

Color transparency and Landshoff multiple-scattering processes

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(Received 29 April 1991)

In the following, the leading and nonleading contributions to fixed angle hadron-hadron elastic scattering are included in a qualitative analysis of $T(s)$, the transparency of pN quasielastic scattering. A simple model is given to predict the center-of-mass energies at which a target nuclei will become transparent to hard- and triple- (Landshoff-) scattered protons.

I. INTRODUCTION

Color transparency is based on the concept that the effective size of the collection of quarks involved in an elastic hadronic scattering process can be much smaller than normal hadronic size consisting of typical distance scales of ~ 1 fm. Smaller objects have smaller color moments and are less likely to interact strongly with a background, hence, their effective transparency. It has been shown that the tightly bound part of the hadronic Fock state contributes to the leading (in s) behavior of elastic hadronic scattering [1,2], from which predictions of color transparency in quasielastic processes naturally follow.

II. TWO LEADING ELASTIC PROCESSES

To leading order in Q , the wave functions of the external hadrons in fixed angle ($s/t \sim 1$, $s \rightarrow \infty$) elastic hadron-hadron scattering consist of the minimum number of constituent quarks; other contributions are either nonleading or can be factorized out of the wave function. Soft-gluon radiation can be factorized via the eikonal approximation, hard radiation costs negative powers of Q , and problems with collinearities can be controlled [3]. Using these hadrons, there are two types of processes that are important (leading or nearly leading in Q) contributions to wide-angle elastic hadron-hadron scattering. In purely hard, or single scattering [2,4], the hadrons and the interaction region are effectively pointlike; that is, have a typical distance scale $O(1/Q)$. The other type, Landshoff [5], or multiple scattering, has the hadrons effectively like pancakes—Lorentz flattened in the direction of motion. Before radiative corrections and consequent Sudakov effects, the radii of the pancakes are not small and unconstrained up to the typical hadronic scale, $r_h \approx 1$ fm. Since protons appear to prefer to exist as 1-fm objects, with no other considerations one would assume that Landshoff protons are like typical high-momenta, on-shell protons. The momentum exchange occurs along a matchsticklike shape aligned out of the plane of scattering consisting of a minimum number of hard subprocesses involving the constituent quarks. For example, there would be three $qq \rightarrow qq$ hard subprocesses in elastic pp scattering.

When Sudakov effects are taken into account,

Landshoff protons become matchsticks themselves, their length out of the scattering plane now scaling as a negative power of Q , $Q^{-0.70}$ for baryons. After this inclusion, the power dependences of single and Landshoff scattering are comparable. For example, if we write $d\sigma/dt(pp \rightarrow pp) \propto s^{-n}$, we have $n_{\text{single}} = 10$ and $n_{\text{Landshoff}} = 9.59$ [1,2,4,6]. The Landshoff contribution to $d\sigma/dt(pp \rightarrow pp, \theta = \pi/2)$ has been explicitly calculated for two choices of hadronic wave function [7]. The results imply that the Landshoff contribution is probably smaller than the hard-scattering contribution for measurable s , $s \lesssim 250 \text{ GeV}^2$, but is a significant fraction ($\gtrsim 5\%$) of the measured cross section for $s \gtrsim 20 \text{ GeV}^2$. That it is smaller is not surprising; only a specialized subset of the overall internal momenta space contributes to Landshoff scattering. For $s \lesssim 400 \text{ GeV}^2$, there is a further complication that cutoff dependences are large, implying that end-point contributions (small x) are non-negligible. Until a sensible phenomenology is developed, predictions of the size of multiple-scattering amplitudes cannot be made to a 50% accuracy at useful energies. In the following, we will discuss qualitative differences between the two processes in a nuclear background.

