Nonlinear realization of heavy fermions and heavy-top-quark effects in bosonic vertices

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(Received 24 May 1991)

We show that in the standard model, when the mass of the top quark becomes very large, compared to other mass scales and external momenta, constraints develop in the field system. The top-quark field becomes nonlinearly realized. $SU(2)\otimes U(1)$ remains a symmetry at the S-matrix level. A general formulation for the one-light-particle-irreducible Green's functional Γ_{1LPI} and the effective Lagrangian L_{eff} is presented in the setting of the external field technique and derivative expansion. The bosonic part of Γ_{1LPI} and L_{eff} is explicitly constructed. It encompasses all the top-quark effects for all low-energy bosonic processes. (Vertices with external fermions as well will be reported shortly.) Examples are given to show that our approach easily reproduces known results. Wess-Zumino terms due to the top quark are also given.

I. INTRODUCTION

Two of the present authors (H.S. and Y.-P.Y.) wrote a brief article [1] outlining a field-theoretical method by which one can succinctly collect *all* the virtual top-quark effects in *all* the low-energy processes in the form of an effective Lagrangian. In that Letter, results of the purely bosonic vertices were displayed in an ungauged model with one quark doublet. Since then, the lower limit [2] of the top-quark mass has been repeatedly raised, whence this previously exploratory program has gained even more phenomenological significance.

We have in the mean time put in the gauge fields of the standard electroweak model [3] and completed a major portion of this program. Here is then the first of a series of articles in which we shall detail our endeavor and elucidate the conciseness and efficiency of this procedure. Processes with external bosons only will be considered here. Results with external fermions as well will be reported soon. As a reassuring check of our formalism, we find full agreement with the known calculations, to which they have been compared so far.

As practitioners in radiative corrections will appreciate, one is not *a priori* guaranteed large effects in hunting for processes which may seem particularly sensitive to certain parameters, such as a large top-quark mass. It apparently takes insight, guesswork, fortitude, and even a little luck. If possible, it would help if one could devise a program to perform a single comprehensive calculation, but which would incorporate all the virtual top-quark effects for all the low-energy processes. After this was carried through, one could then scan the results and isolate the processes which are most interesting.

That such a method exists can be inferred from the nonlinear σ model, where it has been shown that all the large Higgs-boson-mass effects can be isolated [4]. It will become clear that a parallel development in methodology to locate the top-quark effects is at hand. The initial point for this is to argue that in the heavy top-quark limit, the top-quark field is nonlinearly realized.

Heuristically, it may help to recall the physical mechanism for the nonlinear realization in the scalar sector: the self-coupling of the Higgs physical field σ and the unphysical pseudo Goldstone field $\boldsymbol{\phi}$ in the standard model gives a potential term $\approx\lambda(\sigma^2+\boldsymbol{\phi}^2)^2/8$, which in turn leads to the mass squared of the Higgs field $M_H^2\approx\lambda v^2$. Here $v=\langle\sigma\rangle$ is the vacuum expectation value. Because v is fixed by the weak scale to be ≈ 250 GeV, a heavy Higgs boson corresponds to a large positive λ. In the mathematical limit that λ tends to be extremely large, the potential is locked in step to become unphysically large. Unless a constraint is developed, excitation becomes impossible. We have (Ref. [4])

$$\sigma^2+\boldsymbol{\phi}^2=v^2 \quad\text{or}\quad \sigma=\sqrt{v^2-\boldsymbol{\phi}^2}\ , \tag{1.1}$$

which is the condition for a nonlinear realization.

