Electroweak calculations in the presence of nonperturbative quark self-energies

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Electroweak three- and four-point functions with two external quark legs are evaluated in the presence of arbitrarily momentum-dependent quark self-energy contributions of nonperturbative origin, such as those arising from QCD vacuum condensates. Gauge-parameter independence of $O(e^2)$ on-mass-shell quark two-point functions is recovered in the presence of such self-energies, provided electroweak three- and four-point functions retain consistency with the Ward identities arising from the Becchi-Rouet-Stora-Tyutin symmetries of the electroweak Lagrangian. These identities necessarily lead to departures from purely perturbative electroweak Feynman rules; four-point functions that to lowest order are entirely one-particle reducible in perturbative electroweak theory are seen to acquire one-particle-irreducible components of comparable magnitude in the presence of contributions to quark self-energies external to electroweak theory. Such additional contributions to three- and four-point functions, required for the gauge-parameter independence of on-shell quark two-point functions, do not contribute to Landau-gauge amplitudes. In the chiral limit, such self-energy contributions are also shown to induce gauge-parameter-independent Yukawa couplings to on-shell quarks. In the limit of a vanishing Higgs-field momentum, this Yukawa coupling is at most of order Σ^3/M_Z^2, where Σ is the scale of the nonperturbative dynamical quark mass.

I. INTRODUCTION

In calculating any electroweak process involving hadrons, it is important to note that the underlying field theory acting on basis quarks is not just $SU(2)_L \times U(1)$ but the full $SU(3)_c \times SU(2)_L \times U(1)$ gauge theory of the standard model. Moreover, the full standard-model vacuum necessarily permits perturbative $SU(2)_L \times U(1)$ interactions to couple directly to nonperturbative vacuum expectation values [e.g., $\langle 0|{:}\psi(x)\bar{\psi}(y){:}|0\rangle$] that occur in the presence of vacuum condensates [1]. Such nonperturbative contributions to quark two-point functions (which are believed to be responsible for constituent-quark masses [2, 3]) certainly occur, but are not easily incorporated into electroweak calculations.

Suppose the quark two-point function acquires a mass contribution $\Sigma(p^2)$ from sources external to perturbative electroweak theory. Terms proportional to $\not{p}$ in Σ are assumed to have already been absorbed in wave-function renormalization factors (Z_2). Consequently, we will regard Σ as a purely Dirac scalar contribution to the fermion propagator:

$$S(p)=[m_L-\not{p}-\Sigma(p^2)]^{-1} . \tag{1.1}$$

Such contributions in empirical hadronic electroweak processes are expected to arise from QCD-vacuum condensates. For example, in the limit of Lagrangian chiral symmetry $(m_L=0)$, the dimension-three QCD-vacuum condensate $\langle 0|{:}\bar{\psi}(0)\psi(0){:}|0\rangle$ $(\equiv\langle\bar{q}q\rangle)$ yields a self-energy mass contribution [3, 4]

$$-\Sigma(p^2)=[g_s^2|\langle\bar{q}q\rangle|(3+a)]/(9p^2+g_s^2am|\langle\bar{q}q\rangle|/p^2) , \tag{1.2}$$

where a is the QCD covariant-gauge parameter, and where $m[=-\Sigma(m^2)=|g_s^2\langle\bar{q}q\rangle/3|^{1/3}]$ is the gauge-parameter-independent (GPI) mass [2, 4, 5], generated by the chiral noninvariance of the QCD vacuum.

In the present paper, we will consider $\Sigma(p^2)$ to be an arbitrary nonperturbative external contribution to the Dirac-scalar portion of the quark self-energy. This contribution is not expected to respect the $SU(2)_L \times U(1)$ symmetry of the electroweak Lagrangian [e.g., the QCD order-parameter $\langle\bar{q}q\rangle$ is not invariant under $SU(2)_L$-symmetry transformations]. Hence, such external self-energy contributions within electroweak field-theoretical processes are expected to lead to calculational inconsistencies. Earlier work [6] has indeed shown that on-shell electroweak-mediated self-energies in the presence of a quark-condensing vacuum, such as that of QCD, exhibit explicit dependence on electroweak gauge parameters unless the quark mass is completely insensitive to the $\langle\bar{q}q\rangle$ condensate, an assumption inconsistent with having constituent-quark masses arise from the chiral noninvariance of the QCD vacuum [7]. Since on-shell Feynman amplitudes must be gauge-parameter independent for those amplitudes to be meaningful, we can only conclude that a naive application of electroweak Feynman rules is incorrect in the presence of such externally generated quark self-energies [6].

Our approach to the problem delineated above is to obtain electroweak three- and four-point Green's functions in the presence of nonperturbative fermion two-point-function contributions external to $SU(2)_L \times U(1)$ through straightforward use of $SU(2)_L \times U(1)$ Ward identities [8]. These Green's functions reduce to those obtained from conventional electroweak Feynman rules in the limit that externally generated quark self-energy contributions vanish. We then test our approach by examining the electroweak gauge-parameter dependence of on-shell self-energy amplitudes generated in the presence of these arbitrarily momentum-dependent external contributions $\Sigma(p^2)$ to the quark inverse propagator, and, as an application of some physical interest, we obtain from $\Sigma(p^2)$ a (one-loop) induced Higgs coupling to quarks in the chiral limit.

In Sec. II we derive and examine relevant Ward identities of spontaneously broken $SU(2)_L \times U(1)$ gauge theory in the presence of external contributions to quark self-energies. By making use of the Becchi-Rouet-Stora-Tyutin (BRST) invariances of the electroweak Lagrangian, we rederive well-known Ward identities for self-energy corrections to the three-point vertices $\bar{\psi}A\psi$, $\bar{\psi}Z\psi, \bar{\psi}W\psi$. We then employ these same methods to demonstrate how four-point $\bar{\psi}AA\psi$, $\bar{\psi}ZZ\psi$, and $\bar{\psi}WW\psi$ vertices, ordinarily one-point-reducible (1PR) quantities to tree order, acquire one-point-irreducible (1PI) components in the presence of external contributions to quark self-energies. We also find that remaining four-point functions involving two external quark lines (i.e., four-point couplings to scalar and ghost fields) can remain 1PR quantities as long as the quark propagator and $\bar{\psi}Z\psi$, $\bar{\psi}W\psi$ three-point functions include external self-energy effects. The Ward identities we obtain are shown to allow the retention of the usual Feynman rules for Yukawa couplings.

In Sec. III we test the results of Sec. II by examining the electroweak gauge-parameter independence of the simplest possible on-mass-shell amplitude, the electroweak contributions to the quark's two-point function evaluated on the quark mass shell. The use of uncorrected (or the improper use of corrected) three- and four-point functions in the presence of external quark self-energies leads to a gauge-dependent on-shell amplitude. We explicitly show that the three- and four-point functions from Sec. II must be utilized in order to recover the gauge-parameter independence expected for any on-mass-shell Feynman amplitude.

In Sec. IV we consider spontaneously broken $SU(2)_L \times U(1)$ field theory in the limit of Lagrangian chiral symmetry, i.e., the limit in which there are no Lagrangian Yukawa couplings. We show that the four-point functions of Sec. II imply that, even in the chiral limit, a Yukawa coupling is induced if quark propagators have Dirac-scalar components arising from sources external to electroweak theory, such as the chiral-noninvariant QCD vacuum. We demonstrate that this induced Yukawa-interaction three-point function is gauge-parameter independent when external quark lines are taken to be on their mass shells. We then consider the (current) quark mass induced by this Yukawa interaction, and show that its magnitude is negligible ($\lesssim 10^{-2}$ MeV).

Finally, we argue in Sec. V that three- and four-point functions need not be corrected in the presence of arbitrary external contributions to quark self-energies as long as calculations are performed in Landau gauge, as the corrections such contributions impose upon three- and four-point functions are annihilated by transverse projection operators. For practical purposes, this is the most important consequence of the present work. It tells us that for a physical (on-shell) amplitude, the result obtained in a *Landau-gauge* calculation employing "naive" Feynman rules (except for external contributions to the quark propagator) is indeed the correct gauge-invariant result. Similar conclusions have been reached in early studies involving dynamical chiral-symmetry breaking [9], corresponding to the chiral limit of our more general treatment.

We wish to stress that the problems addressed in our paper are not just of a formal nature, but carry important physical implications. In the (semiperturbative) calculation of many electroweak processes involving quarks, the dynamically induced (QCD) quark self-energy plays an essential role, facilitating the avoidance of spurious infrared enhancements in some cases or unphysical helicity suppressions in others. In the absence of a formalism of the type presented here, reliable (i.e., gauge-parameter-independent) predictions for such electroweak processes are impossible unless the full $SU(3)_c \times SU(2)_L \times U(1)$ theory, including nonperturbative quantum vacuum effects, are treated in a complete and consistent fashion. The results we present are also applicable to any chiral gauge theory with externally induced chiral-symmetry breaking.