III. COLOR TRANSPARENCY

In the center-of-mass frame, the external particles are Lorentz flattened. We have

$$r_{\text{transverse}} \approx r_h \approx 1 \text{ fm}, \quad (1)$$

$$r_{\text{longitudinal}} \approx \frac{r_h m_{\text{target}}}{\langle |p_{\text{longitudinal}}| \rangle} \approx \frac{r_h m_N}{Q}.$$

The transverse extent of the elastic-scattering process is the relevant quantity for color transparency. If the scattering process was hard, that is, the momenta transfer occurred within a region of extent $O(1/Q)$, where $Q \equiv \sqrt{2|p_{\text{quark;c.m.}}^{-2}|}$, the outgoing quarks are initially separated by transverse distances of $O(1/Q)$, as shown schematically in Figs. 1(a) and 1(b). For high enough Q , the quarks traverse the nucleus as a tightly packed color singlet much less likely to interact with the other nucleons than a typical hadron. The collection of outgoing quarks is more color transparent to the rest of the nucleus. Somewhere outside the nucleus, the quarks reform

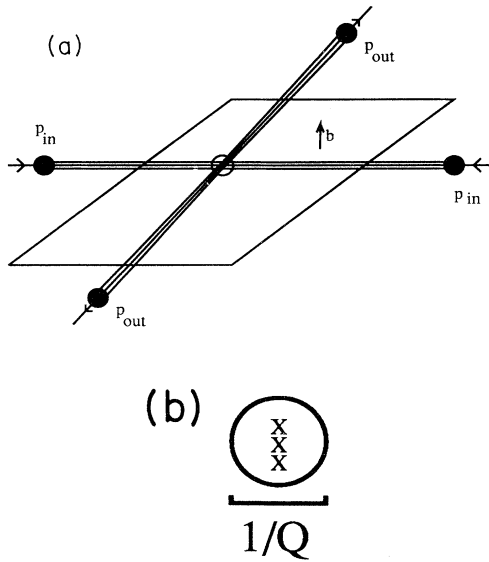


FIG. 1. (a) The typical form of a single-scattering process for baryons. Here dimensional counting rules are obeyed, for example, $d\sigma/dt(pp;elastic) \propto s^{-10}$. (b) The interaction region proportional of a single-scattering process. Each \times signifies a hard $qq \rightarrow qq$ scattering.

into a hadron.

A measurable quantity $T(s)$, the transparency, is defined by

$$T(s) = \frac{1}{Z} \frac{(d\sigma/dt)(pA \rightarrow pp(A-1))}{(d\sigma/dt)(pp \rightarrow pp)} \Big|_{s/t \sim 1}, \quad (2)$$

where Z is the atomic number the target. In the following, we shall work at $\pi/2$ center-of-mass scattering angle. In the asymptotic limit, the escaping partons have effectively zero extent; hence, the perturbative prediction is that $T(s) \rightarrow \infty$ as $s \rightarrow \infty$. If an experiment truly probes the perturbative regime, that is, where the factorization proofs of single- and multiple-scattering amplitudes make

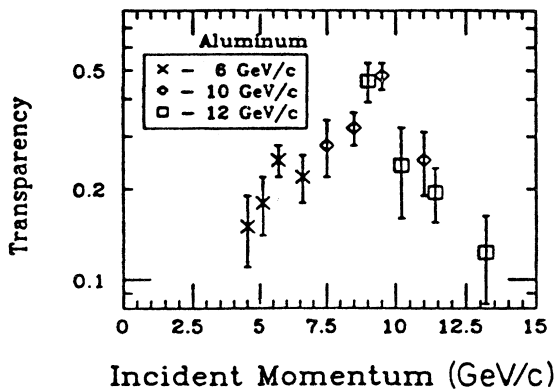


FIG. 2. Measured transparency as a function of scattering angle as taken from Ref. [8].

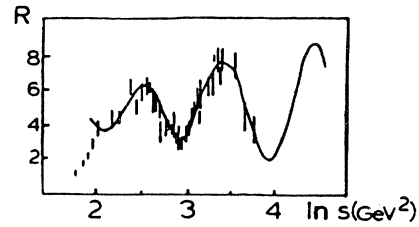


FIG. 3. pp elastic scattering data taken from Ref. [10], with $R(s) = d\sigma/dt|_{\theta=\pi/2} \times 10^{-8} s^{10} s_0^{-8}$, where $s_0 \equiv 1 \text{ GeV}^2$. The fit is also taken from Ref. [10].

sense, one would expect that $T(s)$ should gradually increase with increasing s towards 1.

