

Possibility of measuring the magnetic moment of the W boson at heavy-ion colliders

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We consider the possibility of measuring the magnetic moment at the W boson at a heavy-ion collider. We find that the cross section for the process $Pb + Pb \rightarrow \gamma\gamma \rightarrow e^\pm \bar{\nu}_e \mu^\mp \nu_\mu$ is very sensitive to this parameter and that a 3σ measurement of 35% seems possible at the CERN Large Hadron Collider and a precision of 8% or so at the Superconducting Super Collider, given an integrated luminosity of 0.1 pb^{-1} .

I. INTRODUCTION

The trilinear gauge vertices of the standard model (SM) offer a very powerful probe into the gauge structure of the model. Up to now, this sector is largely unexplored. In the SM, the $\gamma W^+ W^-$ and the $Z W^+ W^-$ vertices are very well defined [1] and any perturbation from these values can ruin crucial properties such as renormalizability. In an effective Lagrangian formalism, the most general $\gamma W^+ W^-$ vertex can be parametrized into 7 form factors [2]. However, requiring CP to be an exact symmetry and considering operators of dimension 4 or less reduces this number to 2. The relevant part of the Lagrangian, now very similar to the SM, is

$$\mathcal{L} = -ie A^\mu (W^{-\mu\nu} W_\nu^+ - W^{+\mu\nu} W_\nu^-) + ie \kappa_\gamma F^{\mu\nu} W^{+\nu} W^{-\nu}, \quad (1)$$

which leads to a $\gamma W^+ W^-$ vertex of the form

$$-ie [(\kappa_\gamma k_1 - k_2)^\alpha g^{\mu\nu} + (k_2 - k_3)^\mu g^{\nu\alpha} + (k_3 - \kappa_\gamma k_1)^\nu g^{\mu\alpha}], \quad (2)$$

where $k_1^\mu, k_2^\nu, k_3^\alpha$ are the incoming momenta of the γ, W^+, W^- , respectively. κ_γ is conventionally referred to as the anomalous magnetic moment of the W boson. In the SM, at the tree level, $\kappa_\gamma \equiv 1$. In the static limit, it will receive small contributions from higher orders [3]. As we have written it above, the $\gamma W^+ W^-$ vertex leads to a magnetic moment of the form $\mathcal{M} = (1 + \kappa_\gamma)(e/2M_W)$ and to an electric quadrupole moment of the form $Q = -e\kappa_\gamma/M_W^2$.

Although κ_γ is very well defined in the SM, its value is, in principle, free in composite models [4]. One must then rely on experimental bounds on different parameters [5] that depend on κ_γ or on unitarity [6] to constrain the models. Changing κ_γ will have different effects on different observables. For example, at the CERN collider LEP 200 ($e^+e^- \rightarrow W^+W^-$) the total production cross section, the angular distributions (W 's and decay products), and the energy distributions (leptonic) are all sensitive to κ_γ .

There exist proposals to build heavy-ion colliders of very high energy: in the range of 3.5 TeV/nucleon at the CERN Large Hadron Collider (LHC) and 8 TeV/nucleon at the Superconducting Super Collider (SSC). Such high-energy ions can become excellent sources of photons [7]. Considering coherent emission, one gains a factor of $Z_1^2 \times Z_2^2$ (charges of the ions considered) over processes where the initial-state photons are emitted incoherently. This coherence effect will hold for photons of energy up to $\sim \gamma/R$ where γ and R , are the Lorentz contraction factor in the process, and the radius of the ion considered, respectively. The electric charge factor is of the order of 10^7 for the ions under consideration (Pb, for example) and can compensate for the α^4 factor in the cross section. This electric charge enhancement due to heavy ions can also compensate the loss of logarithmic enhancement present in an e^+e^- machine, for example, until one reaches extremely high c.m. energy at the e^+e^- collider. Recently, some groups [8] have considered the possibility of using such heavy-ion colliders to produce supersymmetric particles or an intermediate-mass Higgs boson.

A. The process $^{206}\text{Pb} + ^{206}\text{Pb} \rightarrow \gamma\gamma \rightarrow e^\pm \bar{\nu}_e \mu^\mp \nu_\mu$

In this paper, we want to consider the possibility of measuring κ_γ at a heavy-ion collider. The basic process is

$$\gamma\gamma \rightarrow W^+ W^-.$$

The main advantage of this particular process is that the $\gamma W^+ W^-$ vertex enters twice in the amplitude; this will lead to a term proportional to κ_γ^4 in the cross section. In order to get a rough estimate of the sensitivity of this process to κ_γ , one can give equal weight to all diagrams. One then finds that a relative deviation of Δ in κ_γ will lead to a relative deviation of $\sim 2\Delta$ in the cross section. Since we are concerned with κ_γ only, there is no need to modify the four-point vertex from its SM value because this vertex does not contribute to κ_γ , although it does contribute to the λ term [9].

