Spontaneous-compactification effects on SO(10) grand unification with $SU(2)_L \times SU(2)_R \times SU(4)_C$ intermediate symmetry

P. K. Patra* and M. K. Parida
Physics, Department, North-Eastern Hill University, Bijni Complex, Laitumkhrah, Shillong 793 003, India
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The impact of five-dimensional operators, which might originate from compactification of extra dimensions, is investigated on SO(10) grand unification with Pati-Salam (PS) symmetry as the intermediate gauge group. When the PS group is left-right symmetric, the resulting equation for $\sin^2\theta_W$ is noted to be independent of the parameter (ϵ) of the nonrenormalizable Lagrangian, although the unification mass (M_U) does depend upon it. In addition to solutions of the type obtained by Shafi and Wetterich, we find new predictions with a much larger grand-unification mass, consistent with a larger compactification scale. When parity and $SU(2)_R$ breaking are decoupled, the equation for $\ln(M_U/M_W)$ is independent of ϵ, but $\sin^2\theta_W$ does depend upon it. The most interesting predictions include observable n-$\bar{n}$ oscillations, rare kaon decays, and small neutrino masses, that are, however, measurable in the laboratory for ν_μ and ν_τ, corresponding to the low intermediate scale $M_C \sim 10^5$–10^6 GeV. In such cases M_U is large and the solutions are consistent with a larger compactification scale.

I. INTRODUCTION

Kaluza-Klein-type unification of other basic forces with gravitation has received considerable attention over the past years [1]. Originally proposed to unify gravity and electromagnetism, the Kaluza-Klein framework with several modifications has been extended to encompass the standard and grand-unification symmetries including supergravity and superstrings. Methods have been devised to stabilize the vacuum corresponding to the observed four-dimensional Universe obtained after the compactification of extra dimensions. Several attractive modifications of the model have been incorporated to compute the coupling constants from extra dimensions and also to generate the chiral fermions occurring as different generations. In theories employing spontaneous compactification, nonrenormalizable higher-dimensional operators involving gauge and Higgs fields, but scaled by suitable powers of the compactification mass (M_G), occur very frequently [2,3]. Without invoking the idea of dimensional reduction such operators scaled by the Planck mass ($M_{\rm Pl}=10^{19}$ GeV) can also be generated by quantum gravity as correction to the renormalizable Lagrangian [4]. Shafi and Wetterich (SW) [3] and Hill [4] have noted that the presence of such five-dimensional operators could make drastic modifications to the usual predictions of grand unified theories (GUT's). While Hill [4] has noted that the origin of such operators, scaled by $M_{\rm Pl}=10^{19}$ GeV, could be due to the effects of quantum gravity, SW have emphasized such operators to be originating from the compactification of extra dimensions with the compactification mass (M_G) 1–2 orders lower than $M_{\rm Pl}$. A brief review of recent applications in GUT's has been reported in Ref. [5], where it has been shown that the presence of such an operator permits the single intermediate symmetry $SU(2)_L\times U(1)_R\times SU(4)_C$ to survive down to a scale as low as $M_C\sim 10^5$ GeV, thus allowing rare kaon decays to be observable by low-energy experiments. Such predictions are accompanied by large proton lifetimes and small neutrino masses. In the context of the minimal SU(5) model it has also been observed that the presence of five- and six-dimensional operators can make the proton extremely stable [6]. At first SW [3] estimated the modifications of the SO(10) predictions with a single Pati-Salam (PS) intermediate symmetry [7] $SU(2)_L\times SU(2)_R\times SU(4)_C$ $(g_{2L}=g_{2R})(\equiv G_{224P})$ where both parity (P) and $SU(2)_R$ break at the intermediate scale M_C:

$$\mathrm{SO}(10)\underset{M_U}{\overset{54}{\longrightarrow}}G_{224P}\underset{M_C}{\overset{126}{\longrightarrow}}SU(2)_L\times U(1)_Y\times SU(3)_C \underset{M_W}{\overset{10}{\longrightarrow}}U(1)_{\rm em}\times SU(3)_C\ . \quad (1)$$

They noted that solutions with a compactification scale $M_G\sim 10^{17}$ GeV allow a proton lifetime enhancement by a factor 10–100 compared to the existing lower limit for $M_C\simeq 10^{13}$ GeV, and $\sin^2\theta_W\simeq 0.22$.

In this paper we note that the equation for $\sin^2\theta_W$ in Eq. (1) is independent of the parameter (ϵ) of the nonrenormalization term although $\ln(M_U/M_W)$, and α_G ($=g_0^2/4\pi$, g_0=bare-GUT-coupling constant) do depend upon it. In addition to solutions of the SW type [3], other solutions, corresponding to much larger unification mass and τ_p with $M_G\sim M_{\rm Pl}$ are also allowed.

As the primary objective of this paper we then show that, when compactification effects through the five-dimensional operator are included in the new SO(10) model with separate P- and $SU(2)_R$-breaking scales [8], an intermediate scale as low as $M_C\sim 10^5$–10^6 GeV, is possible in the scenario:

$$\mathrm{SO}(10)\underset{M_U}{\overset{210}{\longrightarrow}}\mathrm{SU}(2)_L\times\mathrm{SU}(2)_R\times\mathrm{SU}(4)_C$$
$$\times(g_{2L}\neq g_{2R})(\equiv G_{224})$$
$$\underset{M_C}{\overset{126}{\longrightarrow}}\mathrm{SU}(2)_L\times\mathrm{U}(1)_Y\times\mathrm{SU}(3)_C$$
$$\underset{M_W}{\overset{10}{\longrightarrow}}\mathrm{U}(1)_{\mathrm{em}}\times\mathrm{SU}(3)_C \ . \tag{2}$$

The model does not possess the well-known domain-wall problem [9] and provides the minimal SO(10) scenario obtained so far for observable n-$\bar{n}$ oscillations, rare kaon decays, and small neutrino masses, but proton decay far beyond the observable limit. The other solutions permitted in this model correspond to larger values of M_C and smaller unification mass. In particular the SW-type modification of M_U, τ_p, and $\sin^2\theta_W$ is allowed for $M_C \sim 10^{11}$–10^{12} GeV and $M_G \sim 10^{17}$ GeV. For such solutions, although proton decay is predicted to be observable in foreseeable future, no other low-energy signature of quark-lepton unification seems to be possible. In this case the equations for $\ln(M_U/M_W)$ and α_G are noted to be independent of the corresponding parameter (ϵ), but $\sin^2\theta_W$ does depend upon it.