Recent data [8] are shown in Fig. 2 for $\theta = \pi/2$. A possible oscillation is observed with rising s , which is similar in position and shape to one observed in the pp elastic-scattering cross section at $\theta = 90^\circ$ and shown in Fig. 3. The transparency is decreasing for $s \gtrsim 9 \text{ GeV}^2$, indicating that the most naive expectations of perturbative quantum chromodynamics (PQCD) are thwarted and something else is occurring. It has been postulated that differences in transparency between single- and multiple-scattering

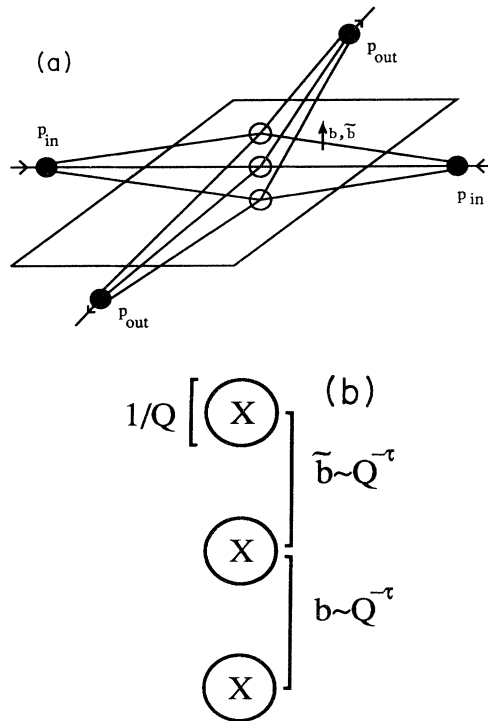


FIG. 4. (a) The Landshoff process for pp elastic scattering. Though subject to Sudakov suppression, these are the leading-order processes (in s) in wide-angle elastic hadron-hadron scattering. For example, for pp scattering we have $d\sigma/dt \propto s^{-9.59}$. (b) The interaction region of a Landshoff-, or triple-, scattering process. Each \times signifies a hard $qq \rightarrow qq$ scattering. In the case of baryon-baryon scattering, as shown here, $\tau = 0.70$. For mesons, $\tau = 0.64$.

contributions and their availability for mutual interference are the source of the structure in $T(s)$ [9].

A picture of Landshoff scattering is shown in Figs. 4(a) and 4(b). Without the inclusion of radiative corrections, the transverse separation of the scattering centers [b in Figs. 4(a) and 4(b)] does not scale with Q . As Ralston and Pire state [10], one could take this argument to its extreme and “visualize the independent scattering of plane waves occurring on opposite sides of the lab.” If an oncoming proton quasielastically scatters off a proton in the nucleus by a multiple-scattering process of this type, the outgoing collection of quarks would have the transverse separation typical of constituent quarks in a hadron, hadronize within the nucleus, and have a mean free path much shorter than the distance necessary to escape from the nucleus without further interaction. Thus, the proton would be “almost certain to collide by conventional soft interactions” [11] and the contribution of Landshoff processes would be filtered out of the elastic-scattering amplitude, leaving only the contributions of traditional single-scattering processes transparent. This picture would imply that, except collective nuclear effects, the structure observed in $T(s)$ has its source only in the structure of its denominator, which could be then modeled as interference between single- and multiple-scattering processes. That is, the transparency can be approximated in terms of single- and multiple-scattering amplitudes by

$$T(s) \approx \frac{|A_{\text{single}}|^2}{|A_{\text{single}} + A_{\text{multiple}}|^2}. \quad (3)$$

In this way, the data can be explained. Multiply scattered quarks are not as big as 1 fm, however, and this picture needs to be modified.

