

**QCD sum rules and the properties of  $\rho$ - and  $\phi$ -meson excited states**

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The recently proposed method of extracting more detailed information on  $\rho$ - and  $\phi$ -meson properties from the Laplace QCD sum rules is extended to include also the lowest-lying radial excitations in both channels. The approach is based on a simultaneous numerical matching of the corresponding sum rule and its derivative in a suitable range of the sum-rule parameter  $M$  as well as on the use of smooth parametrizations for the hadronic cross sections  $\sigma(e^+e^- \rightarrow 2\pi, 4\pi)$  and  $\sigma(e^+e^- \rightarrow K\bar{K})$ . The predicted masses, widths, and some other decay characteristics are close to the experimental values.

**I. INTRODUCTION**

The QCD sum rules<sup>1</sup> (SR's) represent one of few methods capable of making predictions in the low- and medium-energy hadron physics starting basically from the QCD Lagrangian. The only phenomenological input parameters are the values of two or three quark and gluon condensates characterizing the properties of the physical vacuum. The field of application of the SR's has been extended remarkably during the past ten years.<sup>2,3</sup> A number of successful predictions for the masses and leptonic coupling constants of the resonance ground states have been obtained in the pioneering work of Shifman, Vainshtein, and Zakharov.<sup>1</sup> In the subsequent development of the method the rough model of the hadronic spectral density in the form of the  $\delta$  function (lowest-lying resonance) plus a quark-antiquark continuum has become a stable ingredient of the analyses. As was pointed out by Das *et al.*<sup>4</sup> recently, one can obtain in addition to the vector-meson masses rather accurate predictions for their widths if the  $\delta$ -function model for the corresponding cross section is replaced by a smooth parametrization. The numerical procedure used to match the sum rule in a relatively wide interval of the parameter  $M$  has been elaborated further in Ref. 5, where together with the realistic values of the  $\rho$ - and  $\phi$ -meson masses and widths the ratios of the coupling constants defined within the vector-meson dominance model (VDM) have also been obtained. The only somewhat problematic result was a rather low  $\rho$ -meson mass of about 740 MeV. Because in the numerical analysis of the SR's the medium-energy region gives non-negligible contribution, one

should check whether or not the neglect of possible structure from this region is the reason for the inconsistency. The investigation of this possibility and its consequences are the subject of the present paper. It has turned out that the introduction of one fixed higher  $\rho$ -meson pole to the spectral function together with a choice of a more suitable interval for the parameter  $M$  have shifted the  $\rho$ -meson mass to its experimental value. Then in addition to the first SR its derivative has also been optimized with respect to the variations of higher  $\rho$ -state parameters yielding a prediction for the latter. The reason for a simultaneous matching of both sum rules was to restrict the possibly large set of solutions as well as to increase the numerical stability of the procedure. The details of the method are given together with the description of the  $e^+e^- \rightarrow 2\pi, 4\pi$  cross-section models in Sec. II A. The results of the analysis in the  $\rho$ -meson channel can be found in Sec. II B. Section III contains a brief discussion of the kaon form-factor parametrization used in the hadronic spectral function in the  $\phi$ -meson sum rules as well as the obtained properties of the excited  $\phi$ -meson state. Conclusions are given in Sec. IV.

**II. PROPERTIES OF THE  $\rho$ -MESON EXCITED STATE**

**A. The cross-section model and the method for the sum-rule match**

It would be useful prior to the description of our method of analysis to recall briefly the  $\rho$ -meson sum rule and its derivative with respect to  $M^2$ .<sup>1</sup> They are of the well-known form

$$\frac{1}{\pi} \int_{4m_\pi^2}^{\Lambda_0^2} \text{Im}\Pi^h(s) e^{-s/M^2} ds + \frac{1}{\pi} \int_{\Lambda_0^2}^{\infty} \text{Im}\Pi^q(s) e^{-s/M^2} ds = \frac{M^2}{8\pi^2} \left[ 1 + \frac{\alpha_s}{\pi} + \frac{C_4 \langle O_4 \rangle}{M^4} + \frac{C_6 \langle O_6 \rangle}{2M^6} \right], \tag{1a}$$

