freedom are clearly present in baryon wave functions, the process $\gamma q \rightarrow qG$ is expected to play an important role in photoexcitation and other electromagnetic processes involving baryon resonances. In Sec. II we derive the photon-gluon transition operator for the process $\gamma q \rightarrow qG$ in the nonrelativistic limit; this operator provides the corrections to the electromagnetic current at $O(\sqrt{\alpha_s})$. Furthermore, q^3 and q^3G states are strongly mixed in physical baryon resonances because of the quark-gluon coupling, but the isospin and flavor structure remain unchanged, and therefore the ratios of magnetic moments will be the same as in the conventional q^3 quark model, namely, $\mu_p/\mu_n=-\frac{3}{2}$. This property has been noted in the literature [4,5] for the ratio of proton and neutron magnetic moments. In Sec. III we show that a corresponding result is true in general for other baryon states in the **56** multiplet. So, the success of the q^3 quark model in describing the magnetic moments is preserved when q^3G components are incorporated in their wave functions.

In Sec. IV we suggest that there should be a gluonic partner state of the nucleon, with the same flavor and isospin structure, and the same ratio of magnetic moments as the proton and neutron. The ratio of photoproduction amplitudes of this state from p and n is also predicted to be $-\frac{3}{2}$. This follows because of strong mixing between the ^{4}**8** and ^{2}**8** wave functions, so that the Barnes-Close selection rule (namely that ^{4}**8** hybrid states are not photoproduced from protons) does not apply. This is consistent with the photoproduction data on the Roper resonance and resurrects the possibility [2,3] that the Roper is a hybrid state. If this is indeed the case, we expect associated $P_{31}(1550)$ and $P_{13}(1540)$, especially $P_{31}(1550)$, states to occur, and we predict their photoexcitation amplitudes which can be tested in CEBAF experiments. If this picture is indeed correct, the calculated photoproduction amplitudes also suggest that the resonance $P_{11}(1710)$, rather than the Roper resonances, is the q^3 radial excitation of the nucleon: this assignment may help to avoid problems with potential models for which the low mass of $P_{11}(1440)$ has been criticized.

II. THE PHOTON-GLUON TRANSITION OPERATOR

The photon-gluon transition amplitude (Fig. 1) for a bound system can be written by analogy with the QED Compton amplitude as [6]

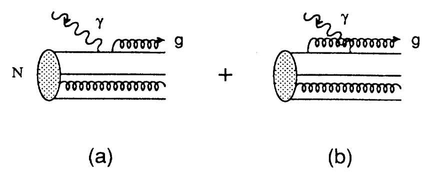


FIG. 1. The diagram for the process $\gamma q \to qG$.

$$M_{fi}=\delta(E_f+\omega_g-E_i-\omega)\sum_j \left[\frac{\langle f,\mathbf{k}_g\epsilon_g|H_{\rm QCD}|j\rangle\langle j|H_{\rm em}|i,\mathbf{k}\epsilon\rangle}{E_f+\omega_g-E_j} + \frac{\langle f,\mathbf{k}_g\epsilon_g|H_{\rm em}|j,\mathbf{k}_g\epsilon_g\,\mathbf{k}\epsilon\rangle\langle j,\mathbf{k}_g\epsilon_g\,\mathbf{k}\epsilon|H_{\rm QCD}|i,\mathbf{k}\epsilon\rangle}{E_i-\omega_g-E_j} \right], \tag{2.1}$$

where $\mathbf{k}_g$ and ϵ_g are the gluon momentum and polarization vectors, which when combined with three quarks form the hybrid baryon state. The total energy of the final state $|f,\mathbf{k}_g\epsilon_g\rangle$ is $E_f+\omega_g$. The quark-gluon interaction $H_{\rm QCD}$ and electromagnetic interaction can be written as

$$H_{\rm QCD}=-\sum_i g_s\frac{\lambda_i^a}{2}\boldsymbol{\alpha}_i\cdot\mathbf{A}_G^a+\cdots \tag{2.2}$$

and

$$H_{em}=-\sum_i e_i\boldsymbol{\alpha}_i\cdot\mathbf{A}(\mathbf{r}_i)\ , \tag{2.3}$$

respectively, where λ_i^a and $\mathbf{A}_G^a$ are SU(3)-color matrices and gluon fields, and $\boldsymbol{\alpha}_i$ are the Dirac matrices. The electromagnetic field $\mathbf{A}$ in Eq. (2.3) has the form

$$\mathbf{A}_{\mathbf{k}}(r_i)=\sqrt{4\pi}\left[\frac{1}{2\omega}\right]^{1/2}\epsilon e^{i\mathbf{k}\cdot\mathbf{r}_i}\ . \tag{2.4}$$

Following the procedure adopted by Brodsky and Primack [7], one can rewrite Eq. (2.1) as

$$M_{fi}=\delta(E_f+\omega_g-E_i-\omega)\times\sum_j\left[\frac{\langle f,\mathbf{k}_g\epsilon_g|h_{\rm QCD}|j\rangle\langle j|h_{em}|i\rangle}{E_f+\omega_g-E_j}+\frac{\langle f,\mathbf{k}_g\epsilon_g|h_{em}|j\rangle\langle j|h_{\rm QCD}|i\rangle}{E_i-\omega_g-E_j}\right], \tag{2.5}$$

where

$$h_{\rm QCD}=\sum_i g_s\frac{\lambda_i^a}{2}(\omega_g\mathbf{r}_i\cdot\mathbf{A}_G^a-\mathbf{r}_i\cdot\mathbf{A}_G^a\,\alpha_i\cdot\mathbf{k}_g) \tag{2.6}$$

and

$$h_{em}=\sum_i e_i(\omega\mathbf{r}_i\cdot\mathbf{A}-\mathbf{r}_i\cdot\mathbf{A}\,\boldsymbol{\alpha}_i\cdot\mathbf{k})\ . \tag{2.7}$$

