PHYSICAL REVIEW D VOLUME 44, NUMBER 8 15 OCTOBER 1991

Strong-coupling solution for a fermion-pair model

S. Capstick, R. E. Cutkosky, and M. A. Joensen
Physics Department, Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213
(Received 17 June 1991)

We consider a model for the condensation of fermion pairs interacting through a generalized Coulomb potential. The model gives a new outlook on chiral condensation and chiral-symmetry breaking. For strong coupling, this model exhibits a new dynamical symmetry associated with helicity conservation, allowing construction of a strong-coupling solution. The symmetry occurs for confining as well as nonconfining potentials, and is independent of the number of colors. Near the strong-coupling limit, for small fermion masses, the low-lying states are characterized by local charge neutralization and correspond to a pseudoscalar collective excitation. The number of flavors determines the symmetry group and also affects details of the approach to the strong-coupling limit.

The model of fermions interacting through a generalized Coulomb potential provides a convenient system for investigating the condensation of fermion pairs and breaking of chiral symmetry in QED and QCD. In the continuum theory, this model has been studied using variational methods applied in superconductivity theory [1] and also using the Bethe-Salpeter equation [2]. One question of interest is whether the existence and properties of any condensation depend on the momentum-transfer dependence of the Coulomb potential, specifically, on whether it is confining or nonconfining. The results have been somewhat equivocal and contradictory. Lattice studies have also been carried out, especially for strong-coupling QED [3] and also in QCD [4] with various approximations, with emphasis on determining the existence and structure of phase transitions. These investigations also lead to somewhat uncertain results, because of finite-size effects and other lattice artifacts, and because the massless limit is not accessible.

In our fermion-pair model we start from the continuum theory and obtain a system with a finite number of degrees of freedom by imposing a momentum cutoff in a finite domain. Studies of the Coulomb-gauge pure-glue theory show that the structure of the vacuum and of low-lying states is dominated by the Coulomb interaction between color charges [5]. We incorporate this feature of QCD by considering the fermions to interact through a general nonretarded interaction in which the fermion vertex includes the Dirac matrix γ^0.

To construct our model, we use a variational formulation that allows for an arbitrary distribution of fermion pairs. The pairs are colorless and have zero net momentum [6]. We shall show that in the massless limit the Coulomb term in the Hamiltonian has a new symmetry of dynamical origin. This is analogous to the dynamical symmetry arising from the momentum-space structure of the ordinary Coulomb potential, which leads to degeneracies of the nonrelativistic hydrogen atom. In our pair-interaction model, the symmetry is independent of the momentum dependence and arises from helicity conservation. Exploitation of the helicity symmetry of our pairing model leads to an exact solution for the lowest multiplet of the symmetry group in the strong-coupling limit $\gamma \to \infty$, where γ denotes the interaction strength. This lowest multiplet consists of states with vanishing local charge density. This new symmetry was not noticed in our previous work [6] because of a mistaken term in the Hamiltonian. For large coupling strengths there are significant cancellations between large positive and negative terms that were not taken into account correctly, leading to incorrect numerical results in some cases.

For a finite number n of colors, we do not find any qualitative change in the character of the solutions for any finite value of the interaction constant γ. In particular, there is no finite value γ_χ at which pairs become fully condensed or at which the chiral properties of the solutions are altered. In our previous work [6] we found that a condensation would occur for some critical finite value γ_C when $n \to \infty$. This shows that large-n results are not a reliable guide. On the other hand, we do find that if $\gamma \to \infty$, chiral symmetry is "almost spontaneously broken" in the sense that the low-lying states, in which the charge is neutralized locally, correspond to collective excitations associated with a pseudoscalar quantum of mass $M \sim \sqrt{m/\gamma}$, where m is the fermion mass.

Our finite volume for discretizing the momentum $\mathbf{p}$ is the unit hypersphere, in which we construct spinor normal modes. The magnitude of momentum is $p_\kappa = \kappa + \frac{1}{2}$, where κ is a positive integer restricted to $\kappa \leq \Lambda$, and the free-particle energy is $E_\kappa = (m^2 + p_\kappa^2)^{1/2}$. The total degeneracy for each helicity $h = \pm\frac{1}{2}$ is $\nu_\kappa = n\Delta_\kappa$, where $\Delta_\kappa = \kappa(\kappa+1)$. At first, the fermions have a single flavor.

