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Further remarks on quantization of massive chiral electrodynamics in four dimensions

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We show that our conclusions about the consistency of massive chiral electrodynamics in four dimensions were based on the use of a peculiar, although popular, expression of the chiral triangle amplitude for massless fermions, viz., a Bose-symmetric sum of Dolgov-Zakharov poles. If the Feynman triangular amplitude is instead computed by means of standard ultraviolet regulators, no decoupling of the unphysical degree of freedom occurs , thereby jeopardizing perturbative unitarity. Our present analysis raises severe doubts about the possibility of a consistent treatment of anomalous theories in a perturbative context.

In Ref. [1] we proposed a model in which a massive Abelian vector field interacts with a chiral left spinorial current in the usual four-dimensional Minkowski space-time. The model is described by the Lagrangian density (the treatment we report here is simpler than the one in Ref. [1], although completely equivalent from a physical viewpoint)

$$\mathcal{L}_0=-\frac{1}{4}F_{\mu\nu}F^{\mu\nu}+\bar{\psi}(\not\partial+ie\not A P_L)\psi + \frac{m^2}{2}A_\mu A^\mu - m\eta\partial_\mu A^\mu \tag{1}$$

P_L being the left projection operator $(1+\gamma_5)/2$, ψ a Dirac spinor, A_μ the Proca potential, $F_{\mu\nu}$ the related field tensor, and η an auxiliary field [2].

At the classical level the fermionic left current

$$J_L^\mu = ie\bar{\psi}\gamma^\mu P_L\psi \tag{2}$$

is conserved, the field A_μ is transverse $\partial^\mu A_\mu=0$ and the ghost field η completely decoupled. At the quantum level the left current is no longer conserved, owing to the chiral anomaly

$$\partial_\mu J_L^\mu = \frac{e^3}{48\pi^2}F_{\mu\nu}\tilde{F}^{\mu\nu} \tag{3}$$

and a problem arises concerning η, which interacts with A_μ via the chiral anomaly [see Eq. (3)].

Nevertheless, in the generating functional

$$W[J_\mu]=\mathcal{N}^{-1}\int dA_\mu d\bar{\psi}\,d\psi\,d\eta \times \exp\left[i\int d^4x(\mathcal{L}_0+J^\mu A_\mu)\right], \tag{4}$$

J_μ being an external source, we can perform the integration over the η field and get

$$W=\mathcal{N}^{-1}\int dA_\mu^\perp d\bar{\psi}\,d\psi \times \exp\left[i\int d^4x[\mathcal{L}_0(A_\mu^\perp)+J^\mu A_\mu^\perp)]\right], \tag{5}$$

where $A_\mu^\perp$ is defined as $(g_{\mu\nu}-\partial_\mu\partial_\nu\partial^{-2})A^\nu$.

Then our previous conclusion about consistency was based on the claim [3] that the one-loop chiral triangle amplitude in a theory with massless fermions is given by the Bose-symmetric combination of Dolgov-Zakharov (DZ) poles [4], viz.,

$$T^{\mathrm{DZ}}_{\sigma\rho\mu}(k_1,k_2,k_3) = -\frac{e^3}{12\pi^2}\epsilon_{\zeta\tau\sigma\rho}k_1^\zeta k_2^\tau\frac{k_{3\mu}}{k_3^2} - \frac{e^3}{12\pi^3}\epsilon_{\zeta\tau\rho\mu}k_2^\zeta k_3^\tau\frac{k_{1\sigma}}{k_1^2} - \frac{e^3}{12\pi^2}\epsilon_{\zeta\tau\mu\sigma}k_3^\zeta k_1^\tau\frac{k_{2\rho}}{k_2^2}. \tag{6}$$

