PHYSICAL REVIEW D VOLUME 44, NUMBER 7 1 OCTOBER 1991

Unification of forces and flavors for three families

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(Received 22 May 1991)

Electroweak, strong, and horizontal interactions are unified in a simple group with an anomaly-free representation which does not include mirror fermions or exotic quarks. ± 1 charged, and neutral exotic leptons are needed in the model, but they acquire heavy masses as a consequence of the survival hypothesis, and also mix with the known leptons producing seesaw and universal seesaw mechanisms in a natural way. Masses for fermions in the third family arise at the tree level via a BCS (flavor-democracy) mass matrix. Masses for other known quarks and leptons can be generated by radiative corrections.

I. INTRODUCTION

One of the major theoretical puzzles in particle physics nowadays is the so-called flavor problem which is the collection of at least three related problems. The first one is the fermion mass hierarchy problem which has several aspects; namely the smallness of the neutrino masses compared with the other known fermions; the three orders of magnitude between the known charged leptons; the five (or more) orders of magnitude between the electron, the lightest charged fermion, and the yet to be discovered t quark; the large isospin splitting in the t-b system; etc. The second is the family problem, which is the lack of information about the total number of families in nature. The third problem is the lack of explanation for the small values of the mixing angles of the elementary fermions (where the Cabibbo-Kobayashi-Maskawa angles are only one sector of them).

The standard model (SM) defined by the $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ local gauge group for strong and electroweak interactions does not provide information about these problems. So, any explanation for them should imply physics beyond the SM.

It has been established in the context of the SM that the constraints from high-precision charged- and neutral-current experiments are enough to directly establish the canonical (left-handed doublet, right-handed singlet) assignments for the three families of fermions [1], implying in particular the existence of the top quark and of ν_τ. There is also experimental evidence that a fourth light neutrino is ruled out [2] which probably means that there are only three generations of quarks and leptons (three families). These two experimental facts constitute our basic scenario, together with the widely accepted hypothesis that known quarks have fractional electric charges 2/3 (up sector) and $-1/3$ (down sector); and that $SU(3)_c \otimes U(1)_Q$, where $U(1)_Q$ stands for the quantum electrodynamics Abelian factor, is an exact symmetry in nature.

There are many ideas in the literature based upon the former constraints which try to solve the flavor problem regarding new physics. For example, compositeness may be an explanation, or perhaps one should look for residual effects (radiative corrections) of theories such as extended technicolor, grand unified theories (GUT's), supersymmetry, etc.

In what follows we present in detail the analysis we have done on a new model proposed recently [3] based upon the ideas of the grand unification of flavors and forces; we especially focus on the particular version of the model which points toward the solution of the flavor problem. Even though our model may find its deepest roots in the three-family extension of the Pati-Salam model [4,5] (which was one inspiration of the present work), it has substantial differences with its ancestor and it exceeds it in several aspects. The most outstanding difference between the two models is the fact that, contrary to the Pati-Salam-type models, the model presented here does not need mirror fermions in order to be renormalizable. We elaborate more on these differences in the following section and in Appendix A.

The rest of the paper is as follows. In Sec. II we present the model (the local gauge group and its fermionic content) and calculate the Weinberg angle at the GUT scale. In Sec. III we break the original symmetry down to $SU(3)_c \otimes U(1)_Q$. In Sec. IV we present our analysis for the fermion masses in the context of the model. Our conclusions are presented in Sec. V. Several appendixes at the end of the paper deal with technical details implicit in the main text.

II. THE MODEL

A. The gauge group

We use $G = SU(6)_L \otimes SU(6)_c \otimes SU(6)_R \times Z_3$ as the gauge group which unifies nongravitational interactions with families [3], where $\otimes$ indicates a direct product, $\times$ a semidirect one, and $Z_3 \equiv (1, P, P^2)$ is the three-element cyclic group acting upon $[SU(6)]^3$ such that if (A,B,C) is a representation of $[SU(6)]^3$ with A a representation of the first factor, B of the second and C of the third, $P(A,B,C)=(B,C,A)$ and then $Z_3(A,B,C) = (A,B,C)+(B,C,A)+(C,A,B)$ is a singlet under Z_3. G is then simple, and it is characterized by one single gauge coupling constant g.

$SU(6)_c$ is the color group which consists of three hadronic and three leptonic colors. $SU(6)_c$ is broken down to $SU(3)_c \otimes U(1)_{Y_{(B-L)}}$, where $U(1)_{Y_{(B-L)}}$ is defined below, through the chain

$$SU(6)_c \to SU(4)_c \otimes SU(2)' \to SU(3)_c \otimes U(1)_{Y_{(B-L)}} , \tag{1}$$

where $SU(4)_c$ is a vectorlike four color gauge group [the same one which appears as a subgroup in GUT SO(10)] and $SU(2)'$ is a subgroup which appears in an intermediate step. T'_Z, the diagonal generator of $SU(2)'$, complements the role of $(B-L) \in SU(4)_c$ algebra, in order to define a new baryon$-$lepton number in $SU(6)_c$:

$$Y_{(B-L)} = (B-L) + 2T'_Z . \tag{2}$$

$Y_{(B-L)}$ in the fundamental representation of $SU(6)_c$ is then given by a 6×6 diagonal matrix with entries $\mathrm{diag}(1/3, 1/3, 1/3, -1, 1, -1)$.

We may write the 35 gauge fields of $SU(6)_c$ in the following way:

$$\begin{aligned}\tfrac{1}{2}\lambda_\alpha A_\alpha &= \sum_{\alpha\neq\beta} \frac{1}{\sqrt{2}} \lambda^\alpha_\beta G^\beta_\alpha + \tfrac{1}{2}\lambda_3 A_3 + \tfrac{1}{2}\lambda_8 A_8 + \tfrac{1}{2}\lambda_{15} A_{15} \\ &\quad + \tfrac{1}{2}\lambda_{24} A_{24} + \tfrac{1}{2}\lambda_{35} A_{35} \\ &= \frac{1}{\sqrt{2}} \begin{bmatrix} D^1 & G^1_2 & G^1_3 & \bar{X}_1 & \bar{Y}_1 & \bar{Z}_1 \\ G^2_1 & D^2 & G^2_3 & \bar{X}_2 & \bar{Y}_2 & \bar{Z}_2 \\ G^3_1 & G^3_2 & D^3 & \bar{X}_3 & \bar{Y}_3 & \bar{Z}_3 \\ X_1 & X_2 & X_3 & D^4 & P^-_1 & P^0 \\ Y_1 & Y_2 & Y_3 & P^+_1 & D^5 & P^+_2 \\ Z_1 & Z_2 & Z_3 & \bar{P}^0 & P^-_2 & D^6 \end{bmatrix} ,\end{aligned} \tag{3}$$

where λ_i, $i=3,5,8,15,24,35$ are the diagonal generators of SU(6),

$$\lambda_3 = \mathrm{diag}(1,-1,0,0,0,0),$$
$$\lambda_8 = \mathrm{diag}(1,1,-2,0,0,0)/\sqrt{3} ,$$
$$\lambda_{15} = \mathrm{diag}(1,1,1,-3,0,0)/\sqrt{6} ,$$
$$\lambda_{24} = \mathrm{diag}(1,1,1,1,-4,0)/\sqrt{10} ,$$
$$\lambda_{35} = \mathrm{diag}(1,1,1,1,1,-5)/\sqrt{15} ,$$

and

$$(\lambda^\alpha_\beta)_{ij} = \delta_{\alpha i}\delta_{\beta j} .$$

The diagonal entries in Eq. (3) are

$$D^i = G^i_i + B_{Y_{(B-L)}}/\sqrt{30} + B_{Y'}/\sqrt{20} + B_{Y''}/\sqrt{12} \quad \text{for } i=1,2,3,$$
$$D^4 = -3B_{Y_{(B-L)}}/\sqrt{30} - 3B_{Y'}/\sqrt{20} + B_{Y''}/\sqrt{12} ,$$
$$D^5 = 3B_{Y_{(B-L)}}/\sqrt{30} - 2B_{Y'}/\sqrt{20} - 2B_{Y''}/\sqrt{12} ,$$
$$D^6 = -3B_{Y_{(B-L)}}/\sqrt{30} + B_{Y'}/\sqrt{5} - B_{Y''}/\sqrt{3} ,$$

where $B_{Y_{(B-L)}}$ is the gauge boson associated with the hypercharge $Y_{(B-L)}$, and $B_{Y'}$ and $B_{Y''}$ are two gauge bosons associated with U(1) factors in $SU(6)_c$ but not in $SU(3)_c \otimes U(1)_{Y_{(B-L)}}$. In the right-hand side of Eq. (3) we have renamed most of the gauge fields. G^i_j for $i,j=1,2,3$ are the $SU(3)_c$ gauge bosons, X_i, Y_i and Z_i are leptoquarks gauge bosons with electrical charges $-2/3, 1/3$, and $-2/3$, respectively; $P^\pm_a, a=1,2$ and P^0 are (complex) dilepton gauge bosons with electrical charges as indicated. The role of these gauge fields will become obvious when we display the fermion content in the next subsection. The generators λ_α of SU(6) are normalized to $\mathrm{tr}\lambda_\alpha\lambda_\beta = 2$. The $SU(3)_c$ gauge coupling constant is $g_3 = g$.

Notice that our color group $SU(6)_c$ is vectorlike, contrary to the three-family extension of the Pati-Salam model [5] where the color group is the chiral one, $SU(6)_{cL} \otimes SU(6)_{cR}$. Even though $SU(6)_c \subset SU(6)_{cL} \otimes SU(6)_{cR}$, this difference has important consequences as we will see in what follows.