We use a set of operators [6] $M(\kappa, h)^\dagger$ that contain products of one fermion and one antifermion creation operator. We have modified our previous definition by inserting an extra factor $\sqrt{\nu_\kappa}$ into $M(\kappa, h)$, and by changing the sign of $M(\kappa-)$ so that the parity operator $\mathcal{P}$ gives $\mathcal{P}M(\kappa, h) = M(\kappa, -h)\mathcal{P}$. Applied to the perturbative vacuum, the operator $M(\kappa, h)^\dagger$ creates a pair in which two particles with specific values of κ and h have been combined to give a total momentum of zero as well as no net color. We consider linear combinations of the states

$$|k_1, \ldots, k_{2\Lambda}\rangle = \prod_i C_i(k_i)[M(\kappa_i, h_i)^\dagger]^{k_i}|0\rangle , \qquad (1)$$

where the product extends over the 2Λ values of κ and h with $\kappa \leq \Lambda$. This defines a class of variational approximations to the full Hamiltonian, because there are also other states which are colorless and have a total momentum of zero. However, in the strong-coupling limit, we can show that inclusion of other states could not reduce the Coulomb energy. As special cases, the basis (1) includes states that have an exponentiated form, which are often used in superconductivity theory and which have also been used in some previous work on this problem[1]. Between the states given by (1), the commutator is

$$[M(\kappa,h), M(\lambda,l)^\dagger] = [\nu_\kappa - 2N(\kappa,h)]\delta_{\kappa\lambda}\delta_{hl}\,, \qquad (2)$$

where N is the number of pairs. For each pair of values of κ and h, the commutator algebra of M, $M^\dagger$, and N is closed.

The effective Coulomb Hamiltonian for massless fermions in this basis is [6]

$$\begin{aligned} H_C = \gamma \sum_{\kappa\lambda} A_{\kappa\lambda}\ \{\ &\Delta_\lambda[N(\kappa+) + N(\kappa-)] \\ &- n^{-1}[:N(\kappa+)N(\lambda+): + :N(\kappa-)N(\lambda-): + M(\kappa+)^\dagger M(\lambda+) + M(\kappa-)^\dagger M(\lambda-)]\} \\ + \gamma \sum_{\kappa\lambda} B_{\kappa\lambda}\{&\Delta_\lambda[n\Delta_\kappa - N(\kappa+) - N(\kappa-)] \\ &+ n^{-1}[2N(\kappa+)N(\lambda-) - M(\kappa+)^\dagger M(\lambda-)^\dagger - M(\kappa+)M(\lambda-)]\}\,, \end{aligned} \qquad (3)$$

where $n\gamma = g^2 n_g/(2\pi)^2$ (the group dimensionality is n_g) and where the coefficients for fermion scattering and pair-creation terms are, respectively,

$$\begin{aligned} A_{\kappa\lambda} &= \sum_K G_K \frac{[(\kappa+\lambda+1)^2 - (K+1)^2](K+1)}{4\Delta_\kappa\Delta_\lambda}\,, \\ B_{\kappa\lambda} &= \sum_K G_K \frac{[(K+1)^2 - (\kappa-\lambda)^2](K+1)}{4\Delta_\kappa\Delta_\lambda}\,. \end{aligned} \qquad (4)$$

Here G_K is the generalized Coulomb potential for momentum transfer K, and $1 \leq K \leq \Lambda$. We assume that $G_K \geq 0$. In the sums, K, κ, and λ satisfy SU(2) triangular and evenness conditions.

In the previous calculations [6], for $\kappa = \lambda$ the terms $M(\kappa,h)^\dagger M(\lambda,h)$ were replaced by $\nu_\lambda N(\lambda,h)$. This is a valid large-n approximation if N is kept fixed, and the discussion of the $n \to \infty$ limit in our previous work is therefore correct, but for finite n it is inconsistent with (2) and leads to incorrect results except for the "superconfining" potential $G_K \sim \delta_{K1}$. We have also included in H_C the constant term $n\gamma \sum B_{\kappa\lambda}\Delta_\lambda\Delta_\kappa$ that arises from fluctuations in the vacuum charge density. This does not change the structure of the Hamiltonian, but the strong-coupling properties are exhibited more clearly.