II. ELECTROWEAK WARD IDENTITIES

For the quantum electrodynamics (QED) subgroup of electroweak theory, the Ward identities relating three- and four-point Green's functions to two-point fermion Green's function are obtained by requiring invariance of all renormalized Green's functions under the relevant BRST transformations. For example, to obtain the relationship between the quark-antiquark-photon ($\bar{\psi}A\psi$) three-point function and the externally generated self-energy in Eq. (1.1), we begin by requiring BRST invariance of the corresponding three-point function involving the photon's Faddeev-Popov ghost ($c^A, \bar{c}^A$) [10]:

$$
\begin{aligned}
0 &= \delta^{\mathrm{BRST}} \langle \psi(x) \bar{c}^A(y) \bar{\psi}(z) \rangle \\
&= -i\lambda e Q \langle \psi(x) c^A(x) \bar{c}^A(y) \bar{\psi}(z) \rangle + (\lambda/\alpha) \langle \psi(x) \partial^\mu A_\mu(y) \bar{\psi}(z) \rangle \\
&\quad + i\lambda e Q \langle \psi(x) c^A(z) \bar{c}^A(y) \bar{\psi}(z) \rangle + \text{contributions from } Z \text{ and } W \text{ sectors} \\
&= -i\lambda e Q \langle \psi(x) \bar{\psi}(z) \rangle \langle c^A(x) \bar{c}^A(y) \rangle + (\lambda/\alpha) \partial_y^\mu \langle \psi(x) A_\mu(y) \bar{\psi}(z) \rangle \\
&\quad + i\lambda e Q \langle \psi(x) \bar{\psi}(z) \rangle \langle c^A(z) \bar{c}^A(y) \rangle + \text{higher-order contributions in } e \text{ and } g \; . \qquad (2.1)
\end{aligned}
$$

Note that the parameter λ in (2.1) is a Grassmann variable (α is the QED covariant-gauge parameter). In obtaining the final line of (2.1), we note that the lowest contributing order in electroweak couplings to (2.1) is insensitive to non-Abelian contributions from the embedding of QED into $SU(2)_L \times U(1)$. Application of the D'Alembertian operator in the y variable yields

$$(1/\alpha)\Box_y \partial_y^\mu \langle \psi(x) A_\mu(y) \bar{\psi}(z) \rangle = ieQ[\langle \psi(x)\bar{\psi}(z)\rangle \delta^4(x-y) - \langle \psi(x)\bar{\psi}(z)\rangle \delta^4(z-y)] + \text{higher-order contributions in } e \text{ and } g \; . \quad (2.2)$$

If we define our momentum-space Green's functions to be related to coordinate-space Green's functions via

$$\langle \psi(x)\bar{\psi}(z)\rangle = (2\pi)^{-8} \int d^4p \int d^4q \, e^{-iq\cdot x} e^{ip\cdot z} G_{\psi\bar{\psi}}(q;p) \; , \quad (2.3a)$$

$$\langle \psi(x) A^\mu(y) \bar{\psi}(z)\rangle = (2\pi)^{-12} \int d^4q \int d^4k \int d^4p \, e^{-iq\cdot x} e^{ik\cdot y} e^{ip\cdot z} G^\mu_{\psi A \bar{\psi}}(q;k,p) \; , \quad (2.3b)$$

where momenta before the semicolon are outgoing and momenta after the semicolon are incoming, we then find that

$$(-ik_\mu k^2/\alpha) G^\mu_{\psi A\bar{\psi}}(q;k,p) = ieQ[G_{\psi\bar{\psi}}(q-k;p) - G_{\psi\bar{\psi}}(q;p+k)] \; . \quad (2.4)$$

Furthermore, if we define truncated momentum-space Green's functions via

$$G_{\psi\bar{\psi}}(q-k;p) \equiv S(p)\delta^4(q-k-p) \; , \quad (2.5a)$$

$$G^\mu_{\psi A\bar{\psi}}(q;k,p) = S(q)\Delta_A^{\mu\nu}(k)\Gamma^\nu_{\bar{\psi}A\psi}(q;k,p) \times S(p)\delta^4(q-k-p) \; , \quad (2.5b)$$

and make use of the Ward identity for the full photon propagator

$$k_\mu \Delta_A^{\mu\nu}(k) = \alpha k^\nu / k^2 \; , \quad (2.6)$$

we then obtain the well-known relation between two- and three-point functions that is upheld to all orders of QED considered in isolation:

$$k_\nu \Gamma^\nu_{\bar{\psi}A\psi}(q;k,p) = -eQ[S^{-1}(q) - S^{-1}(p)] \; . \quad (2.7)$$

Equation (2.7) demonstrates how an external momentum-dependent contribution to the fermion propagator (1.1) necessarily alters the off-shell coupling of fermions to photons. The contribution (2.7), of course, vanishes with vanishing photon momentum, consistent with the definition of electric charge.

We now derive the QED Ward identity relating the $\psi A A \bar{\psi}$ four-point function to the two- and three-point functions of (2.7) by considering the following BRST variation:

$$\begin{aligned} 0 &= \delta^{\text{BRST}} \langle \psi(x) A_\mu(y) \bar{c}^{\,A}(w) \bar{\psi}(z)\rangle \\ &= -i\lambda eQ \langle \psi(x) A_\mu(y) \bar{\psi}(z)\rangle \langle c^A(x) \bar{c}^{\,A}(w)\rangle - \lambda \langle \psi(x)\bar{\psi}(z)\rangle (\partial_y)_\mu \langle c^A(y)\bar{c}^{\,A}(w)\rangle \\ &\quad + (\lambda/\alpha)\partial_w^\nu \langle \psi(x) A_\mu(y) A_\nu(w) \bar{\psi}(z)\rangle + i\lambda eQ \langle \psi(x) A_\mu(y)\bar{\psi}(z)\rangle \langle c^A(z)\bar{c}^{\,A}(w)\rangle \\ &\quad + \text{higher-order contributions in } e \text{ and } g \; . \end{aligned} \quad (2.8)$$

If we ignore the higher-order contributions to (2.8) and apply a D'Alembertian operator in the variable w, we obtain

$$\frac{1}{\alpha}\Box_w \partial_w^\nu \langle \psi(x) A_\mu(y) A_\nu(w) \bar{\psi}(z)\rangle = ieQ\langle \psi(x) A_\mu(y)\bar{\psi}(z)\rangle \delta^4(x-w) + (\partial_y)_\mu [\langle \psi(x)\bar{\psi}(z)\rangle \delta^4(y-w)] - ieQ \langle \psi(x) A_\mu(y) \bar{\psi}(z)\rangle \delta^4(z-w) \; . \quad (2.9)$$

The four-point function in (2.9) has a connected and a disconnected piece:

$$\langle \psi(x) A_\mu(y) A_\nu(w)\bar{\psi}(z)\rangle = \langle \psi(x) A_\mu(y) A_\nu(w) \bar{\psi}(z)\rangle_c + \langle \psi(x)\bar{\psi}(z)\rangle \langle A_\mu(y) A_\nu(w)\rangle \; . \quad (2.10)$$

Equation (2.9) can be used to generate a momentum-space Ward identity through use of (2.5), (2.10), and the momentum-space connected Green's functions

$$\begin{aligned} &\langle A^\mu(y) A^\nu(w)\rangle = (2\pi)^{-8} \int d^4p \int d^4q \, e^{-ip\cdot y} e^{iq\cdot w} G^{\mu\nu}_{AA}(p;q) \; , \\ &G^{\mu\nu}_{AA}(p;q) \equiv \Delta_A^{\mu\nu}(q)\delta^4(p-q) \; , \\ &\langle \psi(x) A^\mu(y) A^\nu(w)\bar{\psi}(z)\rangle_c = (2\pi)^{-16} \int d^4l \int d^4k \int d^4q \int d^4p \, e^{-il\cdot x} e^{ik\cdot y} e^{iq\cdot w} e^{ip\cdot z} G^{\mu\nu}_{\psi AA\bar{\psi}}(l;k,q,p) \; , \end{aligned} \quad (2.11)$$

and the truncated Green's function

$$G^{\mu\nu}_{\psi AA\bar{\psi}}(l;k,q,p) = S(l)\Delta_A^{\mu\rho}(k)\Delta_A^{\nu\sigma}(q)\Gamma^{\tau\eta}_{\bar{\psi}AA\psi}(l;k,q,p) g_{\rho\tau} g_{\sigma\eta} S(p)\delta^4(l-k-q-p) \; . \quad (2.12)$$

Upon substitution of (2.6) and (2.10)–(2.12) into (2.9), we obtain the following Ward identity [$l=k+q+p$]:

$$q_\sigma \Gamma^{\tau\sigma}_{\bar\psi AA\psi}(l;k,q,p) = -eQ[S^{-1}(l)S(l-q)\Gamma^{\tau}_{\bar\psi A\psi}(l-q;k,p) - \Gamma^{\tau}_{\bar\psi A\psi}(l;k,p+q)S(p+q)S^{-1}(p)] . \quad (2.13)$$

In (2.13), we may use (2.7) to replace $S^{-1}(l)$ and $S^{-1}(p)$ with the expressions

$$\begin{aligned} -eQS^{-1}(l) &= q_\nu \Gamma^{\nu}_{\bar\psi A\psi}(l;q,l-q) - eQS^{-1}(l-q) , \\ eQS^{-1}(p) &= eQS^{-1}(p+q) + q_\nu \Gamma^{\nu}_{\bar\psi A\psi}(p+q;q,p) , \end{aligned} \quad (2.14)$$

so as to obtain

$$\begin{aligned} q_\sigma \Gamma^{\tau\sigma}_{\bar\psi AA\psi}(l;k,q,p) = {} & q_\sigma \{\Gamma^{\sigma}_{\bar\psi A\psi}(l;q,l-q)S(l-q)\Gamma^{\tau}_{\bar\psi A\psi}(l-q;k,p) + \Gamma^{\tau}_{\bar\psi A\psi}(l;k,p+q)S(p+q)\Gamma^{\sigma}_{\bar\psi A\psi}(p+q;q,p)\} \\ & + eQ[\Gamma^{\tau}_{\bar\psi A\psi}(l;k,p+q) - \Gamma^{\tau}_{\bar\psi A\psi}(l-q;k,p)] . \end{aligned} \quad (2.15)$$

This last identity reveals the structure of the $\bar\psi AA\psi$ four-point function in the presence of external contributions to the quark self-energy. The curly bracketed term on the right-hand side of (2.15) corresponds to the one-point-*reducible* contributions one would obtain in field theory from appropriately dressed three-point vertices and two-point propagators arising from the QED Lagrangian. The final square-bracketed term on the right-hand side of (2.15) can easily be shown to vanish in the absence of external momentum-dependent contributions to the fermion self-energy. However, in the presence of an external momentum-dependent contribution to the self-energy, as proposed in (1.1), the final term in (2.15) corresponds to a distinct 1PI contribution to the truncated four-point Green's function, as indicated schematically in Fig. 1. This additional contribution is essential for gauge-parameter independence of the on-shell quark self-energy, as will be demonstrated in Sec. III.