One may then take the nonrelativistic limit of these results. h_{em} in the nonrealtivistic limit [7,8] is

$$h_{em}^{\rm NR}=\sum_{i=1}^{3}\left[e_i\mathbf{r}_i\cdot\mathbf{E}_i+i\frac{e_i}{2m_i^*}(\mathbf{p}_i\cdot\mathbf{k}\mathbf{r}_i\cdot\mathbf{A}_i+\mathbf{r}_i\cdot\mathbf{A}_i\mathbf{p}_i\cdot\mathbf{k})-\mu_i\boldsymbol{\sigma}_i\cdot\mathbf{B}_i-\frac{1}{2}\left[2\mu_i-\frac{e_i}{2m_i^*}\right]\boldsymbol{\sigma}_i\cdot\left[\mathbf{E}_i\times\frac{\mathbf{p}_i}{2m_i^*}-\frac{\mathbf{p}_i}{2m_i^*}\times\mathbf{E}_i\right]+e_i\phi_i\right]+\sum_{i<j}\frac{1}{4M_T}\left[\frac{\boldsymbol{\sigma}_i}{m_i^*}-\frac{\boldsymbol{\sigma}_j}{m_j^*}\right]\cdot(e_j\mathbf{E}_j\times\mathbf{p}_i-e_i\mathbf{E}_i\times\mathbf{p}_j)\ . \tag{2.8}$$

We divide $h_{\rm QCD}$ into terms containing creation operators for TE and TM gluons. Because the lowest-lying hybrid baryon states are expected to have positive parity, the relevant valence gluon for positive-parity baryons is TE. The corresponding TE transition operator in the nonrelativistic limit is the color-magnetic interaction

$$h_{\rm QCD}^{\rm TE}=\sum_j ig_s\frac{\lambda_i^a}{2}\frac{1}{2m_j}\boldsymbol{\sigma}_j\cdot(\mathbf{k}_g\times\mathbf{A}_G^a)=\sum_j\boldsymbol{\sigma}_j\cdot\mathbf{J}_j^{\rm TE}\ , \tag{2.9}$$

where we define

$$\mathbf{J}_j^{\rm TE}=ig_s\frac{\lambda_i^a}{2}\frac{1}{2m_j}(\mathbf{k}_g\times\mathbf{A}_G^a)\ .$$

One can formally write Eq. (2.5) as

$$M_{fi}=\delta(E_f+\omega_g-E_i-\omega)\times\sum_j\left[\left\langle f,\mathbf{k}_g\epsilon_g\left|h_{\rm QCD}\frac{1}{E_f+\omega_g-\hat{H}}\right|j\right\rangle\langle j|h_{em}|i\rangle+\left\langle f,\mathbf{k}_g\epsilon_g\left|h_{\rm em}\frac{1}{E_i-\omega_g-\hat{H}}\right|j\right\rangle\langle j|h_{\rm QCD}|i\rangle\right], \tag{2.10}$$

where $\hat{H}$ is the nonrelativistic Hamiltonian for a three-quark system:

$$\hat{H}=\sum_i\frac{\mathbf{p}_i^2}{2m_i}+\sum_{i,j}V(\mathbf{r}_i-\mathbf{r}_j)\ . \tag{2.11}$$

Using the closure relation

$$\sum_j|j\rangle\langle j|=I\ , \tag{2.12}$$

we have

$$M_{fi}=\delta(E_f+\omega_g-E_i-\omega)\times\left[\left\langle f,\mathbf{k}_g\epsilon_g\left|h_{\rm QCD}\frac{1}{E_f+\omega_g-\hat{H}}h_{\rm em}+h_{\rm em}\frac{1}{E_i-\omega_g-\hat{H}}h_{\rm QCD}\right|i\right\rangle\right]\ . \tag{2.13}$$

On expanding the operators $1/(E_f-\omega_g-\hat{H})$ and $1/(E_i-\omega_g-\hat{H})$, we find

$$M_{fi}=\delta(E_f+\omega_g-E_i-\omega)\times\sum_n(-1)^n\left[\left\langle f,\mathbf{k}_g\epsilon_g\left|h_{\rm QCD}\frac{(E_f-\hat{H})^n}{\omega_g^{n+1}}h_{\rm em}-h_{\rm em}\frac{(\hat{H}-E_i)^n}{\omega_g^{n+1}}h_{\rm QCD}\right|i\right\rangle\right]; \tag{2.14}$$

now note that

$$(\hat{H}-E_i)h_{\rm QCD}|i\rangle=[\hat{H},h_{\rm QCD}]|i\rangle$$

and

$$\langle f|h_{\rm QCD}(E_f-\hat{H})=\langle f|[\hat{H},h_{\rm QCD}]\ , \tag{2.15}$$

which leads to our general expression for the transition operator,

$$\mathcal{O}=\sum_n\frac{(-1)^n}{\omega_g^{n+1}}[[\hat{H},\ldots[\hat{H},h_{\rm QCD}]]_n,h_{\rm em}]\ . \tag{2.16}$$

If the binding potential in Eq. (2.11) is relatively weak, only the first few terms in the summation of Eq. (2.16) are required for accurate results, as the remaining terms are higher-order relativistic corrections. For TE gluons, the lowest-order transition operator is

$$\mathcal{O}^{\rm TE}=\frac{1}{\omega_g}[h_{\rm QCD}^{\rm TE},h_{\rm em}]\ , \tag{2.17}$$

which gives

$$\mathcal{O}^{\rm TE}=\sum_j ig_s\frac{e_j\lambda_j}{4m_j^2}\boldsymbol{\sigma}_j\cdot\left[\frac{(\mathbf{k}_g\times\mathbf{A}_G)}{\omega_g}\times(\mathbf{k}\times\mathbf{A})\right]+O\left[\frac{1}{m_j^3}\right]=-i\sum_j\frac{e_j}{m_j}\left[\boldsymbol{\sigma}_j\times\frac{\mathbf{J}_j^{\rm TE}}{\omega_g}\right]\cdot\mathbf{B}+O\left[\frac{1}{m_j^3}\right]\ . \tag{2.18}$$

The transition operator in Eq. (2.18) correlates the magnetic interactions of gluons and photons. Similarly, one can derive the transition operator for TM gluons, which is

$$\mathcal{O}^{\rm TM}=\frac{1}{\omega_g}\left[\left[h_{\rm QCD}-\frac{1}{\omega_g}[\hat{H},h_{\rm QCD}]\right],h_{\rm em}\right]=\sum_j g_s\frac{e_j\lambda_j^a}{2m_j}\mathbf{A}(\mathbf{r}_j)\cdot\mathbf{A}_G^a+O\left[\frac{1}{m_j^2}\right]\ . \tag{2.19}$$

This is a photon-gluon analogue of the Thompson term in QED Compton scattering, and correlates the electric dipole interactions of gluons and photons.