In a heavy-ion collision, the total cross section for this process is given by

$$\sigma = \int dx_1 dx_2 f(x_1) f(x_2) \int d\hat{s} \delta(x_1 x_2 S - \hat{s}) \hat{\sigma}(\gamma\gamma \rightarrow W^+ W^-), \quad (3)$$

where $S, (\hat{s})$ is the usual Mandelstam variable for the ion-ion ($\gamma\gamma$) collision, $x \equiv E_\gamma/E_{\text{ion}}$ and $f(x)$ is the spectrum of the coherent photons (i.e., the number of photons with momentum fraction x) coming from the heavy ions.

Since the W bosons have a lifetime of $\sim 3 \times 10^{-25}$ sec, only their decay products are observed. We want to consider the full production and decay chain of the W 's as a function of κ_γ . Furthermore, since we are in a hadronic environment, it seems clear that we should consider leptonic decay modes of the W bosons in order to have a signal as clean as possible. We should also consider different-lepton signals in order to reduce backgrounds such as $\gamma\gamma \rightarrow l^+l^-$. We will then consider the process $\gamma\gamma \rightarrow e^+ \nu_e \mu^- \bar{\nu}_\mu$. The total amplitude is made of 13 Feynman diagrams. We used a spinor technique [10] to calculate it; the result is too large to be given here. From recent experiments [11], it seems that the top quark is too heavy to provide a decay channel for the W boson. We expect then $\sigma(\gamma\gamma \rightarrow e^+ \nu_e \mu^- \bar{\nu}_\mu) \simeq (\frac{1}{9})(\frac{1}{9})\sigma(\gamma\gamma \rightarrow W^+W^-)$. The photonic production cross section $\hat{\sigma}(\gamma\gamma \rightarrow W^+W^-)$ is very well known [12]:

$$\hat{\sigma}(\gamma\gamma \rightarrow W^+W^-) = \frac{4\pi\alpha^2}{\hat{s}} \sqrt{1-4r} \left[\frac{1}{r}(2+1.5r+6r^2) + \frac{6r-12r^2}{\sqrt{1-4r}} \ln \frac{1-\sqrt{1-4r}}{1+\sqrt{1-4r}} \right] \quad (4)$$

with $r \equiv M_W^2/\hat{s}$. Note that $\hat{\sigma} \rightarrow 8\pi\alpha^2/M_W^2 \simeq 93$ pb as $\hat{s} \rightarrow \infty$, and rises very fast above threshold. We verified our production and decay amplitude via this equation and the previous branching ratios. The agreement was within a few percent; this gave us some confidence in our amplitude. In our numerical simulations, we used the narrow-width approximation on one of the W bosons; the other one was not required to be produced on shell. This procedure gives us a very good idea on the amount of smearing (from near-shell W production) one can expect in the different distributions.

The photon spectrum $f(x)$ is first estimated within the framework of the equivalent photons or Weizsäcker-Williams approximation [13]. In this framework, it is given by

$$f(x) = \frac{\alpha}{\pi} \int_{x^2 M^2}^{\infty} dq^2 \frac{|F(q^2)|^2}{q^2 x} \left[1 - \frac{x^2 M^2}{q^2} \right], \quad (5)$$

where M is the mass of the ion and q^2 is the momentum transfer of the process. $F(q^2)$ is the Fourier transform (FT) of the nuclear charge density:

$$F(q^2) = \frac{4\pi}{q} \int_0^\infty r \rho(r) \sin(qr) dr, \quad (6)$$

where one assumes a spherical nucleus with a normalized charge: $\int_0^\infty d^3r \rho(r) = 1$. At this point, one can use different approximations regarding the nuclear charge distribution or its FT. Soff *et al.* [14] use a *hard sphere* model with $\rho(r) = \Theta(r-R)$, which leads to Bessel functions in the FT. Dress, Zeppenfeld, and Ellis (DZE) [8] assume a Fermi parametrization of the form $\rho = \rho_0 \{1 + \exp[(r-c)/a]\}^{-1}$. This leads to $|F(q^2)|^2 \simeq \exp(-q^2/Q_0^2)$ where one fits Q_0 as well as possible for a given r and a . We first use that spectrum and reparametrize it for ^{238}U : $c = 7.425$ fm, $a = 0.6$ fm, and $Q_0 = 53$ MeV. Using these parameters and the amplitude consisting of 13 Feynman diagrams, we find that our cross section for the process $^{238}\text{U} + ^{238}\text{U} \rightarrow \gamma\gamma \rightarrow W^+W^-$ agrees with the result of Soff *et al.* [14] within a few percent, once the branching ratios are taken into account. This check gave us confidence in the amplitude and indicates that the particular form of the charge distribution one assumes for the nucleus does not have any important

effect. The average radius seems to be the important parameter, at least at high energies and for massive particles. At low energies however (100 GeV/nucleon), the charge distribution does seem to have an important effect [15].