In deriving Ward identities for Z- and W-coupled Green's functions analogous to (2.7) and (2.15), one must take into consideration nonvanishing Yukawa couplings to the Higgs sector. Consequently both the physical Higgs field (ϕ) and unphysical scalar partners ($\chi_3,\chi^\pm$) to the Z and W will appear in relevant Ward identities. As in the previous section, we begin by requiring BRST invariance of an appropriately chosen three-point function:

$$\begin{aligned} 0 &= \delta^{\rm BRST}\langle \psi(x)\bar c^{\,Z}(y)\bar\psi(z)\rangle \\ &= -i\lambda(a-b\gamma_5)\langle \psi(x)c^Z(x)\bar c^{\,Z}(y)\bar\psi(z)\rangle \\ &\quad + \lambda\langle \psi(x)\left[\frac{1}{\alpha_Z}\partial^\mu Z_\mu(y) + M_Z\chi_3(y)\right]\bar\psi(z)\rangle \\ &\quad + i\lambda\langle \psi(x)c^Z(z)\bar c^{\,Z}(y)\bar\psi(z)\rangle(a+b\gamma_5) , \end{aligned} \quad (2.16)$$

where α_Z is the ('t Hooft–Feynman) gauge parameter of the Z, and where a and b characterize the tree-level $Zq\bar q$ vertex [10]:

$$(\Gamma^{\mu}_{\bar\psi Z\psi})^{\rm tree}_{\binom{\rm up}{\rm dn}} \equiv \gamma^\mu(a \mp b\gamma_5) , \quad (2.17a)$$

$$a_{\binom{\rm up}{\rm dn}} = \frac{eM_Z^2}{2M_W\sqrt{M_Z^2-M_W^2}}\left[(\pm)\frac{1}{2} + \begin{bmatrix}-\frac{4}{3}\\ +\frac{2}{3}\end{bmatrix}\frac{M_Z^2-M_W^2}{M_Z^2}\right] , \quad (2.17b)$$

$$b_{\binom{\rm up}{\rm dn}} = \frac{eM_Z^2}{4M_W\sqrt{M_Z^2-M_W^2}} . \quad (2.17c)$$

Upon application of the operator $(\Box_y + \alpha_Z M_Z^2)$ to both sides of (2.16), we find in momentum space (to lowest order in electroweak couplings) that

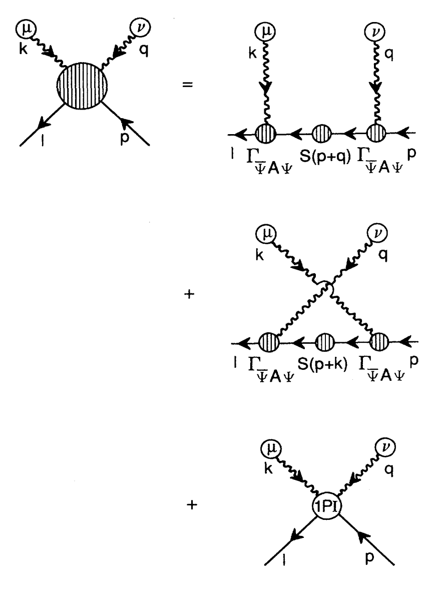

FIG. 1. 1PR and 1PI components of the $\bar\psi AA\psi$ truncated momentum-space Green's function. Shaded circles represent corrections to three-point vertices and the quark propagator following from inclusion of external nonperturbative contributions to the quark self-energy.

$$\frac{1}{\alpha_Z}(ik_\mu)(-k^2+\alpha_Z M_Z^2)G^{\mu}_{\psi Z\bar\psi}(q;k,p)+M_Z(-k^2+\alpha_Z M_Z^2)G_{\psi\chi_3\bar\psi}(q;k,p)$$
$$=i(a-b\gamma_5)G_{\psi\bar\psi}(q-k;p)-iG_{\psi\bar\psi}(q;p+k)(a+b\gamma_5)\ . \quad (2.18)$$

To express (2.18) in terms of truncated Green's functions, we utilize (2.5), (2.12), and

$$G_{\psi\chi_3\bar\psi}(q;k,p)=S(q)\Delta_{\chi_3}(k)\Gamma_{\bar\psi\chi_3\psi}(q;k,p)S(p)\delta^4(q-k-p)\ , \quad (2.19a)$$

$$G^{\mu}_{\psi Z\bar\psi}(q;k,p)=S(q)\Delta_Z^{\mu\nu}(k)g_{\nu\tau}\Gamma^{\tau}_{\bar\psi Z\psi}(q;k,p)S(p)\delta^4(q-k-p)\ , \quad (2.19b)$$

as well as the Z- and χ_3-propagator identities [following from BRST invariance of $\langle Z_\mu(x)\bar c^{\,Z}(y)\rangle$, $\langle \chi_3(x)\bar c^{\,Z}(y)\rangle$]

$$(-k^2+\alpha_Z M_Z^2)\Delta_{\chi_3}(k)=1\ , \quad (2.20a)$$

$$k_\mu\Delta_Z^{\mu\nu}(k)=\alpha_Z k^\nu/(k^2-\alpha_Z M_Z^2)\ , \quad (2.20b)$$

in order to obtain

$$-ik_\nu\Gamma^{\nu}_{\bar\psi Z\psi}(q;k,p)+M_Z\Gamma_{\bar\psi\chi_3\psi}(q;k,p)$$
$$=iS^{-1}(q)(a-b\gamma_5)-i(a+b\gamma_5)S^{-1}(p)\ . \quad (2.21)$$

Equation (2.21) is easily seen to be satisfied by the tree-level vertices (Feynman rules) of SU(2)×U(1). We see from (2.21), as in (2.7), that external contributions to the fermion propagator (1.1) necessarily affect $\psi\bar\psi Z$ and $\psi\bar\psi\chi_3$ three-point Green's functions.

To obtain Z-sector analogs of (2.15) relating four-point Green's functions to two- and three-point Green's functions, we follow the procedures delineated prior to (2.15) to obtain the following Ward identity from BRST invariance of the $\langle \psi(x)Z_\mu(y)\bar c^{\,z}(w)\bar\psi(z)\rangle$ four-point Green's function:

$$-iq_\nu\Gamma^{\mu\nu}_{\bar\psi ZZ\psi}(l;k,q,p)+M_Z\Gamma^{\mu}_{\bar\psi Z\chi_3\psi}(l;k,q,p)=iS^{-1}(l)(a-b\gamma_5)S(p+k)\Gamma^{\mu}_{\bar\psi Z\psi}(p+k;k,p)$$
$$-i\Gamma^{\mu}_{\bar\psi Z\psi}(l;k,p+q)S(p+q)(a+b\gamma_5)S^{-1}(p)-\frac{ik^\mu}{\alpha_Z}\Gamma_{\bar\psi\bar c c\psi}(l;k,q,p). \quad (2.22)$$

Similarly, one obtains from BRST invariance of $\langle \psi(x)\chi_3(w)\bar c^{\,Z}(y)\bar\psi(z)\rangle$ the momentum-space Ward identity $[l-q=p+k]$:

$$-iq_\mu\Gamma^{\mu}_{\bar\psi\chi_3 Z\psi}(l;k,q,p)+M_Z\Gamma_{\bar\psi\chi_3\chi_3\psi}(l;k,q,p)=iS^{-1}(l)(a-b\gamma_5)S(l-q)\Gamma_{\bar\psi\chi_3\psi}(l-q;k,p)$$
$$+2b\frac{\alpha_Z M_Z^2-k^2}{m_\phi^2-(k+q)^2}\Gamma_{\bar\psi\phi\psi}(l;k+q,p)$$
$$-i\Gamma_{\bar\psi\chi_3\psi}(l;k,p+q)S(p+q)(a+b\gamma_5)S^{-1}(p)+M_Z\Gamma_{\bar\psi\bar c c\psi}(l;k,q,p)\ . \quad (2.23)$$

In obtaining (2.23) we have utilized both the tree-level relationship $2b\langle\phi\rangle=M_Z$ and the tree-level expression for the ϕ propagator, as is appropriate for the (lowest nontrivial) order of electroweak coupling in which we are working. Equations (2.22) and (2.23) can be employed to eliminate $\Gamma^{\mu}_{\bar\psi\chi_3 Z\psi}$, thereby yielding the relationship

$$q_\nu k_\mu\Gamma^{\mu\nu}_{\bar\psi ZZ\psi}(l;k,q,p)+M_Z^2\Gamma_{\bar\psi\chi_3\chi_3\psi}(l;q,k,p)=-S^{-1}(l)(a-b\gamma_5)S(p+k)k_\mu\Gamma^{\mu}_{\bar\psi Z\psi}(p+k;k,p)$$
$$+k_\mu\Gamma^{\mu}_{\bar\psi Z\psi}(l;k,p+q)S(p+q)(a+b\gamma_5)S^{-1}(p)+(k^2/\alpha_Z)\Gamma_{\bar\psi\bar c c\psi}(l;k,q,p)$$
$$+iS^{-1}(l)(a-b\gamma_5)S(l-k)M_Z\Gamma_{\bar\psi\chi_3\psi}(l-k;q,p)$$
$$+2bM_Z\frac{\alpha_Z M_Z^2-q^2}{(m_\phi^2-(k+q)^2)}\Gamma_{\bar\psi\phi\psi}(l;k+q,p)-iM_Z\Gamma_{\bar\psi\chi_3\psi}(l;q,p+k)$$
$$\times S(p+k)(a+b\gamma_5)S^{-1}(p)+M_Z^2\Gamma_{\bar\psi\bar c c\psi}(l;q,k,p)\ . \quad (2.24)$$

We note from Bose symmetry that

$$q_\nu k_\mu\Gamma^{\mu\nu}_{\bar\psi ZZ\psi}(l;k,q,p)=k_\nu q_\mu\Gamma^{\mu\nu}_{\bar\psi ZZ\psi}(l;q,k,p)\ , \quad (2.25a)$$

$$\Gamma_{\bar\psi\chi_3\chi_3\psi}(l;q,k,p)=\Gamma_{\bar\psi\chi_3\chi_3\psi}(l;k,q,p)\ . \quad (2.25b)$$

Moreover, careful consideration of the four-point $\bar{\psi}\bar{c}c\psi$ Green's function shows it to be invariant under exchange of "inbound" ghost momenta:

$$\Gamma_{\bar{\psi}\bar{c}c\psi}(l;k,q,p)=\Gamma_{\bar{\psi}\bar{c}c\psi}(l;q,k,p) \ . \tag{2.25c}$$

This last property, directly follows from the momentum-exchange symmetry of the $\bar{c}\phi c$ vertex, as can be verified by explicit construction of the (uncorrected) $\bar{\psi}\bar{c}c\psi$ four-point function from electroweak vertices [as in (2.27) below].