For massless fermions, helicity is conserved and $Z = \sum_\kappa [N(\kappa+) - N(\kappa-)]$ remains constant, so Z is a good quantum number. The parity operation gives $Z \to -Z$. The Hamiltonian conserves parity, although the basis states (1) do not separately have definite parities.

In the expression (3) for H_C, it is not obvious that the SU(2) algebra of the operators M, $M^\dagger$, and N might be useful for discussion of the consequences of helicity conservation. We can obtain a more useful and transparent form for H_C if we introduce the "twisted Dirac sea" $|-\rangle$, which is the same as $|0\rangle$ for the positive-helicity states, but for negative helicity has all the negative-energy states unfilled and all the positive-energy states filled (for $\kappa \leq \Lambda$). Then the $M(\kappa-)$, acting on $|-\rangle$, create hole-antihole pairs. We can define a similar state $|+\rangle$ by interchanging these assignments. The states $|\pm\rangle$ have $Z = \pm\Omega$, where $\Omega = \sum_\kappa \nu_\kappa$. Helicity conservation ensures that the charge-density operator annihilates $|\pm\rangle$, so these states have zero total Coulomb energy.

To obtain angular momentum commutation relations of standard form in which the state $|+\rangle$ appears as a highest weight state, we write

$$\begin{aligned} N(\kappa-) &= s_\kappa - L_{\kappa z}\,, & N(\kappa+) &= s_\kappa + K_{\kappa z}\,, \\ M(\kappa-)^\dagger &= L_{\kappa-}\,, & M(\kappa-) &= L_{\kappa+}\,, \\ M(\kappa+)^\dagger &= K_{\kappa+}\,, & M(\kappa+) &= K_{\kappa-}\,, \end{aligned} \qquad (5)$$

where the subscripts $\pm$ on the L's and K's denote raising/lowering operators for spin $s_\kappa = \frac{1}{2}\nu_\kappa$, while the subscript z denotes the third component of angular momentum. If we substitute these expressions into (3) and use the commutation relations, terms linear in the operators cancel. After some algebra, we obtain

$$\begin{aligned} H_C = 2\gamma n^{-1} &\sum_{\kappa<\lambda} A_{\kappa\lambda}\,[2s_\kappa s_\lambda - \mathbf{K}_\kappa\cdot\mathbf{K}_\lambda - \mathbf{L}_\kappa\cdot\mathbf{L}_\lambda] \\ +2\gamma n^{-1} &\sum_{\kappa\lambda} B_{\kappa\lambda}\,[s_\kappa s_\lambda - \mathbf{K}_\kappa\cdot\mathbf{L}_\lambda]\,. \end{aligned} \qquad (6)$$

In the sum involving A, the diagonal terms $\kappa = \lambda$ have also canceled.

We see that from (6) that H_C is invariant under the infinitesimal rotations generated by the vector operator $\mathbf{J} = \sum_\kappa(\mathbf{K}_\kappa + \mathbf{L}_\kappa)$. Since $J_z = Z$, we can consider $\mathbf{J}$ to be a "helicity-spin" operator. If the free-particle energies are neglected, the eigenstates of H_C form degenerate multiplets having fixed values of J and different values of Z. In the multiplet $J = \Omega$ in which the $\mathbf{K}_\kappa$ and $\mathbf{L}_\kappa$ are maximally aligned, the summands in (6) all vanish, and the Coulomb energy is zero, as already noted for $Z = \pm\Omega$. However, in states with $J < \Omega$, at least one of the summands will be positive, giving an energy of order γ.

The Hamiltonian H_C, as well as $\mathbf{J}$, also commutes with an operator $\mathcal{Q}$ that satisfies $\mathcal{Q}^2 = 1$ and $\mathcal{Q}|\pm\rangle = |\pm\rangle$, and that acting on the pair operators gives $\mathcal{Q}\mathbf{K}_\kappa = \mathbf{L}_\kappa\mathcal{Q}$. The parity operator is $\mathcal{P} = \mathcal{R}_x\mathcal{Q}$, where $\mathcal{R}_x$ gives a 180° rotation around the x axis of the helicity-spin space. The intrinsic parity of a multiplet is therefore $P_0 = (-1)^J Q$, where $Q = \pm 1$.

For fixed $\Omega - J$, the bracketed factors in Eq. (6) are of order n, and the leading n-dependent factors cancel. This gives a new large-n limit. The $n \to \infty$ limit in our previous study gave a stable ground state only for $\gamma < \gamma_C$, where γ_C depended on Λ and on the form of G_K. The large-n behavior obtained here is different, indicating that the $n \to \infty$ and $\gamma \to \infty$ limits cannot be interchanged.