We now turn to the original parametrization of DZE for ^{206}Pb . We present in Fig. 1 the cross section for the process $^{206}\text{Pb} + ^{206}\text{Pb} \rightarrow \gamma\gamma \rightarrow e^+ \nu_e \mu^- \bar{\nu}_\mu$ versus the energy per nucleon; if one were to consider also the charge conjugate of the final state, one could multiply the answer by two. Note that the cross section rises very fast as a function of the energy per nucleon and that the LHC program would really have to work at 3.5 or 4 TeV/nucleon. In Fig. 2, we give different distributions for the positron at an energy of 3.5 TeV/nucleon; identical distributions are obtained for the muon. Note that the peaks in the

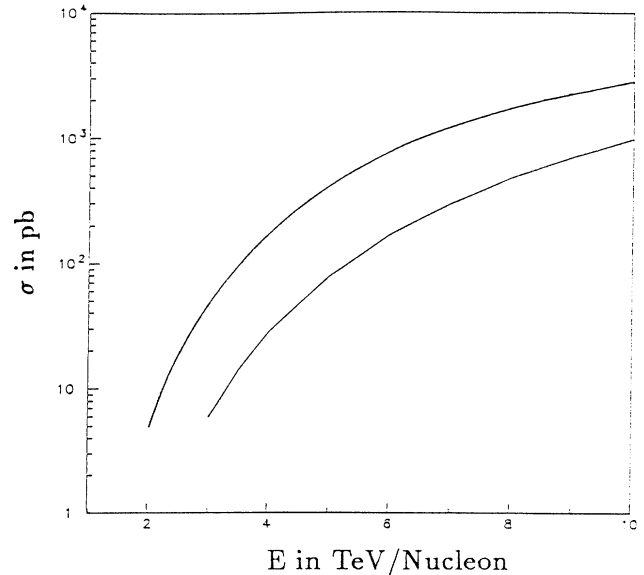


FIG. 1. $\sigma(^{206}\text{Pb} + ^{206}\text{Pb} \rightarrow \gamma\gamma \rightarrow e^+ \nu_e \mu^- \bar{\nu}_\mu)$ as a function of the energy per nucleon for $\kappa_\gamma = 1$. The upper line uses the DZE spectrum; the lower line uses the Cahn-Jackson spectrum.

transverse momentum and energy distributions occur at values relatively easy to observe and relatively large cuts can be imposed without losing very much of the signal. The angular distribution must be symmetric since we do not know which ion the photons come from. This distribution is slightly peaked in the forward-backward direction, but again fairly large cuts could be imposed without much loss in the signal. Note also that the charged leptons have a slight preference to come out back to back. The shapes of all these distributions are rather insensitive to κ_γ ; they are shifted as a whole when one varies κ_γ . This behavior has its roots in the integration over x_1, x_2, \hat{s} . Unfortunately, it also means that one would most likely rely on the total cross section for a measurement of κ_γ . In Fig. 3, we plot the invariant-mass distribution of the charged lepton pair. The peak at $\sim M_W/2$ is typical and

