

Vacuum saturation for $\Delta I = \frac{3}{2}$ components of nonleptonic s -wave weak decays

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The hadronic vacuum-saturation prescription predicts $\Delta I = \frac{3}{2}$ $K \rightarrow 2\pi$, $\Lambda \rightarrow N\pi$, $\Sigma \rightarrow N\pi$, $\Xi \rightarrow \Lambda\pi$, and $\Omega \rightarrow \Xi\pi$ s -wave amplitudes that are compatible with experiment.

It is well appreciated that (Cabibbo-suppressed) $\Delta S = 1$ weak nonleptonic decays are $\Delta I = \frac{1}{2}$ enhanced. While this $\Delta I = \frac{1}{2}$ rule is important to understand, the much smaller $\Delta I = \frac{3}{2}$ transitions should also give us clues about the underlying quark weak dynamics. Empirically these smaller $\Delta I = \frac{3}{2}$ amplitudes cause deviations from the $\Delta I = \frac{1}{2}$ branching ratios which are of the order of 10% for kaon $K \rightarrow \pi\pi$ and hyperon $B \rightarrow B'\pi$ s -wave decays, but 40% for $\Omega \rightarrow \Xi\pi$ weak decays.

In this paper we show that the usual "vacuum-saturation" prescription for hadron intermediate states (or equivalently the quark spectator model) correctly predicts the small $\Delta I = \frac{3}{2}$ components of $K \rightarrow \pi\pi$, $\Lambda \rightarrow N\pi$, $\Sigma \rightarrow N\pi$, $\Xi \rightarrow \Lambda\pi$ s -wave decays (both in magnitude and sign) and also predicts the much larger $\Delta I = \frac{3}{2}$ part of $\Omega \rightarrow \Xi\pi$ decays. Our $\Delta S = 1$ weak Hamiltonian density will always be of the hadronic current-current $V - A$ Cabibbo form [1]

$$H_w = (G_F/2\sqrt{2})(J_\mu^\dagger J^\mu + J^\mu J_\mu^\dagger), \quad (1a)$$

$$J_\mu = c_1(V - A)_\mu^{1-i2} + s_1(V - A)_\mu^{4-i5}, \quad (1b)$$

where s_1, c_1 are the sine and cosine of the Cabibbo angle $\theta_C \approx 13.1^\circ$.

We begin with the pure $\Delta I = \frac{3}{2}$ $K^+ \rightarrow \pi^+\pi^0$ decay. Inserting the intermediate vacuum states $|0\rangle\langle 0|$ in between the JJ currents of H_w in (1a), gives the vacuum-saturation (VS) amplitude (sometimes called the factorization model)

$$\langle \pi^+\pi^0 | H_w^{\text{PV}} | K^+ \rangle_{\text{VS}} = (G_F/2\sqrt{2})s_1c_1 \langle \pi^+ | -A^{1+i2} | 0 \rangle \times \langle \pi^0 | V^{4-i5} | K^+ \rangle \quad (2a)$$

$$\approx -i(G_F/2\sqrt{2})s_1c_1f_\pi(m_K^2 - m_\pi^2) \approx -i1.9 \times 10^{-8} \text{ GeV}. \quad (2b)$$

Here $\langle P^i | A_\mu^j | 0 \rangle = -if_P\delta^{ij}p_\mu$ and $\langle P^f | V_\mu^j | P^i \rangle = if^{fji}(p^f + p^i)_\mu$ (ignoring $O(\epsilon^2)$ Ademollo-Gatto corrections [2] in the latter) along with $f_\pi \approx 93$ MeV are used to find (2b). Note that in (2a) only the $J^\dagger J$ part of H_w enters. On the other hand, the JJ^\dagger part would play a role if one were to add a $|K^+\pi^+\rangle\langle K^+\pi^+|$ intermediate-state contribution (although technically this is no longer "vacuum" saturation), which would double the numerical answer in (2b) since

$$\begin{aligned} & \langle \pi^+\pi^0 | V^{4-i5} | K^+\pi^+ \rangle \langle K^+\pi^+ | A^{1+i2} | K^+ \rangle \\ & = \langle \pi^0 | V^{4-i5} | K^+ \rangle \langle \pi^+ | A^{1+i2} | 0 \rangle. \end{aligned} \quad (3)$$