The paper is organized in the following manner. In Sec. II we derive modifications of the gauge coupling constants at the GUT scale for Eq. (2) due to suitable five-dimensional operator and mention those obtained by SW [3] for Eq. (1). In Sec. III we show how the formulas for $\sin^2\theta_W$, $\ln(M_U/M_W)$, and α_G^{-1} obtained as solutions to the renormalization-group equations (RGE's), depend upon the parameter (ϵ) of the nonrenormalizable Lagrangian. In Sec. IV we present numerical solutions and point out the implications of the GUT predictions on low-energy experiments. In Sec. V, the results of this paper are briefly discussed and conclusions are stated.

II. MODIFICATION OF GAUGE COUPLING CONSTANTS AND GUT BOUNDARY CONDITIONS

In this section we mention the modification of the gauge coupling constants and the boundary conditions at the GUT scale as derived by SW [3] in Eq. (1). We also derive the corresponding new modifications for Eq. (2) in the presence of a five-dimensional operator. In all cases the GUT boundary condition is expressed in a generalized form

$$\alpha_{2L}(M_U)(1+\epsilon_{2L})=\alpha_{2R}(M_U)(1+\epsilon_{2R})$$
$$=\alpha_{4C}(M_U)(1+\epsilon_{4C})=\alpha_G \ , \tag{3}$$

where $\alpha_i(\mu)=g_i^2(\mu)/4\pi$, g_i being the gauge coupling constant for the gauge group G_i in G_{224} with $i=2L, 2R, 4C$. The parameter ϵ_i is related to the nonrenormalizable SO(10) Lagrangian ($\mathcal{L}_{NR}$) as will be defined shortly in the respective cases. After GUT symmetry is broken spontaneously in all such cases $\mathcal{L}_{NR}$ is absorbed in the renormalizable kinetic-energy term ($\mathcal{L}_R$) of the gauge fields having the residual gauge symmetry when the gauge fields are suitably rescaled and coupling constants modified [3,4].

A. Modifications with G_{224P} intermediate symmetry

SW [3] considered the five-dimensional operator, appropriate for Eq. (1), occurring in the SO(10)-invariant nonrenormalizable Lagrangian

$$\mathcal{L}_{NR}=-\frac{\eta}{2M_G}\mathrm{Tr}(F_{\mu\nu}\Phi_{(54)}F^{\mu\nu}) \ , \tag{4}$$

where M_G is the compactification scale. Using the vacuum expectation value (VEV) as

$$\langle\Phi_{(54)}\rangle=\frac{\phi_0}{\sqrt{30}}\mathrm{diag}(1,1,1,1,1,1,-\tfrac{3}{2},-\tfrac{3}{2},-\tfrac{3}{2},-\tfrac{3}{2}) \ , \tag{5}$$

necessary for spontaneous symmetry breaking (SSB) at the first stage of the chain, they obtained the ϵ_i parameters occurring in Eq. (3):

$$\epsilon_{2L}=\epsilon_{2R}=-\tfrac{3}{2}\epsilon, \quad \epsilon_{4C}=\epsilon \ ,$$
$$\epsilon=\frac{1}{\sqrt{30}}\frac{\eta\phi_0}{M_G} \ . \tag{6}$$

Thus, SW [3] demonstrated that, as in the case of SU(5), the five-dimensional operator contributes to the modification of the GUT boundary conditions.

B. Modifications with G_{224} intermediate symmetry

We now derive the modifications of the GUT boundary conditions for Eq. (2) where the four-index antisymmetric tensor **210** drives SSB at the first stage. We follow the convention [10,11] in which $i,j=1,2,\ldots,6(7,8,9,10)$ denote the SO(6) [SO(4)] indices, and use the representation of generators by 16×16 matrices [10]. Using $\Gamma_i, i=1,2,\ldots,10$, as the matrices defined in Ref. [10], the 45 generators are given by $\frac{1}{2}\sigma^{ij}=(1/4i)[\Gamma_i,\Gamma_j]$, $i,j=1,2,\ldots,10$. Representing the 45 gauge bosons by the two-index antisymmetric tensor $W_\mu^{i,j}$, the gauge-boson matrix is 16×16,

$$W_\mu=\tfrac{1}{4}\sum_{i,j=1}^{10}\sigma^{ij}W_\mu^{ij} \ , \tag{7}$$

where every gauge boson occurs repeatedly in more than one matrix element. This is in contrast with the SU(N)-gauge-boson matrix where every gauge boson occurs in only one matrix element of the corresponding $N\times N$ matrix. Following the usual definition

$$F_{\mu\nu}=\partial_\mu W_\nu-\partial_\nu W_\mu-ig[W_\mu,W_\nu] \ , \tag{8}$$

with W_μ given by Eq. (7) and the expression for the kinetic energy,

$$\mathcal{L}_R=-\tfrac{1}{4}\sum_{m=1}^{45}(V_{\mu\nu}^{(m)}V^{(m)\mu\nu}) \ ,$$
$$V_{\mu\nu}^{(m)}=\partial_\mu V_\nu^{(m)}-\partial_\nu V_\mu^{(m)}-ig[V_\mu^{(m)},V_\nu^{(m)}] \ , \tag{9}$$

where $V_\mu^{(m)}$ ($m=1,2,\ldots,45$) represents 45 components of W_μ^{ij}, the corresponding expression in terms of $F_{\mu\nu}$ that reduces to (9) in the case of SO(10) is [12]

$$\mathcal{L}_R=-\tfrac{1}{8}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) \ . \tag{10}$$

To the renormalizable Lagrangian given in (10) we add the nonrenormalizable term containing the five-dimensional operator

$$\mathcal{L}_{NR}=-\frac{\eta}{8M_G}\mathrm{Tr}(F_{\mu\nu}\Phi_{(210)}F^{\mu\nu}) \ , \tag{11}$$

where

$$\Phi_{(210)}=\tfrac{1}{24}\sum_{ijkl}\Gamma_i\Gamma_j\Gamma_k\Gamma_l\phi^{ijkl}, \quad i,j,k,l=1,2,\ldots,10 \ .$$