IV. INCLUSION OF SUDAKOV EFFECTS

Because Landshoff processes include nonsoft transverse-momenta scales, $Q_1 > O(Q^0)$, they are susceptible to gluon radiation and corresponding double, or Sudakov, logarithms [12] that effectively push the independent scatterings together, though not quite to a “pointlike” $1/Q$ separation. It has been shown [1] that the transverse separation of the hard-scattering centers out of the plane of scattering, b in Fig 4(a), scales as a function of Q . What is meant by this statement is that the amplitude for a multiple-scattering process is proportional to an integral over b of a sharply peaked function; that is, $A \propto \int db f(b, Q)$. This result follows from factorization and the application of the renormalization group to each consequent part of the amplitude. Details are given in Ref. [1]. As a function of b , $f(b, Q)$ is roughly Gaussian and peaked at the value $b = \Lambda_{\text{QCD}}^{-1} (Q/\Lambda_{\text{QCD}})^{-\pi}$, $0 < \pi < 1$, where $\pi_{\text{baryon}} \approx 0.70$ and $\pi_{\text{meson}} \approx 0.64$. The width of the Gaussian is approximately $\frac{1}{3} \ln Q$ for baryons and $\frac{1}{2} \ln Q$ for mesons. In the following, we set $\Lambda_{\text{QCD}} = 0.20$ GeV. It is the purpose of this paper to suggest the various regions in Q and A where the different processes are transparent and what signature might exist for each.

Roughly speaking, the scattered quarks can be considered reformed into an ordinary proton when their transverse extent is the normal hadronic radius r_h . If the quarks are out of the nucleus by this time, then the reconstituted proton was transparent.

If we call the momenta of one of the quarks after the scattering p_μ , we have, from the uncertainty principle,

$$T_{\text{had}} \sim \frac{r_h}{\langle |\mathbf{p}_T| \rangle}. \quad (4)$$

In Landshoff processes, the various components of momenta are characterized by their scaling properties with respect to Q . For the i th hadron, we have

$$\begin{aligned} p_i \cdot v_i' &\propto Q, \\ p_i \cdot v_i &\propto 1/Q, \\ p_i \cdot \eta_i &\propto 1, \\ p_i \cdot \hat{b} &\approx \frac{1}{b} \sim \Lambda_{\text{QCD}} \left[\frac{Q}{\Lambda_{\text{QCD}}} \right]^\pi, \end{aligned} \quad (5)$$

where v_i^μ is a lightlike vector collinear to the i th hadron, $v_i'^\mu$ is the lightlike vector satisfying $v_i \cdot v_i' = 1$, and η_i^μ is a spacelike unit vector transverse to v_i and v_i' in the plane of scattering. For example, if one labels the scattering plane as the YZ plane, a hadron moving in the \hat{z} direction would have $v_\mu = (\hat{t} + \hat{z})/\sqrt{2}$, $v_\mu' = (\hat{t} - \hat{z})/\sqrt{2}$, and $\eta_\mu = \hat{y}$.

In the interaction region, momenta of $O(Q)$ span the plane of scattering, allowing no extra logarithmic singularities from momenta integrals over the plane of scattering to bring about Sudakov factors favoring regions in momenta space where $p_i \cdot v_i$ or $p_i \cdot \eta_i$ are not of $O(1/Q)$ or $O(1)$, respectively. That is, $p_i \cdot v_i$ and $p_i \cdot \eta_i$ cannot be treated equivalently to $p_i \cdot \hat{b}$; they are softer in the leading-order process. The largest transverse-momenta component for the leading contribution is out of the plane of scattering, in the \hat{b} direction. We have $\langle \mathbf{p}_T \rangle \approx \hat{b}/b$ or $\langle |\mathbf{p}_T| \rangle \approx \Lambda_{\text{QCD}} (Q/\Lambda_{\text{QCD}})^\pi$ and

$$t_{\text{had}} \sim \frac{r_h}{\Lambda_{\text{QCD}}} \left[\frac{Q}{\Lambda_{\text{QCD}}} \right]^{-\pi}. \quad (6)$$

The time for a quark of forward momentum $\sim Q$ to escape the nucleus is given by

$$t_{\text{escape}} \approx \frac{r_{\text{longitudinal}}}{\langle p_{\text{longitudinal}} \rangle} \approx \frac{\sqrt{2} r_h A^{1/3} m_h}{Q_{\text{c.m.}}^2}, \quad (7)$$

assuming that in the rest frame $r_{\text{longitudinal}} \approx r_h A^{1.3}$. Our simple model will be that if $t_{\text{escape}} < t_{\text{had}}$, we have transparency, otherwise not. At a critical value of A , we are on the border between these two regions, $t_{\text{escape}} = t_{\text{had}}$, and we have

$$A_{3;\text{crit}}(Q) \sim \left[\frac{Q}{\Lambda_{\text{QCD}}} \right]^{3(2-\tau)} \left[\frac{\Lambda_{\text{QCD}}}{\sqrt{2} m_h} \right]^3. \quad (8)$$

Note that the sign of the power is positive with $3(2-\tau_{\text{baryon}}) \approx 4.08$. As Q increases, more of the Periodic

Table becomes transparent to multiple-scattering processes.