We construct the free-particle energy term H_F of the Hamiltonian using Dirac spinors for zero mass, even if the mass is not zero, so that the Coulomb energy (6) remains simple and helicity-nonconservation effects only arise from the mass term H_m of $H_F = H_0 + H_p + H_m$. The constant term $H_0 = \sum_\kappa 2\nu_\kappa E_\kappa$ is the energy of the states $|\pm\rangle$. We also find

$$H_p = \sum_\kappa 2p_\kappa(K_{\kappa z} - L_{\kappa z})\,,$$
$$H_m = -\sum_\kappa 2m(K_{\kappa x} + L_{\kappa x}) = -2mJ_x\,. \qquad (7)$$

Under $\mathcal{Q}$, H_p is odd while H_m is even. The SU(2) helicity symmetry is broken by the kinetic energy term H_p to the U(1) chiral symmetry. This is broken in turn by H_m. Different values of Z are coupled by H_m, giving rise to wave functions $\Psi(Z)$. The overall parity of a state is the product of P_0 and the parity of $\Psi(Z)$. The energy H_F by itself can be diagonalized by rotations in the x-z plane, applied separately for each κ, giving energies spaced by $2E_\kappa$. These rotations give H_C a much more complicated form, which we have checked by direct calculation using finite-mass spinors. In this more usual basis, the symmetry generator $\mathbf{J}$ is very complicated, and the helicity-symmetry of H_C is not at all evident.

If γ is very large, we can estimate the energy of the lowest states by treating H_F as a perturbation. We first consider the case $m = 0$, so Z is still a good quantum number. For each Z, there is a lowest energy E_Z, which we can consider as an effective potential in the helicity space. Only states with $Q = -1$ and $J = \Omega - 1$ are coupled to the ground state by H_p. The matrix elements are proportional to $[(\Omega^2 - Z^2)/\Omega]^{1/2}$, giving, for the lowest energy, $E_Z - H_0 \sim -(\Omega^2 - Z^2)/(\gamma\Omega)$. If $m > 0$, the states with different Z are coupled by J_x, which acts as a difference operator roughly proportional to Ω. Approximating Z as a continuous variable, we obtain a harmonic-oscillator equation in Z. The frequency is $\omega \sim \sqrt{m/\gamma}$. The ground state has even parity and the $\Psi(Z)$ alternate in parity, corresponding to a pseudoscalar quantum. For the validity of this harmonic-oscillator approximation, we need to require $1 \ll \langle Z^2\rangle \ll \Omega^2$, which implies that $\Omega^{-2} \ll m\gamma \ll \Omega^2$.

For large γ, and as we have also checked numerically for smaller γ, the effective potential E_Z has a single minimum at $Z = 0$. The lowest-energy state (the vacuum) is therefore always unique for any γ. These results for the single-flavor model imply that the vacuum is not parity doubled when $m = 0$ and that there is no spontaneous breaking of chiral symmetry for any finite strength of the Coulomb potential. However, chiral symmetry is "almost spontaneously broken" in the sense that the effective potential becomes flat in the limit $\gamma \to \infty$. Although $\langle Z\rangle = 0$, $\langle Z^2\rangle$ becomes large in this limit when $m > 0$. There is also a pseudoscalar boson with a mass proportional to $\sqrt{m}$. In the low-lying states with multiple pseudoscalar particles, charges cancel locally on a distance scale given by Λ^{-1} in units of the hyperspherical radius. In the higher states with energies of order γ, however, charges are separated by distances of the order of the radius. The equal spacing of the harmonic-oscillator energies indicates that the pseudoscalar particle is weakly interacting and the S-wave scattering length vanishes. Note that these results did not depend on the confining or nonconfining character of G_K, but only on the overall strength being very large.

Other studies of similar or related models [1] that have arrived at different conclusions about a chiral condensate may have been affected by insufficiently accurate variational approximations or by some other numerical artifact. The vanishing of the total Coulomb energy in the lowest multiplet depends on exact cancellations, and numerical calculations that are not able to exploit these cancellations can be expected to give unreliable results. In our previous work [6], we found that a standard variational approximation, which used an exponential form for the wave function, was not accurate for small Z. Similar difficulties may occur in the ladder approximation to the Bethe-Salpeter equation [2].