If we subtract from (2.24) the version of (2.24) we would have upon exchanging k and q, we can make use of (2.25) and a judicious rearrangement of terms to obtain the identity

$$\begin{aligned}0=&-S^{-1}(l)(a-b\gamma_5)S(p+k)[k_\mu\Gamma^\mu_{\bar{\psi}Z\psi}(p+k;k,p)+iM_Z\Gamma_{\bar{\psi}\chi_3\psi}(p+k;k,p)]\\&+[k_\mu\Gamma^\mu_{\bar{\psi}Z\psi}(l;k,p+q)+iM_Z\Gamma_{\bar{\psi}\chi_3\psi}(l;k,p+q)]S(p+q)(a+b\gamma_5)S^{-1}(p)\\&+S^{-1}(l)(a-b\gamma_5)S(p+q)[iM_Z\Gamma_{\bar{\psi}\chi_3\psi}(p+q;q,p)+q_\mu\Gamma^\mu_{\bar{\psi}Z\psi}(p+q;q,p)]\\&-[iM_Z\Gamma_{\bar{\psi}\chi_3\psi}(l;q,p+k)+q_\mu\Gamma^\mu_{\bar{\psi}Z\psi}(l;q,p+k)]S(p+k)(a+b\gamma_5)S^{-1}(p)\\&+\frac{k^2-q^2}{\alpha_Z}\Gamma_{\bar{\psi}\bar{c}c\psi}(l;k,q,p)+2bM_Z\frac{(k^2-q^2)}{m_\phi^2-(k+q)^2}\Gamma_{\bar{\psi}\phi\psi}(l;k+q,p) \ .\end{aligned} \tag{2.26}$$

We now apply (2.21) to all square-bracketed terms in (2.26). The only surviving terms in (2.26) then yield the relationship depicted graphically in Fig. 2:

$$\Gamma_{\bar{\psi}\bar{c}c\psi}(l;k,q,p)=-\frac{2b\alpha_Z M_Z}{m_\phi^2-(k+q)^2}\Gamma_{\bar{\psi}\phi\psi}(l;k+q,p) \ . \tag{2.27}$$

The result (2.27) demonstrates the anticipated insensitivity of three- and four-point functions not involving fermions to external fermion two-point function contributions, a property we have found consistently to be upheld. For example, we see from (2.27) that, to lowest contributing order in electroweak couplings, the tree-level $\bar{c}\phi c$ vertex $(-2b\alpha_Z M_Z)$ is impervious to external contributions to the fermion propagator. Such is not necessarily the case for three- and four-point functions that *do* involve fermions; sensitivity of such vertices to external contributions to S^{-1} [e.g., Σ in (1.1)] is evident in (2.21).

We now substitute (2.27) into both (2.22) and (2.23). We also use (2.21) to replace, respectively, factors of $iS^{-1}(l)(a-b\gamma_5)$ and $-i(a+b\gamma_5)S^{-1}(p)$ common to the right-hand sides of both (2.22) and (2.23) with

$$-iq_\nu\Gamma^\nu_{\bar{\psi}Z\psi}(l;q,p+k)+M_Z\Gamma_{\bar{\psi}\chi_3\psi}(l;q,p+k)+i(a+b\gamma_5)S^{-1}(p+k)$$

and

$$-iq_\nu\Gamma^\nu_{\bar{\psi}Z\psi}(p+q;q,p)+M_Z\Gamma_{\bar{\psi}\chi_3\psi}(p+q;q,p)-iS^{-1}(p+q)(a-b\gamma_5) \ .$$

The substitution into (2.22) yields the following result after some algebraic rearrangement:

$$\begin{aligned}&-i[q_\nu\Gamma^{\mu\nu}_{\bar{\psi}ZZ\psi}(l;k,q,p)]+M_Z\{\Gamma^\mu_{\bar{\psi}Z\chi_3\psi}(l;k,q,p)\}\\&\quad=-i[q_\nu\Gamma^\nu_{\bar{\psi}Z\psi}(l;q,p+k)S(p+k)\Gamma^\mu_{\bar{\psi}Z\psi}(p+k;k,p)+q_\nu\Gamma^\mu_{\bar{\psi}Z\psi}(l;k,p+q)S(p+q)\Gamma^\nu_{\bar{\psi}Z\psi}(p+q;q,p)\\&\qquad+4bq^\mu M_Z(m_\phi^2-(k+q)^2)^{-1}\Gamma_{\bar{\psi}\phi\psi}(l;k+q,p)]\\&\qquad+M_Z\{\Gamma_{\bar{\psi}\chi_3\psi}(l;q,p+k)S(p+k)\Gamma^\mu_{\bar{\psi}Z\psi}(p+k;k,p)+\Gamma^\mu_{\bar{\psi}Z\psi}(l;k,p+q)S(p+q)\Gamma_{\bar{\psi}\chi_3\psi}(p+q;q,p)+2ib(k^\mu+2q^\mu)\\&\qquad\times(m_\phi^2-(k+q)^2)^{-1}\Gamma_{\bar{\psi}\phi\psi}(l;k+q,p)\}+i(a+b\gamma_5)\Gamma^\mu_{\bar{\psi}Z\psi}(p+k;k,p)-i\Gamma^\mu_{\bar{\psi}Z\psi}(l;k,p+q)(a-b\gamma_5) \ .\end{aligned} \tag{2.28}$$

In our arrangement of the right-hand side of (2.28), the terms enclosed by square brackets correspond to the 1PR contributions to the $\bar{\psi}ZZ\psi$ Green's function delineated in Fig. 3. Similarly, the terms in curly brackets correspond to the 1PR contributions to the $\bar{\psi}Z\chi_3\psi$ Green's function. The final unbracketed terms in (2.28) vanish at the tree level, but become nonvanishing 1PI contribution in the presence of external contributions to the quark self-energy. By choosing (2.28) and (2.15) to be consistent in the limit $M_Z\to 0, a\to eQ, b\to 0[\bar{\psi}ZZ\psi\to\bar{\psi}AA\psi]$, we find that (Fig. 3)

$$\begin{aligned}q_\nu\Gamma^{\mu\nu}_{\bar{\psi}ZZ\psi}(l;k,q,p)=&[q_\nu\Gamma^\nu_{\bar{\psi}Z\psi}(l;q,p+k)S(p+k)\Gamma^\mu_{\bar{\psi}Z\psi}(p+k;k,p)+\Gamma^\mu_{\bar{\psi}Z\psi}(l;k,p+q)S(p+q)q_\nu\Gamma^\nu_{\bar{\psi}Z\psi}(p+q,q,p)\\&+\Gamma_{\bar{\psi}\phi\psi}(l;q+k,p)[m_\phi^2-(q+k)^2]^{-1}(4bM_Z q^\mu)]\\&+\Gamma^\mu_{\bar{\psi}Z\psi}(l;k,p+q)(a-b\gamma_5)-(a+b\gamma_5)\Gamma^\mu_{\bar{\psi}Z\psi}(p+k;k,p) \ ,\end{aligned} \tag{2.29}$$

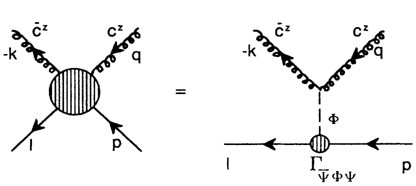

FIG. 2. The $\bar{\psi} \bar{c}^z c^z \psi$ truncated momentum-space Green's function to lowest contributing order (tree-order) in electroweak coupling.

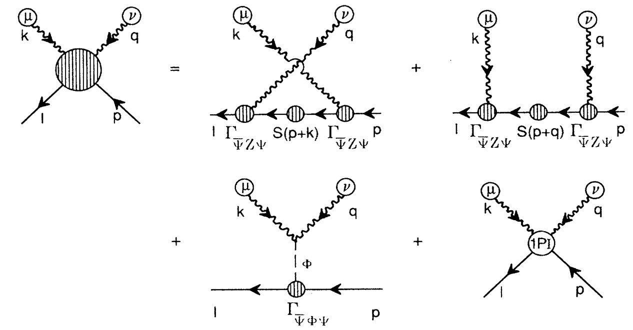

FIG. 3. 1PR and 1PI components of the $\bar{\psi} Z Z \psi$ truncated momentum-space Green's function.

in which case the $\bar{\psi} Z \chi_3 \psi$ Green's function is purely 1PR:

$$\Gamma^{\mu}_{\bar{\psi} Z \chi_3 \psi}(l;k,q,p)=\{\Gamma_{\bar{\psi}\chi_3\psi}(l;q,p+k)S(p+k)\Gamma^{\mu}_{\bar{\psi}Z\psi}(p+k;k,p)+\Gamma^{\mu}_{\bar{\psi}Z\psi}(l;k,p+q)S(p+q)\Gamma_{\bar{\psi}\chi_3\psi}(p+q;q,p)$$
$$+\Gamma_{\bar{\psi}\phi\psi}(l;q+k,p)[m_\phi^2-(q+k)^2]^{-1}[2ib(k^\mu+2q^\mu)]\} \; . \tag{2.30}$$

We note, however, that our decision to include the 1PI contributions proportional to b in the $\bar{\psi} Z Z \psi$ Green's function (2.29) involves some arbitrariness, such contributions could have been partitioned differently without spoiling the correspondence between the $\bar{\psi} Z Z \psi$ and $\bar{\psi} A A \psi$ Green's functions in the $b \rightarrow 0$ limit.