$$\frac{1}{\pi} \int_{4m_\pi^2}^{\Lambda_0^2} \text{Im}\Pi^h(s) s e^{-s/M^2} ds + \frac{1}{\pi} \int_{\Lambda_0^2}^{\infty} \text{Im}\Pi^q(s) s e^{-s/M^2} ds = \frac{M^4}{8\pi^2} \left[ 1 + \frac{\alpha_s}{\pi} - \frac{C_4 \langle O_4 \rangle}{M^4} - \frac{C_6 \langle O_6 \rangle}{M^6} \right] \tag{1b}$$

where  $\text{Im}\Pi(s)$  is the imaginary part of the invariant function of the correlator built from two electromagnetic currents with the  $\rho$ -meson quantum numbers. The hadronic spectral function has been divided into the low-energy part  $\text{Im}\Pi^h/\pi$ , which may up to  $s=2 \text{ GeV}^2$  be safely approximated by the two-pion and the four-pion contributions, and the high-energy part, conventionally approximated by the quark-antiquark spectral density

$$\frac{1}{\pi} \text{Im}\Pi^q = \frac{1}{8\pi^2} \left[ 1 + \frac{\alpha_s(M^2)}{\pi} \right]. \quad (2)$$

Note that the thresholds  $\Lambda_0$  and  $\Lambda'_0$  will be considered as the free parameters in the numerical procedure.<sup>4</sup> The vacuum expectation values of the operators of dimension 4 and 6 (higher-dimensional operators in the expansion have been neglected as usual) are defined in a standard way through the quark and gluon vacuum condensate

$$C_4 \langle O_4 \rangle = \frac{\pi^2}{3} \langle 0 | (\alpha_s/\pi) G_{\mu\nu}^a G^{\mu\nu,a} | 0 \rangle + 4\pi^2 (m_u \langle 0 | \bar{u}u | 0 \rangle + m_d \langle 0 | \bar{d}d | 0 \rangle), \quad (3a)$$

$$C_6 \langle O_6 \rangle = \frac{-896\pi^3}{81} \alpha_s \langle 0 | \bar{q}q | 0 \rangle^2, \quad \alpha_s(\mu) = 0.7 \text{ for } \mu = 0.2 \text{ GeV}, \quad (3b)$$

where (3b) has been obtained using the vacuum-saturation hypothesis.<sup>1</sup> We shall adhere to the standard values of the condensates

$$\langle 0 | (\alpha_s/\pi) G_{\mu\nu}^a G^{\mu\nu,a} | 0 \rangle = 0.012 \text{ GeV}^4, \quad \langle 0 | \bar{q}q | 0 \rangle \equiv \langle 0 | \bar{u}u | 0 \rangle \equiv \langle 0 | \bar{d}d | 0 \rangle = (-0.25 \text{ GeV})^3. \quad (4)$$

As for the phenomenological part of the SR's (1a) and (1b), one should find a simple, but theoretically well justified parametrization, for the two-pion and the four-pion contributions to  $\text{Im}\Pi^h(s)$ , which dominate in the  $I=J=1$  channel. The two-pion part can be expressed in terms of the pion electromagnetic form factor  $F_\pi(s)$  as

$$\frac{1}{\pi} \text{Im}\Pi^{2\pi}(s) = \frac{1}{48\pi^2} \left[ 1 - \frac{4m_\pi^2}{s} \right]^{3/2} |F_\pi(s)|^2. \quad (5)$$

We shall use an analytic model for  $F_\pi(s)$  formulated in the variable  $q$  free of the two-pion cut:

$$F_\pi(s) = \frac{a_0 + a_2 q^2}{(q - q_\rho)(q + q_\rho^*)} + \frac{c_{\rho'}}{(q - q_{\rho'})(q + q_{\rho'}^*)}, \quad q = \frac{1}{2}(s - 4m_\pi^2)^{1/2}. \quad (6)$$

The model was successfully applied in our previous analysis of the  $\rho$ -meson SR.<sup>5</sup> The only difference is the addition of the second term describing the contribution of one higher  $\rho$  resonance represented by a conjugate pair of poles at  $q = q_{\rho'}$  and  $q = -q_{\rho'}^*$ . The first term depends not only upon the analogous pair of the  $\rho$ -meson poles but also upon two arbitrary real coefficients  $a_0$  and  $a_2$ , which can be used to ensure the correct normalization  $F_\pi(s=0)=1$  as well as the equality between the residue

of the pion form-factor VMD model and the residue of the  $\rho$ -meson term of Eq. (6) in the limit  $\Gamma_\rho \rightarrow 0$  (Ref. 5) through the relations

$$a_2 = 1 + \frac{c_\rho}{(1+q_{\rho 0}^2)} - \frac{c_{\rho'}}{(i-q_{\rho'})(i+q_{\rho'}^*)}, \quad (7)$$

$$a_0 = a_2 + (i-q_\rho)(i+q_\rho^*) \left[ 1 - \frac{c_{\rho'}}{(i-q_{\rho'})(i+q_{\rho'}^*)} \right].$$