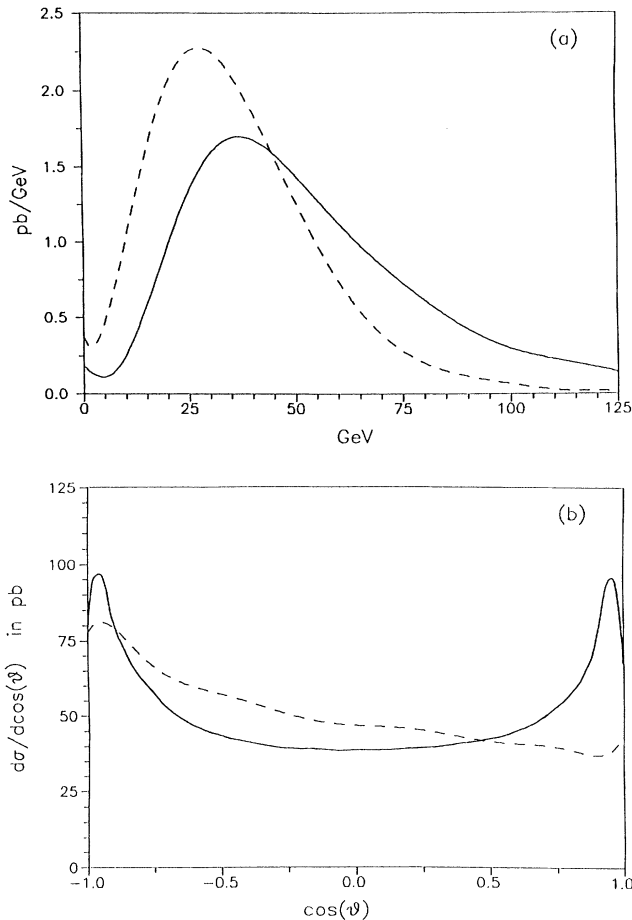


FIG. 2. Different distributions for the same process as in Fig. 1 at 3.5 TeV/nucleon. $\kappa_\gamma = 1$ and we used the DZE spectrum. (a) The dotted line corresponds to the transverse momentum and the solid line corresponds to the energy distribution. (b) The solid line corresponds to the angular distribution with respect to the beam axis and the dotted line corresponds to the angular distributions as a function of the angle between the muon and the positron.

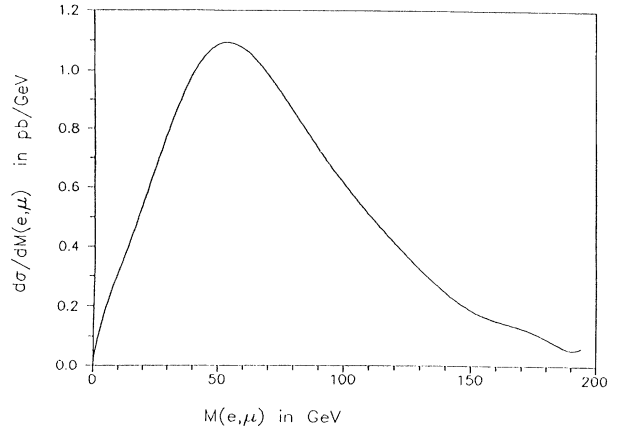


FIG. 3. Invariant-mass distribution of the $e^+\mu^-$ pair in the process $^{206}\text{Pb} + ^{206}\text{Pb} \rightarrow \gamma\gamma \rightarrow e^+\nu_e\mu^-\bar{\nu}_\mu$ at 3.5 TeV/nucleon. $\kappa_\gamma = 1$ and we used the DZE spectrum.

can be used to reduce greatly some backgrounds.

Furthermore, these distributions do not change very much when going from 3.5 TeV/nucleon to 8 TeV/nucleon; an overall rescaling describes well the new distributions although the angular distribution is slightly more peaked in the forward-backward directions. The preference for the leptons to come out back to back is also slightly more pronounced at 8 TeV/nucleon.

The most important question is to know how sensitive the total cross section is to κ_γ . In Fig. 4, we plot the ratio

$$R \equiv \frac{\sigma(\gamma\gamma \rightarrow e^+\mu^-\nu_e\bar{\nu}_\mu)_{\kappa_\gamma \neq 1}}{\sigma(\gamma\gamma \rightarrow e^+\mu^-\nu_e\bar{\nu}_\mu)_{\kappa_\gamma = 1}} \quad (7)$$

as a function of κ_γ . We see that the total cross section is indeed very sensitive to κ_γ . Changing κ_γ by 10% can increase the cross section by 25% or decrease it by 20%. This curve is rather insensitive to the energy per nucleon; one simply gains in rate by going to higher energies.

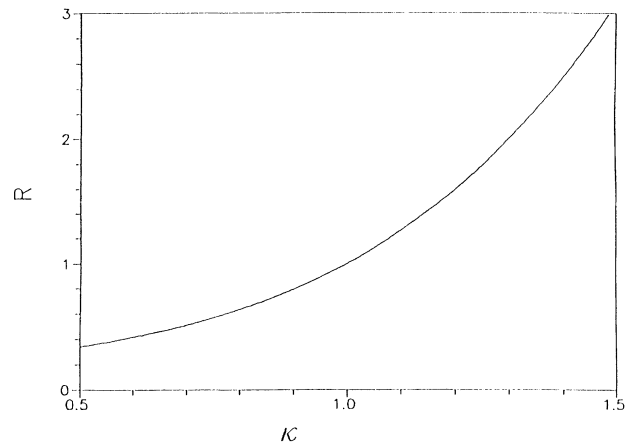


FIG. 4. R vs κ_γ . See text for definition of R . We used the DZE spectrum.