III. THE GLUONIC COMPONENTS IN NUCLEON WAVE FUNCTIONS

Let ϕ, χ, and ψ denote flavor, spin, and color wave functions for three quarks and let superscripts S, ρ, λ, and a denote the permutation symmetry. [$S(a)$ is totally symmetric (antisymmetric) under any exchange among the three quarks, and $\lambda(\rho)$ is symmetric (antisymmetric) under the (12) exchange.] The nonstrange q^3 states are the usual totally symmetric combinations [9]

$$|N_0\rangle=\frac{1}{\sqrt{2}}(\phi^\rho\chi^\rho+\phi^\lambda\chi^\lambda)\psi^a \tag{3.1}$$

and

$$|\Delta_0\rangle=\phi^S\chi^X\psi^a\ . \tag{3.2}$$

The totally antisymmetric q^3G states transform as a **70** of SU(6), and the nonstrange members are explicitly

$$|{}^2N_g\rangle=\tfrac{1}{2}[(\phi^\rho\chi^\rho-\phi^\lambda\chi^\lambda)\psi^\rho-(\phi^\rho\chi^\lambda+\phi^\lambda\chi^\rho)\psi^\lambda]|G\rangle\ , \tag{3.3}$$

$$|{}^4N_g\rangle=\frac{1}{\sqrt{2}}(\phi^\lambda\psi^\rho-\phi^\rho\psi^\lambda)\chi^S|G\rangle\ , \tag{3.4}$$

and

$$|\Delta_g\rangle=\frac{1}{\sqrt{2}}(\chi^\lambda\psi^\rho-\chi^\rho\psi^\lambda)\phi^S|G\rangle\ . \tag{3.5}$$

The superscripts 2, 4 denote the total quark spin as $2S+1$.

Because of the quark-gluon vertex [Eq. (2.9) for the TE gluon], the nucleon wave functions should be linear combinations of q^3 and q^3G components, and are given by

$$|N\rangle=|N_0\rangle+\sum_{N_g}\frac{1}{E_{N_0}-E_{N_g}}|N_g\rangle\langle N_g|h_{\rm QCD}^{\rm TE}|N_0\rangle\ , \tag{3.6}$$

where E_{N_0} (E_{N_g}) denotes the unperturbed energy of the q^3 (q^3G) basis state, and $E_{N_0}-E_{N_g}=-\omega_g$. Defining

$$\delta=\sqrt{6}\left\langle \psi^{\rho}, G \left|\left| \frac{\mathbf{J}_3^{\mathrm{TE}}}{\omega_g} \right|\right| \psi^{a} \right\rangle , \tag{3.7}$$

we have

$$\langle {}^2N_g | h_{\mathrm{QCD}}^{\mathrm{TE}} | N_0 \rangle = \langle {}^4N_g | h_{\mathrm{QCD}}^{\mathrm{TE}} | N_0 \rangle = \delta . \tag{3.8}$$

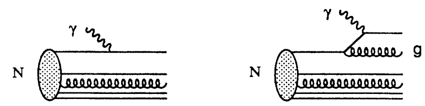
See the Appendix for more detail. To leading order in δ the normalized nucleon wave function is therefore

$$|N\rangle = \frac{1}{\sqrt{1+2\delta^2}}[|N_0\rangle - \delta(|{}^4N_g\rangle + |{}^2N_g\rangle)] . \tag{3.9}$$

The magnetic interaction for such a quark-gluon system is

$$\mathbf{J}_t^{\mathrm{em}}\cdot\mathbf{B} = \sum_j \frac{e_j}{2m_j}\left[\boldsymbol{\sigma}_j + i2\left[\boldsymbol{\sigma}_j \times \frac{\mathbf{J}_j^{\mathrm{TE}}}{\omega_g}\right]\right]\cdot\mathbf{B} , \tag{3.10}$$

where the first term is the magnetic interaction [Fig. 2(a)]



(a) (b)

FIG. 2. The electromagnetic interaction for quark-gluon systems: (a) the photon-quark interaction and (b) gluon radiation in the process $\gamma q \rightarrow qG$.

(as in Ref. [4]), and the second term gives a correlation [see Eq. (2.18)] between the radiated gluon and absorbed photon [Fig. 2(b)]; this contribution was absent in Ref. [4]. For neutrons and protons, this gives

$$\mu = \mu_0 + \mu_1 , \tag{3.11}$$

where

$$\mu_0 = \frac{1}{\sqrt{1+2\delta^2}}\left[\left\langle N_0 \left| \sum_j \frac{e_j}{2m_j}\sigma_j^z \right| N_0 \right\rangle + \delta^2 \left(\left\langle {}^4N_g \left| \sum_j \frac{e_j}{2m_j}\sigma_j^z \right|^4 N_g \right\rangle + \left\langle {}^2N_g \left| \sum_j \frac{e_j}{2m_j}\sigma_j^z \right|^2 N_g \right\rangle + 2\left\langle {}^2N_g \left| \sum_j \frac{e_j}{2m_j}\sigma_j^z \right|^4 N_g \right\rangle \right)\right] \tag{3.12}$$

and

$$\mu_1 = \frac{-\delta}{\sqrt{1+2\delta^2}}\left[\left\langle {}^2N_g \left| \sum_j \frac{e_j}{m_j} i \left[\boldsymbol{\sigma}_j \times \frac{\mathbf{J}_j^{\mathrm{TE}}}{\omega_g}\right] \right| N_0 \right\rangle + \left\langle {}^4N_g \left| \sum_j \frac{e_j}{m_j} i \left[\boldsymbol{\sigma}_j \times \frac{\mathbf{J}_j^{\mathrm{TE}}}{\omega_g}\right] \right| N_0 \right\rangle\right] . \tag{3.13}$$

The matrix element

$$\left\langle N_0 \left| \sum_j \frac{e_j}{m_j} i(\boldsymbol{\sigma}_j \times \mathbf{J}_j^{\mathrm{TE}}) \right|^2 N_g \right\rangle$$

represents the radiation of both gluon and photon, i.e., the process $\gamma G q \rightarrow q$, which vanishes for the operator

$$\sum_j \frac{e_j}{m_j} i [\boldsymbol{\sigma}_j \times \mathbf{J}_j^{\mathrm{TE}}] .$$

The proton magnetic moment is

$$\mu_0^p = \mu_0 \left[\frac{1+\frac{4}{3}\delta^2}{1+2\delta^2}\right] \tag{3.14}$$

[Ref. [4] quotes $\mu_0(1-\frac{2}{3}\delta^2)$, which is an $O(\delta^2)$ approximation to Eq. (3.11)], and

$$\mu_1^p = -\mu_0 \left[\frac{\frac{2}{3}\delta^2}{1+2\delta^2}\right] . \tag{3.15}$$

Hence to $O(\delta^2)$, the magnetic moment of the proton is

$$\mu^p = \mu_0^p + \mu_1^p = \mu_0 \left[\frac{1+\frac{2}{3}\delta^2}{1+2\delta^2}\right] \approx \mu_0(1-\tfrac{4}{3}\delta^2) . \tag{3.16}$$

The relation $\mu^n = -\frac{2}{3}\mu^p$ is preserved when the q^3G states are incorporated, as has been noted previously [4,5]. Similarly, for the Δ states,

$$|\Delta\rangle = \frac{1}{\sqrt{1+\delta^2}}(|\Delta_0\rangle + \delta|\Delta_g\rangle) ; \tag{3.17}$$

the corresponding magnetic moment is

$$\mu^{\Delta^{++}} = \mu_0^{\Delta^{++}} + \mu_1^{\Delta^{++}} \approx 2\mu_0(1-\tfrac{4}{3}\delta^2) , \tag{3.18}$$

so that the ratio $\mu^{\Delta^{++}}/\mu^p = 2$ is preserved to $O(\delta^2)$.