The following notation has been used in Eqs. (6) and (7) ( $m_\pi=1$ ):

$$i = \sqrt{-1}, \quad q_{\rho 0} = \frac{1}{2}(m_\rho^2 - 4), \quad c_V = -\frac{m_V^2 g_V}{4}, \quad (8)$$

$$g_V \equiv \frac{f_{V\pi\pi}}{f_V}, \quad V = \rho, \rho',$$

where the coupling constants  $f_{V\pi\pi}$  and  $f_V$  are defined in the VMD model and characterize the transitions  $V \rightarrow \pi^+\pi^-$  and  $V \rightarrow \gamma$ . The constant numerator of the  $\rho'$  term in (6) has been identified with  $c_{\rho'}$  using again the connection between the corresponding residues of the VMD model and the parametrization (6). Since the  $\rho$  meson gives the dominant contribution to the pion form-factor behavior, we have introduced the power  $q$  dependence (Taylor series) in the numerator of the  $\rho$ -meson term to ensure its adequate description. Contrary to our previous work<sup>5</sup> we do not impose the threshold behavior of  $\text{Im}F_\pi(s)$  because its inclusion does not influence the result appreciably. Consequently, only the quadratic term, yielding a more general imaginary part of  $F_\pi(s)$  than a possible linear term, is present in the numerator.

The second contribution to the low-energy part of the spectral function comes from the state of four pions. We shall somewhat simplify the situation and assume the existence of one higher  $\rho$  resonance coupled to the effective four-pion channel instead of to the experimentally known  $2\pi^0\pi^+\pi^-$  and  $2\pi^+2\pi^-$  ones (see also Sec. II B). Then the four-pion part of the hadronic spectral function is given by

$$\frac{1}{\pi} \text{Im}\Pi^{4\pi}(s) = \frac{1}{16\pi^3 \alpha^2} s \sigma(e^+e^- \rightarrow 4\pi), \quad (9)$$

where  $\alpha$  is the fine-structure constant. For the cross section  $\sigma(e^+e^- \rightarrow 4\pi)$  we adopt the Breit-Wigner parametrization of the form

$$\sigma(e^+e^- \rightarrow 4\pi) = \frac{12\pi}{s} \frac{B_{\rho'} m_{\rho'}^2 \Gamma_{\rho'}^2}{(m_{\rho'}^2 - s)^2 + m_{\rho'}^2 \Gamma_{\rho'}^2}, \quad (10)$$

where

$$B_{\rho'} = B(\rho' \rightarrow e^+e^-) B(\rho' \rightarrow 4\pi) \quad (11)$$

is the product of the  $e^+e^-$  and  $4\pi$  branching ratios of the  $\rho'$  resonance. Finally, the hadronic spectral function on the left-hand side of the SR's is equal to the sum of the Eqs. (5) and (9).

The two sum rules (1a) and (1b) with all components defined by the relations (2)–(11) were studied numerically by the following procedure. In the first step one fixes the

parameters  $m_{\rho'}$ ,  $\Gamma_{\rho'}$  and  $B_{\rho'}$  of the excited state at some trial values in the first SR and varies  $m_{\rho}$ ,  $\Gamma_{\rho}$ ,  $f_{\rho\pi\pi}/f_{\rho'}$  and  $\Lambda_0$  to minimize the sum of squares

$$F_1 = \sum_{i=1}^N \left[ \frac{R_1(M_i^2)}{L_1(M_i^2)} - 1 \right]^2, \quad (12)$$

which measures the total deviation of the ratio  $R_1/L_1$  from unity at a suitable interval of  $M$  broken into  $N$  discrete points. In the second step one optimizes the second SR (quantity  $F_2$  analogous to  $F_1$ ) by varying the parameters of the  $\rho'$  meson and the threshold  $\Lambda'_0$  while the parameters of the  $\rho$  resonance are kept fixed at their optimal values from the previous step. Then one switches back to the first SR, fixes  $\rho'$  parameters at their new values and minimizes  $F_1$  starting from the updated  $\rho$  parameters, etc. The procedure is terminated when the two sets of parameters converge to some stable values and simultaneously both of the sum rules are matched to a high degree of accuracy that can be measured by the quantities