Wetterich [12] has shown how light generations of fermions can be obtained from a six-dimensional SO(12) gauge theory which might originate from pure gravity in eighteen dimensions coupled to Majorana-Weyl spinors. In this theory the Higgs representations **210**, **126**, and **10** of SO(10) necessary for spontaneous symmetry breaking at different stages of the chain in Eq. (2) have been demonstrated to emerge from suitable SO(12) representations possessing a nonvanishing coupling to the spinors [12]. With SO(10) gauge symmetry preserved after compactification of extra dimensions at a scale M_G, the five-dimensional operator in Eq. (11) is expected to occur as a nonrenormalizable term in the Lagrangian. When

$$\mathrm{SO}(10)\underset{M_U}{\rightarrow}G_{224} \ ,$$

with P broken at $\mu\sim M_U$, ϕ^{ijkl} assumes a VEV in the direction $\langle\phi^{78910}\rangle=\phi_0\neq0$. Using the normalized VEV

$$\begin{aligned}\langle\phi_{(210)}\rangle&=\frac{\phi_0}{\sqrt{32}}\Gamma_7\Gamma_8\Gamma_9\Gamma_{10}\langle\phi^{78910}\rangle\\&=\frac{\phi_0}{\sqrt{32}}\mathrm{diag}(-1,-1,-1,-1,-1,-1,-1,-1,1,1,1,1,1,1,1,1)\end{aligned} \tag{12}$$

in (11) we compute the ϵ_i parameters occurring in Eq. (3),

$$\begin{aligned}&\epsilon_{2L}=-\epsilon_{2R}=\epsilon, \quad \epsilon_{4C}=0 \ ,\\&\epsilon=\frac{\eta\phi_0}{4\sqrt{2}M_G}=\frac{\eta}{8}\left[\frac{3}{2\pi\alpha_G}\right]^{1/2}\frac{M_U}{M_G} \ ,\end{aligned} \tag{13}$$

where we have used the relation between the superheavy-gauge-boson masses (M_U) and $\phi_0, M_U=(4\pi\alpha_G/3)^{1/2}\phi_0$. Compared to Eq. (1), we note that in Eq. (2), ϵ_{2R} and ϵ_{2L} are opposite in sign, and the $\mathrm{SU}(4)_C$ coupling at the GUT scale does not receive any modification due to $\mathcal{L}_{NR}$. When ϵ_i parameters, given by Eqs. (6) and (13), are used in Eq. (3) it is clear that the GUT boundary conditions in the two cases are significantly different. Such a difference in the boundary conditions reflects in the resulting solutions of the RGE's as demonstrated in Secs. III and IV.

III. FORMULAS FOR ELECTROWEAK-MIXING ANGLE, UNIFICATION MASS, AND GUT COUPLING

In this section, at first we obtain analytic expressions for the electroweak mixing ($\sin^2\theta_W$), unification mass $[\ln(M_U/M_W)]$, and the bare-GUT-coupling constant ($\alpha_G=g_0^2/4\pi$) as solutions to the renormalization-group equations (RGE's) for the gauge coupling constants under the generalized boundary conditions given by Eq. (3). Using the boundary conditions for Eqs. (1) and (2), specified by SW and in Sec. II, we then show how certain formulas are independent of ϵ.

The gauge coupling constants $g_i(\mu),[\alpha_i(\mu)=g_i^2(\mu)/4\pi]$ in the two cases satisfy the following forms of the RGE's in the two mass ranges.

$M_W\le\mu\le M_C$

$$\begin{aligned}&\frac{1}{\alpha_i(M_W)}=\frac{1}{\alpha_i(\mu)}+\frac{a_i}{2\pi}\ln\frac{\mu}{M_W}, \quad i=Y,2L,3C \ ,\\&a_Y=\tfrac{41}{10}, \quad a_{2L}=-\tfrac{19}{6}, \quad a_{3C}=-7 \ .\end{aligned} \tag{14}$$

$M_C\le\mu\le M_U$

$$\frac{1}{\alpha_i(M_C)}=\frac{1}{\alpha_i(\mu)}+\frac{a_i}{2\pi}\ln\frac{\mu}{M_C}, \quad i=2L,2R,4C \ . \tag{15}$$

In the chain in Eq. (1) parity (P) is left unbroken down to the scale M_C and the Higgs sector maintains left-right symmetry having the decomposition under G_{224P} as $\phi(2,2,1)\oplus\Delta_L(3,1,10)\oplus\Delta_R(1,3,\overline{10})$, in the mass range $M_C\le\mu\le M_U$. With three fermion generations the coefficients occurring in Eq. (15) in this case, are

$$a'_{2L}=a'_{2R}=\tfrac{11}{3}, \quad a'_{4C}=-\tfrac{14}{3} \ . \tag{16}$$

It was noted in Ref. [8] that the G_{224} singlet, $\eta(1,1,1)\subset\mathbf{210}$, is odd under P when all couplings in the SO(10)-invariant Lagrangian are real. In the chain in Eq. (2), when $\mathrm{SO}(10)\rightarrow G_{224}$ due to $\langle\eta\rangle\neq0$, P breaks at $\mu\sim M_U$, and the presence of the Higgs-scalar coupling of the type $\mathbf{210}\times\overline{\mathbf{126}}\times\mathbf{126}$ allows the left-handed triplet $\Delta_L(3,1,10)\subset\mathbf{126}$ to becoming superheavy ($\mu\sim M_U$) whereas the right-handed triplet $\Delta_R(1,3,\overline{10})$ remains light ($\mu\sim M_C$). This has been realized within the constraint of minimal fine-tuning of parameters. The Higgs-scalar multiplets contributing to the RGE's in the mass range $M_C\le\mu\le M_U$ are $\phi(2,2,1)\oplus\Delta_R(1,3,\mathbf{10})$ leading to the values of the coefficients

$$a'_{2L}=-3,\quad a'_{2R}=\tfrac{11}{3},\quad a'_{4C}=-\tfrac{23}{3}\ . \tag{17}$$