However, the above "disconnected" amplitudes (disc, adopting the nomenclature of Ref. [3]) must be supplemented by "connected" amplitudes. Similar to Ref. [3], we find the latter by the usual current-algebra soft-pion method coupled with the chiral-symmetry relation $[Q_S, H_w] = -[Q, H_w]$, leading to

$$\langle \pi^+\pi^0 | H_w | K^+ \rangle_{\text{conn}} \approx (i/2f_\pi) \langle \pi^+ | H_w | K^+ \rangle, \quad (4)$$

where only the π^0 momentum is taken soft. If the vacuum and $K^+\pi^+$ intermediate states (the latter simply doubling the vacuum contribution) are inserted in the reduced amplitude $\langle \pi^+ | H_w | K^+ \rangle$ of (4), one finds

$$\begin{aligned} & \langle \pi^+ | H_w | K^+ \rangle_{W \text{ pole}} \\ & \approx (G_F/\sqrt{2})s_1c_1 \langle \pi^+ | A^{1+i2} | 0 \rangle \langle 0 | A^{4-i5} | K^+ \rangle \\ & \approx (G_F/\sqrt{2})s_1c_1f_\pi f_K m_K^2, \end{aligned} \quad (5)$$

corresponding to a W^+ pole interacting with π^+ and K^+ axial-vector currents. Here $2p_\pi p_K = p_\pi^2 + p_K^2 \approx m_K^2$ using momentum conservation for the overall $K^+ \rightarrow \pi^+\pi^0$ transition of (4) with $p_{\pi^0}^2 = 0$. Then substituting (5) into (4) gives [4]

$$\langle \pi^+\pi^0 | H_w | K^+ \rangle_{\text{conn}} \approx i(G_F/2\sqrt{2})s_1c_1f_K m_K^2, \quad (6)$$

which is roughly the same as (2b) except for a sign change and $f_\pi \rightarrow f_K$. Therefore, the connected amplitude (6) essentially cancels against the nonvacuum intermediate-state contribution of $|K^+\pi^+\rangle\langle K^+\pi^+|$ in (3), leaving only the VS amplitude (2b) which, incidentally, is very close to experiment [5,6]:

$$|\langle \pi^+\pi^0 | H_w | K^+ \rangle|_{\text{expt}} = (1.834 \pm 0.007) \times 10^{-8} \text{ GeV}. \quad (7)$$

With hindsight, combining the hadronic current-current H_w and the notion of crossing with disconnected and connected saturation states gives the net amplitude

$$M = M_{\text{disc}} + M_{\text{conn}} \approx 2M_{\text{VS}} - M_{\text{VS}} = M_{\text{VS}}, \quad (8)$$

where M_{VS} corresponds to our VS amplitude in (2). The latter in fact is compatible with experiment for $K^+ \rightarrow \pi^+\pi^0$ decay. Note that this VS hadronic current-current approach to the $\Delta I = \frac{3}{2}$ components of H_w is in-

dependent of the details of the underlying (and more complex) quark model. A crossing-type approach to the quark model might include a Fierz reshuffling of the quark fields in different hadrons, ignoring the complications of confinement. We shall not do this, but instead consider only the consequences of vacuum saturation for the hadronic current-current H_w of Eq. (1).