$SU(6)_L \otimes SU(6)_R \otimes U(1)_{Y_{(B-L)}}$ is postulated to be, at the G scale, the gauge group which unifies electroweak and horizontal interactions for three families of quarks and leptons. This group is the left-right-symmetric extension of the $SU(6)_L \otimes U(1)_Y$ family unification group [6].

The horizontal interactions arise from a chiral gauge group $G_{HL} \otimes G_{HR}$ which, at an intermediate step we take to be either $SU(3)_{HL} \otimes SU(3)_{HR}$ or $SU(2)_{HL} \otimes SU(2)_{HR}$. In the first case the families are in the fundamental representation of $G_{HL} \otimes G_{HR}$ and the embedding of $SU(2)_L \otimes SU(3)_{HL} \otimes SU(2)_R \otimes SU(3)_{HR}$ into $SU(6)_L \otimes SU(6)_R$ is a special maximal [7] one. In the second case the families are in the adjoint representation of $G_{HL} \otimes G_{HR}$ and the embedding of $SU(2)_L \otimes SU(2)_{HL} \otimes SU(2)_R \otimes SU(2)_{HR}$ into $SU(6)_L \otimes SU(6)_R$ is a special, not maximal one. In both cases the interactions mediated by the gauge fields W^i_L and W^i_R associated with the generators of $SU(2)_L$ and $SU(2)_R$ are universal, i.e., family independent.

The 35 $SU(6)_{L(R)}$ generators may be written in an $SU(2)_{L(R)} \otimes SU(3)_{HL(HR)}$ basis, which shows explicitly the universality of $SU(2)_{L(R)}$:

$$\sigma_i \otimes I_3/\sqrt{3}, \quad I_2 \otimes \lambda_j/\sqrt{2}, \quad \sigma_i \otimes \lambda_j/\sqrt{2}\ , \tag{4}$$

where σ_i, $i=1,2,3$ are the Pauli matrices, λ_j, $j=1,2,\ldots,8$, are the SU(3) matrices in the Gell-Mann basis, and I_2 and I_3 are the 2×2 and 3×3 unit matrices, respectively. g_{2L} and g_{2R}, the gauge coupling constants for $SU(2)_L$ and $SU(2)_R$, respectively, are given by $g_{2L}=g_{2R}=g/\sqrt{3}$ (see Appendix B).

From Eq. (4) we read immediately the $SU(2)_{L(R)}$ and $SU(3)_{HL(HR)}$ diagonal generators, elements of $SU(6)_{L(R)}$; they are

$$T_Z=\mathrm{diag}(1,-1,1,-1,1,-1)/2\sqrt{3}\ ,$$

$$T_3=\mathrm{diag}(1,1,-1,-1,0,0)/2\sqrt{2}\ ,$$

$$T_8=\mathrm{diag}(1,1,1,1,-2,-2)/2\sqrt{6}\ .$$

The special maximal embedding of $SU(2)_H \subset SU(3)_H$ is achieved by using as generators of $SU(2)_H$ the set $(\lambda_1+\lambda_6)/2, (\lambda_2+\lambda_7)/2$, and $(\lambda_3+\sqrt{3}\lambda_8/2)$. We could also use the rotated set $(\lambda_2,\lambda_5,\lambda_7)$, or any other appropriate set of generators. The diagonal generators for $SU(2)_{L(R)}$ and $SU(2)_{HL(HR)}$ are T_Z as above and $T'_{ZH}=\mathrm{diag}(1,1,0,0,-1,-1)/2\sqrt{2}$.

A few branching rules for $SU(6) \to SU(2)\otimes SU(3)_H \to SU(2)\otimes SU(2)_H$ irreducible representations (irreps) are

$$(6)\to(2,3)\to(2,3)\ ,$$

$$(15)\to(1,6)\oplus(3,\bar{3})\to(1,1)\oplus(1,5)\oplus(3,3)\ ,$$

$$(21)\to(3,6)\oplus(1,\bar{3})\to(3,1)\oplus(3,5)\oplus(1,3)\ ,$$

$$(35)\to(3,1)\oplus(1,8)\oplus(3,8)$$

$$\to(3,1)\oplus(1,3)\oplus(1,5)\oplus(3,3)\oplus(3,5)\ ,$$

where (n,m) refers to $(SU(2)_{L,R}, SU(3)_{HL,HR})$ irreps for the first descendent step, and to $(SU(2)_{L,R}, SU(2)_{HL,HR})$ for the second one.

The electric charge operator in the context of this model is

$$Q=T_{ZL}+T_{ZR}+\tfrac{1}{2}Y_{(B-L)}\ , \tag{5}$$

which is seen to acquire components from the three SU(6) factors in G.

B. The fermionic content

The ordinary (known) fermions in our model are included in

$$\psi(108)=Z_3\psi(6,1,\bar{6})_L=\psi(6,1,\bar{6})_L+\psi(1,\bar{6},6)_L+\psi(\bar{6},6,1)_L$$

with the particle content

$$\psi(\bar{6},6,1)_L=\begin{pmatrix} q_{1,r}^{-1/3} & q_{1,y}^{-1/3} & q_{1,b}^{-1/3} & E_1^- & L_1^0 & T_1^- \\ q_{1,r}^{2/3} & q_{1,y}^{2/3} & q_{1,b}^{2/3} & E_1^0 & L_1^+ & T_1^0 \\ q_{2,r}^{-1/3} & q_{2,y}^{-1/3} & q_{2,b}^{-1/3} & E_2^- & L_2^0 & T_2^- \\ q_{2,r}^{2/3} & q_{2,y}^{2/3} & q_{2,b}^{2/3} & E_2^0 & L_2^+ & T_2^0 \\ q_{3,r}^{-1/3} & q_{3,y}^{-1/3} & q_{3,b}^{-1/3} & E_3^- & L_3^0 & T_3^- \\ q_{3,r}^{2/3} & q_{3,y}^{2/3} & q_{3,b}^{2/3} & E_3^0 & L_3^+ & T_3^0 \end{pmatrix}_L\ , \tag{6}$$

where the rows (columns) represent color (flavor) degrees of freedom,

$$\psi(1,\bar{6},6)_L=\begin{pmatrix} q_1^{r,1/3} & q_1^{r,-2/3} & q_2^{r,1/3} & q_2^{r,-2/3} & q_3^{r,1/3} & q_3^{r,-2/3} \\ q_1^{y,1/3} & q_1^{y,-2/3} & q_2^{y,1/3} & q_2^{y,-2/3} & q_3^{y,1/3} & q_3^{y,-2/3} \\ q_1^{b,1/3} & q_1^{b,-2/3} & q_2^{b,1/3} & q_2^{b,-2/3} & q_3^{b,1/3} & q_3^{b,-2/3} \\ E_1^+ & F_1^0 & E_2^+ & F_2^0 & E_3^+ & F_3^0 \\ S_1^0 & L_1^- & S_2^0 & L_2^- & S_3^0 & L_3^- \\ T_1^+ & N_1^0 & T_2^+ & N_2^0 & T_3^+ & N_3^0 \end{pmatrix}_L\ , \tag{7}$$

where now the rows (columns) represent flavor (color) degrees of freedom. $\psi(6,1,\bar{6})_L$ represents 36 exotic Weyl leptons, 9 with positive electric charges, 9 with negative (the charge conjugated to the positive ones) and 18 are neutrals. As a matter of convention we have put in $\psi(1,\bar{6},6)_L$ the fields charge conjugated for the electrical charged fields in $\psi(\bar{6},6,1)_L$ but not for the neutral ones due to the possible existence of Majorana fields. To be precise, the charge-conjugated fields should be identified only at the end, when the mass matrices get diagonalized.

As can be seen, $\psi(108)$ does not contain exotic quarks. The known leptons $(\nu_e, e^-, \nu_\mu, \mu^-, \nu_\tau, \tau^-)$ and the known quarks (u,d,c,s,t,b) are linear combinations of the leptons and quarks in $\psi(\bar{6},6,1)_L+\psi(1,\bar{6},6)_L$, but cannot be pinpointed from the beginning, because above the unification scale all fermions (known and exotic) look alike, except for their electric charges.

For further reference let us write the quantum numbers for $\psi(\overline{6},6,1)_L$ and $\psi(1,\overline{6},6)_L$ with respect to the SM group. For $\psi(\overline{6},6,1)_L$ they are $3(3,2,1/3)\oplus 3(1,2,-1)\oplus 3(1,2,1)\oplus 3(1,2,-1)$, while for $\psi(1,\overline{6},6)_L$ they are $3(\overline{3},1,-4/3)\oplus 3(\overline{3},1,2/3)\oplus 6(1,1,2)\oplus 9(1,1,0)\oplus 3(1,1-2)$, where the numbers between brackets label the $(\mathrm{SU}(3)_c,\mathrm{SU}(2)_L,\mathrm{U}(1)_Y)$ irreps. As can be seen, the exotic leptons in $\psi(\overline{6},6,1)_L+\psi(1,\overline{6},6)_L$, which are left over after the ordinary ones are defined as certain linear combinations of E's and T's, can be arranged as vectorlike representations of $\mathrm{SU}(2)_L\otimes \mathrm{U}(1)_Y$. Also, for completeness, the $\mathrm{SU}(2)_L\otimes\mathrm{SU}(2)_R\otimes\mathrm{U}(1)_{Y_{(B-L)}}$ content of the exotic leptons in $\psi(6,1,\overline{6})_L$ is 9(2,2,0).

