COMMENTS

Comments are short papers which criticize or correct papers of other authors previously published in the **Physical Review**. *Each Comment should state clearly to which paper it refers and must be accompanied by a brief abstract. The same publication schedule as for regular articles is followed, and page proofs are sent to authors.*

Comment on "Statistics transmutation in Maxwell-Chern-Simons theories"

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It is shown that transmutation of statistics fails in Maxwell-Chern-Simons field theory for precisely the same reasons it fails in Chern-Simons field theory.

The idea of fractional statistics had its origins in general considerations of quantum mechanics in two spatial dimensions [1]. The possibility that it could be extended to the domain of quantum field theory has received much consideration of late. In particular, Semenoff [2] claimed to have achieved this goal, though this was subsequently shown [3] to be incorrect. Recently, Matsuyama [4] has sought to revive such claims by extending Semenoff's construction to the full Maxwell-Chern-Simons theory.

It is not difficult to show that the results of Ref. [4] are untenable since they are subject to the same criticism as Ref. [2]. In particular, it can be seen that the result depends crucially upon the alleged identity

$$\nabla_i \Omega(\mathbf{x}) = -\tfrac{1}{2}\epsilon_{ij}\nabla_j \ln x^2 , \qquad (1)$$

where $\epsilon_{ij} = -\epsilon_{ji}$ $(\epsilon_{12} = +1)$ and

$$\Omega(\mathbf{x}) = \arctan y/x .$$

The validity of (1) would allow one to write, for example,

$$-\nabla_i \int \Omega(\mathbf{x}-\mathbf{x}')j^0(x')d^2x' = \epsilon_{ij}\nabla_j \int \ln|\mathbf{x}-\mathbf{x}'| j^0(x')d^2x' , \qquad (2)$$

which is crucial to the manipulations carried out in Ref. [4].

This issue was, however, resolved in Ref. [3] by the demonstration in an explicit calculation for a c-number charge density $j^0(x)$ that Eq. (2) implied a contradiction. In particular, the choice

$$j^0(x) = \begin{cases} 1/\pi R^2, & r<R, \\ 0, & r>R, \end{cases}$$

gave the results

$$-\epsilon_{ij}\nabla_j \int \ln|\mathbf{x}-\mathbf{x}'| j^0(\mathbf{x}')d^2x' = (\epsilon_\phi)_i \times \begin{cases} 1/r, & r>R, \\ r/R^2, & r<R, \end{cases}$$

and

$$\nabla_i \int \Omega(\mathbf{x}-\mathbf{x}')j^0(\mathbf{x}')d^2x' = \begin{cases} (\epsilon_\phi)_i(1/r), & r>R, \\ (\epsilon_\phi)_i r/R^2 + \left[\dfrac{2r}{R^2}\right]\phi(\epsilon_r)_i, & r<R, \end{cases}$$

where ϵ_ϕ and ϵ_r are the usual unit vectors in the plane. Evidently, the integration over the inverse tangent gives an unusual multivalued contribution in the radial direction for $r>R$, thereby disproving relation (2).

It is not difficult to establish why this consistency must occur in general. Relation (2) is easily seen to be correct for $r \neq 0$ by direct differentiation, but it *cannot* be generally true as a simple argument shows. Since

$$\nabla^2 \ln x^2 = 4\pi\delta(\mathbf{x}) ,$$

insertion of (1) requires that

$$\nabla \times \nabla\Omega = 2\pi\delta(\mathbf{x}) , \qquad (3)$$

a result which is recognized in Ref. [4]. As pointed out there, such an equation must be interpreted as a distribution. To this end one multiplies (3) by a function $f(r)$, which is well behaved at $r=\infty$ (e^{-r^2} is a possible choice) and integrates the product of this function with Eq. (3). This yields, upon integration by parts and use of the fact that one can choose $f(r)$ such that

$$\nabla \times \nabla f = 0 ,$$

the contradiction

$$2\pi f(0) = 0 .$$

In other words, the functions Ω which satisfy (3) in the context distribution theory do not exist.

One concludes that the transmutation claimed in Ref. [4] fails in the same way as does that of Semenoff. Finally, the author wishes to emphasize that the support of transmutation attributed to him in Ref. [4] is without basis. The publication cited [5] established the possibility of fractional charge, but no transmutation, exotic statistics, or spin and statistics connection was either suggested or claimed in that work.

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