The corresponding substitutions of (2.27) and the above-described versions of (2.21) into (2.23) yield, after suitable algebraic rearrangement,

$$-i\{q_\mu \Gamma^{\mu}_{\bar{\psi}\chi_3 Z\psi}(l;k,q,p)\}+M_Z[\Gamma_{\bar{\psi}\chi_3\chi_3\psi}(l;k,q,p)]$$
$$=-i\{q_\nu \Gamma^{\nu}_{\bar{\psi}Z\psi}(l;q,p+k)S(p+k)\Gamma_{\bar{\psi}\chi_3\psi}(p+k;k,p)$$
$$+q_\nu\Gamma_{\bar{\psi}\chi_3\psi}(l;k,p+q)S(p+q)\Gamma^{\nu}_{\bar{\psi}Z\psi}(p+q;q,p)+2ib(q^2+2k\cdot q)\Gamma_{\bar{\psi}\phi\psi}(l;k+q,p)[m_\phi^2-(k+q)^2]^{-1}\}$$
$$+M_Z[\Gamma_{\bar{\psi}\chi_3\psi}(l;q,p+k)S(p+k)\Gamma_{\bar{\psi}\chi_3\psi}(p+k;k,p)+\Gamma_{\bar{\psi}\chi_3\psi}(l;k,p+q)S(p+q)\Gamma_{\bar{\psi}\chi_3\psi}(p+q;q,p)$$
$$-(2bm_\phi^2/M_Z)\Gamma_{\bar{\psi}\phi\psi}(l;k+q,p)[m_\phi^2-(k+q)^2]^{-1}]+i(a+b\gamma_5)\Gamma_{\bar{\psi}\chi_3\psi}(l-q;k,p)$$
$$-i\Gamma_{\bar{\psi}\chi_3\psi}(l;k,p+q)(a-b\gamma_5)+2b\Gamma_{\bar{\psi}\phi\psi}(l;k+q,p) \; . \tag{2.31}$$

The curly bracketed terms on both sides of (2.31) are equal. The terms within square brackets on the right-hand side of (2.31) are the 1PR contributions to the $\bar{\psi}\chi_3\chi_3\psi$ Greens' function delineated in Fig. 4. The remaining unbracketed terms on the right-hand side of (2.31) separately vanish at the tree level. Sufficient freedom remains in our system to permit the maintenance of the tree-level relationship between $\bar{\psi}\phi\psi$ and $\bar{\psi}\chi_3\psi$ vertices,

$$\Gamma_{\bar{\psi}\phi\psi}(l;k+q,p)=-\frac{i}{2}[\gamma_5\Gamma_{\bar{\psi}\chi_3\psi}(l-q;k,p)+\Gamma_{\bar{\psi}\chi_3\psi}(l;k,p+q)\gamma_5] \; , \tag{2.32}$$

thereby retaining consistency with having ϕ and χ_3 generated from the neutral component of the original scalar-field blet. Application of (2.32) to the unbracketed terms in (2.31) yields cancellation of all such terms with the coefficient b. The remaining unbracketed a terms correspond to a possible 1PI contribution to the $\bar{\psi}\chi_3\chi_3\psi$ Green's function (Fig. 4):

$$\Gamma_{\bar{\psi}\chi_3\chi_3\psi}(l;k,q,p)=\Gamma_{\bar{\psi}\chi_3\psi}(l;q,p+k)S(p+k)\Gamma_{\bar{\psi}\chi_3\psi}(p+k;k,p)+\Gamma_{\bar{\psi}\chi_3\psi}(l;k,p+q)S(p+q)\Gamma_{\bar{\psi}\chi_3\psi}(p+q;q,p)$$
$$-(2bm_\phi^2/M_Z)[m_\phi^2-(k+q)^2]^{-1}\Gamma_{\bar{\psi}\phi\psi}(l;k+q,p)+\frac{ia}{M_Z}[\Gamma_{\bar{\psi}\chi_3\psi}(l-q;k,p)-\Gamma_{\bar{\psi}\chi_3\psi}(l;k,p+q)] \; . \tag{2.33}$$

Corresponding Ward identities for the W sector are listed below [i,I denote, respectively, the bottom and top members of the $SU(2)_L$ fermion doublet]:

$$-ik_\mu\Gamma^{\mu}_{\bar{\psi}_i W^- \psi_I}(q;k,p)+M_W\Gamma_{\bar{\psi}_i\chi^-\psi_I}(q;k,p)=(ig/2\sqrt{2})[S_i^{-1}(q)(1-\gamma_5)-(1+\gamma_5)S_I^{-1}(p)] \; , \tag{2.34}$$

$$\Gamma_{\bar{\psi}_I \bar{c}^+ c^- \psi_I}(l;k,q,p)=\Gamma_{\bar{\psi}_I \bar{c}^- c^+ \psi_I}(l;k,q,p)=-\frac{\alpha_W g M_W/2}{m_\phi^2-(k+q)^2}\Gamma_{\bar{\psi}_I \phi \psi_I}(l;k+q,p)\ , \tag{2.35}$$

$$\begin{aligned} q_\nu[\Gamma^{\mu\nu}_{\bar{\psi}_I W^- W^+ \psi_I}(l;k,q,p)+\Gamma^{\mu\nu}_{\bar{\psi}_I W^+ W^- \psi_I}(l;k,q,p)]=&\, q_\nu\{\Gamma^{\nu}_{\bar{\psi}_I W^+ \psi_i}(l;q,p+k)S_i(p+k)\Gamma^{\mu}_{\bar{\psi}_i W^- \psi_I}(p+k;k,p) \\ &+\Gamma^{\mu}_{\bar{\psi}_I W^+ \psi_i}(l;k,p+q)S_i(p+q)\Gamma^{\nu}_{\bar{\psi}_i W^- \psi_I}(p+q;q,p) \\ &+2gM_W g^{\mu\nu}[m_\phi^2-(k+q)^2]^{-1}\Gamma_{\bar{\psi}_I \phi \psi_I}(l;k+q,p)\} \\ &+(g/2\sqrt{2})[\Gamma^{\mu}_{\bar{\psi}_I W^+ \psi_i}(l;k,p+q)(1-\gamma_5) \\ &-(1+\gamma_5)\Gamma^{\mu}_{\bar{\psi}_i W^- \psi_I}(p+k;k,p)]\ , \end{aligned} \tag{2.36}$$

$$\begin{aligned} \Gamma^{\mu}_{\bar{\psi}_I W^- \chi^+ \psi_I}(l;k,q,p)+\Gamma^{\mu}_{\bar{\psi}_I W^+ \chi^- \psi_I}(l;k,q,p)=&\,\Gamma_{\bar{\psi}_I \chi^+ \psi_i}(l;q,p+k)S_i(p+k)\Gamma^{\mu}_{\bar{\psi}_i W^- \psi_I}(p+k;k,p) \\ &+\Gamma^{\mu}_{\bar{\psi}_I W^+ \psi_i}(l;k,p+q)S_i(p+q)\Gamma^{\mu}_{\bar{\psi}_i \chi^- \psi_I}(p+q;q,p) \\ &+ig(k+2q)^\mu[m_\phi^2-(k+q)^2]^{-1}\Gamma_{\bar{\psi}_I \phi \psi_I}(l;k+q,p)\ , \end{aligned} \tag{2.37}$$

$$\begin{aligned} \Gamma_{\bar{\psi}_I \chi^+ \chi^- \psi_I}(l;k,q,p)+\Gamma_{\bar{\psi}_I \chi^- \chi^+ \psi_I}(l;k,q,p)=&\,\Gamma_{\bar{\psi}_I \chi^+ \psi_i}(l;q,p+k)S_i(p+k)\Gamma_{\bar{\psi}_i \chi^- \psi_I}(p+k;k,p) \\ &+\Gamma_{\bar{\psi}_I \chi^+ \psi_i}(l;k,p+q)S_i(p+q)\Gamma_{\bar{\psi}_i \chi^- \psi_I}(p+q;q,p) \\ &-2(gm_\phi^2/2M_W)[m_\phi^2-(k+q)^2]^{-1}\Gamma_{\bar{\psi}_I \phi \psi_I}(l;k+q,p)\ . \end{aligned} \tag{2.38}$$

Our final comments in this section are interpretive. In the presence of an external self-energy contribution to S_i and S_I, we can retain a distinction between the pole of the propagator (1.1) and the Lagrangian quark mass resulting from Yukawa interactions with the vacuum expectation value $\langle \phi \rangle$. This latter mass characterizes the lowest-order contribution to the three-point function

$$[\Gamma_{\bar{\psi}_I \phi \psi_I}(l;k+q,p)]_{\text{tree}}=-2m_I b/M_Z\ . \tag{2.39}$$

None of the identities we have derived relating three-point functions to two-point functions precludes the maintenance of (2.39) in the presence of self-energy mass contributions external to SU(2)×U(1). In other words, (2.39) is uncorrected by externally generated self-energies to lowest contributing order in the electroweak coupling. If Eq. (2.32) is to be upheld, however, we see that $\Gamma_{\bar{\psi}\chi_3\psi}$ must also be unaffected by externally generated mass contributions to the self-energy. Moreover, the absence of 1PI contributions to (2.38) requires the following analog to (2.32):

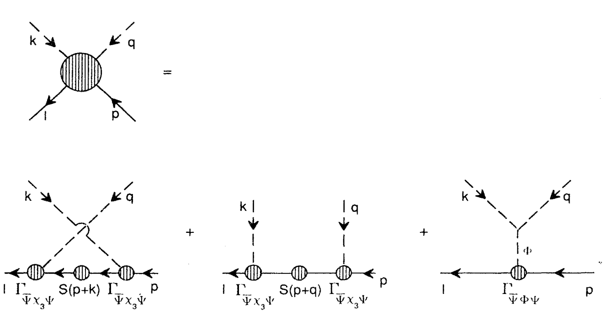

FIG. 4. 1PR components of the $\bar{\psi}\chi_3\chi_3\psi$ truncated momentum-space Green's function.