As pointed out above, there are some states with a total momentum of zero which are omitted from our basis. They could be constructed from pairs with a nonzero net momentum. However, there can be no other states with a lower Coulomb energy, because the Coulomb energy operator is semidefinite positive. Furthermore, enlarging the basis would not raise the minimum energies E_Z. The limiting E_Z for $\gamma \to \infty$ might possibly be reduced further if the kinetic energy could be lowered by inclusion of more complicated states, although this seems unlikely. The main effect of a larger basis would be that additional states, corresponding to bosons with nonzero momenta, would appear in the excitation spectrum. Unfortunately, states with more complicated internal momentum distributions bring in many new algebraic complications. Another source of additional configurations arises from the flavor degree of freedom. We shall consider a model with several flavors next, because it corresponds more closely to the physical situation. This will also provide a simplified way to study the effect of additional states. If only flavor-singlet pair configurations are considered, the results are not changed from those already obtained, except that the number of flavors also enters into the degeneracy factors.

For f different flavors, operators that are bilinear in fermion operators acquire two flavor indices. We temporarily consider only a single value of momentum and helicity. For M_{ab} and $M^\dagger_{ab}$, the first index labels the flavor of the fermion and the second labels the flavor of the antifermion. We define operators n_{ab} for fermions and $\bar{n}_{ab}$ for antifermions, where the first index labels a creation operator and the second an annihilation operator. There are $(2f)^2$ operators, of which the $2f$ diagonal operators n_{aa} and $\bar{n}_{aa}$ commute and satisfy the constraint $\sum(n_{aa} - \bar{n}_{aa}) = 0$, corresponding to the SU$(2f)$ algebra. Some of the commutation relations are

$$[M_{cd}, M^\dagger_{ab}] = \delta_{ca}\delta_{bd}\nu - \delta_{bd}n_{ac} - \delta_{ca}\bar{n}_{bd}\,,$$
$$[n_{cd}, M_{ab}] = -\delta_{ca}M_{db}\,, \tag{8}$$
$$[n_{cd}, n_{ab}] = \delta_{da}n_{cb} - \delta_{cb}n_{ad}\,.$$

Consider $N_{ab} = n_{ab} + \bar{n}_{ba}$; the traceless combinations $N_{ab} - \delta_{ab}N_0/f$, where $N_0 = \sum N_{aa}$, form the SU(f) algebra. The flavor-singlet combinations N_0, $M_0 = \sum M_{aa}$, and $M^\dagger_0$ are equivalent to the previous SU(2) algebra.

The Coulomb Hamiltonian has the same form as in (3), except for sums over the flavor indices:

$$\begin{aligned} H_C = \gamma \sum_{\kappa\lambda} A_{\kappa\lambda} \Bigg(& \tfrac{1}{2}\Delta_\lambda \sum_{ah}[n_{aa}(\kappa,h) + \bar{n}_{aa}(\kappa,h)] - \frac{1}{2n}\sum_{abh}[:n_{ab}(\kappa,h)n_{ba}(\lambda,h): + :\bar{n}_{ab}(\kappa,h)\bar{n}_{ba}(\lambda,h):] \\ & -\frac{1}{n}\sum_{abh} M_{ab}(\kappa,h)^\dagger M_{ab}(\lambda,h)\Bigg) \\ +\gamma\sum_{\kappa\lambda} B_{\kappa\lambda}\Bigg[& \Bigg(fn\Delta_\lambda\Delta_\kappa - \tfrac{1}{2}\Delta_\lambda \sum_{ah}[n_{aa}(\kappa,h) + \bar{n}_{aa}(\kappa,h)]\Bigg) + \frac{1}{n}\sum_{abh} n_{ab}(\kappa,h)\bar{n}_{ab}(\lambda,-h) \\ & -\frac{1}{n}\sum_{ab}[M_{ab}(\kappa+)^\dagger M_{ba}(\lambda-)^\dagger + M_{ab}(\kappa+)M_{ba}(\lambda-)]\Bigg]\,. \end{aligned} \tag{9}$$