A top quark carries a Yukawa coupling

$$H(\bar{t}_L,\bar{b}_L)\begin{pmatrix}\phi^0\\ \phi^-\end{pmatrix}t_R\ , \tag{1.2}$$

with the bottom quark and the complex scalar field. L and R refer to left and right handed, respectively. The top-quark mass is $\approx Hv/\sqrt{2}$ and hence a heavy top quark carries a large coupling constant H. Here, while we cannot use the argument of boundedness of the potential from above as before, the fact remains that the potential energy will be unbounded when H becomes very large, and instability will be induced unless constraints develop. Indeed, the equations of motion for the top-quark field mandate that such constraints result (Ref. [1]):

$$(\bar{t}_L, \bar{b}_L) = \begin{bmatrix} \phi^0 \\ \phi^- \end{bmatrix} = 0 \quad \text{or} \quad \bar{t}_L = -\bar{b}_L \phi^- / \phi^0 \tag{1.3}$$

and

$$t_R = 0 \ . \tag{1.4}$$

Note that these are SU(2)-invariant conditions, which entail a nonlinear realization [5] again.

One can easily show that the tree equations of motion, together with the nonlinear constraints of Eqs. (1.3) and (1.4) generate all the non-negligible contributions to the tree graphs. The constraints correspond to replacing the heavy-quark propagator $1/(m_t^2+p^2)$ by $1/m_t^2$, where p is the propagating momentum.

A more concise way to express this is to impose the constraints on the classical Lagrangian. Because of its origin, this tree effective Lagrangian contains graphs which can be separated into two halves by cutting an internal top-quark line. It is therefore one-particle irreducible only with respect to light line (1LPI).

We can pursue this approach much further to include loop effects. A convenient quantity for generalization is the generating functional $\Gamma_{\rm 1LPI}$ for one-light-particle-irreducible Green's functions $\Gamma_{\rm 1LPI}^{(n)}$, which in principle can be constructed to any loop order, but we shall concretely do it to one-loop order in this article. To commence this construction, one elementary but crucial observation is that, if we choose to perform our calculation in the symmetric phase, the underlying symmetry is there for all values of H. Of course, we will be interested in the case when H is large. We can shift the scalar field to give the Higgs field its vacuum expectation value afterwards. Because we shall use a large Yukawa coupling proportional to the top-quark mass as an expansion parameter, it is natural to perform a derivative expansion on $\Gamma_{\rm 1LPI}$. We shall show that (1) at a given loop order, the parts of $\Gamma_{\rm 1LPI}$ which have top-quark effects cannot have more than a certain maximum number of external light-quark lines and covariant derivatives, (2) out of them, one can construct only a finite number of local vertices, which are SU(2)⊗U(1) invariant, together with an entity which is on-shell invariant, and (3) the coefficients for these local vertices can be and will be determined.

One may at this point raise the issue that strictly speaking $\Gamma_{\rm 1LPI}$ so constructed is good only for $m_{b,W,Z}$, $m_t \gg p_{\rm ext}$, because graphs with purely light particles are also included in the derivative expansion. This is true; but if one's focus is only the m_t dependence, $\Gamma_{\rm 1LPI}$ will provide that correctly. Nonetheless, we come away with a sense of discomfort if we stop here, because ideally we would like to obtain an effective Lagrangian which is valid for $m_t \gg p_{\rm ext}$ and light masses, such that all the physical analytical requirements are respected. We have in mind, for example, that light-particle thresholds will appear as cuts in amplitudes when we reach the proper kinematic regions.

This in fact can be accomplished if one now extracts, for example, $L_{\rm eff}^{\rm 1\,loop}$ from $\Gamma_{\rm 1LPI}^{\rm 1\,loop}$. Basically, what needs to be done is to subtract out all the one-loop effects generated by $L_{\rm eff}^{\rm tree}$ to the same maximum order in external momentum derivatives as in $\Gamma_{\rm 1LPI}$. If we now use $L_{\rm eff} \approx L_{\rm eff}^{\rm tree} + L_{\rm eff}^{\rm 1\,loop}$ to perform a calculation for a low-energy process with $m_t \gg p_{\rm ext}$, we shall discover that the correct analyticity is there. Furthermore, the heavy top-quark effects will be explicitly displayed in $L_{\rm eff}^{\rm 1\,loop}$. The divergence due to internal integrations in constructing one-loop graphs from $L_{\rm eff}^{\rm tree}$ will be canceled by the counter terms which are automatically generated in $L_{\rm eff}^{\rm 1\,loop}$.