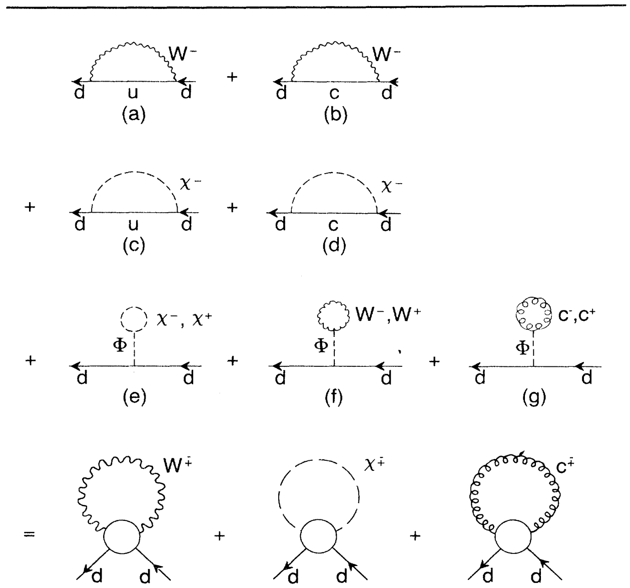

FIG. 5. Purely perturbative electroweak contributions to the d-quark two-point function in the absence of external nonperturbative contributions to the quark inverse propagator.

$$\Gamma_{\bar{\psi}_I\phi\psi_I}(l;k+q,p)=(-i/2\sqrt{2})[(1+\gamma_5)\Gamma_{\bar{\psi}_i\chi^-\psi_I}(p+k;k,p)-\Gamma_{\bar{\psi}_I\chi^+\psi_i}(l;k,p+q)(1-\gamma_5)]\ . \tag{2.40}$$

If $\Gamma_{\bar{\psi}\phi\psi}$ is to be unaffected by external self-energy mass contributions, (2.40) is upheld provided $\Gamma_{\bar{\psi}\chi^-\psi}$, $\Gamma_{\bar{\psi}\chi^+\psi}$ are similarly unaffected. Moreover, the 1PI contributions to (2.33), corresponding to those terms on the right-hand side with coefficient a, are also seen to vanish.

Thus external self-energy contributions, particularly those originating from the coupling of electroweak interactions to an SU(2)×U(1) noninvariant QCD vacuum, are seen to modify only the $\bar{\psi}A\psi$, $\bar{\psi}Z\psi$, $\bar{\psi}W^{\pm}\psi$ subset of electroweak three-point functions. These modifications are then seen to generate additional self-energy-sensitive contributions to the $\bar{\psi}AA\psi$, $\bar{\psi}ZZ\psi$, $\bar{\psi}W^{\mp}W^{\pm}\psi$ subset of electroweak four-point functions that would not be expected from the purely 1PR contributions to those functions anticipated from the Feynman rules.

III. ON-MASS SHELL GAUGE INDEPENDENCE OF QUARK TWO-POINT FUNCTIONS

In any perturbative quantum field theory, gauge-parameter independence of physical (i.e., on-mass-shell) Green's functions is essential for the gauge invariance of physically measurable processes. This on-shell gauge-parameter independence also characterizes two-point functions. For example, the α_W gauge parameter of *purely perturbative* $SU(2)_L\times U(1)$ theory enters the d-quark two-point function through the α_W sensitive contributions of Fig. 5. The gauge-parameter dependence of Fig. 10's first two contributions (henceforth labeled Δ_a and Δ_b) can be ascertained by utilizing only the gauge-parameter-sensitive piece of the W propagator $[(k_\sigma k_\tau/M_W^2)/(k^2-\alpha_W M_W^2)]$; the remaining contributions $\Delta_c-\Delta_g$ may be evaluated directly from $SU(2)_L\times U(1)$ electroweak Feynman rules [10].

It is then a straightforward exercise to show that the α_W dependence of the purely perturbative d-quark two-point function vanishes on the d-quark mass shell:

$$\lim_{p\to m_d}\frac{\partial}{\partial\alpha_W}\left[\sum_{i=a}^{g}\Delta_i(p)\right]=0\ . \tag{3.1}$$

(A corresponding demonstration of the vanishing of α_z dependence on shell is presented in detail in Ref. [6].)

In the final line of Fig. 5, the α_W-dependent contributions to the d-quark two-point function are shown to correspond to the electroweak four-point functions $\langle\bar{d}W^+W^-d\rangle$, $\langle\bar{d}\chi^+\chi^-d\rangle$, $\langle\bar{d}\bar{c}^+c^-d\rangle$ with external W, χ, and c lines respectively contracted into α_W-dependent propagators. These purely perturbative functions are all 1PR, as is evident from severing the α_W-sensitive propagator lines in drawings a–g. In the presence of additional nonperturbative contributions to quark self-energies, as denoted by $\Sigma(p^2)$ in (1.1), the same four-point functions occurring in Fig. 5 acquire Σ-sensitive contributions, as discussed in the previous section. Indeed, the three gauge parameters of electroweak theory enter quark self-energies through the α-sensitive contributions of Fig. 6, the α_Z-sensitive contributions of Fig. 7, and the α_W-sensitive contributions of Fig. 8. The (now shaded) four-point function of Fig. 6 corresponds to tying together the photon legs of Fig. 1; correspondingly, Fig. 7 constructed from looping the nonfermion legs of Figs. 2, 3, and 4.

For example, the α-sensitive contribution to Fig. 6 [generated through the $(1-\alpha)k_\tau k_\sigma/(k^2)^2$ portion of the photon propagator] is proportional to

$$\Delta_6=(1-\alpha)\bar{u}(p)\int\frac{d^4k}{(2\pi)^4}\frac{k_\tau k_\sigma}{(k^2)^2}\Gamma^{\tau\sigma}_{\bar{\psi}AA\psi}(p;k,-k,p)u(p)\ . \tag{3.2}$$

Upon substitution of (2.13) into (3.2) we find that

$$\Delta_6=eQ(1-\alpha)\int\frac{d^4k}{(2\pi)^4(k^2)^2}\bar{u}(p)[S^{-1}(p)S(p+k)k_\tau\Gamma^{\tau}_{\bar{\psi}A\psi}(p+k;k,p)-k_\tau\Gamma^{\tau}_{\bar{\psi}A\psi}(p;k,p-k)S(p-k)S^{-1}(p)]u(p)\ . \tag{3.3}$$

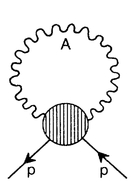

FIG. 6. α-sensitive contribution to the quark two-point function in the presence of external nonperturbative contributions to the quark inverse propagator. The four-point function in the figure is the same as that of Fig. 1.

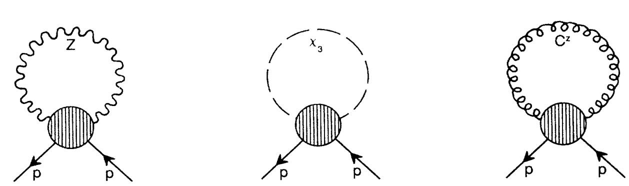

FIG. 7. α_Z-sensitive contributions to the quark two-point function in the presence of external nonperturbative contributions to the quark inverse propagator. The four-point functions in the figure are those of Figs. 2, 3, and 4.

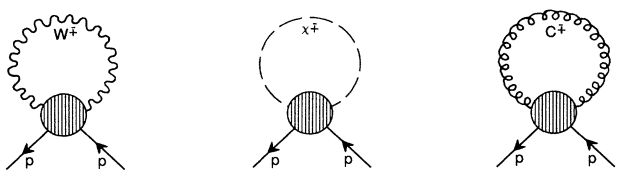

FIG. 8. α_W-sensitive contributions to the quark two-point function in the presence of external nonperturbative contributions to the quark inverse propagator.

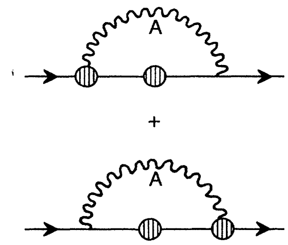

FIG. 9. Usual incorporation of perturbative vertex and propagator corrections into radiative corrections to the fermion two-point function.

On the quark-mass shell, $S^{-1}(p)u(p)$ and $\bar{u}(p)S^{-1}(p)$ are *defined* to be zero, provided the quark mass is identified with the pole of the quark propagator (1.1):

$$m_{\text{pole}} \equiv m_L - \sum(m_{\text{pole}}^2) \ . \tag{3.4}$$

Consequently, $\Delta_6=0$; the QED four-point function we obtained in Fig. 1 ensures the retention of an α-independent quark two-point function.