In order to show that our $\Delta I = \frac{3}{2}$ VS prescription (2) or (8) for $K^+ \rightarrow \pi^+ \pi^0$ is quite general, we first extend it to the $\Delta I = \frac{3}{2}$ part of the $K_S \rightarrow \pi^+ \pi^-, \pi^0 \pi^0$ decays. In contrast with the pure $\Delta I = \frac{3}{2}$ transition $K^+ \rightarrow \pi^+ \pi^0$, the $K_S \rightarrow \pi\pi$ decays have both $\Delta I = \frac{1}{2}, \frac{3}{2}$ contributions, which are coupled through final-state interactions:

$$M_{+-} = \langle \pi^+ \pi^- | H_w | K_S \rangle = a_{1/2} e^{i\delta_0} + \frac{2}{3} a_{3/2} e^{i\delta_2}, \quad (9a)$$

$$M_{00} = \langle \pi^0 \pi^0 | H_w | K_S \rangle = a_{1/2} e^{i\delta_0} - \frac{4}{3} a_{3/2} e^{i\delta_2}. \quad (9b)$$

The experimental magnitudes [5] $|M_{+-}| = (39.11 \pm 0.09) \times 10^{-8}$ GeV and $|M_{00}| = (37.14 \pm 0.17) \times 10^{-8}$ GeV along with the experimental phase shifts [7] $\delta_0 - \delta_2 \approx 55^\circ$ allow one to solve Eqs. (9) for $a_{1/2}$ and $a_{3/2}$. The resulting two quadratic equations lead to the $\Delta I = \frac{1}{2}, \frac{3}{2}$ amplitudes

$$\begin{aligned} a_{1/2} &= (38.5 \pm 0.1) \times 10^{-8} \text{ GeV}, \\ a_{3/2} &= (1.75 \pm 0.18) \times 10^{-8} \text{ GeV}, \end{aligned} \quad (10)$$

where the relative sign $a_{3/2}/a_{1/2} > 0$ is required by (9).

In order to compare (10) with the relatively real VS amplitudes and to remove normalization effects, we define the real analogues of (9): $a_{+-} = a_{1/2} + \frac{2}{3} a_{3/2}$ and $a_{00} = a_{1/2} - \frac{4}{3} a_{3/2}$. For the definition of H_w in (1) only the $K_S \rightarrow \pi^+ \pi^-$ decay will contribute to the $\Delta I = \frac{3}{2}$ VS amplitude, giving

$$\begin{aligned} \langle \pi^+ \pi^- | K_w | K_S \rangle_{\text{VS}} &= -i(G_F/\sqrt{2})s_1 c_1 f_\pi (m_K^2 - m_\pi^2) \\ &\approx -i3.9 \times 10^{-8} \text{ GeV}. \end{aligned} \quad (11)$$

The VS prescription in (11) is the same as used in (2). This VS result compares well with the experimental amplitude found from (10) based only on $K_S \rightarrow \pi\pi$ decay data when the dominant $a_{1/2}$ part is subtracted out:

$$a_{+-} - a_{00} = 2a_{3/2} = (3.5 \pm 0.4) \times 10^{-8} \text{ GeV}. \quad (12)$$

Additionally, $a_{3/2}$ in (10), found only from $K_S \rightarrow \pi\pi$ decay amplitudes, is close to what one finds from $K^+ \rightarrow \pi^+ \pi^0$ in (7) since $a_{3/2} = |M_{+0}|$.

With the good agreement between (2b) and (7), as well as between (11) and (12), we move on to the baryon decays. We test our VS prescription here for the s -wave parity-violating (PV) component amplitude A of baryon decays $B \rightarrow B'\pi$ defined from the transition $\langle \pi B' | H_w | B \rangle = \bar{u}_B (iA + \gamma_5 B) u_B$. With the states normalized covariantly ($\bar{u}u = 2m_B$), the most recent data compilation [5] gives the s -wave amplitudes listed in Table I, where, e.g., $\Lambda^0 \rightarrow p\pi^-$ is denoted as Λ_-^0 . This table has only slight alterations from the Particle Data Group (PDG) results of 17 years ago [8]. It has long been realized that these amplitudes are dominated by an H_w

TABLE I. Experimental [5] s -wave $B \rightarrow B'\pi$ decay amplitudes.