Generally, one can prove that the magnetic moments of the **56** multiplet of SU(6) are all multiplied by a factor of $(1-\frac{4}{3}\delta^2)$ when the q^3G basis states are incorporated in their wave functions. The underlying physical reason is that the isospin and flavor structure of the SU(6) representation is unaltered by the gluonic degrees of freedom, because the quark-gluon vertex of Eq. (2.9) depends only on spin and color operators. Therefore the addition of gluonic degrees freedom to q^3 systems should not affect the ratios of magnetic moments. The successes of the quark model in describing static electromagnetic couplings are therefore preserved in this approach. The gluons do, however, lead to extra states which can be distinguished from the q^3 SU(6)×O(3) states. We will discuss this in the next section.

IV. EXCITED STATES WITH GLUONIC DEGREES OF FREEDOM

Given our approximations, there is a state whose wave function is

$$|N',J=\tfrac{1}{2}\rangle=\left[\frac{2}{1+2\delta^2}\right]^{1/2}[\delta|N_0\rangle+\tfrac{1}{2}(|^2N_g\rangle+|^4N_g\rangle)]\ , \tag{4.1}$$

which is the orthogonal partner of the ground state of Eq. (3.9). The corresponding magnetic moments for these states are

$$\mu_{N'}=\begin{cases}\frac{4}{3}\mu_0\ , & I_z=+\frac{1}{2}\ ,\\ -\frac{8}{9}\mu_0\ , & I_z=-\frac{1}{2}\ .\end{cases} \tag{4.2}$$

Note that $\mu_{p'}/\mu_{n'}=-\frac{3}{2}$ for this state. Similarly, the state

$$|\Delta',J=\tfrac{3}{2}\rangle=\frac{1}{\sqrt{1+\delta^2}}(|\Delta_g\rangle-\delta|\Delta_0\rangle) \tag{4.3}$$

is a partner of the Δ resonance $P_{33}(1232)$, and has a magnetic moment of

$$\mu_{\Delta++'}\approx\tfrac{2}{3}(1+\tfrac{2}{3}\delta^2)\mu_0\ . \tag{4.4}$$

We therefore take the linear combinations of the unmixed hybrid states to be

$$|N',J=\tfrac{3}{2}\rangle=\frac{1}{\sqrt{2}}(|^4N_g\rangle+|^2N_g\rangle)\ , \tag{4.5}$$

$$|N'',J=\tfrac{1}{2},\tfrac{3}{2}\rangle=\frac{1}{\sqrt{2}}(|^2N_g\rangle-|^4N_g\rangle)\ , \tag{4.6}$$

$$|N'',J=\tfrac{5}{2}\rangle=|^4N_g\rangle\ , \tag{4.7}$$

and

$$|\Delta',J=\tfrac{1}{2}\rangle=|\Delta_g\rangle\ . \tag{4.8}$$

To study the transition properties of these states, we give the matrix elements of the operator

$$\mathcal{O}_z^{\rm TE}=\sum_j\frac{e_j}{m_j}i\left[\sigma_j\times\frac{\mathbf{J}_j^{t\rm E}}{\omega_g}\right]$$

between the states $|N_0\rangle$ and $|N_g\rangle$ in Table I. It is interesting to note that

$$\langle{}^4N_g|\mathcal{O}_z^{\rm TE}|N_0\rangle=0\ , \tag{4.9}$$

which was first obtained by Barnes and Close in Ref. [4] in their calculation of magnetic dipole transitions between these states. To make this result more transparent one can write the matrix element of Eq. (4.9) explicitly as

$$\langle{}^4N_g|\mathcal{O}_z^{\rm TE}|N_0\rangle=\left\langle{}^4N_g\left|\sum_j\frac{e_j}{m_j}i\left[\sigma_j\times\frac{\mathbf{J}_j^{\rm TE}}{\omega_g}\right]\right|N_0\right\rangle$$
$$=3\left\langle{}^4N_g\left|\frac{e_3}{m_3}i\left[\sigma_3\times\frac{\mathbf{J}_3^{\rm TE}}{\omega_g}\right]\right|N_0\right\rangle \tag{4.10}$$

TABLE I. Transition matrix elements for the operator $\mathcal{O}^{\rm TE}$ between the state $|N_0\rangle$ and hybrid states. The full matrix elements are obtained by multiplying the entries in this table by a factor of $\mu_0\delta$. In all cases $A_{3/2}=\sqrt{3}A_{1/2}$, characteristic of magnetic dipole transition.

States	$A^p_{1/2}$	$A^n_{1/2}$
$\|^2N_g,J=\frac{1}{2}\rangle$	$\frac{2}{3}$	$-\frac{2}{9}$
$\|^2N_g,J=\frac{3}{2}\rangle$	$-\frac{1}{3}\sqrt{2}$	$\frac{1}{9}\sqrt{2}$
$\|^4N_g,J=\frac{1}{2}\rangle$	0	$-\frac{2}{9}$
$\|^4N_g,J=\frac{3}{2}\rangle$	0	$-\frac{2}{9}\sqrt{5}$
$\|^4N_g,J=\frac{5}{2}\rangle$	0	0
$\|^2\Delta_g,J=\frac{1}{2}\rangle$	$\frac{2}{9}$	$\frac{2}{9}$
$\|^2\Delta_g,J=\frac{3}{2}\rangle$	$-\frac{1}{9}\sqrt{2}$	$-\frac{1}{9}\sqrt{2}$

because of the permutation symmetry. On substituting the explicit expressions for $|N_0\rangle$ and $|^4N_g\rangle$ into Eq. (4.10), we find

$$\langle{}^4N_g|\mathcal{O}_z^{\rm TE}|N_0\rangle$$
$$=i\frac{3}{m_3}\langle\phi^\lambda|e_3|\phi^\lambda\rangle\left\langle\chi^S G\psi^\rho\left|\left[\sigma_3\times\frac{\mathbf{J}_3^{\rm TE}}{\omega_g}\right]\right|\chi^\lambda\psi^a\right\rangle\ . \tag{4.11}$$