TABLE I. 3σ limits on κ_γ , one could set by a measurement of the cross section $^{206}\text{Pb} + ^{206}\text{Pb} \rightarrow \gamma\gamma \rightarrow e^\pm \nu_e \mu^\mp \nu_\mu$, for different beam energies and different integrated luminosities. The upper value results from the use of the DZE spectrum (the ions can collide head on) and the lower value is from the Cahn-Jackson spectrum (the ions do not collide head on). NB indicates that no bound in the range $0 \leq \kappa_\gamma \leq 2$ could be obtained.

$\int L dt$	3.0 TeV/nucleon	3.5 TeV/nucleon	8 TeV/nucleon
0.1 pb ⁻¹	NB-1.29 NB	$\sim 0.5-1.22$ NB- ~ 1.5	0.92-1.06 0.84-1.11
1.0 pb ⁻¹	0.84-1.11 NB ~ 1.27	0.90-1.08 $\sim 0.5-1.19$	0.98-1.02 0.95-1.04

In Table I, we give the range on κ_γ one could probe by measuring the cross section for the process $^{206}\text{Pb} + ^{206}\text{Pb} \rightarrow \gamma\gamma \rightarrow e^\pm \bar{\nu}_e \mu^\mp \nu_\mu$ at different integrated luminosities. The error we consider here is due to the statistical error in the measurement of the cross section. This is to be compared to the results expected from different accelerators. At LEP II [16], the angular distribution of the W bosons seems to be the most sensitive observable to κ_γ . The main problems are identification and reconstruction of the W bosons. It has been estimated that one could measure $|\Delta\kappa_\gamma| \sim 0.2$, where $\Delta\kappa_\gamma \equiv \kappa_\gamma - 1$. At the Fermilab Tevatron [17], assuming a misidentification probability of $\frac{1}{200}$ Cortes *et al.* estimate that a measurement of $|\Delta\kappa_\gamma| \sim 2$ is possible through $p\bar{p} \rightarrow W\gamma X$. Recently, a more complete analysis has been done by Baur and Berger. They assumed a photon-jet misidentification probability of 5×10^{-3} and an integrated luminosity of 100 pb^{-1} . They consider $p\bar{p} \rightarrow W^\pm \gamma$, $W^\pm \rightarrow e^\pm \nu_e$ and obtain $|\Delta\kappa_\gamma| \sim 1$. Baur and Zeppenfeld

[18] considered the measurement of κ_γ at the LHC and SSC through associated W production. Assuming a misidentification probability of 10^{-4} and an integrated luminosity of 10^4 pb^{-1} , they show that a measurement of $|\Delta\kappa_\gamma| \sim 0.2-0.5$ at 99.9% C.L. or better, is possible. The same authors have recently considered an $e-p$ collider [19]. With an integrated luminosity of 1000 pb^{-1} they obtain $|\Delta\kappa_\gamma| \sim 0.4-0.5$ at DESY HERA energies through single W production. In the long-range planning, it has been shown [20] that an $e-\gamma$ collider could allow a measurement $|\Delta\kappa_\gamma| \sim 0.08$ through single W production.

B. The process $\gamma\gamma \rightarrow \tau^+ \tau^- \rightarrow e^+ \nu_e \bar{\nu}_\mu \mu^- \bar{\nu}_\mu \nu_\tau$

These results are encouraging but one must consider some backgrounds. Our signal is $^{206}\text{Pb} + ^{206}\text{Pb} \rightarrow \gamma\gamma \rightarrow W^+ W^- \rightarrow e^+ \nu_e \mu^- \nu_\mu$ $^{206}\text{Pb} + ^{206}\text{Pb}$. There is an important background to consider: $\gamma\gamma \rightarrow \tau^+ \tau^- \rightarrow e^+ \nu_e \bar{\nu}_\tau \mu^- \bar{\nu}_\mu \nu_\tau$. This process leads to the same leptonic signal and its production cross section is so much larger than the production cross section for the W bosons that one has to worry about it and try to get rid of it.

Since we do not know the energy of the photons and we integrate over their momentum, it seems clear that we need boost-invariant cuts. We will consider the transverse momentum of a given lepton ($P_{T,l}$) and their relative angle in the transverse plane ($\theta_{R,T}$). We will also impose a cut on their invariant mass; since the signal and background come from particles of vastly different masses, this cut might also help us. The first step is to consider the production cross section $\gamma\gamma \rightarrow \tau^+ \tau^-$. This cross section is given by

$$\sigma(\gamma\gamma \rightarrow \tau^+ \tau^-) = \frac{\pi\alpha^2}{\hat{s}^2} \int \left[\frac{\hat{s}(m^2 - \hat{t}) - 3m^4 - \hat{t}^2}{(\hat{t} - m^2)^2} + \hat{t} \rightarrow \hat{u} \right] + \frac{2(\hat{s}m^2 - 4m^4)}{(\hat{t} - m^2)(\hat{u} - m^2)} d\hat{t},$$

where $\hat{s}, \hat{t}, \hat{u}$ are the usual Mandelstam variables of the subprocess and

$$m^2 - \frac{\hat{s}}{2} \left[1 + \left(1 - \frac{4m^2}{\hat{s}} \right)^{1/2} \right] \leq \hat{t} \leq m^2 - \frac{\hat{s}}{2} \left[1 - \left(1 - \frac{4m^2}{\hat{s}} \right)^{1/2} \right].$$