Weak transitions		$10^6 A$
(Λ_-^0)	$\Lambda \rightarrow p\pi^-$	0.323 ± 0.002
(Λ_0^0)	$\Lambda \rightarrow n\pi^0$	-0.237 ± 0.003
(Σ_0^+)	$\Sigma^+ \rightarrow p\pi^0$	-0.326 ± 0.011
(Σ_+^+)	$\Sigma^+ \rightarrow n\pi^+$	0.014 ± 0.003
(Σ^-)	$\Sigma^- \rightarrow n\pi^-$	0.427 ± 0.002
(Ξ^-)	$\Xi^- \rightarrow \Lambda\pi^-$	-0.450 ± 0.002
(Ξ_0^0)	$\Xi^0 \rightarrow \Lambda\pi^0$	0.344 ± 0.006

transforming as $\Delta I = \frac{1}{2}$, with the small observed $\Delta I = \frac{3}{2}$ s -wave components given by the combinations of the A 's which filter out the much larger $\Delta I = \frac{1}{2}$ parts:

$$A(\Lambda_-^0) + \sqrt{2}A(\Lambda_0^0) = (-0.012 \pm 0.005) \times 10^{-6}, \quad (13a)$$

$$A(\Sigma_+^+) - A(\Sigma^-) - \sqrt{2}A(\Sigma_0^+) = (0.048 \pm 0.016) \times 10^{-6}, \quad (13b)$$

$$A(\Xi^-) + \sqrt{2}A(\Xi_0^0) = (0.036 \pm 0.009) \times 10^{-6}. \quad (13c)$$

We suggest that these $\sim 4\%$ $\Delta I = \frac{3}{2}$ amplitudes are completely governed by the vacuum-saturated (VS) part of the current-current Hamiltonian density (1). Since the neutral pion decays $\Lambda \rightarrow n\pi^0$, $\Sigma^+ \rightarrow p\pi^0$, $\Xi^0 \rightarrow \Lambda\pi^0$ and even the charged pion decay $\Sigma^+ \rightarrow n\pi^+$ have no VS components, for H_w defined in (1), we need only consider the vacuum saturation of the π^- amplitudes in (13). The s -wave PV VS amplitudes of interest are

$$\begin{aligned} \langle \pi^- p | H_w^{PV} | \Lambda \rangle_{\text{VS}} &= (G_F/2\sqrt{2})s_1 c_1 \langle \pi^- | -A^{1-i2} | 0 \rangle \langle p | V^{4+i5} | \Lambda \rangle \\ &= -i(G_F/2)(\frac{3}{2})^{1/2} s_1 c_1 f_\pi (m_\Lambda - m_p) \bar{u}_p u_\Lambda, \end{aligned} \quad (14a)$$

$$= -i(G_F/2)(\frac{3}{2})^{1/2} s_1 c_1 f_\pi (m_\Lambda - m_p) \bar{u}_p u_\Lambda, \quad (14b)$$

as well as

$$\langle \pi^- n | H_w^{PV} | \Sigma^- \rangle_{\text{VS}} = -i(G_F/2)s_1 c_1 f_\pi (m_\Sigma - m_n) \bar{u}_n u_\Sigma, \quad (15a)$$

$$\langle \pi^- \Lambda | H_w^{PV} | \Xi^- \rangle_{\text{VS}} = i(G_F/2)\sqrt{\frac{3}{2}}s_1 c_1 f_\pi (m_\Xi - m_\Lambda) \bar{u}_\Lambda u_\Xi \quad (15b)$$

where we use Cartesian phases of the baryon states in (14) and (15) as well as in Table I. Then the s -wave VS amplitudes of (14) and (15) generate the $\Delta I = \frac{3}{2}$ amplitude combinations

$$\begin{aligned} A(\Lambda_-^0) + \sqrt{2}A(\Lambda_0^0) &= -(G_F/2)(\frac{3}{2})^{1/2} s_1 c_1 f_\pi (m_\Lambda - m_p) \\ &\approx -0.026 \times 10^{-6}, \end{aligned} \quad (16a)$$