Notice that the representation $\psi(108)$ is free of anomalies. As a matter of fact the sector $\psi(6,1,\overline{6})_L$ demanded by the Z_3 symmetry cancels the anomalies arising from the fermions in $\psi(\overline{6},6,1)_L+\psi(1,\overline{6},6)_L$ without the introduction of mirror fermions. This way of canceling the anomalies makes a conspicuous difference between our model and the three-family extension of the Pati-Salam one [4,5]. Not only the new sector $\psi(6,1,\overline{6})$ avoids the introduction of mirror fermions, but it mixes in a natural way with the ordinary leptons, producing seesaw mechanisms [8,9]. That is, the heavy particles needed for generating the seesaw mechanism [8] and the universal seesaw mechanism [9] are present as a consequence of the symmetries of the model. Furthermore, as we will see later, the mixing of the several leptons in $\psi(108)$ is also natural. (By natural we mean that the mixings are produced by the minimal Higgs sector required to break the symmetries present in G.)

C. The Weinberg angle

Since G is simple, the Weinberg angle can be calculated. For a simple group the Weinberg angle at the unification scale M is given by [10]

$$\sin^2\theta_W(M)=\mathrm{tr}(T_{ZL}^2)/\mathrm{tr}(Q^2)\ , \tag{8}$$

where the traces can be computed using any representation (irreducible or not) of the simple group. In this way the Weinberg angle is well defined and is a unique value for the entire group, independent of a particular representation.

When we calculate the traces for $\psi(108)$ we get the value $\sin^2\theta_W(M)=9/23$. This value that represents the Weinberg angle for G can be double checked by calculating the traces for $\psi(18)=Z_3\psi(1,1,6)$ (an unphysical sector) or either for $\psi(105)=Z_3\psi(1,1,35)$, the gauge-boson sector. In Appendix B we rederive this value in a different way.

If the gauge group is not simple, then the Weinberg angle cannot be calculated at all. If we start with a semisimple group, product of equal factors, and make it simple by the introduction of one appropriate discrete symmetry, then the Weinberg angle should be calculated using any representation of the simple group and it is wrong to calculate it with a particular representation of the semisimple group which is not completely reducible to irreps of the simple one. For example, in our case, the traces calculated with $\psi(\overline{6},6,1)_L+\psi(1,\overline{6},6)_L$ give the value 9/28 which is not related to the Weinberg angle for G. As we show in Appendix A, this last value is related to the three-family extension of the Pati-Salam model.

At this point we may emphasize this difference between our model and the three-family extension of the Pati-Salam model. In the last one $\sin^2\theta_W(M)=9/28$ at the GUT scale [5], a value slightly smaller than in the present model. This difference will obviously manifest itself in the comparison of the phenomenology of these two models (e.g., the proton lifetime).

III. THE STAGES OF SYMMETRY BREAKING

Our next step is to break the symmetry and give masses to the particles in this model. We assume that this is done by the introduction of appropriate elementary Higgs scalars, which trigger the spontaneous breaking of the symmetry.

Our goal is to break the symmetry G down to $\mathrm{SU}(3)_c\otimes\mathrm{U}(1)_Q$. Our analysis suggests the symmetry-breaking chain

$$\begin{aligned}
G &\xrightarrow{M} G_{\mathrm{SP}}(\equiv \mathrm{SP}(6)_L\otimes\mathrm{SU}(4)_C\otimes\mathrm{SU}(2)'\otimes\mathrm{SP}(6)_R)\\
&\xrightarrow{M'} \mathrm{SU}(3)_c\otimes\mathrm{SU}(2)_L\otimes\mathrm{SU}(2)_R\otimes\mathrm{U}(1)_{Y_{(B-L)}}\\
&\xrightarrow{M_R} \mathrm{SU}(3)_c\otimes\mathrm{SU}(2)_L\otimes\mathrm{U}(1)_Y\\
&\xrightarrow{M_L} \mathrm{SU}(3)_c\otimes\mathrm{U}(1)_Q\ ,
\end{aligned}$$

where SP(6) stands for the simplectic group of dimension 6. Other intermediate steps such as

$$\begin{aligned}
G_{\mathrm{SP}} &\to \mathrm{SP}(6)_L\otimes\mathrm{SU}(3)_c\otimes\mathrm{U}(1)_{Y_{(B-L)}}\otimes\mathrm{SP}(6)_R\\
&\to \mathrm{SU}(3)_{LH}\otimes\mathrm{SU}(2)_L\otimes\mathrm{SU}(3)_c\otimes\mathrm{U}(1)_{Y_{(B-L)}}\otimes\mathrm{SU}(3)_{HR}\otimes\mathrm{SU}(2)_R\\
&\to \mathrm{SU}(2)_{LH}\otimes\mathrm{SU}(2)_L\otimes\mathrm{SU}(3)_c\otimes\mathrm{U}(1)_{Y_{(B-L)}}\otimes\mathrm{SU}(2)_{HR}\otimes\mathrm{SU}(2)_R
\end{aligned}$$

can also be included, suggesting the existence of a cascade decay of G.

The hierarchy $M \geq M' \geq M_R \gg M_L \sim 10^2$ GeV's, combined with the renormalization-group equations for this multistage descendant [four or more stages compared with two for SU(5) or three for SO(10)] is enough to cope with experimental results such as proton stability, the value for $\sin^2\theta_W(M_L)$, suppression of flavor-changing neutral currents, etc. Precise values for M, M', M_R, and $\sin^2\theta_W(M_L)$ are hard to pin down due to the multistage descendant and the fairly extensive particle spectrum around the GUT scale. In order to assure consistency with low-energy phenomenology we should assume $M_R \geq 10$ TeV's.

Even though there are many possible ways to break G down to $SU(3)_c \otimes U(1)_Q$, we will base our approach in two assumptions. (i) We use only Higgs bosons which develop vacuum expectation values (VEV's) and also couple, via Yukawa-type terms, to $\psi(108)$, the fermionic representation in the model. In this way we break the symmetry and give masses to the particles at the same time. (ii) Once condition (i) is satisfied, we look for the most economical and simplest possible combination of Higgs scalars and VEV's. These two assumptions produce what we call the minimal version of the model.

Now we outline the stages of the symmetry breaking. First we use $\langle \phi_1 \rangle = M$, where

$$\begin{aligned}\phi_1 = \phi_1(675) &= Z_3\phi_1(\overline{15},1,15)\\ &= \phi_1(\overline{[a,b]},1,[A,B])\\ &\quad + \phi_1(1,[\alpha,\beta],\overline{[A,B]})\\ &\quad + \phi_1([a,b],\overline{[\alpha,\beta]},1)\ . \end{aligned} \tag{9}$$

$a,b,c,\ldots; A,B,C,\ldots; \alpha,\beta,\gamma\ldots$ refer to $SU(6)_L$, $SU(6)_R$, and $SU(6)_c$ tensor indices respectively, and $[..]$ stands for antisymmetric permutation of the indices inside the brackets. The VEV's in ϕ_1 are in the directions $[a,b]=[1,6]=-[2,5]=\pm[3,4]$, $[A,B]$ similar to $[a,b]$ and $[\alpha,\beta]=[5,6]$.

$\langle \phi_1 \rangle$ breaks G down to G_{SP}, suggesting $SP(6)_L \otimes SP(6)_R \otimes U(1)_{(B-L)}$ as the group which unifies horizontal and electroweak interactions [11] at the scale M'.

Next let $\langle \phi_2 \rangle = M'$, where

$$\begin{aligned}\phi_2 = \phi_2(675) &= Z_3\phi_2(\overline{15},1,15)\\ &= \phi_2(\overline{[a,b]},1,[A,B])\\ &\quad + \phi_2(1,[\alpha,\beta],\overline{[A,B]})\\ &\quad + \phi_2([a,b],\overline{[\alpha,\beta]},1) \end{aligned} \tag{10}$$

with VEV's in the directions $[a,b]=[1,2]=[3,6]=-[4,5]$, $[A,B]$ similar to $[a,b]$ and $[\alpha,\beta]=[4,5]$.

Now $\langle \phi_2 \rangle$ breaks G_{SP} down to $SU(2)_L \otimes SU(3)_c \otimes U(1)_{Y_{(B-L)}} \otimes SU(2)_R$. Therefore the net effect of $\langle \phi_1 \rangle + \langle \phi_2 \rangle$ is to break G down to the left-right-symmetric extension of the SM.

As mentioned before the Higgs bosons and VEV's described above constitute the most economical and simplest way to break G down to the left-right-symmetric extension of the SM. Taking a detour we can see, for example, that the intermediate step $SU(2)_{LH} \otimes SU(2)_L \otimes SU(3)_c \otimes U(1)_{Y_{(B-L)}} \otimes SU(2)_{HR} \otimes SU(2)_R$ can be reached by the introduction of new scalars ϕ_2' instead of ϕ_2, where (Ref. [6]) $\phi_2' = \phi_2'(315) = Z_3\phi_2'(105,1,1)$.

Moreover, using only $\phi_1 + \phi_2$ there are several, more complicated ways to reach the left-right extension of the SM. Taking, for example, $[\alpha,\beta]=[4,5]=[5,6]$ in both steps above achieves the same breaking, but gives different masses (and mixing) to the gauge bosons associated with the broken generators. That we can orient the vacuum in a particular direction in order to break G in the simplest way possible is due to the freedom we have arising from the horizontal symmetry available.