For single-scattering processes, we calculate $A_{\text{crit}}(Q^2)$ in exactly the same manner. This process has picked out the transversely tightly bound three-quark Fock state with the transverse momenta soft, $Q_1 \sim m_h$. The escape time is unchanged from the triple-scattering case, but the hadronization time is longer. We have

$$T_{\text{had}} \sim \frac{r_h}{m_h} \rightarrow A_{1;\text{crit}}(Q^2) = \left[\frac{Q^2}{\sqrt{2}m_h^2} \right]^3. \quad (9)$$

$$T_{\text{Landshoff}}(s) = \frac{\left| \sum_{IJK=1}^3 \int d_3x \int_0^{1/Q_{\text{crit}}(Z)} db d\tilde{b} \tilde{H}_{IJK}(Q, x_i) \tilde{U}_{IJK}(b, \tilde{b}) \phi_p^4(x_i, 1/b) e^{-S_{IJK}(b, \tilde{b}, x_i)} \right|^2}{\left| \sum_{IJK=1}^3 \int d_3x \int_0^{1/\Lambda} db d\tilde{b} \tilde{H}_{IJK}(Q, x_i) \tilde{U}_{IJK}(b, \tilde{b}) \phi_p^4(x_i, 1/b) e^{-S_{IJK}(b, \tilde{b}, x_i)} \right|^2}, \quad (10)$$

where x_i is the momenta fraction of the i th quark and $d_3x \equiv \int_0^1 dx_1 dx_2 dx_3 \delta(1-x_1-x_2-x_3)$. The I, J , and K are color flow indices. U_{IJK} is the contribution of soft-gluon exchanges and the overall Dirac traces. H_{IJK} is the hard-scattering function, to lowest order it is the product of three Born-scattering subdiagrams $H_I^{\text{Born}} H_J^{\text{Born}} H_K^{\text{Born}}$. The impact parameters b and \tilde{b} are shown in Fig. 4(b). The function $e^{-S_{IJK}}$ is the Sudakov factor, a result of radiative corrections and their renormalization-group treatment.

To approximate the above expression, we take the following steps.

(i) We make a change of variables to $\zeta \equiv -\ln b \Lambda$ and $\tilde{\zeta} \equiv -\ln \tilde{b} \Lambda$.

(ii) The soft-gluon-exchange function $U(\zeta, \tilde{\zeta})$ is to lowest order in $\alpha_s(Q^2)$ independent of ζ and $\tilde{\zeta}$ and is taken outside the ζ and $\tilde{\zeta}$ integrals.

(iii) From its evolution equation, we know the hadronic wave function $\phi_p(x_i, \zeta, \tilde{\zeta})$ has a very slowly varying dependence in ζ and can be taken as a constant over a wide range in this variable.

(iv) To leading order in $\ln Q/\Lambda$, the Sudakov factor does not depend on the color flow; that is, $S_{IJK} = S + O(\ln \ln Q/\Lambda)$.

(v) Many terms are identical in the numerator and denominator of $T(s)$, become multiplicative in this approximation, and cancel out.

What remains is the ratio of $\int d\zeta d\tilde{\zeta} \exp(-S)$ over different ranges in integration:

$$T(s) \approx \frac{\int_0^{\zeta_{\text{crit}}(Z)} d\zeta d\tilde{\zeta} e^{-S(\zeta, \tilde{\zeta})}}{\int_0^\infty d\zeta d\tilde{\zeta} e^{-S(\zeta, \tilde{\zeta})}}. \quad (11)$$

V. CALCULATION OF $T_{\text{Landshoff}}(S)$

We shall now calculate the asymptotic value of $T(s)$, defining asymptopia of wide-angle elastic hadronic scattering as the region where triple scattering is dominant. From Ref. [7], asymptopia starts at the order of $s \sim 400 \text{ GeV}^2$.