There is another technical point which we wish to touch upon briefly before elaboration later. As we shall repeatedly emphasize, the construction of $\Gamma_{\rm 1LPI}$ and $L_{\rm eff}$ is feasible only because SU(2)⊗U(1) remains a symmetry. To respect this, the gauge conditions we shall use are chosen to be SU(2)⊗U(1) covariant in the background gauge formulation [6,7]. In fact, because of great number of terms, we find it most convenient to use the background field technique throughout, coupled with the derivative expansion. This will be seen to reduce greatly the number of explicit *processes* we need to evaluate in order to determine all the coefficients of the monomials which make up $\Gamma_{\rm 1LPI}$ and $L_{\rm eff}$.

From the discussion above, we repeat that the effective theory we are deriving here far exceeds traditional usage. Usually, when one applies an effective Lagrangian, the understanding is either to stop after the tree level, as in soft-pion physics, or to use experimental input to fix new counterterms, as in chiral perturbation theory at one loop. For us, because there is an underlying full theory, the effective Lagrangian can be constructed systematically to any loop order. The number of input parameters remains the same as in the full theory. Some of the advantages of this approach over direct calculation from the full theory are that (a) the calculation becomes simpler, and (b) the heavy-particle dependence is explicitly displayed *before any labor is committed.* This is accomplished without any cost to renormalizability, analyticity, and unitarity.

The plan of the paper is as follows. In the next section, we shall introduce our notation. Solutions to the classical equations of motion will be expanded in powers of $1/H$. We shall see that the constraints follow immediately. The SU(2)⊗U(1) transformation properties of the solutions will be examined; particularly, we shall show that the $1/H$ term of the solution t_R $(\bar{t}_R^{(1)})$, which will appear later on in loop corrections, is an on-shell singlet in SU(2).

In Sec. III, we shall introduce background fields to prepare for loop calculations. We shall write down a set of gauge conditions, which will preserve the SU(2)⊗U(1) symmetry. We shall perform a loop analysis. Because the background fields we are going to use are the zeroth-order approximation to the classical equations, but not the exact solutions, there will be terms which are linear in quantum fluctuations. From this, we shall see that the term $\bar{t}_R^{(1)}$ mentioned earlier will accompany these linear fluctuations. It will appear in processes where there are external fermions. We will draw an important conclusion that only the S-matrix elements will be invariant under $SU(2)\otimes U(1)$ transformations. The one-light-particle-irreducible generating functional Γ_{1LPI} will be introduced here.

In Sec. IV, we do a power counting on one-particle-irreducible graphs with at least a top-quark internal line. We shall do a derivative expansion with respect to external momenta and establish a formula, to show how many external derivatives we need to keep for a given external line configuration and a given loop order before the residual terms become negligible. Combining with results of the last section, we know the possible structural dependence of Γ_{1LPI} on the background fields. We would like to draw attention to the fact that our Γ_{1LPI} contains also contributions from pure light diagrams, up to the same maximum order in external derivatives we are keeping to locate heavy-top-quark effects. Thus, our results are uniformally valid only in the region $p_{\rm ext} \ll m_t$ and m_b. Although by itself this already constitutes a new result, relevant for a hypothetical fourth nondegenerate quark doublet, it is definitely at variance with our present objectice as an effective Lagrangian. Fortunately, this shortcoming will be quickly remedied by the construction of $L_{\rm eff}$.

We shall develop further calculation techniques for one-loop bosonic vertices in Sec. V. We shall introduce Green's functions for fermions in the presence of external scalars. As will become clear, because we shall need only a small number of charged scalars to determine the bosonic parts of Γ_{1LPI}, the external scalars in the fermionic Green's functions will be taken neutral. This simplifies the group algebra.