This expression can be integrated to give

$$\sigma(\gamma\gamma \rightarrow \tau^+ \tau^-) = \frac{4\pi\alpha^2}{\hat{s}} \left[\left(1 + \frac{1}{x} - \frac{1}{2x^2} \right) 2 \ln(\sqrt{x} + \sqrt{x-1}) - \left(1 - \frac{1}{x} \right)^{1/2} \left(1 + \frac{1}{x} \right) \right], \quad (8)$$

where $x \equiv \hat{s}/4m^2$. This is our benchmark. Since we will be dealing with a six-body final state and we will have to integrate over the momenta of the initial particles, we resorted to Monte Carlo techniques to handle the multidimensional integrations. Since the width of the τ 's is very small, we will consider on-shell production only. The (matrix element)² of the production and decay of the τ 's is given by

$$|\mathcal{M}(\gamma\gamma \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau e^+ \nu_e \bar{\nu}_\tau)|^2 = |\mathcal{M}(\gamma\gamma \rightarrow \tau\bar{\tau})|^2 \times |\mathcal{M}(\bar{\tau} \rightarrow e^+ \nu_e \bar{\nu}_\tau)|^2 \times |\mathcal{M}(\tau \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau)|^2.$$

We neglect spin correlations. Presumably, these are very small effects and we could certainly get rid of them by loosening the cuts that we will impose to get rid of the τ background. One can first integrate out the momenta of the neutrinos. One is then left with an expression of the form

$$\begin{aligned}
|M(\gamma\gamma \rightarrow \tau\bar{\tau})|^2 \propto & \frac{32m_\tau^2(k_1 \cdot P^+ - P^+ \cdot P^-) + 32(k_1 \cdot P^-)(k_1 \cdot P^+) - 64m_\tau^2(m_\tau^2 - k_1 \cdot P^-)}{4(k_1 \cdot P^-)^2} \\
& + \frac{32m_\tau^2(k_2 \cdot P^+ - P^+ \cdot P^-) + 32(k_2 \cdot P^-)(k_2 \cdot P^+) - 64m_\tau^2(m_\tau^2 - k_2 \cdot P^-)}{4(k_2 \cdot P^-)^2} \\
& + \frac{16m_\tau^2[2(k_2 + k_1) \cdot P^- - (k_1 + k_2) \cdot P^+] + (P^+ \cdot P^-) - (k_1 \cdot k_2) - m_\tau^2}{2(k_1 \cdot P^-)(k_2 \cdot P^-)} \\
& + \frac{32(P^+ \cdot P^-)[(k_1 + k_2) \cdot P^- - (k_1 \cdot k_2)]}{2(k_1 \cdot P^-)(k_2 \cdot P^-)}
\end{aligned}$$

and

$$|M(\bar{\tau} \rightarrow e^+ \nu_e \bar{\nu}_\tau)|^2 \propto (P^+ \cdot q^+)(3m_\tau^2 - 4P^+ \cdot q^+)$$

and

$$|M(\tau \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau)|^2 \propto (P^- \cdot q^-)(3m_\tau^2 - 4P^- \cdot q^-),$$

where k_1, k_2 are the photon momenta and the other momenta refer to charged particles. We will not worry here about absolute normalization; this will become clear shortly. Equation (3) then leads to the total cross section. We used the DZE spectrum. We verified that our Monte Carlo routine gave us the same answer as the analytical expression, once the branching ratios and normalization factors were taken into account. The next step is to vary some cuts and see which ones are more efficient in reducing the τ background. Since the τ production is several orders of magnitude larger than our signal, we simply require the background to be gone completely. We want a cut that does not allow any background event to pass.