Using the RGE's (14) and (15) and the boundary conditions in the generalized form (3), we follow the standard procedure for obtaining formulas for $\ln(M_U/M_W)$, $\sin^2\theta_W$, and α_G^{-1}, through the combinations $e^{-2}(M_W)-\frac{8}{3}g_{3C}^{-2}(M_W)$, $e^{-2}(M_W)-\frac{8}{3}g_{2L}^{-2}(M_W)$, and $e^{-2}(M_W)=\frac{5}{3}g_Y^{-2}(M_W)+g_{2L}^{-2}(M_W)$:

$$\ln\frac{M_U}{M_W}=\frac{2\pi}{D}\{[2+\epsilon_{2L}+\epsilon_{2R}+\tfrac{2}{3}(1+\epsilon_{4C})]\alpha_s^{-1}-(1+\epsilon_{4C})\alpha^{-1}\}$$
$$+\frac{1}{D}[(1+\epsilon_{4C})(\tfrac{5}{3}a_Y+a_{2L}-\tfrac{2}{3}a_{3C}-a'_{2L}-a'_{2R})-(2+\epsilon_{2L}+\epsilon_{2R})(a_{3C}-a'_{4C})]\ln\frac{M_C}{M_W}\ , \tag{18}$$

$$\sin^2\theta_W=\frac{1}{D}\Bigg[a'_{4C}(1+\epsilon_{2L})-a'_{2L}(1+\epsilon_{4C})+\frac{\alpha}{\alpha_s}[a'_{2L}(1+\epsilon_{2R})+\tfrac{2}{3}a'_{2L}(1+\epsilon_{4C})-(a'_{2R}+\tfrac{2}{3}a'_{4C})(1+\epsilon_{2L})]$$
$$+\frac{\alpha}{2\pi}\{[a_{3C}(a'_{2R}+\tfrac{2}{3}a'_{4C})-\tfrac{5}{3}a_Ya'_{4C}](1+\epsilon_{2L})+(a_{2L}a'_{4C}-a'_{2L}a_{3C})(1+\epsilon_{2R})$$
$$+[a'_{2L}(\tfrac{5}{3}a_Y-\tfrac{2}{3}a_{3C})-a_{2L}a'_{2R}](1+\epsilon_{4C})\}\ln\frac{M_C}{M_W}\Bigg]\ , \tag{19}$$

$$\frac{1}{\alpha_G}=\frac{1}{D}\left[\frac{a'_{4C}}{\alpha}-\frac{a'_{2L}+a'_{2R}+\frac{2}{3}a'_{4C}}{\alpha_s}-\frac{1}{2\pi}[a'_{4C}(a_{2L}+\tfrac{5}{3}a_Y)-a_{3C}(a'_{2L}+a'_{2R}+\tfrac{2}{3}a'_{4C})]\ln\frac{M_C}{M_W}\right]\ , \tag{20}$$

where

$$D=a'_{4C}(2+\epsilon_{2L}+\epsilon_{2R})-(a'_{2L}+a'_{2R})(1+\epsilon_{4C})\ . \tag{21}$$

A. Formulas with G_{224P} intermediate symmetry

Using the coefficients from Eqs. (14) and (16), and the values of ϵ_{2L}, ϵ_{2R}, and ϵ_{4C} from Eq. (6) in Eqs. (18) and (21), we obtain $D=-\frac{50}{3}(1-\frac{2}{5}\epsilon)$ and the formulas

$$\ln\frac{M_U}{M_W}=\frac{3\pi}{25-10\epsilon}\left[\left[\frac{1}{\alpha}-\frac{8}{3\alpha_s}\right]+\left[\frac{1}{\alpha}+\frac{7}{3\alpha_s}\right]\epsilon\right]-\frac{17-18\epsilon}{50-20\epsilon}\ln\frac{M_C}{M_W}\ , \tag{22}$$

$$\frac{1}{\alpha_G}=\frac{1}{50-20\epsilon}\left[\frac{14}{\alpha}+\frac{38}{3\alpha_s}+\frac{56}{3\pi}\ln\frac{M_C}{M_W}\right]\ . \tag{23}$$

It is clear that $\ln(M_U/M_W)$ and α_G^{-1} are dependent upon ϵ. But while computing $\sin^2\theta_W$ from Eq. (19), we find

$$a'_{4C}(1+\epsilon_{2L})-a'_{2L}(1+\epsilon_{4C})=-(50-20\epsilon)/6,\quad a'_{2L}(1+\epsilon_{2R})+\tfrac{2}{3}a'_{2L}(1+\epsilon_{4C})-(a'_{2R}+\tfrac{2}{3}a'_{4C})\times(1+\epsilon_{2L})=(50-20\epsilon)/9\ ,$$

and

$$[a_{3C}(a'_{2R}+\tfrac{2}{3}a'_{4C})-\tfrac{5}{3}a_Ya'_{4C}](1+\epsilon_{2L})+(a_{2L}a'_{4C}-a'_{2L}a_{3C})(1+\epsilon_{2R})$$
$$+[a'_{2L}(\tfrac{5}{3}a_Y-\tfrac{2}{3}a_{3C})-a_{2L}a'_{2R}](1+\epsilon_{4C})=\tfrac{22}{9}(50-20\epsilon)\ .$$

Thus, the factor $(50-20\epsilon)$ occurs in the numerator and the denominator D in Eq. (19), making it independent of ϵ, yielding

$$\sin^2\theta_W=\frac{1}{2}-\frac{\alpha}{3\alpha_s}-\frac{11\alpha}{3\pi}\ln\frac{M_C}{M_W}\ . \tag{24}$$

We show in Sec. IV that Eqs. (22)–(24), in addition to predicting solutions of the SW type with $M_G\sim10^{17}$ GeV, also predict new types of solutions with a large M_U and $M_G\sim M_{\rm Pl}$.

B. Formulas with G_{224} intermediate symmetry

Using the coefficients from Eqs. (14) and (17) and the values of ϵ_{2L}, ϵ_{2R}, and ϵ_{4C} from Eq. (13) in Eqs. (18)–(21), we obtained D to be independent of ϵ, $D=-16$:

$$\ln\frac{M_U}{M_W}=\frac{2\pi}{16}\left[\frac{1}{\alpha}-\frac{8}{3\alpha_s}\right]-\frac{19}{48}\ln\frac{M_C}{M_W}\ , \tag{25}$$

$$\frac{1}{\alpha_G}=\frac{1}{48}\left[\frac{23}{\alpha}-\frac{40}{3\alpha_s}-\frac{533}{6\pi}\ln\frac{M_C}{M_W}\right]\ , \tag{26}$$

$$\sin^2\theta_W = \frac{7}{24} + \frac{2}{9}\frac{\alpha}{\alpha_s} - \left[\frac{23}{48} - \frac{5}{18}\frac{\alpha}{\alpha_s}\right]\epsilon - \frac{\alpha}{2\pi}\left[\frac{193}{72} - \frac{533}{144}\epsilon\right]\ln\frac{M_C}{M_W} . \quad (27)$$

Thus ϵ dependence occurs only for $\sin^2\theta_W$. For $\ln(M_U/M_W)$ the ϵ dependence occurring in the numerator of Eq. (18) cancels out. As will be shown in Sec. IV, these equations yield interesting solutions with $M_C \sim 10^5$–10^6 GeV leading to new observable predictions at low energies.