Using all the tools developed, we shall explicitly construct the bosonic parts of Γ_{1lPI} at one loop in Sec. VI. The results agree with those published by Steger, Flores, and Yao (Ref. [1]) some time go and extend to include gauge bosons.

In Sec. VII, we shall subtract from Γ_{1LPI} all the terms up to fourth order in external momenta, which are generated in one-loop order by $L_{\rm eff}^{\rm tree}$. If we now use $L_{\rm eff}$ to calculate, the results are exact for $m_t \gg p_{\rm ext}$. Furthermore, all the analyticity requirements are met.

As is well known, each fermion in a gauge theory gives rise to Wess-Zumino [8] terms. If uncompensated by other members of a multiplet, they will give rise to anomalies and spoil the renormalizability [9] of the theory. We shall discuss these Wess-Zumino terms due to the top quark in Sec. VIII, just to make the bosonic analysis complete.

Section IX is reserved to deal with three examples. The ρ parameter [10] and $H \to 2\gamma$ [11] will be used to illustrate that our method easily reproduces known results. As another example, the $W^+ \to \phi^+ + \phi'^{0\dagger}(\phi^0 = \phi'^0 + v/\sqrt{2})$ will demonstrate that our effective Lagrangian reproduces the correct analytic amplitude. Further physical applications will be reported elsewhere, to limit the size of this article.

Some brief concluding remarks are made in Sec. X, basically to recapitulate the essential elements of our work.

II. TREE-LEVEL CONSIDERATION OF CONSTRAINTS

We want to derive some constraints on our field system when the top quark receives a heavy mass because of strong coupling, i.e., when $m_t = Hv/\sqrt{2} \gg m, p_{\rm ext}$, where m is any other mass scale in the system and $p_{\rm ext}$ denotes external momenta. (At the tree level, all internal momenta are of course expressible as external momenta, but the theory of our effective Lagrangian is formulated to apply to cases with loops as well.)

Let us introduce source terms into the Lagrangian

$$L = L_{\rm lin} + (\bar\eta_R b_R + \bar\eta_L b_L + \bar\phi J_\phi + {\rm H.c.}) + J^a_\mu A^{a\mu} + K_\mu B^\mu . \tag{2.1}$$

Because we do not allow the production of top quarks due to our external momentum requirement, we need not introduce source functions for them. We shall neglect mixing of the top-bottom quark family with other low-mass families. Its restoration is trivial. Also, we need not write down the lepton members, because they do not enter into our present discussion. Then,

$$L_{\rm lin} = L_{\rm fermions} + L_{\rm Higgs} + L_{\rm gauge\ fields} + L_{\rm Yukawa} + L_{\rm gf} , \tag{2.2}$$

with

$$L_{\rm fermions} = -(\bar t \quad \bar b)_L \frac{1}{i}\gamma^\mu D_\mu \begin{pmatrix} t \\ b \end{pmatrix}_L - \bar t_R \frac{1}{i}\gamma^\mu D_\mu t_R - \bar b_R \frac{1}{i}\gamma^\mu D_\mu b_R , \tag{2.3}$$

$$L_{\rm Higgs} = -(D_\mu\phi)^\dagger(D^\mu\phi) - \frac{\lambda}{2}(\bar\phi\phi)^2 - \mu^2(\bar\phi\phi) , \tag{2.4}$$

$$L_{\rm gauge\ fields} = -\tfrac14 G^a_{\mu\nu}G^{a\mu\nu} - \tfrac14 F_{\mu\nu}F^{\mu\nu} , \tag{2.5}$$

$$L_{\rm Yukawa} = -\left[H(\bar t \quad \bar b)_L \begin{pmatrix} \phi^0 \\ \phi^- \end{pmatrix} t_R + h(\bar t \quad \bar b)_L \begin{pmatrix} -\phi^+ \\ \phi^{0\dagger} \end{pmatrix} b_R + {\rm H.c.} \right] , \tag{2.6}$$