To calculate $T(s)$ assuming Landshoff-scattering processes dominate, we use the leading-order factorized form for these amplitudes given by Ref. [1]. We assume this form for both pp and pN scattering, with the difference between numerator and denominator the inclusion of a cutoff for large b in the pN scattering to model the opacity of transversely large collections of quarks. We have

This expression can be well approximated by saddle-point (SP) techniques with

$$S(\zeta, \tilde{\zeta}) \approx S_0(\zeta_{\text{SP}}, \tilde{\zeta}_{\text{SP}}) + a_3 [(\zeta - \zeta_{\text{SP}})^2 + (\tilde{\zeta} - \tilde{\zeta}_{\text{SP}})^2], \quad (12)$$

where $1/\sqrt{a_3}$ is the width of the Gaussian, e^{-S} , and is given by

$$\begin{aligned} a_3 &\equiv \frac{1}{2} \frac{\partial^2 S_{IJK}}{\partial \zeta^2} \Big|_{\zeta = \zeta_{\text{SP}}} \\ &= \frac{c}{\tau^2 \ln Q/\Lambda} \\ &\quad + O((\ln Q/\Lambda)^{-2}, [(\ln Q/\Lambda)(\ln \ln Q/\Lambda)]^{-1}). \end{aligned} \quad (13)$$

Some details of this calculation are given in Refs. [1] and [7]. We then have

$$T_{\text{Landshoff}}(s) \approx \left[\frac{1 - \text{erf}(\sqrt{a_3}(\zeta_{\text{crit};3} - \zeta_{\text{SP};3}))}{1 - \text{erf}(-\sqrt{a_3}\zeta_{\text{SP};3})} \right]^4, \quad (14)$$

where $\text{erf}(x)$ is the error function.

Values for $T_{\text{Landshoff}}(s)$ for $Z = 13$, an aluminum target, are shown in Fig. 5. The rise of T from 0 to 1 is slow with rising energy and is over many decades of s . This rise is centered at the expected value of $\ln s/s_0 \approx 15$ ($s_0 \equiv 1 \text{ GeV}^2$) where we have $\zeta_{\text{crit}} \approx \zeta_{\text{SP}}$. Note that, from Eqs. (6) and (8), we have

$$\begin{aligned} \zeta_{3;\text{crit}} &= \frac{\tau}{2-\tau} \ln(\sqrt{2} A^{1/3} m_h/\Lambda), \\ \zeta_{1;\text{crit}} &= \ln(\sqrt{2} A^{1/6} m_h/\Lambda). \end{aligned} \quad (15)$$

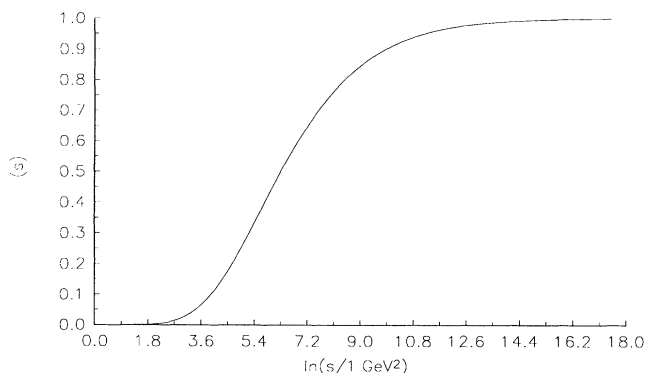


FIG. 5. The asymptotic PQCD prediction for the transparency: the transparency of Landshoff-scattered protons.

The purely PQCD calculation of the transparency of the Landshoff processes gives $T=0$ for any foreseeable measurable center-of-mass energy in agreement with the postulate of Ralston and Pire. That is to say, while the transverse size of the Landshoff scattered proton decreases with rising Q , it decreases too slowly to forestall hadronization within the nucleus. For currently available center-of-mass energies, this calculation implies that the nucleus is an efficient filter of multiply scattered objects.