[The other terms in Eq. (4.10) vanish because the wave functions belong to different permutation symmetries.] Note that this matrix element is proportional to $\langle\phi^\lambda|e_3|\phi^\lambda\rangle$, and hence the fact that

$$\langle\phi^\lambda|e_3|\phi^\lambda\rangle=0 \tag{4.12}$$

for protons immediately gives the Barnes-Close selection rule [Eq. (4.9)]. This also shows an equivalence between the Barnes-Close selection rule and the Moorhouse selection rule [10] for P-wave baryons, which also follows from Eq. (4.12). The Moorhouse selection rule, however, is broken by relativistic effects, in particular by the nonadditive term in Eq. (2.8). For this reason one might also expect this to be the case for transitions between the states $|N_0\rangle$ and $|N_G\rangle$, for which there should be two transition operators: (1) the magnetic dipole transition which is calculated by Barnes and Close [4], and (2) the photon-gluon transition operator shown in Table I. The relativistic corrections to the magnetic dipole transition are well understood [8], but relativistic corrections to the operator $\mathcal{O}^{\rm TE}$ require further investigation.

The photoproduction amplitudes are determined by the electromagnetic current$\mathbf{J}_t^{\rm em}$ of Eq. (3.10). On substituting the explicit expression for $\mathbf{A}$ [Eq. (2.4)], we find that the helicity amplitude can be written as

$$A_\lambda=3\sqrt{\pi k}\,\mu_0\left\langle N,J,\lambda\left|e_3\left[\sigma_3^+ + 2i\left[\sigma_3\times\frac{\mathbf{J}_3^{\rm TE}}{\omega_g}\right]^+\right]e^{ikz_3}\right|N,J=\tfrac{1}{2},\lambda-1\right\rangle , \qquad (4.13)$$

where $\sigma^+=\sigma_x+i\sigma_y$. The explicit expressions for helicity amplitudes A_λ to $O(\delta^2)$ for the states defined in Eqs. (4.1)–(4.8) are given in Table II.

For the Δ resonance, the helicity amplitude $A_{1/2}$ is

$$A_{1/2}=-\frac{e}{2m_q}\frac{2\sqrt{2}}{3}\left[\frac{1+\frac{2}{3}\delta^2}{\sqrt{(1+2\delta^2)(1+\delta^2)}}\right]$$
$$\approx-\frac{e}{2m_q}\frac{2\sqrt{2}}{3}(1-\tfrac{5}{6}\delta^2) . \qquad (4.14)$$

Substitution of the mangetic moment of the proton,

$$\mu=\frac{e}{2m_q}(1-\tfrac{4}{3}\delta^2) ,$$

into Eq. (4.14) gives the result quoted in Table II.

Notice that the helicity amplitudes for the partner state of the nucleon defined in Eq. (4.1) satisfy the relation

$$A_{1/2}^p=-\tfrac{3}{2}A_{1/2}^n . \qquad (4.15)$$

This follows because the isospin (or flavor) and spin correlation in the state $|N',J=\frac{1}{2}\rangle$ is the same as in the nucleon, and therefore the ratio of magnetic moments and the $M1$ transition amplitudes from p and n to the partner states are the same. This also shows that it may be premature to take the Barnes-Close selection rule as a signature for the lowest hybrid baryon state, as it neglects any mixing between the states $|^4N_g\rangle$ and $|^2N_g\rangle$. (This was justified in the bag model because the hybrid state $|^4N_g\rangle$ has the lowest energy [3].) Equation (4.14) is consistent with photoproduction data for the Roper resonance $P_{11}(1440)$, which suggests that the Roper resonance may be the proposed *gluonic partner* of the nucleon.

To determine whether the Roper resonance $P_{11}(1440)$ is dominantly a hybrid or a radial excitation one should compare its observed mass and couplings with the predictions of the q^3 quark potential model. The mass of this state is a long-standing puzzle, as it lies below the P-wave baryons. Although the potential model of Isgur and Karl [11] attributes this low mass to an anharmonic perturbation in the spin-independent potential, these corrections are actually too large to be treated perturbatively. Høgaasen and Richard [12] have also shown that the position of the radial excitation of the nucleon should not lie lower than the negative-parity baryon resonances in the quark potential model for a wide range of plausible potentials. The relativized model of Capstick and Isgur [13] may be able to accommodate the light mass of Roper as a conventional q^3 state; however, an *ad hoc* 90-MeV energy shift between the center gravities of $N=1$ and $N=2$ states has to be made in order to fit overall data. In Fig. 3, we show the different spectra expected for dominantly q^3 states and for states with gluonic degrees of freedom. If the Roper resonance is indeed a radial excitation of the nucleon, there should be a corresponding radial excitation state of the $P_{33}(1232)$ in the conventional q^3 spectrum [14]; the candidate state $P_{33}(1600)$ is indeed found experimentally, but needs confirmation [the Particle Data Group (PDG) rate it 2*]. For the spectrum of hybrid states, the pattern of levels would be different; there should exist a P_{31} state below or near the resonance $P_{33}(1600)$ and a P_{13} above the Roper resonance. There is some experimental evidence [15] that $P_{31}(1530)$ and $P_{13}(1540)$ resonances may exist. Experimental clarification of the status of the $P_{31}(1530)$ and $P_{13}(1540)$ is crucial in confirming or refuting the Roper resonance as a gluonic partner of the nucleon as these P_{31} and P_{13} states do not emerge naturally in a q^3 potential model.

TABLE II. Hybrid photoproduction amplitudes from nucleon targets to $\mathcal{O}(\delta^2)$. The full expressions are obtained by multiplying the entries in this table by a factor of $\sqrt{\pi k}\,\mu_0 e^{-k^2/6\alpha^2}$. In all cases, $A_{3/2}=\sqrt{3}A_{1/2}$.