IV. NEW PREDICTIONS, AND THEIR TESTS BY LOW-ENERGY EXPERIMENTS

In this section we obtain numerical solutions for the two SO(10) models, Eqs. (1) and (2), using analytic formulas for $\ln(M_U/M_W)$, $\sin^2\theta_W$, and α_G developed for each case in Sec. III. We discuss some implications of the new GUT predictions for low-energy experiments and the big-bang cosmology of the universe.

A. New predictions with G_{224P} intermediate symmetry

Using $\alpha_s = 0.11$, $\alpha^{-1} = 127.54$, and $M_W = 81$ GeV we computed possible combinations of (M_C, M_U) as a function of ϵ that satisfy Eqs. (22) and (24) with the constraint $M_U \geq 10^{15}$ GeV and $\sin^2\theta_W \simeq 0.22$–$0.24$. Some of our best solutions are presented in Table I. We find that $\sin^2\theta_W$ decreases below (increases beyond) 0.22 (0.24) when $M_C > 10^{14}$ GeV ($< 10^{13}$ GeV). The value of $\sin^2\theta_W$ is not affected by ϵ as shown in Table I and evident from Eq. (24). For a fixed value of M_C in the range 10^{13}–10^{14} GeV, the unification mass is controlled by the parameter ϵ. Solutions already obtained by SW [3] corresponding to the enhancement of τ_p by a factor 10–100 over the conventional SO(10) predictions occur for $\epsilon \simeq 0.01$–0.02. They are consistent with lower values of the compactification scale $\sim 10^{17}$ GeV as noted in Ref. [3]. With such GUT predictions proton decay in the $p \to e^+\pi^0$ mode might be observable in the near future by low-energy experiments with improved accuracy [13].

The new class of solutions that we note in this case are the ones with larger values of the unification mass, $M_U \sim 10^{16}$–10^{18} GeV, and the compactification scale in the range 10^{17}–10^{19} GeV. As reported in Table I with fixed $M_C \simeq 4\times 10^{13}$ GeV, allowed values of M_U cover the range $(1.5\times 10^{16}$–$2\times 10^{18})$ GeV for $\epsilon = 0.04$–0.10. Using the relation $\eta = 2\sqrt{10\pi\alpha_G\epsilon M_G/M_U}$ we find $\eta \simeq 0.2$–0.5 for $M_G = 10^{17}$–10^{19} GeV. In particular the solutions corresponding to a very stable proton with $M_U \simeq 10^{17}$–10^{18} GeV are consistent with larger compactification scale $M_G \simeq 10^{18}$–10^{19} GeV.

As noted by Kibble, Lazaridis, and Shafi [9] several years ago, the domain-wall problem could have been severe if the intermediate scale M_C were less than 10^{12} GeV. But, with the renormalization group permitting $M_C \simeq 10^{14}$ GeV, the problem does not exist in the present model. The Majorana neutrino masses are governed by the seesaw formula for the three generations [14,15]:

TABLE I. Solutions of renormalization-group equations in SO(10) with left-right-symmetric G_{224P} intermediate gauge group [Eq. (1)], in the presence of the five-dimensional operator.

M_C (GeV)	$\sin^2\theta_W$	ϵ	M_U (GeV)	α_G^{-1}	M_G (GeV)	η
4×10^{13}	0.230	0.01	1.4×10^{15}	41.4	10^{17}	1.21
		0.02	3.2×10^{15}	41.6	10^{17}	1.10
		0.04	1.5×10^{16}	41.9	10^{17}	0.44
					10^{18}	4.47
		0.06	7.8×10^{16}	42.2	10^{17}	0.13
					10^{18}	1.32
		0.08	4.1×10^{17}	42.6	10^{18}	0.33
					10^{19}	3.38
		0.10	2.2×10^{18}	42.9	10^{19}	0.79
10^{14}	0.221	0.01	1.1×10^{15}	41.5	10^{17}	1.65
		0.02	2.3×10^{15}	41.6	10^{17}	1.50
		0.04	1.1×10^{16}	42.0	10^{17}	0.61
					10^{18}	6.05
		0.06	5.8×10^{16}	42.3	10^{17}	0.17
					10^{18}	1.78
		0.08	3×10^{17}	42.7	10^{18}	0.45
					10^{19}	4.55
		0.10	1.6×10^{18}	43.1	1.6×10^{18}	0.17
					10^{19}	1.06

$$m_{\nu_i} \simeq \frac{m_i^2}{M_C}, \quad i=1,2,3 , \tag{28}$$

where $m_{\nu_1}=m_{\nu_e}$, $m_{\nu_2}=m_{\nu_\mu}$ and $m_{\nu_3}=m_{\nu_\tau}$. Two different choices for m_i exist in the literature. While an up-quark mass of the ith generation has been used by Gell-Mann, Ramond, and Slansky [14], others have used the corresponding charged-lepton mass [15]. Using $M_C \simeq 10^{13}$–10^{14} GeV, the allowed range from Table I, and $m_1=m_u=5$ MeV, $m_2=m_C=1.5$ GeV, and $m_3=m_t \simeq 100$ GeV, the model predicts

$$m_{\nu_e} \simeq (2\times 10^{-10}\text{–}2\times 10^{-9}) \text{ eV} ,$$
$$m_{\nu_\mu} \simeq (2\times 10^{-3}\text{–}2\times 10^{-2}) \text{ eV} , \tag{29a}$$
$$m_{\nu_\tau} \simeq (0.1\text{–}1) \text{ eV} .$$

But using the charged-lepton masses $m_1=m_e$, $m_2=m_\mu$, $m_3=m_\tau$, the model has the prediction

$$m_{\nu_e} \simeq (2\times 10^{-12}\text{–}2\times 10^{-11}) \text{ eV} ,$$
$$m_{\nu_\mu} \simeq (10^{-7}\text{–}10^{-6}) \text{ eV} , \tag{29b}$$
$$m_{\nu_\tau} \simeq (3\times 10^{-5}\text{–}3\times 10^{-4}) \text{ eV} .$$

Although these masses are too small for laboratory detection, they might be compatible with the values needed to understand the solar-neutrino puzzle [16].

B. New predictions with G_{224} intermediate symmetry

In the absence of the five-dimensional operator and $\mathcal{L}_{NR}$, $\epsilon=0$, and with all superheavy masses the same as M_U, the renormalizable SO(10) model with a single G_{224} intermediate symmetry predicted $\sin^2\theta_W=0.22$, and 0.24, for the combinations $(M_C,M_U)=(0.5\times 10^{13},10^{15})$ GeV, and $(10^{11},6\times 10^{15})$ GeV, respectively, at the one-loop level [8].