$$\begin{aligned} A(\Sigma_+^+) - A(\Sigma^-) - \sqrt{2}A(\Sigma_0^+) &= (G_F/2)s_1 c_1 f_\pi (m_\Sigma - m_n) \\ &\approx 0.031 \times 10^{-6}, \end{aligned} \quad (16b)$$

$$\begin{aligned} A(\Xi^-) + \sqrt{2}A(\Xi_0^0) &= (G_F/2)(\frac{3}{2})^{1/2} s_1 c_1 f_\pi (m_\Xi - m_\Lambda) \\ &\approx 0.030 \times 10^{-6}. \end{aligned} \quad (16c)$$

We find it significant that these three VS predictions (16) are reasonably close to the experimental values (13) both in magnitude and in sign [9].

Finally turning to decuplet $\Omega \rightarrow \Xi\pi$ decays, we note that the observed branching ratio [5] $B(\Omega^- \rightarrow \Xi^0\pi^- / \Xi^-\pi^0) = (2.74 \pm 0.15)$ is significantly removed from the $\Delta I = \frac{1}{2}$ value $B=2$. Nevertheless, as we now demonstrate, our $\Delta I = \frac{3}{2}$ vacuum-saturation scheme continues to hold even for these weak Ω decays. Decomposing these amplitudes into the parity-conserving (PC) p -wave and (PV) d -wave parts, we write $\langle \Xi\pi | H_w | \Omega \rangle = \bar{u}_\Xi (E + i\gamma_5 F) p_\mu^\pi u_\Omega^\mu$, where u^μ is a Rarita-Schwinger bispinor representing the spin- $\frac{3}{2}$ Ω baryon satisfying $p^\Omega u(\Omega) = 0$. Since trace kinematics for the square of $\langle \Xi\pi | H_w | \Omega \rangle$ suppresses the d -wave F part, one can assume [10] $\langle \Xi\pi | H_w | \Omega \rangle \approx E \bar{u}_\Xi p_\mu u^\mu$, so that the rate simplifies to

$$\Gamma_{\Omega\Xi\pi} \approx (p^3 / 24\pi m_\Omega^2) |E|^2 (m_\Omega + m_\Xi)^2, \quad (17)$$

with momentum $p \approx 294$ MeV. For the lifetime [5] $\tau_\Omega = (0.822 \pm 0.012) \times 10^{-10}$ sec, and branching ratios $(23.6 \pm 0.7)\%$ and $(8.6 \pm 0.4)\%$, Eq. (17) translates respectively to the amplitudes $|E(\Omega^- \rightarrow \Xi^0\pi^-)|_{\text{expt}} = (1.33 \pm 0.02) \times 10^{-6} \text{ GeV}^{-1}$ and $|E(\Omega^- \rightarrow \Xi^-\pi^0)|_{\text{expt}} = (0.80 \pm 0.02) \times 10^{-6} \text{ GeV}^{-1}$. The pure $\Delta I = \frac{3}{2}$ part of the experimental PC amplitudes is then

$$\begin{aligned} |E(\Omega^- \rightarrow \Xi^0\pi^-) - \sqrt{2}E(\Omega^- \rightarrow \Xi^-\pi^0)|_{\text{expt}} \\ = (0.20 \pm 0.03) \times 10^{-6} \text{ GeV}^{-1}. \end{aligned} \quad (18)$$