The next step is to break $SU(2)_R \otimes U(1)_{Y_{(B-L)}}$ down to $U(1)_Y$ at an energy scale M_R (see Appendix C). Following our assumptions (i) and (ii) we do it by using

$$\begin{aligned}\phi_3 = \phi_3(675) &= Z_3\phi_3(\overline{15},1,15)\\ &= \phi_3(\overline{[a,b]},1,[A,B])\\ &\quad + \phi_3(1,[\alpha,\beta],\overline{[A,B]})\\ &\quad + \phi_3([a,b],\overline{[\alpha,\beta]},1)\ , \end{aligned} \tag{11}$$

where we take the VEV's in the most general direction available, without breaking $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$, as demanded by low-energy phenomenology. That is, we take

$$\langle \phi_3(\overline{[a,b]},1,[A,B]) \rangle = \langle \phi_3([a,b],\overline{[\alpha,\beta]},1) \rangle = 0$$

and

$$\langle \phi_3(1,[\alpha,\beta],\overline{[A,B]}) \rangle = M_R$$

for $[\alpha,\beta]=[4,6]$ and $[A,B]=[2,4]=[2,6]=[4,6]$.

As we will see in the following section, the Higgs boson and VEV's introduced so far produce masses for all the exotic fermions in the model. That is, when the breaking of G down to $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ is done, the following particles remain massless: the six quarks, three Dirac charged leptons and three Weyl neutral leptons. Those states are identified as the ordinary fermions.

The final stage of the breaking $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \to SU(3)_c \otimes U(1)_Q$, is achieved by using scalars belonging to $\phi_4 = \phi_4(108) = Z_3\phi_4(1,\overline{6},6)$. As we will see in detail in the following section there are 36 different directions for the VEV's in $\langle \phi_4 \rangle$, and in principle, we should take all of them as independent parameters due to the fact that we do not have the freedom to align the vacuum at the last step of the breaking.

As it is clear from the branching rules at the end of Sec. II A, ϕ_4 contains only $SU(2)_L$ doublets and singlets, and it plays the equivalent role of the ordinary Higgs doublet in the standard model.

Finally, let us mention that the Z_3 symmetry is broken by the VEV's of the Higgs scalars at the first step of the symmetry breaking.

IV. MASSES FOR FERMIONS

A. Masses for exotics

The Higgs bosons $\phi_1+\phi_2+\phi_3$ allow for the following G-invariant Yukawa coupling terms:

$$Z_3\psi(6,1,\overline{6})\psi(6,1,\overline{6})\sum_{i=1}^{3} h_i\phi_i(\overline{15},1,15)\ , \tag{12}$$

where naturalness demands that h_i, $i=1,2,3$ be of the same order and not very small to avoid fine-tuning. Let us analyze the mass terms in Eq. (12) generated by the VEV's introduced in Sec. III (see also Appendix D).

A close look to the mass matrices produced by $\langle\phi_1\rangle+\langle\phi_2\rangle$ in Eq. (12) reveals the following.

(i) All the exotic (charged and neutrals) leptons in $\psi(6,1,\overline{6})_L$ get superheavy masses of order M.

(ii) There is no mixing between the leptons in $\psi(6,1,\overline{6})_L$ and the leptons in $\psi(\overline{6},6,1)_L+\psi(1,\overline{6},6)_L$.

(iii) The 9×9 mass matrix for the charged leptons in $\psi(\overline{6},6,1)_L+\psi(1,\overline{6},6)_L$ has rank six. That is, three linear combinations of the charged leptons in (6) and (7) remain massless. We identify them as the known charged leptons.

(iv) The 18×18 mass matrix for the neutral leptons in $\psi(\overline{6},6,1)_L+\psi(1,\overline{6},6)_L$ has rank twelve. That is, six massless Weyl states related to three Dirac neutrinos remain massless.

(v) The six quarks remain massless.

(vi) All the masses generated by $\langle\phi_1\rangle+\langle\phi_2\rangle$ are $\Delta I_W=0$ Dirac-type masses, where I_W stands for the weak isospin related to $SU(2)_L$.

These facts are just the remarkable way how the survival hypothesis [12] works in the context of this specific model. So the masslessness of ordinary fermions is related to the fact that the left-right-symmetric extension of the SM is a chiral model which is not broken by $\langle\phi_1\rangle+\langle\phi_2\rangle$.

The massless leptons produced by $\langle\phi_1\rangle+\langle\phi_2\rangle$ in the notation of Eqs. (6) and (7) are the following.

(a) Massless charged leptons. $(VE_1+V'T_3)/v$; $(VE_3-V'T_2)/v$; $(VE_2+V'T_1)/v$; where $V=Mh_1$, $V'=M'h_2$, and $v=\sqrt{(V^2+V'^2)}$. Notice that their left-handed components belong to $\psi(\overline{6},6,1)_L$, so they are members of $SU(2)_L$ doublets, meanwhile their right-handed components belong to $\psi(1,\overline{6},6)_L$, so they are $SU(2)_L$ singlets, members of $SU(2)_R$ doublets. Because of this the three states above define a basis for the physical states (e,μ,τ). The six massive charged states orthogonal to the massless ones above have masses equal to v. A detailed discussion of this paragraph is presented in Appendix D.

(b) Massless neutral leptons. $(VE_1^0+V'T_3^0)/v$; $(VE_3^0-V'T_2^0)/v$; $(VE_2^0+V'T_1^0)/v$; $(VF_1^0+V'N_3^0)/v$; $(VF_3^0-V'N_2^0)/v$; and $(VF_2^0+V'N_1^0)/v$. Where again the first three are members of $SU(2)_L$ doublets and they define a basis for $(\nu_e,\nu_\mu,\nu_\tau)_L$, meanwhile the last three are $SU(2)_L$ singlets, members of $SU(2)_R$ doublets, so they define a basis for $(\nu_e^c,\nu_\mu^c,\nu_\tau^c)_L$. Again, the twelve massive Weyl states all have masses equal to v and pair to form Dirac masses.

Turning to the mass terms produced by $\langle\phi_3\rangle$, the algebra shows that they generate only Majorana masses for the neutral states in $\psi(1,\overline{6},6)_L$ (where $\nu_{e,L}^c,\nu_{\mu,L}^c,\nu_{\tau,L}^c$ are). That is, $\langle\phi_3\rangle$ produces Majorana masses of order M_R for the three right-handed neutrinos. So, the 18×18 mass matrix produced by $\langle\phi_1\rangle+\langle\phi_2\rangle+\langle\phi_3\rangle$ has rank fifteen, where we identify the eigenvectors of the three zero eigenvalues as a basis for the physical Weyl states $(\nu_e,\nu_\mu,\nu_\tau)_L$.

B. The VEV's of ϕ_4

As mentioned in Sec. III the final step of the breaking is achieved by using scalars $\phi_4=\phi_4(108)=Z_3\phi_4(1,\overline{6},6)=\phi_4(1,\overline{6},6)+\phi_4(\overline{6},6,1)+\phi_4(6,1,\overline{6})$. By arranging ϕ_4 as three 6×6 matrices, their VEV's are in the directions

$$\langle\phi_4(6,1,\overline{6})\rangle=\begin{bmatrix} K_{11} & 0 & K_{13} & 0 & K_{15} & 0\\ 0 & K_{22} & 0 & K_{24} & 0 & K_{26}\\ K_{31} & 0 & K_{33} & 0 & K_{35} & 0\\ 0 & K_{42} & 0 & K_{44} & 0 & K_{46}\\ K_{51} & 0 & K_{53} & 0 & K_{55} & 0\\ 0 & K_{62} & 0 & K_{64} & 0 & K_{66}\end{bmatrix}\ , \tag{13}$$

$$\langle\phi_4(\overline{6},6,1)\rangle=\begin{bmatrix} 0 & 0 & 0 & 0 & K'_{15} & 0\\ 0 & 0 & 0 & K'_{24} & 0 & K'_{26}\\ 0 & 0 & 0 & 0 & K'_{35} & 0\\ 0 & 0 & 0 & K'_{44} & 0 & K'_{46}\\ 0 & 0 & 0 & 0 & K'_{55} & 0\\ 0 & 0 & 0 & K'_{64} & 0 & K'_{66}\end{bmatrix}\ , \tag{14}$$

$$\langle\phi_4(1,\overline{6},6)\rangle=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & 0\\ 0 & K''_{42} & 0 & K''_{44} & 0 & K''_{46}\\ K''_{51} & 0 & K''_{53} & 0 & K''_{55} & 0\\ 0 & K''_{62} & 0 & K''_{64} & 0 & K''_{66}\end{bmatrix}\ , \tag{15}$$

where we have 36 independent parameters, in principle all of them different. Any information about those parameters should come from the Higgs potential.

We realize that to minimize the most general Higgs potential including ϕ_i, $i=1,2,3,4$ is an horrendous task. What we have done is to study the Higgs potential for each step of the symmetry breaking in isolation. We presented in Sec. III our results for the first three steps. We present here the results obtained for the last step of the breaking. Corrections to the results presented in this section are expected to be of order $(M_L/M_R)^2$ and smaller [13].