We now introduce the twisted sea through the replacements $n_{ab} \to \frac{1}{2}n\Delta\delta_{ab} \pm n_{ab}$ for $h = \pm\frac{1}{2}$, and $M^\dagger_{ab} \to M_{ba}$ for negative helicity. The twisted sea $|+\rangle$ is a highest weight state in the SU$(2f)$ representation $\mathcal{F}$ corresponding to the tableau $[(2\Omega)^f]$ and has helicity $Z = f\Omega$. We obtain a Coulomb Hamiltonian which is similar to (6):

$$H_C = 2\gamma n^{-1}\sum_{\kappa<\lambda} A_{\kappa\lambda}\,[2fs_\kappa s_\lambda - \mathcal{K}_\kappa\cdot\mathcal{K}_\lambda - \mathcal{L}_\kappa\cdot\mathcal{L}_\lambda]$$
$$+2\gamma n^{-1}\sum_{\kappa\lambda} B_{\kappa\lambda}\,[fs_\kappa s_\lambda - \mathcal{K}_\kappa\cdot\mathcal{L}_\lambda]\,. \tag{10}$$

The $(4f^2-1)$-dimensional vectors $\mathcal{K}_\kappa$ and $\mathcal{L}_\kappa$ are the generators of SU$(2f)$ in the representation $[(2s_\kappa)^f]$. The A term cancels for $\kappa = \lambda$ because the Casimir operator equals [7] $fs_\kappa(s_\kappa + f)$ with our normalization. The free-particle energy is similar in form to (7). The term H_p breaks the full SU$(2f)$ symmetry to SU$(f) \otimes$ SU(f).

The Coulomb energy vanishes in the fully aligned multiplet $\mathcal{F}$. As indicated above, if we were to consider only pair operators which involved flavor singlets, we would find a helicity-spin multiplet $J = f\Omega$ with zero Coulomb energy. By including more general pair operators, we obtain a zero-energy multiplet from a larger group, which contains many more states.

The approach to the strong-coupling limit for $f > 1$ is much more complicated than for $f = 1$ and depends on the flavor multiplet. In general, the resolution of SU$(2f)$ multiplets into SU$(f) \otimes$ SU(2) multiplets associates each representation of SU(f) with many different values of J in a complicated pattern. For second-order perturbations of $\mathcal{F}$, we need to consider $\mathcal{B} \subset \mathcal{F}\otimes\mathcal{A}$ with $Q = -1$, where $\mathcal{A}$ is the adjoint representation of SU$(2f)$. Explicit general results are known to us only for $f = 2$. The representation $\mathcal{F}$ contains the SU(2) $\times$ SU(2) multiplet (I, J) once, provided $2\Omega - I - J$ is a non-negative even integer [7], and it can be shown that for $I + J \le 2\Omega$, the multiplicity of (I, J) in $\mathcal{B} = [2\Omega, 2\Omega-1, 1]$ is $2 - \delta_{J0} - \delta_{I0} - \delta_{2\Omega,I+J}$. Some additional general observations can also be made. We denote by S and A the singlet and adjoint representations of SU(f). For $J = f\Omega$, only S occurs, and only in $\mathcal{F}$. For $J = f\Omega - 1$, application of the step operators shows that $\mathcal{F}$ contains only A. In $\mathcal{B}$, we find both S and A with $J = f\Omega - 1$. For smaller J, we expect additional multiplets and additional occurrences of S and A, as seen for $f = 2$. In $\mathcal{F}$ there are $f(f-1)$ states having maximal charges that are related to $|\pm\rangle$ by Weyl reflections. These belong to a representation C of SU(f) found in $\mathcal{F}$ with $J = 0$ but not occurring in $\mathcal{B}$ and therefore unperturbed in second order.

The shape of the effective energy functions E_Z for various flavor multiplets cannot be determined without detailed calculations. The E_Z become matrices that describe the coupling, via intermediates in $\mathcal{B}$, of the various J values found in $\mathcal{F}$. The perturbation matrix elements are proportional to $(J^2 - Z^2)^{1/2}$ for $\Delta J = 1$, but for $\Delta J = 0$ they are proportional to Z. This is consistent with the parity rule. In general, the low-dimension multiplets will have many closely spaced diagonal E_Z values, all coupled by H_m, giving many low-lying excited states. We expect the lowest flavor singlet to be a scalar and the lowest A-flavored state to be a pseudoscalar, as they will be associated with even functions of Z. The lowest singlet excitation is probably pseudoscalar, and other low-lying states may correspond to combinations of pseudoscalar quanta.