$$\Delta_j = \frac{1}{N} \sqrt{F_j}, \quad j = 1, 2. \quad (13)$$

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$$\begin{aligned} m_{\rho} &= 755 \text{ MeV}, \quad \Gamma_{\rho} = 158 \text{ MeV}, \quad g_{\rho} = 1.19, \quad g_{\rho'} = 0.22, \quad \Lambda_0 = 2.09 \text{ GeV}, \quad \Delta_1 \cong 10^{-4}, \\ m_{\rho'} &= 1620 \text{ MeV}, \quad \Gamma_{\rho'} = 249 \text{ MeV}, \quad B_{\rho'} = 19 \times 10^{-6}, \quad \Lambda'_0 = 2.02 \text{ GeV}, \quad \Delta_2 \cong 10^{-4}. \end{aligned} \quad (14)$$

The contribution from the region of  $\rho'$  resonance reached (39–41)% of the right-hand side of the SR (1b) at the chosen interval of  $M$ . Note that with a plausible assumption

$$B_{\rho}(e^+e^-)B_{\rho'}(2\pi^0\pi^+\pi^-) \cong B_{\rho'}(e^+e^-)B_{\rho}(2\pi^+2\pi^-)$$

one has for both quantities a value of about  $10^{-5}$ . The convergence of the procedure was rather fast, e.a. stable minima for  $F_1$  and  $F_2$  have been obtained after seven iterations when the table values were used as the initial parameter values. Starting from  $m_{\rho} = 750$  MeV and  $m_{\rho'} = 1500$  MeV led to the same optimal parameters that

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$$\begin{aligned} m_{\rho} &= 759 \text{ MeV}, \quad \Gamma_{\rho} = 156 \text{ MeV}, \quad g_{\rho} = 1.20, \quad g_{\rho'} = 0.20, \quad \Delta_1 = 5 \times 10^{-4}, \\ m_{\rho'} &= 1676 \text{ MeV}, \quad \Gamma_{\rho'} = 246 \text{ MeV}, \quad g_{\rho'} = 19 \times 10^{-6}, \quad \Delta_2 = 2 \times 10^{-5}, \end{aligned} \quad (15)$$

while the largest change induced by shifting the interval of  $M$  by 100–200 MeV corresponded to  $g_{\rho'}$ :  $g_{\rho'} = 0.17$ – $0.27$ . We have verified also that the results are almost insensitive to the variations of  $\pm 100$  MeV in  $\Lambda_0$  and  $\Lambda'_0$ . Our final result for the ranges of the resonance parameters is then

$$\begin{aligned} m_{\rho} &= 753\text{--}759 \text{ MeV}, \quad \Gamma_{\rho} = 156\text{--}160 \text{ MeV}, \quad g_{\rho} = 1.19\text{--}1.20, \\ m_{\rho'} &= 1560\text{--}1676 \text{ MeV}, \quad \Gamma_{\rho'} = 237\text{--}261 \text{ MeV}, \quad g_{\rho'} = 0.17\text{--}0.27, \quad B_{\rho'} = (18\text{--}20) \times 10^{-6}. \end{aligned} \quad (16)$$

## B. Results for the $\rho$ -meson channel

We have used CERN program MIGRAD in the numerical work to minimize  $F_1$  and  $F_2$ . The approximate intervals of the sum-rule variable  $M$  have been chosen in the spirit of the SVZ approach.<sup>1</sup> The lower limit has been determined from the requirement that the contribution from the region of expected higher state (roughly  $1.2 \text{ GeV}^2 \leq s \leq \Lambda_0^2$ ) be sufficiently large, say not less than 20%. The upper limit follows from the fact that the continuum contribution should not exceed (25–30)% if one wants to keep the accuracy of the calculation at the (8–10)% level. Consequently, the initial guess for the interval is  $1.1 \leq M \leq 1.4$  GeV for the first SR and  $1.0 \leq M \leq 1.2$  GeV for the second SR. The stability of the final results will be tested, however, with respect to the variations of these intervals as well as to the variations of the cut-off parameters  $\Lambda_0$  and  $\Lambda'_0$  and the input condensate values.