The most general ϕ_4 potential, Z_3 symmetric, with the discrete symmetry $\phi_4\leftrightarrow-\phi_4$ is

$$V(\phi_4)=Z_3[a\phi_4(\overline{6},1,6)\phi_4(6,1,\overline{6})+b\phi_4(6,1,\overline{6})\phi_4(6,1,\overline{6})\phi_4(\overline{6},1,6)\phi_4(\overline{6},1,6)$$
$$+c\phi_4(\overline{6},1,6)\phi_4(6,1,\overline{6})\phi_4(\overline{6},1,6)\phi_4(6,1,\overline{6})+d\phi_4(\overline{6},1,6)\phi_4(6,1,\overline{6})\phi_4(6,\overline{6},1)\phi_4(\overline{6},6,1)]\ , \tag{16}$$

where a,b,c, and d are real arbitrary parameters. When we substitute the 36 parameters of $\langle\phi_4\rangle$ in $V(\phi_4)$ and differentiate it with respect to each one of them we get, for an extreme, 36 simultaneous equations. Our analysis showed that (1) $\langle\phi_4(6,1,\overline{6})\rangle\neq 0$ only if

$$|K_{11}|=|K_{31}|=|K_{51}|=|K_{13}|=|K_{33}|$$
$$=|K_{53}|=|K_{15}|=|K_{35}|=|K_{55}|\equiv|K_D|\ ,$$

and

$$|K_{22}|=|K_{24}|=|K_{26}|=|K_{42}|=|K_{44}|$$
$$=|K_{46}|=|K_{62}|=|K_{64}|=|K_{66}|\equiv|K_U|\ ,$$

(2) $\langle\phi_4(\overline{6},6,1)\rangle\neq 0$ only if

$$|K'_{24}|=|K'_{26}|=|K'_{44}|=|K'_{46}|=|K'_{64}|=|K'_{66}|\equiv|K_E|\ ,$$

and (3) $\langle\phi_4(1,\overline{6},6)\rangle\neq 0$ only if

$$|K''_{42}|=|K''_{44}|=|K''_{46}|=|K''_{62}|=|K''_{64}|=|K''_{66}|\equiv|K_N|\ ,$$

where $K_D,K_U,K_E,K_N,K'_{15},K'_{35},K'_{55},K''_{51},K''_{53}$, and K''_{55} are functions of a,b,c, and d and they depend upon the particular direction chosen for the minimum of $V(\phi_4)$. For example, in the direction of the minimum where all the 36 parameters are different from zero we have that

$$|K_D|=|K_U|=\sqrt{2/3}|K_E|=\sqrt{2/3}|K_N|=|K_{c1}|/3$$
$$=|K_{c2}|/3=\tfrac{1}{6}\sqrt{-2a/(2b+c+2d)}\ , \tag{17}$$

where

$$|K_{c1}|=\sqrt{K'^2_{15}+K'^2_{35}+K'^2_{55}}$$

and

$$|K_{c2}|=\sqrt{K''^2_{51}+K''^2_{53}+K''^2_{55}}\ .$$

C. Masses for known fermions

With ϕ_4 the Yukawa-type term can be written

$$Z_3[\psi(\overline{6},6,1)_L\psi(1,\overline{6},6)_L]\phi_4(6,1,\overline{6})$$
$$=[\psi(\overline{6},6,1)_L\psi(1,\overline{6},6)_L]\phi_4(6,1,\overline{6})$$
$$+[\psi(6,1,\overline{6})_L\psi(\overline{6},6,1)_L]\phi_4(1,\overline{6},6)$$
$$+[\psi(1,\overline{6},6)_L\psi(6,1,\overline{6})_L]\phi_4(\overline{6},6,1)\ , \tag{18}$$

which leads to the following mass terms for fermions.

(1) $[\psi(6,1,\overline{6})_L\psi(\overline{6},6,1)_L]\langle\phi_4(1,\overline{6},6)\rangle$
$+[\psi(1,\overline{6},6)_L\psi(6,1,\overline{6})_L]\langle\phi_4(\overline{6},6,1)\rangle$.

In the vacuum direction where K_E and K_N are different from zero, and as mentioned at the end of Sec. II B, this term produces mixing of the exotic fermions in $\psi(6,1,\overline{6})_L$ with the ordinary fermions, contributing to the seesaw [8] and universal seesaw mechanisms [9] for neutral and charged leptons. These terms are thus responsible for lowering the known lepton masses compared with the quark masses.

(2) $[\psi(\overline{6},6,1)\psi(1,\overline{6},6)]\langle\phi_4(6,1,\overline{6})\rangle$.

In the vacuum direction where K_U and K_E are different from zero, this term produces four 3×3 mass matrices for ordinary fermions: one for the up quarks, one for the down quarks, one for the known charged leptons and one for known neutral leptons. Each one of those matrices is of the form

$$M_D=C\begin{pmatrix}1&1&1\\1&1&1\\1&1&1\end{pmatrix}\ , \tag{19}$$

where C is a constant proportional to K_U for the up quark and neutrino sectors and to K_D for the down quark and charged lepton sectors. These matrices are BCS-(flavor-democracy-)type mass matrices [14], which are rank one, with the only eigenvalue different from zero associated with the eigenvector $(1,1,1)/\sqrt{3}$. We identify those states as the particles in the third family. In this way we have a conspicuous realization of the horizontal survival hypothesis [15] built in a natural way in the context of this model. As we know, BCS (flavor-democracy) mass matrices for elementary fermions are a very sound starting point for solving the flavor problem [16]. Notice that using different Yukawa couplings h_i, $i=1,2,3,4$ for each Higgs sector ϕ_i does not change the form of matrix (19), which depends only upon h_4.

It seems to us that the vacuum direction where all the 36 parameters K_{ij}, K'_{ij}, and K''_{ij} are different from zero is not the direction chosen by nature. It is more likely that nature has chosen the vacuum direction where $K_D=0$ and the other 27 parameters are different from zero (with the appropriate constraints). If this is the case, then we have the following scenario (which we call the modify horizontal survival hypothesis).

(1) At the tree level only the t quark and ν_τ get masses via BCS-(flavor-democracy-)type mass matrices.

(2) The τ lepton gets its mass at the tree level via a universal seesaw mechanism [9].

(3) The smallness mass of the ν_τ neutrino is explained by the double seesaw mechanism produced by $\langle\phi_3+\phi_4\rangle$.

(4) All the other fermions get their masses as radiative corrections [17].

V. CONCLUSIONS

We have presented here a model based upon a simple gauge group which unifies flavors and nongravitational forces. This model has several notorious advantages with respect to other family GUT's such as SO(18),E_8: namely, it contains exactly three families of quarks and leptons; it does not include unwanted mirror fermions, etc. The exotic leptons introduced in order to have an anomaly-free model are vectorlike representations with respect to the SM, so, according to the survival hypothesis [12] they are very massive; they also mix naturally with the known leptons producing seesaw and universal seesaw mechanisms, two features used frequently in the literature in order to explain the hierarchical spectrum of masses of the known elementary fermions.

To show that this model is a realistic one there is still much tedious work to be done, such as a more careful analysis of the Higgs potential, the calculation of the radiative masses for the tree-level massless fermions, the calculation of the different mass scales via renormalization-group equations, the calculation of the mixing angles, the calculation of the strength of the double seesaw mechanism for ν_τ, etc. Our feeling is that those calculations deserve a try and that most of them are simpler than similar calculations in other family GUT models, due to the fact that our model is more economical than their competitors as far as the number of fermions and gauge bosons are concerned.

As we have discussed in the main text, our model includes in a natural way features such as the survival hypothesis, the seesaw and universal seesaw mechanisms, and the horizontal survival hypothesis (or alternatively the modified horizontal survival hypothesis).

It is also obvious in our model that the gauge bosons are either quark even or quark odd; therefore, they have well-defined values of baryon number (B) and lepton number (L). Thus, proton decay is absent at the tree level in the gauge sector due to the absence of gauge bosons lighter than the proton. So, our model may fit a lower GUT scale ($M \ll 10^{15}$ GeV's) than most of the popular GUT models. This fact may have important consequences for the upcoming Superconducting Super Collider.

Even if our model is not fully realistic, there are still many alternatives of it worth being explored and that may contain realistic physics. We note a few examples.

(i) It could happen that the vacuum in this model is aligned in the direction where all the 36 parameters in $\langle \phi_4 \rangle$ are different from zero. Then the BCS (flavor-democracy) mass matrices generates the horizontal survival hypothesis for the third family with masses of the order of a few GeV's for the fermions. Then the largest mass for the t quark must find an explanation outside the context of GUT's of flavors and forces.

(ii) It could happen that the final stage of the symmetry breaking is achieved by Higgs scalars which do not couple to fermions via Yukawa-type terms [the simplest of them being $\phi_5 = Z_3\phi_5(6,6,1)$]. In this approach the proton is completely stable because its decay is also absent in the Higgs sector. Then masses for the known fermions should be generated as radiative corrections. Again the largest mass for the t quark must find an explanation outside the model.

(iii) Last but not least, it may happen that the final step of the symmetry breaking is not accomplished by Higgs scalars but it is dynamically in its origin. If such is the case then masses for ordinary fermions are generated by the radiative corrections of the extended technicolor forces due to the vacuum condensates (in this model condensates with the exotic leptons are available).

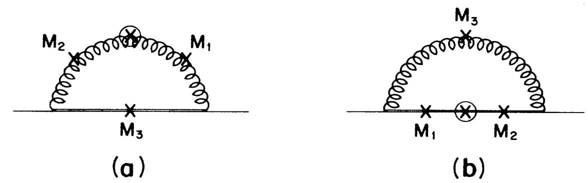

FIG. 1. One-loop diagrams contributing to the mass matrices of ordinary fermions.

To conclude let us say a few words about the radiative corrections produced by the gauge bosons.

The one-loop diagrams contributing to the radiative quark and lepton masses of known particles are depicted in Figs. 1(a) and 1(b). In those diagrams the wavy line refers to a heavy gauge boson, the internal double line to an exotic superheavy lepton, the symbol $\otimes$ stands for a mixing of the gauge bosons or exotic leptons, the symbol $\times$ stands for a mass insertion, and the external lines are related to ordinary quarks and leptons.