Using the same parameters as in Eq. (1) we computed values of (M_C,M_U) as a function of ϵ which are solutions of Eqs. (25) and (27) imposing the constraint $M_U \geq 10^{15}$ GeV, and $\sin^2\theta_W=0.22$–0.24. Some of our solutions with higher (lower) values of M_C are presented in Table II (Table III). In both tables the constancy of M_U and α_G^{-1} with respect to variation in ϵ as evident from Eqs. (25) and (26), are exhibited. But, for a fixed M_C, the value of $\sin^2\theta_W$ is controlled by the parameter ϵ. This is a situation drastically different from Eq. (1) where the unification mass, rather than $\sin^2\theta_W$, is controlled by ϵ. Thus, the solutions in Eqs. (1) and (2) are expected to be

TABLE II. Same as Table I, but for Eq. (2), and higher intermediate scales (M_C).

M_C (GeV)	M_U (GeV)	ϵ	$\sin^2\theta_W$	α_G^{-1}	M_G (GeV)	η
10^{12}	3.2×10^{15}	0.01	0.226	44.9	10^{17}	0.54
					10^{18}	5.43
		0.02	0.223	44.9	10^{17}	1.08
					10^{18}	10.87
10^{11}	7.9×10^{15}	0.01	0.234	46.2	10^{17}	0.21
					10^{18}	2.15
		0.02	0.230	46.2	10^{17}	0.43
					10^{18}	4.31
		0.04	0.223	46.2	10^{17}	0.64
					10^{18}	6.46
10^{10}	1.9×10^{16}	0.02	0.238	47.6	10^{17}	0.17
					10^{18}	1.71
		0.04	0.230	47.6	10^{17}	0.34
					10^{18}	3.41
		0.06	0.223	47.6	10^{17}	0.51
					10^{18}	5.12
10^{8}	4.9×10^{16}	0.04	0.238	48.9	10^{17}	0.13
					10^{18}	1.35
		0.06	0.230	48.9	10^{17}	0.20
					10^{18}	2.03
		0.08	0.222	48.9	10^{17}	0.27
					10^{18}	2.71

very much different for larger values of ϵ. The values of η parameter have been calculated using the formula (13) with different values of M_G and ϵ in each case.

It is clear from Table II that for $M_C \simeq 10^{11}$–10^{12} GeV the model predicts observable proton decay by high-precision–low-energy experiments [13] as they correspond to $M_U = (3.0$–$8) \times 10^{15}$ GeV with $\tau_p \simeq (10^{34\pm 2}$–$10^{36\pm 2})$ yr for $p \rightarrow e^+\pi^0$ mode. They are solutions similar to the SW type in Eq. (1) with the lower values of the compactification scale $M_G \simeq 10^{17}$ GeV. In this case the neutrino masses are two orders of magnitude larger than the values given in Eq. (29). For example, with $M_C \simeq 10^{11}$ GeV and up-quark masses for m_i, Eq. (28) gives $m_{\nu_e} \simeq 2 \times 10^{-7}$ eV, but $m_{\nu_\mu} \simeq 2$ eV, and $m_{\nu_\tau} \simeq 100$ eV, which are detectable by laboratory experiments. When charged-lepton masses are used for m_i, although the predicted neutrino masses are too small, they could still be compatible with values needed to understand the solar-neutrino puzzle. The other class of solutions in the model corresponding to $M_C \simeq 10^8$–10^{10} GeV and $M_U \simeq 2 \times 10^{16}$–$10^{17}$ GeV predict much larger proton lifetime $\tau_p \simeq (10^{37\pm 2}$–$10^{41\pm 2})$ yr and neutrino masses 4–5 orders larger compared to Eq. (29). Using the quark masses for m_i we obtain $m_{\nu_e} \simeq (2 \times 10^{-6}$–$2 \times 10^{-4})$ eV, $m_{\nu_\mu} \simeq (20$ eV–2 keV), and $m_{\nu_\tau} \simeq (0.1$–$10)$ keV; but the model predicts $m_{\nu_e} \simeq (2 \times 10^{-8}$–$2 \times 10^{-6})$ eV, $m_{\nu_\mu} \simeq (10^{-3}$–$10^{-1})$ eV, and $m_{\nu_\tau} \simeq (0.3$–$30)$ eV, when the charged lepton masses are used for m_i. Thus the ν_μ and ν_τ masses are within the detectable range and the solutions in this class are consistent with the compactification scale $M_G \simeq 10^{17}$–10^{18} GeV.

The most interesting solutions in the chain in Eq. (2) are presented in Table III where $\sin^2\theta_W \simeq 0.22$–$0.24$ permit M_C as low as 10^5–10^6 GeV needed for observable signatures of quark-lepton unification by low-energy experiments through $n-\bar{n}$ oscillations [17] with $\tau_{n-\bar{n}} \simeq 10^8$–$10^9$ s, and rare kaon decay, $K_L \rightarrow \bar{\mu}e$ with branching ratio $7 \times (10^{-8}$–$10^{-12})$ [18]. With $M_C \simeq (10^5$–$10^6)$ GeV the predicted neutrino masses are in the range

$$
\begin{aligned}
&m_{\nu_e} \simeq (2 \times 10^{-4}\text{–}2 \times 10^{-3}) \text{ eV} ,\\
&m_{\nu_\mu} \simeq (10\text{–}100) \text{ eV} ,\\
&m_{\nu_\tau} \simeq (3\text{–}30) \text{ keV}
\end{aligned}
\tag{30}
$$

when charged lepton masses are used for m_i; but $m_{\nu_e} \simeq 0.02$–0.2 eV, $m_{\nu_\mu} \simeq 200$ keV–2 MeV, and $m_{\nu_\tau} \simeq (10$–$100)$ MeV when the up-quark masses are used for m_i. Thus the seesaw formula, with m_i as the quark masses, forbids $M_C \simeq 10^5$ GeV as the predicted m_{ν_μ} and m_{ν_τ} violate the existing laboratory limits ($m_{\nu_\mu} \lesssim 250$ keV,

TABLE III. Same as Table II, but for lower intermediate scales.