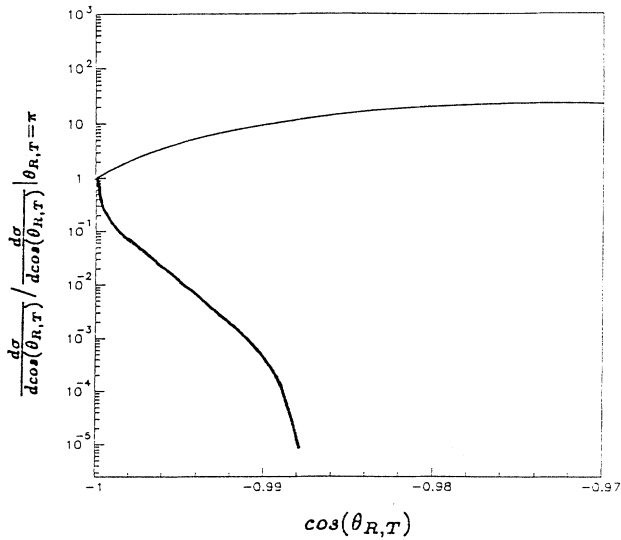


FIG. 5. Normalized cross sections vs $\cos(\theta_{R,T})$. The upper line is our signal and the lower one is the τ background. $\kappa_\gamma = 1$ and we used the DZE spectrum.

There exists such a cut: we require a relatively large invariant mass for the leptons (M_{ll}); we also require a relatively large transverse momentum P_T for each individual lepton. Figure 5 shows that the two leptons will then be back to back in the transverse plane. In that figure, we plot the normalized cross section both for the signal and for the background. Normalized here means that we plot the ratio

$$\frac{d\sigma}{d\cos(\theta_{R,T})} \bigg/ \frac{d\sigma}{d\cos(\theta_{R,T})} \bigg|_{\theta_{R,T}=\pi}$$

versus $\cos(\theta_{R,T})$. We see clearly that a relatively small cut in the relative transverse angle between the leptons can essentially kill this background completely while leaving the W signal relatively unchanged. Admittedly, this cut has a slight dependence on the P_T and M_{ll} cuts that we impose. For example, in Fig. 5, we imposed the cuts $P_T, M_{ll} > 10$ GeV, which leads to $\cos(\theta_{R,T}) \sim -0.99$, while a cut $P_T, M_{ll} > 5$ GeV would lead to $\cos(\theta_{R,T}) \sim -0.965$. This last value is rather extreme, though. In Table II, we give the signal production cross section for different P_T and M_{ll} cuts at different energies; we always imposed $\cos(\theta_{R,T}) > -0.98$ and $10 < \theta_l < 170$.

TABLE II. Cross section for the process $^{206}\text{Pb} + ^{206}\text{Pb} \rightarrow \gamma\gamma \rightarrow e^+ \nu_e \mu^- \bar{\nu}_\mu$ in pb for different cuts on the invariant mass of the lepton pair and their individual transverse momentum. The beam energies are 3.0, 3.5, and 8.0 TeV/nucleon. We used the DZE spectrum at $\kappa_\gamma = 1$. One gains a factor of 2 by including the charge conjugate of the final state.

P_T^{\min} (GeV)	M_{inv}^{\min} (GeV)			
	0	5	10	15
	50			
0	101 1667	*	*	*
5	*	47.5 96.5 1597	46.7 95.8 1583	46.0 94.5 1558
10	*	45.5 91.9 1518	45.1 91.3 1513	44.4 89.9 1498
15	*	41.0 82.7 1380	40.6 82.4 1372	40.0 81.2 1368

Admittedly, this cut would not be adequate for the low P_T and M_{ll} cuts but this table shows clearly that the signal cross section is not very sensitive to these cuts, until we reach rather large values of P_T . From Fig. 3, this behavior was expected. For comparison, we also give the results when only the cut on the lepton angle is imposed.

One could also worry about off-shell production of τ . This is not an issue because the production cross section is reduced by about 12 orders of magnitude for one τ produced far off shell and by more than 22 orders of magnitudes when both τ 's are produced off shell. The first condition is sufficient to bring this potential background to about 5 pb at SSC energies [21]; recall that the signal is more than 1500 pb. The close-to-shell production does not exist, given the very small width of the τ leptons.

II. NUCLEAR OVERLAP

There is still an issue that we have to face: up to now, we have allowed the ions to collide head on. Obviously, this will lead to a very large multiplicity of the events. This is already a serious problem since we want to pick two leptons out of hundreds of particles. More worrisome, there is a background that could mimic perfectly our signal: $gg \rightarrow t\bar{t}$ and then these heavy top quarks decay to W bosons which then decay to our signal. The cuts that worked so well for the τ background are useless here since we deal with a massive fermion. One could argue that our original signal requires no hadronic activity at all but this will most likely not happen if the ions are allowed to collide head on. Therefore, for this argument to be reliable, one must require the ions to miss each other and have no hadronic activity. Only then will we be sure that the signal really comes from W bosons (assuming that the previous cuts are implemented). The whole issue is then to obtain the correct photon spectrum when one requires a rather large impact parameter. In order to take this finite impact parameter into account, one can write the photon spectrum as a function of this impact parameter. This is the procedure used by Papageorgiu and Mueller *et al.* [22]. We will instead use a parametrization derived by Cahn and Jackson [23], which is itself based on a procedure initiated by Baur *et al.* [24]. In this section, we will simply calculate the production cross section of W -boson pairs. As we have seen before, the cuts used to eliminate the background from τ production changes this rate by only a few percent. This result should still hold in the large impact-parameter process. One then writes the cross section as