The picture of chiral-symmetry breaking suggested by our pair interaction model is different from the usual picture. There is no finite value γ_χ at which the structure of low-lying states changes abruptly. Instead, a weakly interacting low-lying pseudoscalar excitation is developed gradually as an asymptotic phenomenon in the strong-coupling regime. The existence of this excitation does not depend on whether the interaction is confining or nonconfining, but only on the strength being large. However, a natural candidate interaction that meets our requirements is provided by a QCD potential that becomes large for small momenta; that is, G_K increases more rapidly than $1/K^2$ for $K < \lambda$, where λ is the scale parameter for QCD. The light pseudoscalar particle that emerges in this model is constructed to have no net charge, but charges are also canceled locally within it. In other states, which lie very high, charges are separated.

The group theory of the new dynamical symmetry present in our model has been essential for our discussion. It has allowed us to derive in a simple way the qualitative properties of the single-flavor version of the model. With several flavors, the symmetry group is richer and more complicated. To determine the relations among excitation energies, extensive numerical calculations will be required, but further use of group-theoretical methods can simplify these calculations.

We thank V. N. Gribov, V. A. Miransky, and K. C. Wang for discussions. This work was supported by the U. S. Department of Energy under Contract No. DE-AC02-76ER-03066. Support for computation at the Pittsburgh Supercomputing Center has been provided through Grant No. PHY870047P.

[1] J. T. Finger and J. E. Mandula, Nucl. Phys. **B199**, 168 (1982); J. Govaerts, J. E. Mandula, and J. Weyers, *ibid.* **B237**, 59 (1984); S. J. Adler and A. C. Davis, *ibid.* **B244**, 469 (1984); A. C. Davis and A. M. Matheson, *ibid.* **B246**, 203 (1984); Phys. Lett. **165B**, 395 (1985); A. Matheson, Phys. Rev. D **38**, 3566 (1988).

[2] V. A. Miransky, Phys. Lett. **91B**, 421 (1980); Nuovo Cimento **90A**, 149 (1985); Zh. Eksp. Teor. Fiz. **88**, 1514 (1985) [Sov. Phys. JETP **61**, 905 (1985)]; V. P. Gusynin, V. A. Kushnir, and V. A. Miransky, Phys. Lett. B **120**, 635 (1989); Y. Dai, C. Huang, and D. Liu, Phys. Rev. D **43**, 1717 (1991).

[3] J. B. Kogut, E. Dagotto, and A. Kocić, Phys. Rev. Lett. **60**, 772 (1988); **62**, 1001 (1989); Nucl. Phys. **B317**, 253 (1989); E. Dagotto, A. Kocić, and J. B. Kogut, *ibid.* **B331**, 500 (1990); S. Hands and R. Wensley, Phys. Rev. Lett. **63**, 2169 (1989); S. Hands and J. B. Kogut, Nucl. Phys. **B335**, 445 (1990); S. Hands, J. B. Kogut, and E. Dagotto, *ibid.* **B333**, 551 (1990); S. Hands, J. B. Kogut, and J. H. Sloan, *ibid.* **B344**, 255 (1990).

[4] D. K. Sinclair and J. B. Kogut, Phys. Lett. B **229**, 93 (1989); J. B. Kogut and D. K. Sinclair, *ibid.* **229**, 107 (1989); Nucl. Phys. **B344**, 238 (1990); R. V. Gavai *et al.*, Phys. Lett. B **241**, 567 (1990); M. Fukugita *et al.*, Phys. Rev. Lett. **65**, 816 (1990); K. M. Bitar *et al.*, Phys. Rev. D **42**, 3794 (1990).

[5] K. C. Wang and R. E. Cutkosky, Phys. Rev. D **36**, 3825 (1987); R. E. Cutkosky and K. C. Wang, *ibid.* **37**, 3024 (1988); R. E. Cutkosky, Czech. J. Phys. B **40**, 252 (1990).

[6] K. C. Wang and R. E. Cutkosky, Phys. Rev. D **42**, 1260 (1990); S. Capstick, R. E. Cutkosky, M. Joensen, and K. C. Wang, *ibid.* **42**, 1270 (1990).

[7] K. T. Hecht and S. C. Pang, J. Math. Phys. **10**, 1571 (1969); S. I. So and D. Strottman, *ibid.* **20**, 153 (1979).