M_C (GeV)	M_U (GeV)	ϵ	$\sin^2\theta_W$	α_G^{-1}	M_G (GeV)	η
10^8	1.2×10^{17}	0.06	0.237	50.3	10^{18}	0.80
					10^{19}	8.04
		0.08	0.229	50.3	10^{18}	1.07
					10^{19}	10.72
		0.10	0.221	50.3	10^{18}	1.34
					10^{19}	13.41
10^7	3×10^{17}	0.08	0.236	51.6	10^{18}	0.42
					10^{19}	4.25
		0.10	0.228	51.6	10^{18}	0.53
					10^{19}	5.32
		0.12	0.220	51.6	10^{18}	0.63
					10^{19}	6.38
10^6	7.6×10^{17}	0.10	0.235	53.0	10^{18}	0.21
					10^{19}	2.11
		0.12	0.226	53.0	10^{18}	0.25
					10^{19}	2.53
10^5	1.9×10^{18}	0.12	0.233	54.4	2×10^{18}	0.20
					10^{19}	1.0
		0.14	0.224	54.4	2×10^{18}	0.23
					10^{19}	1.17

$m_{\nu_\tau} \leq 35$ MeV). This implies that $M_C > 10^6$ GeV, $B(K_L \rightarrow \mu e) \lesssim 7\times 10^{-12}$ and $\tau_{n-\bar{n}} \gtrsim 10^9$ s. But the seesaw formula with m_i as the charged-lepton masses allows $M_C \simeq 10^5$–10^6 GeV since the ν masses do not violate the existing laboratory limits. The unification mass for $M_C \simeq 10^6(10^5)$ GeV is high, $M_U \simeq 10^{17}(10^{18})$ GeV predicting a very stable proton with lifetime $\tau_p \simeq 10^{40}(10^{44})$ yr in the $p \rightarrow e^+\pi^0$ mode. Such high unification masses are consistent with the five-dimensional operator scaled by high compactification masses $M_G \sim 10^{18}$–10^{19} GeV. Note that the simplest Kaluza-Klein model leading to the four-dimensional spacetime as a result of compactification of the extra dimension on a circle yields $M_G = M_{\rm Pl}/2\pi = 1.6\times 10^{18}$ GeV. Most of our solutions with observable low-energy signatures of quark lepton unification are compatible with $\eta \sim 1$ and such a high compactification scale. Since the left-right discrete symmetry (P) is broken at the GUT scale along with SO(10)-gauge symmetry, the model does not in principle possess the domain-wall problem [8].

Some of our solutions are consistent with Majorana neutrino masses of the order 10 keV for ν_μ or ν_τ. It is interesting to note that the end-point β spectrum in Simpson-type experiments [19] has indicated the presence of a heavy neutrino (ν_s) with a mass $\simeq 17$ keV although conclusive evidence is still awaited. In such a case the physical electron-type neutrino predominantly consists of the mass eigenstate of a light neutrino with a small admixture of the heavy-mass eigenstate with $\sin^2\theta \simeq 0.8\%$ (θ=mixing angle). The existing limit on the neutrinoless double-β decay then forbids the heavy neutrino to be of Majorana type [20]. A Dirac type $\nu_\tau \equiv \nu_s$ is consistent with the available data and solution to the solar-neutrino puzzle [16]. The present GUT scenario is not fully consistent with these observations on the heavy neutrino.

In a number of predictions for the neutrino masses in Eq. (2) the cosmological bound $\sum_i m_{\nu_i} \lesssim 65$ eV seems to be violated. This happens, for example, for $M_C \simeq 10^5$–10^8 GeV for m_{ν_μ} and m_{ν_τ} using up-quark masses for m_i in Eq. (28). One procedure to evade the cosmological bound is to make the heavier neutrinos unstable with respect to Majoron emission and decay into the lightest neutrino (ν_e) (Ref. [19]). The Majoron is generated by breaking spontaneously an additional global $U(1)_l$ (l=lepton number) symmetry which must be introduced along with SO(10) to start with and broken at a scale $M \gg M_W$.

V. SUMMARY, DISCUSSION, AND CONCLUSION

Higher-dimensional operators in specific forms, involving gauge and Higgs fields might appear as nonrenormalizable terms in the GUT Lagrangian in four dimensions as a result of compactification of extra dimensions in higher-dimensional theories [1–3,12], or as effects of quantum gravity [4]. It has been shown that [1–6] such terms can be absorbed in the renormalizable-gauge-fields-kinetic energy of the residual gauge group when the grand unifying symmetry is broken spontaneously by the VEV of the Higgs field occurring in the higher-dimensional operator(s). In such cases the gauge coupling constants at the GUT scale are usually modified resulting in the modifications of M_U and $\sin^2\theta_W$. We have demonstrated in this paper that although the gauge couplings are modified, in certain cases, either M_U, or $\sin^2\theta_W$ might remain unaffected by the introduction of the higher-dimensional operator.

Shafi and Wetterich [3] examined the impact of a five-dimensional operator on SO(10) and obtained a factor of 10–100 enhancement in the proton lifetime over the conventional predictions with the left-right-symmetric Pati-Salam group surviving as the intermediate symmetry down to $M_C \simeq 10^{13}$ GeV [Eq. (1)]. Such predictions with observable proton decay in the $p \rightarrow e^+\pi^0$ mode, but no other low-energy signatures of the GUT are consistent only if the compactification scale $M_G \simeq 10^{17}$ GeV, nearly two orders lower than $M_{\rm Pl} = 10^{19}$ GeV. Examining formulas obtained as solutions of RGE's in this case we found that $\sin^2\theta_W$ is independent of ϵ, the parameter in $\mathcal{L}_{NR}$. We find that, in addition to the GUT predictions of the SW [3] type, the model predicts a very stable proton corresponding to large values of grand unification mass consistent with higher compactification scales, $M_G \simeq 10^{18}$–10^{19} GeV. The model does not possess the domain-wall problem in practice with such high values of $M_C \simeq 10^{13}$–10^{14} GeV. Although the predicted values of the neutrino masses are small, they still might be compatible with the values needed to understand the solar-neutrino puzzle [16].