States		$A_{1/2}^p$	$A_{1/2}^n$
P_{11}	$\lvert N'\rangle$	$\frac{4}{3}\sqrt{2}\delta$	$-\frac{8}{9}\sqrt{2}\delta$
	$\lvert N''\rangle$	$\frac{4}{3}\sqrt{2}\delta$	0
P_{13}	$\lvert N'\rangle$	$\frac{2}{3}\delta$	$\frac{2}{9}(\sqrt{10}-1)\delta$
	$\lvert N''\rangle$	$\frac{2}{3}\delta$	$\frac{2}{9}(\sqrt{10}+1)\delta$
F_{15}	$\lvert N'\rangle$	0	0
P_{31}	$\lvert \Delta'\rangle$	$\frac{8}{9}\delta$	$\frac{8}{9}\delta$
P_{33}	$\lvert \Delta\rangle$	$\frac{2}{3}\sqrt{2}(1+\frac{1}{2}\delta^2)$	$-\frac{2}{3}\sqrt{2}(1+\frac{1}{2}\delta^2)$
P_{33}	$\lvert \Delta'\rangle$	$\frac{2}{9}\sqrt{2}\delta$	$\frac{2}{9}\sqrt{2}\delta$

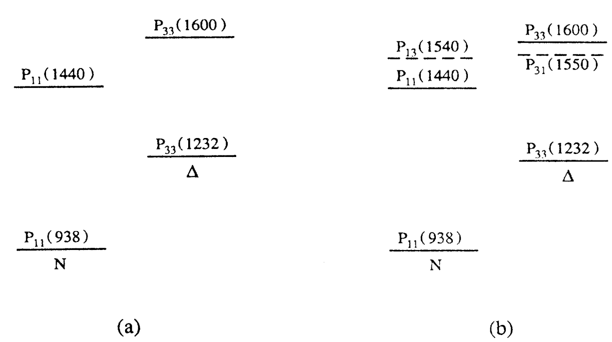


FIG. 3. The assignments for the $P_{11}(1440)$ and $P_{33}(1600)$ "partners" of N and Δ: (a) assuming that the Roper resonance is the radial excitation of the nucleon, and the resonance $P_{33}(1600)$ is a radial excitation of $\Delta(1232)$; (b) the Roper resonance as the gluonic partner of the nucleon. $P_{31}(1550)$ and $P_{13}(1540)$ hybrids are also expected given this assignment.

TABLE III. The experimental data and theoretical predictions for the photoproduction amplitudes for some low-lying resonances. Calculations assuming both qqq assignments and admitting q^3G states are presented. $A_{\rm KI}$ is the result of Koniuk and Isgur, in which they included the QCD mixing effects in the nonrelativistic SHO quark model (see Ref. [21]). $A_{\rm LC}$ is the result of Li and Close [22], in which relativistic effects are consistently included in the potential quark model of Isgur and Karl. A_G is the result of the calculation in which the resonances are assumed to be the states with a gluonic degree of freedom. The phase convention of Koniuk and Isgur [21] is used here. An asterisk denotes a resonance which is not well established, and which has no natural qqq assignment. The experimental data is taken from the Particle Data Group compilation (Ref. [15]).

State	A_J^N	$A_{\rm KI}$	$A_{\rm CL}$	A_G	$A_{\rm exp}$
$P_{33}(1232)$	$A_{1/2}^{p,n}$	-103	-93	-98	-141 ± 5
	$A_{3/2}^{p,n}$	-179	-160	-170	-258 ± 22
$P_{11}(1440)$	$A_{1/2}^{p}$	-24	-80	-61	-69 ± 7
	$A_{1/2}^{n}$	16	60	41	37 ± 29
$P_{31}^*(1550)$	$A_{1/2}^{p,n}$			25	16 ± 16
$P_{13}^*(1540)$	$A_{1/2}^{p}$			19	-14 ± 28
	$A_{1/2}^{n}$			15	?
	$A_{3/2}^{p}$			35	9 ± 27
	$A_{3/2}^{n}$			25	?
$P_{33}(1600)$	$A_{1/2}^{p,n}$	-16	-33	-7	-20 ± 29
	$A_{3/2}^{p,n}$	-46	-64	-12	1 ± 22
$P_{11}(1710)$	$A_{1/2}^{p}$	-47	-16	-44	5 ± 16
	$A_{1/2}^{n}$	-21	-23	0	-5 ± 23
$P_{13}(1720)$	$A_{1/2}^{p}$	-133	-112	15	52 ± 39
	$A_{1/2}^{n}$	57	24	-21	-2 ± 26
	$A_{3/2}^{p}$	46	55	26	-35 ± 24
	$A_{3/2}^{n}$	-10	-15	-36	-43 ± 94

In Table III we quote our numerical estimates of the photoproduction amplitudes of Table II in a simple-harmonic-oscillator (SHO) basis and compare to the data. The mixing parameter δ is chosen to be -0.35 to fit the observed Roper amplitudes ($\delta\approx0.72\sqrt{\alpha_s/2}$ is comparable in the bag model [4]; note that the δ in Ref. [4] is defined with a sign opposite to our convention). An SHO parameter of $\alpha=0.25$ GeV is used, which is known to give reasonable results for the static properties of baryon resonances [16]. This estimate also shows that the data for the $P_{31}(1530)$ are consistent with a hybrid assignment for this resonance. One should also note, however, that the $P_{33}(1600)$ photoproduction amplitudes are problematical in both models; the constraint

$$A_{3/2}=\sqrt{3}A_{1/2} \tag{4.16}$$

predicted by both q^3 and by hybrid assignments is not satisfied by the data, but it can be accommodated by the result of hybrid assignment. Clearly, more precise experimental data will help to establish the nature of the $P_{33}(1600)$.

If the Roper resonance is indeed the gluonic partner of the nucleon, a natural question is where the actual radial excitation lies. The photoproduction amplitudes suggest that the $P_{11}(1710)$ is a possible candidate. Adopting the formalism and parameters used in Ref. [8], the radial photoproduction amplitudes including relativistic effects are

$$A_{1/2}=\frac{\sqrt{3}}{9}\sqrt{\pi}k\,e^{-k^2/6\alpha^2}\begin{cases}\left[\left[\dfrac{k}{\alpha}\right]^2-\dfrac{2}{3}\dfrac{k}{m_q}+\dfrac{1}{36}\dfrac{k}{m_q}\left[\dfrac{k}{\alpha}\right]^2\right] & \text{for } p\,,\\[2ex] -\dfrac{2}{3}\left[\left[\dfrac{k}{\alpha}\right]^2-\dfrac{1}{2}\dfrac{k}{m_q}\right] & \text{for } n\,.\end{cases} \tag{4.17}$$

For the $P_{11}(1710)$ this gives

$$A_{1/2}=\begin{cases}25 & \text{for } p\,,\\ -19 & \text{for } n\end{cases} \tag{4.18}$$

if the $P_{11}(1710)$ is the radial excitation. Comparison with the data in Table III shows that this is in better agreement with experiment than other assignments and also solves the sign problem encountered if we identify the Roper with the radial excitation [17]. However, the mixing between q^3 and q^3G basis states may well be large for the $P_{11}(1710)$, so this conclusion should be regarded as tentative.