A potential problem is a rather large number of physical parameters to be determined from the sum-rule match (three for  $\rho$  meson, four for  $\rho'$  meson plus two thresholds  $\Lambda_0$  and  $\Lambda'_0$ ). We have reduced this number by setting  $\Lambda_0 = \Lambda'_0$  at first stage and by matching the first SR with  $g_{\rho'}$  fixed at different values. The best match has been achieved for  $g_{\rho'} \cong 0.20$ – $0.25$ . Both  $\Lambda_0$  and  $g_{\rho'}$  moved only slightly when they were released to obtain the final results

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agree reasonably with the experimental values<sup>6,7</sup>  $m_{\rho} = 768.1 \pm 0.7$  MeV,  $\Gamma_{\rho} = 152.4 \pm 1.5$  MeV,  $g_{\rho} = 1.22$ ,  $m_{\rho'} = 1700 \pm 20$  MeV,  $\Gamma_{\rho'} = 235 \pm 50$  MeV,  $B_{\rho'} = (11 \pm 1) \times 10^{-6}$ . As for the coupling-constant ratio  $g_{\rho'}$ , it is in rough agreement with the result of the recent Orsay analysis of the pion-form-factor measurement with high statistics.<sup>8</sup> In spite of the fact that the results (14) are rather encouraging one should investigate a question of their stability with respect to the variations of the input condensate values (an uncertainty of a factor 2<sup>1</sup>) and of the interval limits. Taking for example  $2\langle(\alpha_s/\pi)G^2\rangle$  gave

While setting  $B_{\rho'}=0$ ,  $g_{\rho'}=0$  does not spoil the sum-rule match too much (only by a factor 3 in the quantities  $\Delta_1$  and  $\Delta_2$ ), some of the  $\rho$ -meson parameters are shifted away from their realistic values:  $m_\rho=748$  MeV (first SR) and  $\Gamma_\rho=176$  MeV (second SR).

It is evident that the obtained results do have some contact with reality: the  $\rho$ -meson parameters are in satisfactory agreement with the experimental numbers and the properties of our  $\rho'$  meson correspond to the well-established  $\rho(1700)$  resonance, at least to the accuracy inherent to the present approach. This fact stimulated our attempt to go still further—namely, to try to resolve the structure of higher  $\rho$  mesons in greater detail, particularly to investigate whether the SR's admit the existence of the long-discussed  $\rho(1250)$  state. There has been a growing experimental<sup>8</sup> and phenomenological<sup>9</sup> evidence for the existence of this resonance with a higher mass of 1450 MeV, however. Our analysis of this problem by the method described in the present paper (the only difference being a necessity to use the  $\rho$ -meson parameters as the input) led to the conclusion that the SVZ sum rules support a picture of two excited  $\rho$ -meson states below 2 GeV with the masses of 1350–1478 MeV and 1620–1765 MeV.<sup>10</sup>

### III. PROPERTIES OF THE $\phi$ -MESON EXCITED STATE

The analysis in the  $\phi$ -meson channel is very similar to the  $\rho$ -meson case. The theoretical part of the SR's differs, besides an overall factor  $\frac{2}{3}$ , mainly by the presence of the power term of dimension 2.<sup>1</sup> It depends on the running  $s$ -quark mass defined in the minimal-subtraction (MS) renormalization scheme to lowest order as

$$\bar{m}_s(M^2) = \frac{\hat{m}_s}{(\ln M/\Lambda_{\text{MS}})^{4/9}}, \quad (17)$$

where  $\hat{m}_s=267$  MeV is the renormalization-group invariant mass of the strange quark.<sup>11</sup> The condensates of dimensions 4 and 6 have an analogous form as in the previ-

ous section. The value  $\langle 0|\bar{s}s|0\rangle=0.6\langle 0|\bar{u}u|0\rangle$  has been taken for the  $s$ -quark condensate.<sup>11</sup>