As the primary objective of this paper we examined the impact of the five-dimensional operator in SO(10) with single G_{224} intermediate symmetry when parity is broken at the GUT scale such that the model does not have the domain-wall problem [9] [Eq. (2)]. The five-dimensional operator is expected to be present as a nonrenormalizable term in the Lagrangian after compactification of extra dimensions in the higher-dimensional model of Wetterich [12]. In this case purely renormalizable interactions (without the five-dimensional operator) predict $10^{11} \lesssim M_C \lesssim 5\times 10^{13}$ GeV and $6\times 10^{15} \gtrsim M_U \gtrsim 10^{15}$ GeV at the one-loop level such that, except for proton decay, no other interesting GUT signatures are possible at low energies. Including the appropriate five-dimensional operator in $\mathcal{L}_{NR}$ we found that the resulting equations for $\ln(M_U/M_W)$ and α_G are independent of ϵ, although $\sin^2\theta_W$ does depend upon it. As the primary distinguishing feature in the structural form of the equations in the two cases we note that, for a fixed M_C, the values of $\ln(M_U/M_W)$ $\sin^2\theta_W$ are controlled by the parameter ϵ in Eq. (1) [Eq. (2)]. In Eq. (2) the solutions of renormalization-group equations are classified into three categories: (a) solutions of SW type with $M_C \simeq 10^{11}$–10^{12} GeV and $M_G \simeq 10^{17}$ GeV, predicting observable τ_p and neutrino masses 1–3 orders larger than Eq. (1), (b) $M_C \simeq 10^{18}$–10^{12} GeV, τ_p at least 5 orders larger than the experimental lower limit, and neutrino masses 4–5 orders larger than Eq. (1), (c) $M_C \simeq 10^{15}$–10^6 GeV with a very stable proton but experimentally observable n-$\bar{n}$ oscillation, rare kaon decays, and Majorana neutrino masses consistent with higher values of the compactification scales, $M_G \simeq 10^{18}$–10^{19} GeV. Some of our solutions are consistent with 17-keV Majorana neutrino masses for ν_μ or ν_τ. But the heavy neutrino signal indicated in Simpson-type experiment [19] might be a Dirac type ν_τ which in turn might be visualized as a combination of two degenerate Majorana neutrinos [20]. In any case a definite conclusive evidence on massive neutrino spectrum is yet to be confirmed by experiments. The cosmological bound can be evaded in appropriate cases by making the heavy neutrinos unstable with respect to the decay into the lighter neutrinos by the emission of a Majoron [21]. The Majoron can be generated by invoking an additional global symmetry $U(1)_L$ or $U(1)_{B-L}$, where $B(L)$ is the baryon (lepton) number, and breaking it spontaneously at a scale $M \gg M_W$.

Finally we conclude that the impact of the five-dimensional operator causes drastic but very attractive modifications of SO(10) predictions with single Pati-Salam intermediate symmetry when parity and $SU(2)_R$ breakings are decoupled [8].

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*On leave of absence from School of Agricultural Sciences and Rural Development, North-Eastern Hill University, Medziphema 797106, India.

[1] Th. Kaluza, Sitzungsber. Preuss. Akad. Wiss. Math. Phys. **K1**, 996 (1921); O. Klein, Z. Phys. **37**, 895 (1925). For a review of recent works in higher-dimensional theories, see T. Appelquist, A. Chodos, and P. G. O. Freund, *Modern Kaluza-Klein Theories* (Addison-Wesley, Reading, MA, 1987).

[2] T. Appelquist and A. Chodos, Phys. Rev. Lett. **50**, 141 (1983); Phys. Rev. D **28**, 772 (1983).

[3] Q. Shafi and C. Wetterich, Phys. Rev. Lett. **51**, 875 (1984).

[4] C. T. Hill, Phys. Lett. **135B**, 47 (1984).

[5] M. K. Parida and P. K. Patra, Phys. Rev. D **39**, 2000 (1989).

[6] M. K. Parida, P. K. Patra, and A. K. Mohanty, Phys. Rev. D **39**, 316 (1989).

[7] J. C. Pati and A. Salam, Phys. Rev. Lett. **31**, 661 (1973); Phys. Rev. D **10**, 275 (1974).

[8] D. Chang, R. N. Mohapatra, and M. K. Parida, Phys. Rev. Lett. **52**, 1072 (1984); Phys. Rev. D **30**, 1052 (1984).

[9] T. W. B. Kibble, G. Lazaridis, and Q. Shafi, Phys. Rev. D **26**, 435 (1982).

[10] S. Rajpoot, Phys. Rev. D **9**, 2244 (1980).

[11] R. N. Mohapatra and B. Sakita, Phys. Rev. D **21**, 1062 (1980).

[12] The corresponding factor in the case of SU(N) is $\frac{1}{2}$ in Eq. (10), instead of $\frac{1}{8}$ in SO(10). See, for instance, C. Wetterich, Nucl. Phys. **B244**, 359 (1984); **B253**, 366 (1985).

[13] J. C. Pati, A. Salam, and B. V. Sreekanthan, Int. J. Mod. Phys. A **1**, 147 (1986).

[14] M. Gell-Mann, P. Ramond, and R. Slansky, in *Supergravity*, edited by D. Z. Freedman *et al.* (North-Holland, Amsterdam, 1979); T. Yanagida, in *Proceedings of the Workshop on Unified Theory and Baryon Number in the Universe*, Tsukuba, Japan, 1979, edited by O. Sawada and A. Sugamoto (KEK Report No. 79-18, Tsukuba, Japan, 1979).

[15] R. N. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980); Phys. Rev. D **23**, 165 (1981); R. N. Mohapatra, *Unification and Supersymmetry* (Springer, Berlin, 1986); R. N. Mohapatra and P. Pal, *Massive Neutrinos and Astrophysics* (World Scientific, Singapore, 1990).

[16] S. P. Mikheyev and A. Yu. Smirnov, Yad. Fiz. **42**, 1441 (1985) [Sov. J. Nucl. Phys. **42**, 913 (1985)]; Nuovo Cimento **9C**, 17 (1986); L. Wolfenstein, Phys. Rev. D **17**, 2369 (1978).

[17] R. N. Mohapatra and R. E. Marshak, Phys. Rev. Lett. **44**, 1316 (1980).

[18] N. G. Deshpande and R. Johnson, Phys. Rev. D **27**, 1165 (1983).

[19] J. J. Simpson, Phys. Rev. Lett. **54**, 1891 (1985); J. J. Simpson and A. Hime, Phys. Rev. D **39**, 1825 (1989); A. Hime and J. J. Simpson, *ibid.* **39**, 1837 (1989).

[20] M. J. Dugan, G. B. Gelmini, H. Georgi, and L. J. Hall, Phys. Rev. Lett. **54**, 2302 (1985); S. L. Glashow and A. Manohar, *ibid.* **54**, 2304 (1985).

[21] Y. Chikasige, R. N. Mohapatra, and R. D. Peccei, Phys. Lett. **98B**, 265 (1981); K. Sato and N. Terazawa, Phys. Lett. B **185**, 412 (1987).