Were this the case, the surviving transverse sector of our model would be both perturbatively renormalized and unitary. As a matter of fact the propagator of $A_\mu^\perp$ is well behaved in the ultraviolet region on the basis of power counting. From its expression

$$D_{\mu\nu}^\perp = \left[g_{\mu\nu}-\frac{k_\mu k_\nu}{k^2+i\epsilon}\right]\frac{1}{k^2-m^2+i\epsilon}, \tag{7}$$

one realizes that its pole at $k^2=m^2$ describes the three physical degrees of freedom of a Proca field. Of course, at variance with the Proca propagator, $D_{\mu\nu}^\perp$ exhibits also an unphysical pole at $k^2=0$, which is the relic of the η

field. This is the price one has to pay for having good ultraviolet behavior. Nevertheless the DZ expression (6) is purely longitudinal and therefore cannot couple to $A^{\perp}_{\mu}$. Of course $A^{\perp}_{\mu}$ does couple to nonanomalous diagrams; in them however current conservation occurs so that Proca and "transverse" propagators both lead to the same S-matrix elements [2].

This brought us to the conclusion that our model was indeed perturbatively renormalizable and unitary.

Unfortunately the chiral triangle amplitude $T_{\sigma\rho\mu}(k_1,k_2,k_3)$, when computed with standard ultraviolet regulators, has an expression quite different from the one in Eq. (6). To avoid fermion mass terms we have repeated the one-loop calculation in the case of massless fermions, using dimensional regularization and the Breitenlohner-Maison [5] recipe for handling γ_5, with the result [the same technique applied to the axial-vector-vector amplitude exactly reproduces, in the massless fermion limit, the Rosenberg [6] expression, obtained by means of Pauli-Villars regulators; this expression differs from the chiral amplitude of Eq. (8) by the addition of the monomial $(e^3/12\pi^2)\epsilon_{\tau\sigma\rho\mu}(k_1^\tau-k_2^\tau)$]

$$T_{\sigma\rho\mu}(k_1,k_2,k_3)=\frac{e^3}{12\pi^2}\epsilon_{\tau\sigma\rho\mu}[(k_1^\tau-k_2^\tau)k_3^2I_{x_1x_2}+(k_2^\tau-k_3^\tau)k_1^2I_{x_2x_3}+(k_3^\tau-k_1^\tau)k_2^2I_{x_3x_1}]+\frac{e^3}{4\pi^2}\epsilon_{\xi\tau\sigma\rho}k_1^\xi k_2^\tau k_{3\mu}I_{x_1x_2}$$
$$+\frac{e^3}{4\pi^2}\epsilon_{\xi\tau\rho\mu}k_2^\xi k_3^\tau k_{1\sigma}I_{x_2x_3}+\frac{e^3}{4\pi^2}\epsilon_{\xi\tau\mu\sigma}k_3^\xi k_1^\tau k_{2\rho}I_{x_3x_1}\ , \tag{8}$$

the integrals $I_{x_1x_2}$, $I_{x_2x_3}$, and $I_{x_3x_1}$ being defined as

$$I_{x_ix_j}=\int_0^1 2x_ix_j\frac{\delta(1-x_1-x_2-x_3)dx_1dx_2dx_3}{-k_1^2x_2x_3-k_2^2x_1x_3-k_3^2x_1x_2}\ . \tag{9}$$

We notice that $T_{\sigma\rho\mu}$ depends explicitly on two different tensorial structures; only on the submanifold $k_1^2=k_2^2=k_3^2$ is the expression (6) recovered [7]. No DZ pole is present for general kinematical configurations; $T_{\sigma\rho\mu}$ exhibits quite involved analyticity properties in the complex variables k_1^2, k_2^2, and k_3^2, as is expected on general grounds. No cancellation can henceforth occur of the unphysical pole entering the transverse $D^{\perp}_{\mu\nu}$ propagator, which couples in the transverse sector of the model and jeopardizes perturbative unitary.

In conclusion we point out that our present analysis raises severe doubts on the possibility of a consistent treatment of anomalous theories in a perturbative context even in the presence of gauge group functional integration and/or addition of Wess-Zumino terms. As a matter of fact, if in Eq. (4) we perform the gauge transformation $A_\mu \to A_\mu - \partial_\mu\theta$, keeping the "gauge-fixing" term $m\eta\partial A$ unchanged, and thereby integrate over the group variable θ [8], we recover the unitary Proca propagator, but renormalization is lost.

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