We emphasize that this gauge parameter independence relies upon the same QED Ward identity that yields the 1PI contribution discussed immediately after Eq. (2.15). A potential source of confusion about the need for such 1PI contributions is the on-shell α independence of the contributions of Fig. 9, corresponding to purely 1PR contributions to the $\langle \bar{\psi} A A \psi \rangle$ four-point function. The α independence of Fig. 9 considered on shell $[S^{-1}(p)\psi(p)=0]$ is easily verified through use of the three-point function Ward-identity (2.7) and symmetric integration ($\int d^4k\, k_\mu/k^4=0$). Indeed, Fig. 9 is the appropriate realization of Fig. 6 for purely perturbative QED contributions to fermion two-point functions, since the retention of a bare vertex is necessary to avoid double counting the (overlapping) divergences of purely perturbative amplitudes. However, such arguments are no longer appropriate for the nonperturbative corrections external to electroweak theory (such as those arising from the QCD vacuum) responsible for the $\Sigma(p^2)$ of (1.1). Indeed, the Fig. 9 analogues (including scalar partners) for $\alpha_{Z,W}$-sensitive contributions to quark two-point functions do not exhibit gauge-parameter independence in the presence of an externally generated $\Sigma(p^2)$; the on-shell α_W and α_Z independence of quark two-point functions is realized only through careful consideration of the four-point functions of Figs. 7 and 8.

The appropriate Ward identities for the four-point functions of Fig. 7 are Eqs. (2.27), (2.29), and (2.33), with $\Gamma_{\bar{\psi}\phi\psi}$ and $\Gamma_{\bar{\psi}\chi_3\psi}$ constrained to their tree-level values $-2m_u b/M_Z$, $-2im_u b\gamma_5/M_Z$, respectively, for up-quarks, as discussed at the end of Sec. II. The gauge-parameter-dependent part of the Fig. 7 amplitude (Δ_7) is obtained through explicit use of the α_Z dependent portion of the Z propagator $[(k_\sigma k_\tau/M_Z^2)/(k^2-\alpha_Z M_Z^2)]$ for connecting the external Z lines in Fig. 3, as well as the χ_3 and c^Z propagators $[-1/(k^2-\alpha_Z M_Z^2)]$ for connecting the scalar-partner and ghost external lines occurring in Figs. 2 and 4:

$$\begin{aligned}
\Delta_7=\int \frac{d^4k}{(2\pi)^4(\alpha_Z M_Z^2-k^2)M_Z^2}\,\bar{u}(p)\{\tfrac{1}{2}[&-k_\nu\Gamma^\nu_{\bar{\psi}Z\psi}(p;-k,p+k)S(p+k)k_\mu\Gamma^\mu_{\bar{\psi}Z\psi}(p+k;k,p)\\
&+k_\mu\Gamma^\mu_{\bar{\psi}Z\psi}(p;k,p-k)S(p-k)(-k_\nu)\Gamma^\nu_{\bar{\psi}Z\psi}(p-k;-k,p)\\
&+(-2m_u b/M_Z)(1/m_\phi^2)(-4bM_Z k^2)+k_\mu\Gamma^\mu_{\bar{\psi}Z\psi}(p;k,p-k)(a-b\gamma_5)\\
&-(a+b\gamma_5)k_\mu\Gamma^\mu_{\bar{\psi}Z\psi}(p+k;k,p)]\\
+\tfrac{1}{2}[&(-2im_u b\gamma_5)S(p+k)(-2im_u b\gamma_5)\\
&+(-2im_u b\gamma_5)S(p-k)(-2im_u b\gamma_5)-(2bm_\phi^2)(1/m_\phi^2)(-2m_u b)]\\
+(-1)[&(-2b\alpha_Z M_Z^2/m_\phi^2)(-2m_u b)]\}u(p)\\
=\int \frac{d^4k}{(2\pi)^4 M_Z^2}\,\bar{u}(p)&\left\{\frac{-4m_u b^2}{m_\phi^2}\right\}u(p)
\end{aligned} \tag{3.5}$$

in which case

$$\frac{\partial}{\partial \alpha_Z}\Delta_7=0 \ . \tag{3.6}$$

In obtaining the last line of (3.5), we have used repeatedly the three-point function Ward identity (2.21) as well as the on-shell $\bar{u}(p)S^{-1}(p)=S^{-1}(p)u(p)=0$ constraints discussed above.

The on-shell α_W-independence of Fig. 8 is obtained through analogous use of α_W-sensitive four-point functions:

$$\Delta_8=\int \frac{d^4k}{(2\pi)^4(\alpha_W M_W^2-k^2)M_W^2}\bar{u}(p)\{-k_\mu k_\nu[\Gamma^{\mu\nu}_{\bar\psi W^+W^-\psi}(p;k,-k,p)+\Gamma^{\mu\nu}_{\bar\psi W^-W^+\psi}(p;k,-k,p)]$$
$$+M_W^2[\Gamma_{\bar\psi\chi^+\chi^-\psi}(p;k,-k,p)+\Gamma_{\bar\psi\chi^-\chi^+\psi}(p;k,-k,p)]$$
$$-M_W^2[\Gamma_{\bar\psi\bar c^+c^-\psi}(p;k,-k,p)+\Gamma_{\bar\psi\bar c^-c^+\psi}(p;k,-k,p)]\}u(p)\ . \qquad (3.7)$$

One finds from (2.35), (2.36), and (2.38), as well as from judicious application of (2.34) that $(\partial/\partial\alpha_W)\Delta_8=0$ when the $\bar{u}(p)S^{-1}(p)=S^{-1}(p)u(p)=0$ mass-shell condition is imposed.

Thus, we find that the one-loop corrections to quark two-point functions are gauge-parameter independent, both the purely perturbative spontaneously broken SU(2)×U(1) theory and for that same theory augmented with arbitrarily momentum-dependent self-energy contributions $\Sigma(p^2)$ of nonperturbative origin. We stress that gauge-parameter independence in the latter case would *not* have occurred had the 1PI contributions to (2.15), (2.29), and (2.36) been omitted; such contributions, of course, vanish if $\Sigma(p^2)=0$.

IV. INDUCED YUKAWA INTERACTIONS IN THE CHIRAL LIMIT

In the chiral-symmetry limit of the electroweak Lagrangian, primitive Yukawa couplings of ϕ and χ_3 scalar fields necessarily vanish with the vanishing of the Lagrangian quark mass. Consequently, the lowest-order induced Yukawa interaction in the chiral limit arises from the graphs of Fig. 10. The contribution to this amplitude from internal Z lines is given by

$$\bar{u}(p+q)\Gamma^{\rm ind}_{\bar\psi\phi\psi}(p+q;q,p)u(p)=2bM_Z\int\frac{d^4k}{i(2\pi)^4}\bar{u}(p+q)\Gamma^{\mu\nu}_{\bar\psi ZZ\psi}(p+q;k+q,-k,p)$$
$$\times\left[\frac{g_{\nu\tau}-(1-\alpha_Z)k_\nu k_\tau/(k^2-\alpha_Z M_Z^2)}{k^2-M_Z^2}\right]$$
$$\times\left[\frac{g_{\mu\tau}-(1-\alpha_Z)(k_\mu+q_\mu)(k_\tau+q_\tau)/[(k+q)^2-\alpha_Z M_Z^2]}{(k+q)^2-M_Z^2}\right]u(p)\ . \qquad (4.1)$$

The portion of this expression involving the gauge parameter α_Z necessarily contains factors of either $k_\nu\Gamma^{\mu\nu}_{\bar\psi ZZ\psi}(p+q;k+q,-k,p)$ or $(k+q)_\mu\Gamma^{\mu\nu}_{\bar\psi ZZ\psi}(p+q;k+q,-k,p)$ as coefficients of α_Z-sensitive quantities. These factors can be evaluated through use of (2.22), which in the chiral limit simplifies to

$$ik_\nu\Gamma^{\mu\nu}_{\bar\psi ZZ\psi}(p+q;k+q,-k,p)=iS^{-1}(p+q)(a-b\gamma_5)S(p+k+q)\Gamma^{\mu}_{\bar\psi Z\psi}(p+k+q;k+q,p)$$
$$-i\Gamma^{\mu}_{\bar\psi Z\psi}(p+q;k+q,p-k)S(p-k)(a+b\gamma_5)S^{-1}(p)\ . \qquad (4.2)$$

We then see from (4.1) that the right-hand side of (4.2) vanishes between $\bar{u}(p+q)$ and $u(p)$ on-shell external-fermion spinors, as $\bar{u}(p+q)S^{-1}(p+q)$ and $S^{-1}(p)u(p)$ both equal zero for on-shell momenta. A similar argument can be presented for the α_W independence of the induced Higgs coupling.

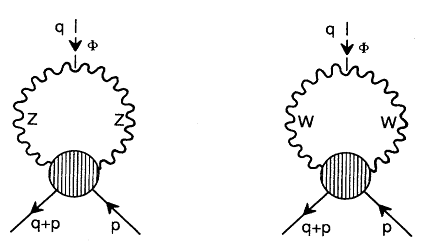

FIG. 10. Induced Yukawa interaction in the chiral limit of electroweak theory.