V. DISCUSSION AND CONCLUSIONS

Our approach to the accommodation of the gluonic degree of freedom in spectroscopy provides a framework which retains the successes of the q^3 quark model in describing the static properties of baryons, in particular the magnetic moments. The presence of the gluonic degrees of freedom may solve the long-standing puzzle of the Roper resonance. We have shown that photoproduction data for the Roper resonance is consistent with an assignment as a gluonic partner of the nucleon. Further studies both in theory and experiment are needed, however. An experimental confirmation of the $P_{31}(1550)$ and $P_{13}(1540)$ will be crucial in confirming this hybrid baryon assignment. A study of strong decays of gluonic states will also be very important. There has been some investigation of the pion decay of q^3G states [18] in the bag model, which concluded that their decay width should be large. However, there are two subprocesses which will contribute to these decays; specifically (1) the process $q^3 \rightarrow q\bar{q}q^3$, which has been studied phenomenologically, and (2) the process $q^3G \rightarrow q\bar{q}q^3$, which has not been investigated in any detail in the quark model. Further studies of these decay amplitudes may help us to understand the role of gluonic degrees of freedom in baryon spectroscopy. The absence of evidence for the $P_{31}(1550)$ in πN scattering should be understood in such an investigation.

The gluonic degrees of freedom may provide an explanation of the observation that the spin content of nucleons is not carried dominantly by valence quarks. However, the correction to the electromagnetic current due to the radiation of polarized gluons must be considered, just as in polarized deep-inelastic scattering. As we have shown in Secs. II and III, this gives an $O(\alpha_s)$ correction to the nucleon magnetic moments. New questions raised by the presence of gluons in nucleon wave functions include the problem of how to satisfy the Drell-Hearn-Gerasimov [19] sum rule, assuming the total spin is not carried by three valence quarks in the nucleon. To answer this question, one has to derive the photon-gluon transition operator at order $O(1/m^3)$, which will also contribute to the study of relativistic corrections to hybrid-baryon photoproduction. This investigation is in progress.

The presence of gluonic degrees of freedom also provides challenges in the study of baryon spectroscopy. If the gluons play an explicit role in the formation of resonances, the effect of the quark-gluon interaction on the spectrum of excited baryon resonances should be investigated more fully. The reassignments of some resonances will certainly require changes in fitted Hamiltonian parameters in the q^3 sector, and offer the possibility that positive and negative parity can be treated in an unified way.

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APPENDIX: THE CALCULATION OF MATRIX ELEMENTS

As the wave function of three quarks must be totally antisymmetric, the transition matrix element for the operator $\mathcal{O}=\sum_j \mathcal{O}_j$ can be written as

$$\langle N|\mathcal{O}|N\rangle = 3\langle N|\mathcal{O}_3|N\rangle \ . \tag{A1}$$

For the operator $\mathcal{O}_3 = e_3\sigma_3^z$, we have

$$\begin{aligned}\langle {}^2N_g, J=\tfrac{1}{2}|e_3\sigma_3^z|{}^2N_g, J=\tfrac{1}{2}\rangle &= \tfrac{1}{4}(\langle\phi^\rho|e_3|\phi^\rho\rangle + \langle\phi^\rho|e_3|\phi^\rho\rangle) \\ &\quad\times(\langle\chi^\rho, G|\sigma_3^z|\chi^\rho, G\rangle + \langle\chi^\lambda, G|\sigma_3^z|\chi^\lambda, G\rangle) \ ,\end{aligned} \tag{A2}$$

where the spin-1 gluon is coupled to the spins of the three valence quarks and gives total spin $\frac{1}{2}$ for this quark-gluon system. One can then write the matrix element $\langle\chi^\rho, G|\sigma_3^z|\chi^\rho, G\rangle$ as

$$\langle\chi^\rho, G|\sigma_3^z|\chi^\rho, G\rangle = \langle\tfrac{1}{2}\tfrac{1}{2}\,1\,0|\tfrac{1}{2}\tfrac{1}{2}\rangle\langle\chi^\rho, G\|\sigma_3^z\|\chi^\rho, G\rangle \ , \tag{A3}$$

where $\langle J_1\, J_{1z}\, J_2\, J_{2z}|J\, J_z\rangle = \langle\frac{1}{2}\frac{1}{2}\,1\,0|\frac{1}{2}\frac{1}{2}\rangle$ is a Clebsch-Gordan coefficient and $\langle\chi^\rho, G\|\sigma_3\|\chi^\rho, G\langle$ is the reduced matrix element of σ_3. Using the formula

$$\begin{aligned}\langle j_1 j_2 J||R_k(1)||j_1' j_2' J'\rangle &= \delta(j_2 j_2')\sqrt{(2J'+1)(2j_1+1)}(-1)^{k+j_2+J'+j_1} \\ &\quad\times\begin{Bmatrix} J & J' & k \\ j_1' & j_1 & j_2\end{Bmatrix}\langle j_1||R_k(1)||j_1'\rangle \ ,\end{aligned} \tag{A4}$$

where $R_k(1)$ is the tensor operator in space j_1 with rank k (which is 1 for the σ spin operator) and $\left\{\begin{smallmatrix} J & J' & k \\ j_1' & j_1 & j_{2l}\end{smallmatrix}\right\}$ is the SU(2) Racah coefficient, we then have

$$\begin{aligned}\langle {}^2N_g, J=\tfrac{1}{2}|e_3\sigma_3^z|{}^2N_g, J=\tfrac{1}{2}\rangle &= -\tfrac{1}{2}(\langle\phi^\rho|e_3|\phi^\rho\rangle + \langle\phi^\rho|e_3|\phi^\rho\rangle) \\ &\quad\times\begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & 1 \\ \frac{1}{2} & \frac{1}{2} & 1\end{Bmatrix}(\langle\chi^\rho|\sigma_3^z|\chi^\rho\rangle + \langle\chi^\lambda|\sigma_3^z|\chi^\lambda\rangle) \ .\end{aligned} \tag{A5}$$