where

$$D_\mu \begin{pmatrix} t \\ b \end{pmatrix}_L = \left[\partial_\mu - ig\frac{\tau^a}{2}A^a_\mu - i\frac{g'}{2}Y_L B_\mu \right] \begin{pmatrix} t \\ b \end{pmatrix}_L , \tag{2.7}$$

$$\begin{aligned} D_\mu t_R &= \left[\partial_\mu - i\frac{g'}{2}Y_{t_R}B_\mu \right] t_R , \\ D_\mu b_R &= \left[\partial_\mu - i\frac{g'}{2}Y_{b_R}B_\mu \right] b_R , \end{aligned} \tag{2.8}$$

$$D_\mu\phi = \left[\partial_\mu - ig\frac{\tau^a}{2}A^a_\mu - i\frac{g'}{2}Y_\phi B_\mu \right]\phi , \tag{2.9}$$

$$\begin{aligned} G^a_{\mu\nu} &= \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\epsilon^{abc}A^b_\mu A^c_\nu , \\ F_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu . \end{aligned} \tag{2.10}$$

The hypercharges are $Y_L=\frac{1}{3}$, $Y_{t_R}=\frac{4}{3}$, $Y_{b_R}=-\frac{2}{3}$, and $Y_\phi=-1$. We will specify the gauge-fixing term in the next section. Suffice it to say here that it will be SU(2)⊗U(1) invariant. As is well known, to generate all the three diagrams, we need only the classical solutions for the Lagrangian of Eq. (2.1). Of particular interest to us is the propagation of the top quark, which obeys the equations

$$-\frac{1}{i}\gamma^\mu\left[D_\mu\begin{bmatrix}t\\b\end{bmatrix}_L\right]^\dagger-H\phi^0 t_R+h\phi^+ b_R=0\ ,$$

and

$$-\frac{1}{i}\gamma^\mu D_\mu t_R-H(\phi^{0\dagger}t_L+\phi^+ b_L)=0\ . \tag{2.11}$$

where the up (down) arrow means that the upper (lower) component of the term should be taken. For fixed b and ϕ, one can expand the solutions is inverse powers of H:

$$t_L=t_L^{(0)}+H^{-1}t_L^{(1)}+\cdots,\quad t_R=t_R^{(0)}+H^{-1}t_R^{(1)}+\cdots\ . \tag{2.12}$$

Substituting these into Eqs. (2.11) and equating powers of H, we obtain

$$t_L^{(0)}=-(\phi^+/\phi^{0\dagger})b_L,\quad t_R^{(0)}=0\ , \tag{2.13}$$

and

$$t_L^{(1)}=0\ ,\quad t_R^{(1)}=\left\{-\frac{1}{i}\gamma^\mu\left[D_\mu\begin{bmatrix}t_L^{(0)}\\b_L\end{bmatrix}\right]^\dagger+h\phi^+ b_R\right\}\Big/\phi^0\ . \tag{2.14}$$

In places where $(\phi^0)^{-1}$ and $(\phi^{0\dagger})^{-1}$ appear, they are defined as expansions around their vacuum expectation value $\langle\phi^0\rangle=\langle\phi^{0\dagger}\rangle=v/\sqrt{2}$. We can rewrite the first part of Eq. (2.13) as

$$(\phi^{0\dagger},\phi^+)\begin{bmatrix}t_L^{(0)}\\b_L\end{bmatrix}=0\ , \tag{2.15}$$