As with the $B \rightarrow B'\pi$ decays, only the π^- decay (but not the π^0 decay) has a vacuum-saturated component, but now it is due to H_w^{PC} . Then the VS analogue of (14) and (15) has a magnitude [11]

$$\begin{aligned} |\langle \pi^- \Xi^0 | H_w^{\text{PC}} | \Omega^- \rangle|_{\text{VS}} \\ = (G_F / 2\sqrt{2}) s_1 c_1 |\langle \pi^- | A^{1-i2} | 0 \rangle \langle \Xi^0 | A^{4+i5} | \Omega^- \rangle| \end{aligned} \quad (19a)$$

$$= (G_F / \sqrt{2}) s_1 c_1 f_\pi \bar{u}_\Xi p_\mu u_\Omega^\mu. \quad (19b)$$

Finally this VS prescription (19) leads to the $\Delta I = \frac{3}{2}, \Omega \rightarrow \Xi\pi$ prediction

$$\begin{aligned} |E(\Omega^- \rightarrow \Xi^0\pi^-) - \sqrt{2}E(\Omega^- \rightarrow \Xi^-\pi^0)|_{\text{VS}} \\ = (G_F / \sqrt{2}) s_1 c_1 f_\pi \approx 0.17 \times 10^{-6} \text{ GeV}^{-1}. \end{aligned} \quad (20)$$

We believe it significant that the $\Omega \rightarrow \Xi\pi, \Delta I = \frac{3}{2}$, VS prediction (20) is also close to experiment (18). Thus the same VS procedure applied to kaon $K \rightarrow 2\pi$ and hyperon s -wave $\Lambda \rightarrow N\pi, \Sigma \rightarrow N\pi, \Xi \rightarrow \Lambda\pi$, decays giving the measured 4% $\Delta I = \frac{3}{2}$ amplitudes (both in magnitude and sign) also predicts the larger 15% observed $\Delta I = \frac{3}{2}$ $\Omega \rightarrow \Xi\pi$ amplitude (20). Such a VS procedure as used in this paper for $\Delta I = \frac{3}{2}$ weak transitions can also be extended to charm-changing decays [12].

It might appear that we have ignored strong-interaction QCD dynamics when obtaining our $\Delta I = \frac{3}{2}$ VS weak interaction results (2), (11), (16), and (20). This is not really the case because we have always dealt with vector and axial-vector currents. Such currents are ‘‘good’’ operators in the Gell-Mann sense of always satisfying $\langle 0 | A_\mu^3 | \pi^0 \rangle = i f_\pi q_\mu$ and, e.g., the nonrenormalization limit $\langle n | V_\mu^{4+i5} | \Sigma^- \rangle = -(p_\Sigma^2 + p^n) \bar{u}_n u_\Sigma$. Detailed QCD dynamics will only tell us how the decay constant f_π is nonperturbatively generated. If this hadronic VS current pattern is broken by Fierz reshuffling of (confined) QCD quark fields, then π^0 transitions via color suppression could also contribute to VS amplitudes. But such terms could spoil the good agreement with experiment now found in (2), (11), (16), and (20).

Our attitude here toward the quark model is that the above VS hadron pattern in (2), (11), (16), and (20) suggests standard quark spectator graphs (which are the analogues of the hadron VS procedure) generate this constant background $\Delta I = \frac{3}{2}$ tree-level ‘‘noise’’ along with small $\Delta I = \frac{1}{2}$ components in the weak Hamiltonian H_w . Normalized to this small VS-quark spectator noise, however, the much larger $\Delta I = \frac{1}{2}$ matrix elements of $H_w^{1/2}$ in the overall nonleptonic $\Delta S = 1$ weak Hamiltonian density [13]

$$H_w = (G_w / 2\sqrt{2}) (J_\mu^+ J^\mu + J^\mu J_\mu^+) + H_w^{1/2} \quad (21)$$

are where detailed quark-model dependence arises. However, there is no consensus at the present time as to the origin of this $\Delta I = \frac{1}{2}$ enhancement for $K_S \rightarrow 2\pi, B \rightarrow B'\pi$, or $\Omega \rightarrow \Xi\pi$ decays [14]. In any case, from the VS studies in this paper we conclude that the $\Delta I = \frac{3}{2}$ part of the nonleptonic weak Hamiltonian is universal and simple, whereas the $\Delta I = \frac{1}{2}$ part is (quark) model dependent and complicated.

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