The mixing between the superheavy particles makes both diagrams finite and one gets the following contribution to the ordinary fermion masses:

$$\delta m = P[M_1^2/(M_1^2 - M_3^2)\ln(M_1/M_3) - M_2^2/(M_2^2 - M_3^2)\ln(M_2/M_3)] , \qquad (20)$$

where the meaning of the masses M_1, M_2, and M_3 is as depicted in the diagrams 1(a) and 1(b), P is a proportionality constant of the order of M_3 for diagram 1(a) times the mixing present in $\otimes$ and of the order of $\sqrt{M_1 M_2}$ for diagram 1(b) times the mixing. Such a mixing is always a function of K_N and K_E.

The mass matrices generated by the first-order radiative corrections for a particular sector are of the BCS (flavor-democracy) type. In this way, radiative masses are generated in a cascade way [18].

ACKNOWLEDGMENTS

This work was supported in part by ICFES in Colombia, and CONACyT in Mexico.

APPENDIX A

In this short appendix we show how the three-family extension of the Pati-Salam model [5] rests heavily on the existence of mirror fermions, and calculate in a simple way the Weinberg angle for such a model. Our aim is to point out the differences between the Pati-Salam-type models and our model and to point to the origin of those differences.

The gauge group for the three-family extension of the

Pati-Salam model is

$$G' = SU(6)_L \otimes SU(6)_R \otimes SU(6)_{CL} \otimes SU(6)_{CR} \times Z_4 .$$

The known fermions are then included in

$$\psi'(72) = \psi_1'(\bar{6},1,6,1) + \psi_2'(1,6,1,\bar{6}) .$$

As far as the particle content of $\psi'(72)$ is concerned it is equivalent to the particle content of $\psi(\bar{6},6,1)+\psi(1,\bar{6},6)$ in Eqs. (6) and (7). It is clear that $\psi'(72)$ is not anomaly-free; neither is it an irrep of G' (it is not Z_4 symmetric). The corresponding irrep of G' is

$$\begin{aligned}\psi'(144) &= Z_4\psi'(\bar{6},1,6,1)\\ &= \psi_1'(\bar{6},1,6,1)+\psi_2'(1,6,1,\bar{6})+\psi_3'(6,1,\bar{6},1)\\ &\quad +\psi_4'(1,\bar{6},1,6) .\end{aligned}$$

Trivially $\psi'(144)$ is free of anomalies. Also $\psi_3'+\psi_4'$ are nothing else but the mirror fermions of $\psi_1'+\psi_2'$.

Now to calculate the Weinberg angle for G' we should calculate the traces for $\psi'(144)$ and plug them into Eq. (8). But as far as the calculation of the Weinberg angle is concerned, it is simpler to use $\psi'(72)$ instead of $\psi'(144)$, due to the fact that the contribution of the mirror fermions $\psi_3'+\psi_4'$ to Eq. (8) amounts only to a factor of 2 in the numerator and another factor of 2 in the denominator which therefore cancel out. When we apply Eq. (8) to $\psi_1'+\psi_2'$ [or equivalently to $\psi(\bar{6},6,1)+\psi(1,\bar{6},6)$] we get $\sin^2\theta_W(M)=9/28$ as was obtained in a different way in Ref. [5].

APPENDIX B

In this appendix we derive g_{2L} and g_{2R} in terms of g. Also the photon field A^μ is calculated as a function of $W_L^{\mu,0}$, $W_R^{\mu,0}$, and $B_{Y_{(B-L)}}^{\mu,0}$ the gauge boson associated with the hypercharge $Y_{(B-L)}$. As a by-product of this calculation the Weinberg angle shows up, and $B_Y^{\mu,0}$, the gauge boson of the Glashow-Weinberg-Salam (GWS) model [19], emerges immediately.

The covariant derivative for this model is given by

$$\begin{aligned}(D_\mu)^{a,\alpha,A}_{b,\beta,B} &= \delta^a_b\delta^\alpha_\beta\delta^A_B\partial_\mu\\ &\quad + ig\delta^\alpha_\beta\delta^A_B(\mathbf{A}^L_\mu\cdot\Lambda_L)^a_b + ig\delta^a_b\delta^A_B(\mathbf{A}^C_\mu\cdot\Lambda_c)^\alpha_\beta\\ &\quad + ig\delta^\alpha_\beta\delta^a_b(\mathbf{A}^R_\mu\cdot\Lambda_R)^A_B ,\end{aligned} \tag{B1}$$

where $\mathbf{A}^L_\mu$, $\mathbf{A}^C_\mu$, $\mathbf{A}^R_\mu$ are the gauge bosons, and Λ_L, Λ_c, and Λ_R are the generators associated with $SU(6)_L$,$SU(6)_c$, and $SU(6)_R$, respectively [in particular $\mathbf{A}^C_\mu\cdot\Lambda_c$ is the 6×6 matrix presented in Eq. (3)].

For the analysis that concern us here it is enough to consider only the following seven gauge fields, out of the 108 contained in G: $W^+_{\mu,L}, W^-_{\mu,L}, W^0_{\mu,L}, W^+_{\mu,R}, W^-_{\mu,R}, W^0_{\mu,R}$, and $B^{\mu,0}_{Y_{(B-L)}}$.

The conventions and normalization conditions stated in the main text imply that we may write

$$W^L\cdot\Lambda_L = \frac{1}{\sqrt{2}}\begin{bmatrix} W_L^0/\sqrt{(6)} & W_L^+/\sqrt{(3)} \\ W_L^-/\sqrt{(3)} & -W_L^0/\sqrt{(6)}\end{bmatrix}\otimes I_3 . \tag{B2}$$

$W^R\cdot\Lambda_R$ can be written in a similar way with the replacement $L\to R$. Finally, from Eq. (3) we read

$$\mathbf{B}^\mu_{Y_{(B-L)}}\cdot\Lambda_c = B^\mu_{Y_{(B-L)}}\,\mathrm{diag}(1,1,1,-3,3,-3)/\sqrt{(60)} . \tag{B3}$$

The interaction Lagrangian of the fermions in $\psi(108)$ with the gauge bosons is given by

$$\begin{aligned}Z_3\bar{\psi}(\bar{6},6,1)D_\mu\gamma^\mu\psi(\bar{6},6,1) = g[&\bar{\psi}(\bar{6},6,1)(\mathbf{B}^\mu_{Y_{(B-L)}}\cdot\Lambda_c)\gamma_\mu\psi(\bar{6},6,1)^T\\ &-\bar{\psi}(\bar{6},6,1)^T(\mathbf{W}^L_\mu\cdot\Lambda^L)\gamma^\mu\psi(\bar{6},6,1)+\bar{\psi}(1,\bar{6},6)(\mathbf{W}^R_\mu\cdot\Lambda^R)\gamma^\mu\psi^T(1,\bar{6},6)\\ &-\bar{\psi}(1,\bar{6},6)^T(\mathbf{B}^\mu_{Y_{(B-L)}}\cdot\Lambda_c)\gamma_\mu\psi(1,\bar{6},6)+\bar{\psi}(6,1,\bar{6})(\mathbf{W}^L_\mu\cdot\Lambda^L)\gamma^\mu\psi(6,1,\bar{6})^T\\ &-\bar{\psi}(6,1,\bar{6})^T(\mathbf{W}^R_\mu\cdot\Lambda^R)\gamma^\mu\psi(6,1,\bar{6})+\cdots] ,\end{aligned} \tag{B4}$$

where the minus signs inside the brackets are due to the fact that, in those terms, the covariant derivative acts upon the complex-conjugate fundamental irrep (instead of the fundamental one).

Using (when necessary) the identity $\bar{\chi}^c_L\gamma^\mu\xi^c_L = -\bar{\xi}_R\gamma^\mu\chi_R$, we extract from the former Lagrangian the following terms.

(1) The interaction Lagrangian which contains $W_L^\mp$ is

$$\begin{aligned}L^L_{W^\pm} = \frac{g}{\sqrt{6}}W^-_{\mu,L}\Bigg[&\sum_{i=1}^3\sum_{\alpha=1}^3 \overline{q^{-1/3}_{i,\alpha,L}}\gamma^\mu q^{2/3}_{i,\alpha,L}\\ &+\sum_{\alpha=1}^3(\overline{E^-_{\alpha,L}}\gamma^\mu E^0_{\alpha,L}+\overline{T^-_{\alpha,L}}\gamma^\mu T^0_{\alpha,L}+\overline{L^0_{\alpha,L}}\gamma^\mu L^+_{\alpha,L})\Bigg]+\text{H.c.}+\cdots\equiv\frac{g}{\sqrt{6}}W_L^-J_L^+ + \text{H.c.} ,\end{aligned} \tag{B5}$$

where α is the color index (three hadronic and three leptonic colors), and i is a flavor index. Since G includes $SU(2)_L\otimes U(1)_Y$, the GWS gauge group, then $L^L_{W^\pm} = g_{2L}W_L^-J_L^+/\sqrt{2}+\text{H.c.}$, where g_{2L} is the gauge coupling constant for $SU(2)_L$. Then we read immediately $g_{2L}=g/\sqrt{3}$ as stated in the main text.

The left-right symmetry of the model implies, in the same way, $g_{2R}=g/\sqrt{3}$.