The lowest-lying contribution to the phenomenological spectral function comes from the state of two kaons. It can be written in terms of the isoscalar kaon form factor  $F_K^s(s)$  with the  $\phi$ - and  $\phi'$ -meson contributions. Our parametrization for  $F_K^s$  is again based on the analytic properties of the form-factor function in the complex  $s$  plane. They consist of pairs of resonance poles and a square-root cut starting at  $s=9m_\pi^2$ . The new variable  $r = \frac{1}{3}(s - 9m_\pi^2)^{1/2}$  removes the cut and  $F_K^s(s)$  can be expressed as

$$F_K^s(s) = \frac{b_0 + b_2 r^2}{(r - r_\phi)(r + r_\phi^*)} + \frac{c_{\phi'}}{(r - r_{\phi'})(r + r_{\phi'}^*)}, \quad (18)$$

$$r_V = r(s = (m_V - i\frac{1}{2}\Gamma_V)^2), \quad V = \phi, \phi'$$

with arbitrary real  $b_0$  and  $b_2$ , which we again exploit to ensure proper normalization  $F_K^s(0)=\frac{1}{2}$  and the equality between residues of the VDM representation for  $F_K^s$  and the formula (18) taken with  $\Gamma_\phi=0$  ( $\Gamma_{\phi'}=0$ ). The corresponding formulas read

$$b_2 = \frac{1}{2} + \frac{c_\phi}{(1+r_\phi^2)} - \frac{c_{\phi'}}{(i-r_{\phi'})(i+r_{\phi'}^*)},$$

$$b_0 = b_2 + (i-r_\phi)(i+r_\phi^*) \left[ \frac{1}{2} - \frac{c_{\phi'}}{(i-r_{\phi'})(i+r_{\phi'}^*)} \right], \quad (19)$$

$$c_V = -\frac{m_V^2 g_V}{9}, \quad g_V \equiv \frac{f_{VK\bar{K}}}{f_V}, \quad V = \phi, \phi'.$$

where  $f_{VK\bar{K}}$  and  $f_V$  correspond to the transitions  $V \rightarrow K\bar{K}$  and  $V \rightarrow \gamma$ , respectively.

The  $\phi$ -meson SR and its derivative have been optimized by the method described in Sec. II A in the same intervals of the sum-rule variable  $M$ . The procedure has converged for the following optimal parameters of the  $\phi$  and  $\phi'$  mesons:

$$m_\phi = 1019.9 \text{ MeV}, \quad \Gamma_\phi = 4.07 \text{ MeV}, \quad g_\phi = 0.38, \quad \Lambda_0 = 1.69 \text{ MeV}, \quad \Delta_2 = 2 \times 10^{-3},$$

$$m_{\phi'} = 1566 \text{ MeV}, \quad \Gamma_{\phi'} = 79 \text{ MeV}, \quad g_{\phi'} = 0.16, \quad \Lambda'_0 = 1.70 \text{ GeV}, \quad \Delta_1 = 2 \times 10^{-4}. \quad (20)$$

Taking into account the same variations as for the  $\rho$ -meson case, one has for the uncertainties of the predicted quantities:

$$m_\phi = 1019\text{--}1024 \text{ MeV}, \quad \Gamma_\phi = 3.8\text{--}4.1 \text{ MeV}, \quad g_\phi = 0.34\text{--}0.38,$$

$$m_{\phi'} = 1486\text{--}1604 \text{ MeV}, \quad \Gamma_{\phi'} = 69\text{--}103 \text{ MeV}, \quad g_{\phi'} = 0.12\text{--}0.19. \quad (21)$$

The obtained  $\phi$ -meson mass and width are quite close to their experimental values.<sup>6</sup> The coupling-constant ratios  $g_\phi$  and  $g_{\phi'}$  are also in agreement with the values extracted from data using generalized VDM kaon form-factor model of Dubnička.<sup>12</sup> The predicted mass and width of the excited state are somewhat smaller than the values  $1680 \pm 50$  MeV and  $150 \pm 50$  MeV of the Particle Data Group.<sup>6</sup>

### IV. SUMMARY AND CONCLUSIONS

The purpose of the present work was to generalize the analysis of the genuine QCD sum rules of SVZ for the  $\rho$  and  $\phi$  mesons in such a manner that the existence of one radial excitation in each channel could be investigated. Note that SVZ in their original work<sup>1</sup> consider a possibility to extract more information about the rich resonance

structure of the individual cross sections to be realistic and desirable. Our analysis was based on the suitable smooth parametrizations of the corresponding spectral functions and on the numerical procedure used to match the two sum rules simultaneously in relatively wide intervals of the sum-rule variable  $M$ . It has turned out that these intervals can be chosen in such a way that the SR's are rather sensitive to the presence of additional poles in

the phenomenological spectral functions. Consequently, in addition to the ground state the properties of one excited state also could be determined. In both cases they are rather close to the experimentally measured characteristics. Our conclusion is therefore that it is possible to extend the applicability of the Borel QCD sum rules also for the investigation of the radial excitations of the light vector mesons.

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