$$\sigma(\text{Pb} + \text{Pb} \rightarrow W^+ W^- + \text{Pb} + \text{Pb}) = \int_{\hat{s}_{\min}}^{\hat{s}_{\max}} \sigma(\gamma\gamma \rightarrow W^+ W^-) \frac{L_0 \Psi(z)}{\hat{s}} d\hat{s}, \quad (9)$$

where $\sigma(\gamma\gamma \rightarrow W^+ W^-)$ is given by Eq. (4) and $L_0 = 16Z^4 \alpha^2 / 3\pi^2$ with Z being the charge of the ion. Furthermore, $z = \sqrt{\hat{s}} R / \gamma_l$ where R is the radius of the ion and γ_l is the usual Lorentz factor. Cahn and Jackson parametrized the total photon spectrum by the function

$\Psi(z) = \sum_{j=1}^3 A_j e^{-b_j z}$ with $A_1 = 1.909$, $A_2 = 12.35$, $A_3 = 46.28$, $b_1 = 2.566$, $b_2 = 4.948$, $b_3 = 15.21$. This particular spectrum leads to the cross section given in Fig. 1. We see that the rate is reduced by a factor 3–4 at the SSC and by a factor 7–8 at the LHC. This leads to limits on κ_γ as shown in Table II. We see clearly that the LHC cannot do very well to measure κ_γ but the SSC remains interesting. These last results did not impose any cuts on the outgoing charged leptons. As seen before, the cuts imposed either to make the particles visible (10° from the beam pipe) or to eliminate the possible backgrounds will not change these results by much. Furthermore, the sensitivity curve on Fig. 4 does not change with this spectrum. It should be pointed out that the Cahn-Jackson spectrum is smaller than other spectra by a factor 3–4 and can be interpreted as some sort of lower limit on our production cross section [25].

Another issue regards the possible systematics involved in the measurement of the cross section. Such a measurement is known to be difficult. However, one could use the τ production cross section as a benchmark and compare it to the cross section of our signal. The single pion and lepton decay modes of the tau appear to be a good candidate for this calibration; one requires one charged pion and one charged lepton. The photon production of the τ leptons can be calculated reliably because the Compton wavelength of this particle is much smaller than the size of the nuclei (the tau is really a particle in this system). The single pion and lepton decay modes seem interesting because one then avoids the photon (or gluon) pair production of the electrons, muons, and pions. The only potential background is now reduced to the production of two pions and two leptons *and* the requirement that one particle of each pair goes down the beam pipe. Presumably, this last requirement will bring this background down to manageable levels.

III. CONCLUSIONS

We have shown that the nuclear charge distribution has little impact on the process at hand when the nuclei are allowed to collide head on; the average radius of the distribution is the important parameter. In the case where the nuclei are not allowed to collide head on, the tail of the nuclear charge distribution could become important and reduce further the photon luminosity: the effective radius used in the Cahn-Jackson spectrum would have to be increased to avoid “strong” processes. This can lead to a very fast reduction in the coherent photon spectrum.

We have also shown that the process $\text{Pb} + \text{Pb} \rightarrow \gamma\gamma \rightarrow e^+ \nu_e \mu^- \bar{\nu}_\mu$ is very sensitive to κ_γ . With an optimistic integrated luminosity of 1 pb^{-1} , a measurement in the 10% range seems possible at LHC and in the 2% range at the SSC. These figures become 30% and 8% with an integrated luminosity of 0.1 pb^{-1} . Using the Cahn-Jackson spectrum, the cross section is reduced by a factor 2–3 at the SSC and about 7 at the LHC. This loosens the bounds on κ_γ by a factor 2 at the SSC and 3–4

at the LHC.

The background from τ pair production can be eliminated completely by cuts on the transverse momentum of each lepton and their relative angle in the transverse plane. Head-on collisions of the ions must be avoided in order to reduce the large multiplicity of the events and, more importantly, to avoid backgrounds from heavy top-quark pair production via gluon fusion. The requirements of no head-on collisions and no hadronic activity produce a clean signal.

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