(2) The Lagrangian which contains the coupling of $W^0_{L,R}$ and $B^0_{Y_{(B-L)}}$ to the neutral fermions is

$$
\begin{aligned}
L^0_{N's}=g\sum_{\alpha=1}^{3}\Bigg[&\frac{W^0_{L,\mu}}{\sqrt{12}}(\overline{L^0_{\alpha,L}}\gamma^\mu L^0_{\alpha,L}-\overline{E^0_{\alpha,L}}\gamma^\mu E^0_{\alpha,L}-\overline{T^0_{\alpha,L}}\gamma^\mu T^0_{\alpha,L})\\
&-\frac{W^0_{R,\mu}}{\sqrt{12}}(\overline{S^0_{\alpha,L}}\gamma^\mu S^0_{\alpha,L}-\overline{F^0_{\alpha,L}}\gamma^\mu F^0_{\alpha,L}-\overline{N^0_{\alpha,L}}\gamma^\mu N^0_{\alpha,L})\\
&-3\frac{B^0_{Y_{(B-L)},\mu}}{\sqrt{60}}(\overline{L^0_{\alpha,L}}\gamma^\mu L^0_{\alpha,L}-\overline{E^0_{\alpha,L}}\gamma^\mu E^0_{\alpha,L}-\overline{T^0_{\alpha,L}}\gamma^\mu T^0_{\alpha,L}\\
&\qquad+\overline{S^0_{\alpha,L}}\gamma^\mu S^0_{\alpha,L}-\overline{F^0_{\alpha,L}}\gamma^\mu F^0_{\alpha,L}-\overline{N^0_{\alpha,L}}\gamma^\mu N^0_{\alpha,L})\Bigg]+\cdots\ .
\end{aligned}
\tag{B6}
$$

According to Eq. (5) the photon field A^μ must be written as $A^\mu=\alpha_1 W_L^{0,\mu}+\alpha_2 W_R^{0,\mu}+\alpha_3 B^{0,\mu}_{Y_{(B-L)}}$. Since it is obvious that $L^0_{N's}$ does not contain A^μ, then we must have that $\alpha_1=\alpha_2=3\alpha_3/\sqrt{5}$, with $|\alpha_1|=3/\sqrt{23}$ in order to have the appropriate normalization. Then the photon field is, up to a phase factor,

$$
A^\mu=\frac{3}{\sqrt{23}}\left[W_L^{0,\mu}+W_R^{0,\mu}+\frac{\sqrt{5}}{3}B^{0,\mu}_{Y_{(B-L)}}\right]\ .
\tag{B7}
$$

Since $A^\mu=\sin\theta_W W_L^{0,\mu}+\cos\theta_W B_Y^{0,\mu}$ we immediately read that $\sin^2\theta_W=9/23$ as calculated in a different way in the main text. Also we have that

$$
B_Y^{0,\mu}=\frac{3}{\sqrt{14}}\left[W_R^{0,\mu}+\frac{\sqrt{5}}{3}B^{0,\mu}_{Y_{(B-L)}}\right]
\tag{B8}
$$

is the gauge boson associated with the hypercharge of the GWS model.

APPENDIX C

In this appendix we carry out explicitly the algebra involved in the breaking of $SU(2)_R\otimes U(1)_{Y_{(B-L)}}$ down to $U(1)_Y$ arising from $\langle\phi_3\rangle$. What we pretend here is to give an example of the kind of algebra implicit in Sec. III. As a bonus we obtain again $B_Y^{\mu,0}$ in a different way. Contrary to the rest of the paper where most of the tensor indices and the traces are implicit, we display here all of them.

We are interested in calculating $\mathrm{tr}(D^\mu\langle\phi_3\rangle)^\dagger(D_\mu\langle\phi_3\rangle)$, where D^μ is the covariant derivative defined in (B1) and ϕ_3 is defined in Eq. (11). Notice that $\langle\phi_3\rangle$ has four tensor indices, i.e.: $\langle\phi_3\rangle^{[\alpha,\beta]}_{[A,B]}$. Then,

$$
\begin{aligned}
\mathrm{tr}(D^\mu\langle\phi_3\rangle)^\dagger(D_\mu\langle\phi_3\rangle)=g^2[&-\langle\phi_3\rangle^{[B,A]}_{[\gamma,\alpha]}(\not{A}^C_\mu)^\gamma_\beta-\langle\phi_3\rangle^{[B,A]}_{[\beta,\gamma]}(\not{A}^C_\mu)^\gamma_\alpha+(\not{A}^R_\mu)^B_D\langle\phi_3\rangle^{[D,A]}_{[\beta,\alpha]}+(\not{A}^R_\mu)^A_D\langle\phi_3\rangle^{[B,D]}_{[\beta,\alpha]}]\\
&\times[(\not{A}^\mu_C)^\alpha_\delta\langle\phi_3\rangle^{[\delta,\beta]}_{[A,B]}+(\not{A}^\mu_C)^\beta_\delta\langle\phi_3\rangle^{[\alpha,\delta]}_{[A,B]}-\langle\phi_3\rangle^{[\alpha,\beta]}_{[A,F]}(\not{A}^\mu_R)^F_B-\langle\phi_3\rangle^{[\alpha,\beta]}_{[F,B]}(\not{A}^\mu_R)^F_A]\ ,
\end{aligned}
\tag{C1}
$$

where $\not{A}^c_\mu=\mathbf{A}^C_\mu\cdot\Lambda_c$ and $\not{A}^R_\mu=\mathbf{A}^R_\mu\cdot\Lambda_R$ and summation over all the indices is understood. There are three different types of terms T_i in (C1): namely,

$$
\begin{aligned}
T_1&=g^2[-\langle\phi_3\rangle^{[B,A]}_{[\gamma,\alpha]}(\not{A}^C_\mu)^\gamma_\beta-\langle\phi_3\rangle^{[B,A]}_{[\beta,\gamma]}(\not{A}^C_\mu)^\gamma_\alpha][(\not{A}^\mu_c)^\alpha_\delta\langle\phi_3\rangle^{[\delta,\beta]}_{[A,B]}+(\not{A}^\mu_c)^\beta_\delta\langle\phi_3\rangle^{[\alpha,\delta]}_{[A,B]}]\\
&=12g^2M_R^2[\mathrm{tr}(M_1\not{A}^\mu_C\not{A}^C_\mu M_1)+\mathrm{tr}(M_1\not{A}^\mu_C M_1\not{A}^{C,T}_\mu)]\ ,
\end{aligned}
\tag{C2}
$$

where we summed over A,B such that $[A,B]=-[B,A]=[2,4]=[2,6]=[4,6]$, and M_1 is the matrix

$$
M_1=\begin{bmatrix}
0&0&0&0&0&0\\
0&0&0&0&0&0\\
0&0&0&0&0&0\\
0&0&0&0&0&1\\
0&0&0&0&0&0\\
0&0&0&-1&0&0
\end{bmatrix}\ ,
\tag{C3}
$$

$$
\begin{aligned}
T_2&=g^2[(\not{A}^R_\mu)^B_D\langle\phi_3\rangle^{[D,A]}_{[\beta,\alpha]}+(\not{A}^R_\mu)^A_D\langle\phi_3\rangle^{[B,D]}_{[\beta,\alpha]}][-\langle\phi_3\rangle^{[\alpha,\beta]}_{[A,F]}(\not{A}^\mu_R)^F_B-\langle\phi_3\rangle^{[\alpha,\beta]}_{[F,B]}(\not{A}^\mu_R)^F_A]\\
&=4g^2M_R^2[\mathrm{tr}(M_2\not{A}^\mu_R\not{A}^R_\mu M_2)+\mathrm{tr}(M_2\not{A}^\mu_R M_2\not{A}^{R,T}_\mu)]\ ,
\end{aligned}
\tag{C4}
$$

where we summed over α,β such that $[\alpha,\beta]=[4,6]=-[6,4]$, and M_2 is the matrix

$$M_2=\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & -1 & 0 & 0 \end{bmatrix}; \tag{C5}$$

and

$$\begin{aligned} T_3 &= g^2[\langle\phi_3\rangle^{[B,A]}_{[\gamma,\alpha]}(\not{\mathbf{A}}^c_\mu)^\gamma_\beta+\langle\phi_3\rangle^{[B,A]}_{[\beta,\gamma]}(\not{\mathbf{A}}^c_\mu)^\gamma_\alpha][\langle\phi_3\rangle^{[\alpha,\beta]}_{[A,F]}(\not{\mathbf{A}}^\mu_R)^F_B\langle\phi_3\rangle^{[\alpha,\beta]}_{[F,B]}(\not{\mathbf{A}}^\mu_R)^F_A]+\text{H.c.} \\ &= -4g^2[(\mathbf{A}^C_\mu\cdot\Lambda_c)^4_4+(\mathbf{A}^C_\mu\cdot\Lambda_c)^6_6]\text{tr}(M_2\not{\mathbf{A}}^\mu_R M_2)+\text{H.c.} \\ &= g^2\left[24\frac{B_{Y_{(B-L)},\mu}}{\sqrt{60}}+4\frac{B_{Y',\mu}}{\sqrt{40}}+4\frac{B_{Y'',\mu}}{\sqrt{24}}\right]\text{tr}(M_2\not{\mathbf{A}}^\mu_R M_2)+\text{H.c.}\ , \end{aligned} \tag{C6}$$

where again we have summed over $[\alpha,\beta]=[4,6]=-[6,4]$, the antisymmetry of M_2 was used and the diagonal elements in Eq. (3) have been written down explicitly.