which is a manifestedly SU(2)-invariant constraint: under SU(2) rotation,

$$\begin{aligned}\phi^0&\to\phi^0+\frac{i}{2}\delta\alpha_3\phi^0+\frac{1}{2}(i\delta\alpha_1+\delta\alpha_2)\phi^-\ ,\\ \phi^-&\to\phi^-+\frac{1}{2}(i\delta\alpha_1-\delta\alpha_2)\phi^0-\frac{i}{2}\delta\alpha_3\phi^-\ ,\\ b_L&\to b_L+\frac{1}{2}(i\delta\alpha_1-\delta\alpha_2)t_L^{(0)}-\frac{i}{2}\delta\alpha_3 b_L\ ,\end{aligned} \tag{2.16}$$

it can be easily checked that, as written in Eq. (2.13),

$$t_L^{(0)}\to t_L^{(0)}+\frac{i}{2}\delta\alpha_3 t_L^{(0)}+\frac{1}{2}(i\delta\alpha_1+\delta\alpha_2)b_L\ , \tag{2.17}$$

which affirms that $(t_L^{(0)},b_L)$ form an isodoublet. We shall discuss the transformation property of $t_R^{(1)}$ shortly.

Now we insert the constraints of Eq. (2.13) into the Lagrangian. This will be called the nonlinear Lagrangian $L_{\rm nl}$. In the following, we shall write interchangeably

$$\tilde t_L=\tilde t_L^{(0)}=\tilde t_{\rm nl}=-(\tilde\phi^+/\tilde\phi^{0\dagger})\tilde b_L\ , \tag{2.18}$$

in which all quantities with tildes are classical solutions for $L_{\rm nl}$ to zeroth order in H:

$$\begin{aligned}L_{\rm nl}=&-(\bar{\tilde t}\ \ \bar{\tilde b})_L\frac{1}{i}\gamma^\mu D_\mu\begin{bmatrix}\tilde t_L\\ \tilde b_L\end{bmatrix}-\bar{\tilde b}_R\frac{1}{i}\gamma^\mu D_\mu\tilde b_R-(D^\mu\tilde\phi)^\dagger(D_\mu\tilde\phi)-\frac{\lambda}{2}(\bar{\tilde\phi}\tilde\phi)^2-\mu^2(\bar{\tilde\phi}\tilde\phi)\\ &-\tfrac14\tilde G^{a\mu\nu}\tilde G^a_{\mu\nu}-\tfrac14\tilde F^{\mu\nu}\tilde F_{\mu\nu}-[h(\bar{\tilde b}_L\tilde\phi^{0\dagger}-\bar{\tilde t}_L\tilde\phi^+)\tilde b_R+\text{H.c.}]\\ &+(\bar\eta_R\tilde b_R+\bar\eta_L\tilde b_L+\bar J_\phi\tilde\phi+\text{H.c.})+J^a_\mu\tilde A^{a\mu}+K_\mu\tilde B^\mu\ .\end{aligned} \tag{2.19}$$

This Lagrangian gives us all the vertices for light particles $\tilde\phi$, $\tilde b$, $\tilde A$, and $\tilde B$. The terms proportional to $\tilde t$ come from diagrams in which the top propagator $1/(p^2+m_t^2)$ shrinks to a point $1/m_t^2$. We will call the collection of (a) diagrams which cannot be separated into two parts by cutting a light internal line, and (b) those which can be separated by cutting a top internal line, one-light-particle-irreducible diagrams. With the sources turned off, $\int d^4x\, L_{\rm nl}$ is in fact the tree-level generating functional for the connected one-light-particle-irreducible Green's functions $\Gamma^{(n)}_{\rm 1LPI}$.

Except for the source terms, $L_{\rm nl}$ is SU(2)⊗U(1) invariant. It is also noted that the corrections to $L_{\rm nl}$ at the tree level will be $O(H^{-1})$. Hence, to detect effects $H^n\ln^m H$ with $m,n\geq 0$, we need to perform loop calculations. A similar situation exists in the strongly coupled Higgs system (Ref. [4]), where the kinetic terms produce extra interaction because of a nonlinear constraint, but explicit strong-coupling dependence shows up only after loop corrections.