For purely hard-scattering processes, we have $b \approx 1/Q$. We find that $\ln Q/\Lambda \equiv \zeta_{\text{crit};1} = \zeta_{\text{SP};1}$ at $\ln s/s_0 \approx 4$, which is at approximately the high end of the current experimental data. If we assume $\Delta b < b$, we have $\Delta \zeta < 1$. If, analogously to the Landshoff case, we assume a Gaussian distribution in ζ with the maximal transverse width of $a_1 \approx \mathcal{C}$, $\mathcal{C} \approx 1$ of the hard-scattered quarks, we can estimate $T_{\text{single}}(s)$. Similarly, we have

$$T_{\text{single}}(s) \approx \left[\frac{1 - \text{erf}(\sqrt{a_1}(\zeta_{\text{crit};1} - \zeta_{\text{SP};1}))}{1 - \text{erf}(-\sqrt{a_1}\zeta_{\text{SP};1})} \right]. \quad (16)$$

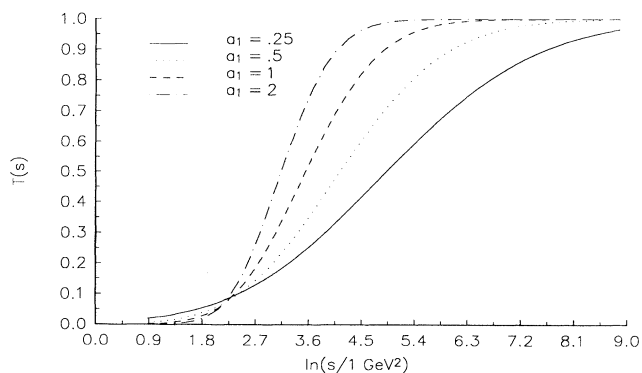


FIG. 6. A prediction of the transparency of hard-scattered protons.

This is shown for an aluminum target in Fig. 6 for various choices of \mathcal{C} . We would predict from this rough calculation that the next generation of experiments would observe a rise in $T(s)$ as the truly perturbative component would begin to contribute. While this analysis sheds no light on the currently observed features in the transparency, from the observed departure from the Glauber region and the decrease in $T(s)$ for incident momenta greater than 8 GeV, we would claim that PQCD has not really yet been tested, that the region of applicability of current perturbative calculations has not been reached. This does not preclude future improvements in calculational technique and factorization, including a systematic treatment of small- x effects. In elastic and quasi-elastic wide-angle hadronic scattering, the contribution of the internal momenta region where transverse and hard scales are comparable may prove to be large and its calculation necessary for a more useful description of these processes.

VI. CONCLUSION

We would conclude that any structure of a perturbative nature that will be observed in $T(s)$ will be a reflection of structure in pp elastic scattering. It has been shown that the coefficient of the energy-dependent phase in the Landshoff amplitude relative to the single-scattering amplitude is too small to explain the oscillations in the data for pp elastic scattering at fixed angle [7]. However, interference oscillations would still be predicted, though the results of Ref. [7] imply they would be of a much longer effective period in $\ln \ln s$, the PQCD prediction of the relative phase being $\exp(i0.5 \ln \ln s)$ rather than $\exp(i50 \ln \ln s)$. If, as Brodsky and de Teramond suggest [13], one interprets the observed oscillations as two $J=L=S=1$, $B=2$ resonance structures, the slowly varying oscillations should be observable at center-of-mass energies just larger than currently observed, away from these resonances. Even if oscillations of the form $\exp(i50 \ln \ln s)$ should be found to persist with rising s , the superposition of the PQCD component on this structure should still be observable since the amplitude of the PQCD oscillations are larger, a result that becomes more firm as s increases into the truly perturbative range $s > 400 \text{ GeV}^2$.

At small angles, scattering occurs along a line rather than a plane and two transverse directions \mathbf{b} exist. If after asymptotic factorization, one found the dominant contributions to come multiple scattering and $b^2 = O(Q^0)$, the small-angle power law would be reproduced. Because there would be no Sudakov suppression, there would be neither an energy-dependent phase nor an interference pattern for smaller scattering angles.

ACKNOWLEDGMENTS

The author would like to thank Gerry Miller, Glennys Farrar, Gerry Bunce, and John Ralston for helpful discussions. This work was supported in part by the U.S. Department of Energy, Contract No. DE-AS06-88ER40423.

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