The next step is to evaluate the traces in T_1, T_2, and T_3. The general result is not very illuminating; so let us work in the approximation $M \geq M' \gg M_R (\gg M_L)$, and concentrate only on the terms which include $W^{\mu,\pm}_R$, $W^{0,\mu}_R$, and $B^{\mu,0}_{Y_{(B-L)}}$. Using (B2) with the replacement $L \to R$, and (B3), we write

$$\sum_{i=1}^{3} T_i = -8\frac{M_R^2 g^2}{\sqrt{2}}\left[W^{0,2}_R+W^+_R W^-_R+\frac{9}{5}B^{0,2}_{Y_{(B-L)}} -\frac{3}{\sqrt{5}}W^0_R B^0_{Y_{(B-L)}} -\frac{3}{\sqrt{5}}B^0_{Y_{(B-L)}}W^0_R\right]+\cdots\ . \tag{C7}$$

Neglecting mixing with extra (heavy) gauge bosons, we may write the following mass matrix for the neutral fields, in the basis $(W^0_R, B^0_{Y_{(B-L)}})$:

$$M^2=-\frac{8M_R^2 g^2}{\sqrt{2}}\begin{bmatrix} 1 & -3/\sqrt{5} \\ -3/\sqrt{5} & 9/5 \end{bmatrix} \tag{C8}$$

which has eigenvalues 0 and $-56\sqrt{2}M_R^2 g^2/5$. The eigenvector associated with the eigenvalue zero is

$$B^{0,\mu}_Y=\frac{3}{\sqrt{14}}\left[W^{0,\mu}_R+\frac{\sqrt{5}}{3}B^{0,\mu}_{Y_{(B-L)}}\right], \tag{C9}$$

which is nothing else but the gauge boson associated with the hypercharge of the GWS model (not broken by $\langle\phi_3\rangle$) obtained in a different way than in Appendix B.

From Eqs. (C7) and (C8) it is also simple to calculate

$$\frac{M^2_{ZR}}{M^2_{WR}}=\frac{28}{5}=\frac{\cos^2\theta_W}{\cos 2\theta_W} \tag{C10}$$

as it should be [20], since the mass terms produced by $\langle\phi_3\rangle$ are of the $\Delta I_R=1$ type.

Finally let us say that the conclusions in this appendix are independent of the way we orient the vacuum at this particular stage of the symmetry breaking. For example, if we orient $\langle\phi_3\rangle$ such that $\langle\phi_3(\overline{[a,b]},1,\underline{[A,B]})\rangle=\langle\phi_3([a,b],\overline{[\alpha,\beta]},1)\rangle=0$, and $\langle\phi_3(1,[4,6],\overline{[4,6]})\rangle=M_R$, our results read

$$T_1'=4g^2M_R^2[\text{tr}(M_1\not{\mathbf{A}}^\mu_c\not{\mathbf{A}}^C_\mu M_1)+\text{tr}(M_1\not{\mathbf{A}}^\mu_c M_1\not{\mathbf{A}}^{C,T}_\mu)]\ , \tag{C11}$$

$$T_2'=4g^2M_R^1[\text{tr}(M_1\not{\mathbf{A}}^\mu_R\not{\mathbf{A}}^R_\mu M_1)+\text{tr}(M_1\not{\mathbf{A}}^\mu_R M_1\not{\mathbf{A}}^{R,T}_\mu)]\ , \tag{C12}$$

$$T_3'=g^2\left[24\frac{B_{Y_{(B-L)},\mu}}{\sqrt{60}}+4\frac{B_{Y',\mu}}{\sqrt{40}}+4\frac{B_{Y'',\mu}}{\sqrt{24}}\right]\text{tr}(M_1\not{\mathbf{A}}^\mu_R M_1) +\text{H.c.} \tag{C13}$$

And after evaluating the traces we get $\sum_{i=1}^3 T_i'=(\sum_{i=1}^3 T_i)/3$; and $M'^2=M^2/3$, without changing the results (C9) and (C10).

APPENDIX D

In this appendix we show how the known leptons appear as linear combinations of the leptons in $\psi(\overline{6},6,1)_L+\psi(1,\overline{6},6)_L$ and how the exotic leptons in the same representation gain masses in the way predicted by the survival hypothesis.

According to Eq. (12) we want to calculate

$$\begin{aligned} M_F &= \psi(\overline{6},6,1)_L^T C\psi(\overline{6},6,1)_L[h_1\langle\phi_1(15,\overline{15},1)\rangle+h_2\langle\phi_2(15,\overline{15},1)\rangle] \\ &\quad+\psi(1,\overline{6},6)_L^T C\psi(1,\overline{6},6)_L[h_1\langle\phi_1(1,15,\overline{15})\rangle+h_2\langle\phi_2(1,15,\overline{15})\rangle] \end{aligned} \tag{D1}$$

under the assumptions $\langle\phi_1\rangle = M, \langle\phi_2\rangle = M'$, and $h_1 \sim h_2 \simeq 1$. In (D1) $C = -C^T$ is the charge-conjugation operator.

When we calculate the traces implicit in (D1) for the directions of the VEV's described in Eqs. (9) and (10) we get

$$\begin{aligned} M_F^{\rm ch} = {} & Mh_1(L_{2,L}^{+;T}CT_{2,L}^{-} - L_{3,L}^{+;T}CT_{1,L}^{-} - L_{1,L}^{+;T}CT_{3,L}^{-} + T_{2,L}^{+;T}CL_{2,L}^{-} - T_{3,L}^{+;T}CL_{1,L}^{-} - T_{1,L}^{+;T}CL_{3,L}^{-}) \\ & + M'h_2(L_{2,L}^{+;T}CE_{3,L}^{-} + L_{1,L}^{+;T}CE_{1,L}^{-} + L_{3,L}^{+;T}CE_{2,L}^{-} + E_{1,L}^{+;T}CL_{1,L}^{-} + E_{2,L}^{+;T}CL_{3,L}^{-} + E_{3,L}^{+;T}CL_{2,L}^{-}) + \text{H.c.} \end{aligned} \tag{D2}$$

The 18×18 mass matrix for the charged fermions in the basis $(E_1^-, E_2^-, E_3^-, L_1^-, L_2^-, L_3^-, T_1^-, T_2^-, T_3^-, E_1^+, E_2^+, E_3^+, L_1^+, L_2^+, L_3^+, T_1^+, T_2^+, T_3^+)$ is of the form

$$M^{\rm ch} = \begin{bmatrix} 0_{9\times9} & M_{9\times9} \\ M_{9\times9}^T & 0_{9\times9} \end{bmatrix}, \tag{D3}$$

where $0_{9\times9}$ is the zero 9×9 matrix, and $M_{9\times9}$ is the matrix

$$M_{9\times9} = \begin{bmatrix} 0 & 0 & 0 & V' & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & V' & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & V' & 0 & 0 & 0 & 0 \\ V' & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -V \\ 0 & 0 & V' & 0 & 0 & 0 & 0 & V & 0 \\ 0 & V' & 0 & 0 & 0 & 0 & -V & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -V & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & V & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -V & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{D4}$$

with $V = Mh_1$ and $V' = M'h_2$. We have to diagonalize $(M^{\rm ch})^2$, but since (D4) is by assumption a real matrix, we can diagonalize the 9×9 matrix $(M_{9\times9})^2$ instead of the 18×18 matrix $(M^{\rm ch})^2$. The algebra shows the following.

(1) $(M_{9\times9})^2$ is a rank six matrix, so it has three eigenvalues equal to zero.

(2) The only eigenvalue different from zero is $(V^2 + V'^2)$ which is six times degenerate.

(3) The three eigenvectors associated with the zero eigenvalues are $(VE_1 + V'T_3)/v, (VE_3 - V'T_2)/v$, and $(VE_2 + V'T_1)/v$; with $v = \sqrt{V^2 + V'^2}$. As stated in the main text these states define a basis for the physical states (e, μ, τ).

(4) The six eigenvectors associated with the nonzero eigenvalue are $(V'E_1 \pm vL_1 - VT_3)/\sqrt{2}v$; $(V'E_2 \pm vL_3 - VT_1)/\sqrt{2}v$; $(V'E_3 \pm vL_2 + VT_2)/\sqrt{2}v$. Since these states are degenerate they are not necessarily the physical states. The physical states are appropriate linear combinations of them. In accord with the survival hypothesis we take as the physical states: $L_1, L_2, L_3, (V'E_1 - VT_3)/v, (V'E_2 - VT_1)/v$ and $(V'E_3 + VT_2)/v$. Now using the previous arguments we reorganize $M_F^{\rm ch}$ in the following way:

$$\begin{aligned} M_F^{\rm ch} = v \Bigg[& L_{1,L}^{+;T}C\frac{V'E_{1,L}^{-} - VT_{3,L}^{-}}{v} + L_{2,L}^{+;T}C\frac{V'E_{3,L} + VT_{2,L}^{-}}{v} + L_{3,L}^{+;T}C\frac{V'E_{2,L}^{-} - VT_{1,L}^{-}}{v} \\ & + \frac{(V'E_{1,L}^{+} - VT_{3,L}^{+})^T}{v}CL_{1,L}^{-} + \frac{(V'E_{3,L}^{+} + VT_{2,L}^{+})^T}{v}CL_{2,L}^{-} + \frac{(V'E_{2,L}^{+} - VT_{1,L}^{+})^T}{v}CL_{3,L}^{-} \Bigg] + \text{H.c.} \end{aligned} \tag{D5}$$

Notice in (D5) that the first three mass terms include only spinors in $\psi(\overline{6},6,1)_L$ and the last three terms include only spinors in $\psi(1,\overline{6},6)_L$. But all the terms are $\Delta I_W = 0$ Dirac mass terms as they should be, since $SU(2)_L$ is not broken by $\langle\phi_1\rangle + \langle\phi_2\rangle$. As mentioned in the main text this is how the survival hypothesis enters in the context of this specific model.

For the neutral states we choose the basis $(E_1^0, E_2^0, E_3^0, S_1^0, S_2^0, S_3^0, T_1^0, T_2^0, T_3^0, F_1^0, F_2^0, F_3^0, L_1^0, L_2^0, L_3^0, N_1^0, N_2^0, N_3^0)$. In this basis the mass matrices for the neutral fermions are identical to (D3) and (D4) and similar conclusions follow.

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