Let us return to the second part of Eq. (2.14), when all fields are replaced by the classical solutions. Under Eq. (2.16), we have

$$\begin{aligned}\tilde t_R^{(1)}\to\tilde t_R^{(1)}+\frac12(i\delta\alpha_1+\delta\alpha_2)&\left\{-\frac{1}{(\tilde\phi^0)^2}\tilde\phi^-\frac{1}{i}\gamma^\mu\left[D_\mu\begin{bmatrix}\tilde t_L\\ \tilde b_L\end{bmatrix}\right]^\dagger+\frac{1}{\tilde\phi^0}\frac{1}{i}\gamma^\mu\left[D\mu\begin{bmatrix}\tilde t_L\\ \tilde b_L\end{bmatrix}\right]_\downarrow+h\left[\frac{\tilde\phi^{0\dagger}}{\tilde\phi^0}+\frac{\tilde\phi^+\tilde\phi^-}{\tilde\phi^{0^2}}\right]\tilde b_R\right\}\\ &=\tilde t_R^{(1)}+\frac12(i\delta\alpha_1+\delta\alpha_2)\frac{1}{\tilde\phi^0}\eta_L\ .\end{aligned} \tag{2.20}$$

In the last step, use has been made of the equation of motion for $\tilde{t}_L$. Now, in evaluating S-matrix elements, we need to put the participating particles on shell and turn off the external sources afterwards. This procedure will obliterate the second term of Eq. (2.20). That is to say, $\tilde{t}_R^{(1)}$ is an isoscalar with respect to the S matrix. This leads to the statement that, after quantum loop corrections, the S matrix is an isosinglet, but the Green's functional can have terms with noninvariant transformation behavior.

[N.B. The discussion is primarily conducted in the symmetrical phase of SU(2)⊗U(1). However, because we will be performing calculations with the background field method, the results can be readily used in the broken phase as well. In the latter case, we can impose the conditions [12]

$$(\partial_\mu \tilde{W}^{\pm\mu} \mp i\frac{g}{2} v\tilde{\phi}^{\pm})|\text{physical states}\rangle = 0 ,$$

$$\left[\partial_\mu \tilde{Z}^{\mu} - \frac{(g^2+g'^2)^{1/2}}{2} v\tilde{\phi}_3\right]|\text{physical states}\rangle = 0 ,$$

$$\partial_\mu \tilde{A}^{\mu}|\text{physical states}\rangle = 0 ,$$

at the S-matrix level, after the generating functional of the next section has been constructed. Here

$$W^{\pm\mu} = (\tilde{A}_1^{\mu} \mp i\tilde{A}_2^{\mu})/\sqrt{2} ,$$

$$\tilde{Z}_\mu = (g\tilde{A}_\mu^3 - g'\tilde{B}_\mu)/\sqrt{g^2+g'^2} ,$$

$$\tilde{A}_\mu = (g'\tilde{A}_\mu^3 + g\tilde{B}_\mu)/\sqrt{g^2+g'^2} ,$$

are, respectively, the charged-vector bosons, the neutral weak-vector boson, and the photon. Also, $\tilde{\phi}^{\pm} = (\tilde{\phi}_1 \mp i\tilde{\phi}_2)/\sqrt{2}$.]

III. GENERAL DISCUSSION OF ONE-LOOP CORRECTIONS

In this section, we want to investigate the structure of the Green's functional under nonlinear SU(2)⊗U(1) transformations. Also, we want to give a discussion of the construction of an effective Lagrangian with loop corrections upon taking the heavy-top-quark-mass limit.