The Racah coefficient [20] $\left\{\begin{smallmatrix} 1/2 & 1/2 & 1 \\ 1/2 & 1/2 & 1\end{smallmatrix}\right\}$ equals $\frac{1}{3}$, and as

$$\langle\phi^\lambda|q^{(3)}|\phi^\lambda\rangle = 0\ (p),\ \ \tfrac{1}{3}\ (n) \ , \tag{A6}$$

$$\langle\phi^\rho|q^{(3)}|\phi^\rho\rangle = \tfrac{2}{3}\ (p),\ \ -\tfrac{1}{3}\ (n) \ , \tag{A7}$$

$$\langle\chi^\rho_{1/2}|\sigma_3^z|\chi^\rho_{1/2}\rangle = 1 \ , \tag{A8}$$

and

$$\langle \chi^{\lambda}_{1/2}|\sigma_3^z|\chi^{\lambda}_{1/2}\rangle = -\tfrac{1}{3} , \tag{A9}$$

we obtain

$$\langle {}^2N_g, J=\tfrac{1}{2}|e_3\sigma_3^z|{}^2N_g, J=\tfrac{1}{2}\rangle = \begin{cases} -\frac{1}{27} & \text{for } p , \\ 0 & \text{for } n . \end{cases} \tag{A10}$$

Similarly,

$$\begin{aligned}\langle {}^4N_g, J=\tfrac{1}{2}|e_3\sigma_3^z|{}^4N_g, J=\tfrac{1}{2}\rangle &= (\langle\phi^\rho|e_3|\phi^\rho\rangle + \langle\phi^\rho|e_3|\phi^\rho\rangle) \\ &\quad\times \begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & 1 \\ \frac{3}{2} & \frac{3}{2} & 1 \end{Bmatrix} \langle \tfrac{1}{2}\tfrac{1}{2}\,1\,0|\tfrac{1}{2}\tfrac{1}{2}\rangle\langle\chi^S\|\sigma_3^z\|\chi^S\rangle \end{aligned} \tag{A11}$$

and

$$\langle\chi^S\|\sigma_3^z\|\chi^S\rangle = \frac{1}{\langle\frac{3}{2}\frac{3}{2}\,1\,0|\frac{3}{2}\frac{3}{2}\rangle} = \sqrt{\tfrac{5}{3}} , \tag{A12}$$

which gives

$$\langle {}^4N_g, J=\tfrac{1}{2}|e_3\sigma_3^z|{}^4N_g, J=\tfrac{1}{2}\rangle = \begin{cases} \frac{5}{27} & \text{for } p , \\ 0 & \text{for } n . \end{cases} \tag{A13}$$

For the matrix element $\langle {}^2N_g, J=\frac{1}{2}|e_3\sigma_3^z|{}^4N_g, J=\frac{1}{2}\rangle$, we have

$$\langle {}^2N_g, J=\tfrac{1}{2}|e_3\sigma_3^z|{}^4N_g, J=\tfrac{1}{2}\rangle = \begin{cases} \frac{4}{27} & \text{for } p, \\ -\frac{4}{27} & \text{for } n . \end{cases} \tag{A14}$$

Defining the tensor T_{1M} by

$$T_{1M} = \sum_m \langle 1\, m\, 1\, M-m|1\, M\rangle \sigma_m J^{\rm TE}_{M-m} , \tag{A15}$$

we have

$$\mathcal{O}_3 = i(\boldsymbol{\sigma}_3\times\mathbf{J}_3^{\rm TE})^z = \sqrt{2}T_{10} . \tag{A16}$$

Using the formula

$$\begin{aligned}\langle j_1 j_2 J\|T_K(k_1,k_2)\|j_1' j_2' J'\rangle &= \sqrt{(2J'+1)(2K+1)} \begin{Bmatrix} J & J' & K \\ j_1 & j_1' & k_1 \\ j_2 & j_2' & k_2 \end{Bmatrix} \\ &\quad\times\sqrt{2j_1+1}\langle j_1\|R_{k_1}\|j_1'\rangle\sqrt{2j_2+1}\langle j_2\|S_{k_2}\|j_2'\rangle , \end{aligned} \tag{A17}$$

where the tensor $T_K(k_1,k_2)$ is defined as

$$T_{KQ}(k_1,k_2) = \sum_{q_1 q_2} R_{k_1 q_1}(1) S_{k_2 q_2}(2)\langle k_1\, q_1\, k_2\, q_2|K\, Q\rangle , \tag{A18}$$

and

$$\begin{Bmatrix} J & J' & K \\ j_1 & j_1' & k_1 \\ j_2 & j_2' & k_2 \end{Bmatrix}$$

is an SU(2) 9-j coefficient, we find

$$\begin{aligned}\langle {}^2N_g, J=\tfrac{1}{2}|i(\boldsymbol{\sigma}_3\times\mathbf{J}_3^{\rm TE})^z|N_0\rangle &= \frac{\sqrt{6}}{2}\delta \begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & 1 \\ \frac{1}{2} & \frac{1}{2} & 1 \\ 1 & 0 & 1 \end{Bmatrix} (\langle\phi^\rho|e_3|\phi^\rho\rangle\langle\chi^\rho|\sigma_3^z|\chi^\rho\rangle \\ &\quad - \langle\phi^\lambda|e_3|\phi^\lambda\rangle\langle\chi^\lambda|\sigma_3^z|\chi^\lambda\rangle) , \end{aligned} \tag{A19}$$

where

$$\begin{Bmatrix} \frac{1}{2} & \frac{1}{2} & 1 \\ \frac{1}{2} & \frac{1}{2} & 1 \\ 1 & 0 & 1 \end{Bmatrix} = \frac{1}{3\sqrt{6}} .$$

Hence we conclude

$$\langle {}^2N_g, J=\tfrac{1}{2}|i(\boldsymbol{\sigma}_3\times\mathbf{J}_3^{\rm TE})^z|N_0\rangle = \begin{cases} \frac{1}{9}\delta & \text{for } p , \\ -\frac{1}{27}\delta & \text{for } n . \end{cases} \tag{A20}$$

For the operator $\boldsymbol{\sigma}_3\cdot\mathbf{J}_3^{\rm TE}$, we find

$$\begin{aligned}\langle {}^2N_g, J=\tfrac{1}{2}|\boldsymbol{\sigma}_3\cdot\mathbf{J}_3^{\rm TE}|N_0\rangle &= \frac{1}{4\sqrt{3}}\delta(\langle\chi^\rho\|\sigma_3\|\chi^\rho\rangle - \langle\chi^\lambda\|\sigma_3\|\chi^\lambda\rangle) = \tfrac{1}{3}\delta . \end{aligned} \tag{A21}$$

The other matrix elements summarized in Tables I and II follow similarly.

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