We note that our application of the on-shell condition $S^{-1}(p)u(p)=0$ corresponds in the chiral limit to having a dynamical quark mass m defined by the quark propagator pole obtained after setting the Lagrangian mass m_L in (3.4) equal to zero. The fermion masses generated through the usual electroweak Yukawa interactions may be regarded as arising from the zero-momentum-transfer Yukawa coupling of the vacuum expectation value $\langle\phi\rangle$ to a massless fermion. In the limit of Lagrangian chiral symmetry, in which Lagrangian Yukawa couplings vanish, any *induced* Yukawa interaction (such as in Fig. 10) necessarily will permit the occurrence of a mass via this zero-momentum-transfer-induced coupling of a massless fermion to $\langle\phi\rangle$:

$$\bar{u}(p)\Gamma^{\text{ind}}_{\bar{\psi}\phi\psi}(p;0,p)u(p) \equiv -2b(m^{\text{ind}})/M_Z$$
$$=(2bM_Z)\int \frac{d^4k}{i(2\pi)^4}\bar{u}(p)[\Gamma^{\mu\nu}_{\bar{\psi}ZZ\psi}(p;k,-k,p)g_{\mu\nu}/(k^2-M_Z^2)^2]u(p)$$
$$+(gM_W/2)\int \frac{d^4k}{i(2\pi)^4}\bar{u}(p)\{[\Gamma^{\mu\nu}_{\bar{\psi}W^+W^-\psi}(p;k,-k,p)+\Gamma^{\mu\nu}_{\bar{\psi}W^-W^+\psi}(p;k,-k,p)]$$
$$\times g_{\mu\nu}/(k^2-M_W^2)^2\}u(p) . \tag{4.3}$$

Let us consider the first integral on the right-hand side of (4.3), corresponding to the first graph of Fig. 10. The four-point function in this integrand may be obtained directly from (2.29) taken in the $\Gamma_{\bar{\psi}\phi\psi}=0$ chiral limit, and is given explicitly in the next section. Moreover, the three-point functions in (2.29) can be evaluated through incorporation of (1.1) into (2.21):

$$\Gamma^{\mu}_{\bar{\psi}Z\psi}(p+k;k,p)=\gamma^{\mu}(a-b\gamma_5)+\frac{k^{\mu}}{k^2}[\Sigma((p+k)^2)(a-b\gamma_5)-(a+b\gamma_5)\Sigma(p^2)] . \tag{4.4}$$

Such substitutions yield the following expression for the Z sector's contribution to the induced fermion Yukawa (current) mass ($m^{\text{ind}}\equiv m_Z^{\text{ind}}+m_W^{\text{ind}}$):

$$-2b(m_Z^{\text{ind}})/M_Z=4bM_Z\int \frac{d^4k}{i(2\pi)^4}\bar{u}(p)\left[\gamma^{\mu}(a-b\gamma_5)\frac{-\not p-\not k+\Sigma[(p+k)^2]}{(p+k)^2-\Sigma^2((p+k)^2)}\gamma_{\mu}(a-b\gamma_5)\right.$$
$$+\frac{1}{k^2}\{\Sigma(p^2)(a-b\gamma_5)-(a+b\gamma_5)\Sigma[(p+k)^2]\}\frac{-\not p-\not k+\Sigma[(p+k)^2]}{(p+k)^2-\Sigma^2[(p+k)^2]}$$
$$\times\{\Sigma[(p+k)^2](a-b\gamma_5)-(a+b\gamma_5)\Sigma(p^2)\}$$
$$\left.+\frac{1}{k^2}\{(a^2+b^2)\Sigma(p^2)-(a^2-b^2)\Sigma[(p+k)^2]\}\right]u(p)/(k^2-M_Z^2)^2 . \tag{4.5}$$

To evaluate (4.5), we first assume that the external self-energy $\Sigma(k'^2)$ falls sufficiently quickly with k'^2 that integrals of the form $\int d^4k'\Sigma(k'^2)F(k')$ can be neglected relative to $\Sigma(p^2)\int d^4k'F(k')$. We utilize the expansion $[k'^2-\Sigma^2(k'^2)]^{-1}=1/k'^2+(1/k'^2)\Sigma^2(k'^2)(1/k'^2)+\cdots$ and then note that the nonleading terms in the expansion, upon integration over k', are suppressed relative to the leading term by factors of Σ^2/M_Z^2. Consequently, we find that the leading contributions to m_Z^{ind} are given by

$$-2b(m_Z^{\text{ind}})/M_Z=4bM_Z\bar{u}(p)\left[(a^2+b^2)\Sigma(p^2)\int\frac{d^4k'}{i(2\pi)^4}\frac{1}{k'^2(k'^2-M_Z^2)^2}\right.$$
$$+\Sigma^2(p^2)\int\frac{d^4k'(a-b\gamma_5)\gamma\cdot k'(a+b\gamma_5)}{i(2\pi)^4k'^2[(k'-p)^2-M_Z^2]^2(k'-p)^2}$$
$$\left.+\int\frac{d^4k'(a+b\gamma_5)\gamma^{\mu}(-\gamma\cdot k')\gamma_{\mu}(a-b\gamma_5)}{i(2\pi)^4k'^2[(k'-p)^2-M_Z^2]^2}\right]u(p)+O(\Sigma^3/M_Z^3) . \tag{4.6}$$

The contribution of the first and third integral in (4.6) cancels upon utilization of the on-shell condition $S^{-1}(p)u(p)=0$ to replace $\not p u(p)$ (from evaluation of the third integral) with $-\Sigma(p^2)u(p)$. The second integral yields a contribution that is of order Σ^3/M_Z^2 in magnitude. Thus we find that the mass generated in the chiral limit by the induced Yukawa interaction of Fig. 10's Z-exchange graph is (at most) of order $m_Z^{\text{ind}}=|\Sigma|^3/M_Z^2\lesssim10^{-2}$ MeV where $|\Sigma|$ is assumed comparable to a dynamical quark mass [see (3.4)] of order 300 MeV arising from the chiral noninvariance of the QCD vacuum [2]. The mass m_W^{ind} induced via Fig. 10's W exchange can also be shown to be characterized by a comparable small upper bound. This result, in and of itself, is disappointing; it might have been interesting had there been a causal connection demonstrated between the relatively small (~5 MeV) up- and down-quark current masses and the $m_{\text{nucl}}/3$ dynamical mass scale characterizing quark masses in static hadron processes.

As a final comment, it should be noted that the induced Yukawa interaction in Fig. 10 is relevant for the decay of the physical Higgs field ϕ into $u\bar{u}$ and $d\bar{d}$ pairs, provided the Higgs field line [assigned zero momentum in (4.3)] is placed on shell. Such a calculation would depend on the actual structure of $\Sigma(p^2)$ and is presently under investigation.

V. SIGNIFICANCE OF LANDAU GAUGE

The incorporation of quark propagators with external mass-self-energy contributions into electroweak theory can be facilitated through consideration of the actual three- and four-point functions that satisfy appropriate Ward identities. The $\langle \bar{\psi} Z \psi \rangle$ three-point function for example is given by (4.4); the $\langle \bar{\psi} ZZ \psi \rangle$ four-point function (Fig. 3) consistent with (2.29), (4.4), and Bose symmetry is given by

$$\begin{aligned}
\Gamma^{\mu\nu}_{\bar{\psi}ZZ\psi}(p+k+q;k,q,p) =& \gamma^\nu(a-b\gamma_5)S(p+k)\gamma^\mu(a-b\gamma_5)+\gamma^\mu(a-b\gamma_5)S(p+q)\gamma^\nu(a-b\gamma_5) \\
&-\left[\frac{2mb}{M_Z}\right]\frac{1}{m_\phi^2-(k+q)^2}(4bM_Z g^{\mu\nu}) \\
&+\frac{q^\nu}{q^2}(\{\Sigma[(p+k+q)^2](a-b\gamma_5)-(a+b\gamma_5)\Sigma[(p+k)^2]\}S(p+k)\gamma^\mu(a-b\gamma_5) \\
&\qquad +\gamma^\mu(a-b\gamma_5)S(p+q)\{\Sigma[(p+q)^2](a-b\gamma_5)-(a+b\gamma_5)\Sigma(p^2)\}) \\
&+\frac{k^\mu}{k^2}(\gamma^\nu(a-b\gamma_5)S(p+k)\{\Sigma[(p+k)^2](a-b\gamma_5)-(a+b\gamma_5)\Sigma(p^2)\} \\
&\qquad +\{\Sigma[(p+k+q)^2](a-b\gamma_5)-(a+b\gamma_5)\Sigma[(p+q)^2]\}S(p+q)\gamma^\nu(a-b\gamma_5)) \\
&+\frac{q^\nu k^\mu}{q^2k^2}(\{\Sigma[(p+k+q)^2](a-b\gamma_5)-(a+b\gamma_5)\Sigma[(p+k)^2]\}S(p+k) \\
&\qquad \times\{\Sigma[(p+k)^2](a-b\gamma_5)-(a+b\gamma_5)\Sigma(p^2)\} \\
&\qquad +\{\Sigma[(p+k+q)^2](a-b\gamma_5)-(a+b\gamma_5)\Sigma[(p+q)^2]\}S(p+q) \\
&\qquad \times\{\Sigma[(p+q)^2](a-b\gamma_5)-(a+b\gamma_5)\Sigma(p^2)\} \\
&\qquad +(a-b\gamma_5)^2\Sigma[(p+k+q)^2]+(a+b\gamma_5)^2\Sigma(p^2) \\
&\qquad -(a^2-b^2)\{\Sigma[(p+q)^2]+\Sigma[(p+k)^2]\}) \ .
\end{aligned} \tag{5.1}$$

Although other kinematic structures may also be consistent with (2.21) and (2.29), those chosen for (4.4) and (5.1) are seen to have all vertex Σ dependence in terms that are annihilated by transverse projection operators. In other words, all Σ-dependent departures from tree-level vertices (including the 1PI contribution to Fig. 3) are seen *to vanish* in a Landau-gauge calculation. This result, which is anticipated in a calculation coupling the nonperturbative $\langle 0|:\psi(x)\bar{\psi}(y):|0\rangle$ vacuum expectation value to $\Delta S=1$ processes [11], implies that a *Landau gauge* calculation with naive electroweak Feynman rules will yield the "correct" (i.e., gauge-invariant) result, even when quark propagators $S(p)$ contain externally generated self-energies of nonperturbative origin, as in (1.1).

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[8] Of course, *perturbative* QCD also yields self-energy contributions external to electroweak theory, but such contributions respect electroweak symmetry and pose no problems in the context of purely perturbative $SU(3)_c$

$\times SU(2)_L \times U(1)$ field theory.
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