First, let us introduce gauge-fixing terms. We shall use the background field method, so that SU(2)⊗U(1) gauge invariance is manifest. The gauge conditions are (Ref. [7]), respectively,

$$G^a \equiv D_\mu^{ab}(\tilde{A})(A^{b\mu} - \tilde{A}^{b\mu}) + i\frac{g}{2}\bar{\tilde{\phi}}\tau^a(\phi - \tilde{\phi}) - i\frac{g}{2}(\bar{\phi} - \bar{\tilde{\phi}})\tau^a\tilde{\phi} ,$$

with

$$D_\mu^{ab}(\tilde{A}) \equiv \partial_\mu\delta^{ab} - g\epsilon^{abc}\tilde{A}_\mu^c ,$$

and

$$G_B \equiv \partial_\mu(B^\mu - \tilde{B}^u) - i\frac{g'}{2}\bar{\tilde{\phi}}(\phi - \tilde{\phi}) + i\frac{g'}{2}(\bar{\phi} - \bar{\tilde{\phi}})\tilde{\phi} . \quad (3.1)$$

Note that we have equated the background fields with the approximate tree classical solutions of the last section. It can be shown that this is a consistent procedure, at least to the one-loop level.

The generating functional is

$$\exp(iW[J,K,J_\phi,\eta,\tilde{A},\tilde{B},\tilde{\phi})] = \int db\, d\bar{b}\, dt\, d\bar{t}\, dA_\mu dB_\mu d\phi\, d\bar{\phi}\, \Delta[A,B,\phi,\bar{\phi};\tilde{A},\tilde{B},\tilde{\phi},\bar{\tilde{\phi}}]\exp\left[i\int dx\, L\right] , \quad (3.2)$$

where Δ is a Faddeev-Popov determinant. L is given by Eqs. (2.1)–(2.6) and

$$L_{\text{gf}} = -\tfrac{1}{2}G^{a\dagger}G^a - \tfrac{1}{2}G_B^\dagger G_B . \quad (3.3)$$

Because the approximate tree classical solutions have well-defined SU(2)⊗U(1) transformation properties and are convenient for expansion in H^{-1}, we make a change of integration variables by shifting

$$b_L = B_L + \tilde{b}_L, \quad b_R = B_R + \tilde{b}_R ,$$

$$t_L = T_L + \tilde{t}_L, \quad t_R = T_R$$

$$\phi = \Phi + \tilde{\phi} ,$$

$$A_\mu = A_\mu^{\text{qf}} + \tilde{A}_\mu ,$$

and

$$B_\mu = B_\mu^{\text{gf}} + \tilde{B}_\mu , \quad (3.4)$$

with $B_{L,R}$, $T_{L,R}$, Φ, A_μ^{qf}, and B_μ^{qf} as quantum fluctuations. Through this, the connected Green's functional becomes

$$W = W[\tilde{A},\tilde{B},\tilde{\phi},\tilde{t},\tilde{b}]$$
$$= W[J,K,J_\phi,\eta] . \quad (3.5)$$

This follows because the tilde quantities are functions of sources via the field equations.

We can determine the loop-corrected classical fields and perform a Legendre transformation to obtain the connected one-light-particle-irreducible (1LPI) generating functional Γ_{1LPI}, which contains all graphs that cannot be separated into two halves by cutting one single A, B, ϕ, or b line. This is in fact what we will construct, but we are skipping a detailed account of the formal aspects of this procedure. (At the tree level, Γ_{1LPI} is just L_{nl} without the sources.) For a while longer, however, we keep working with W.

After substitution of Eq. (3.4), we have

$$L=L_{\rm nl}+\left[\bar{B}_R\left[\frac{\partial L_{\rm lin}}{\partial\bar{b}_R}+\eta_R\right]_{\to\sim}+\bar{B}_L\left[\frac{\partial L_{\rm lin}}{\partial\bar{b}_L}+\eta_L\right]_{\to\sim}+\bar{T}_R\left[\frac{\partial L_{\rm lin}}{\partial\bar{t}_R}\right]_{\to\sim}+\bar{T}_L\left[\frac{\partial L_{\rm lin}}{\partial\bar{t}_L}\right]_{\to\sim}\right.$$
$$\left.+\Phi\left[\frac{\partial L_{\rm lin}}{\partial\bar{\phi}}-\partial_\mu\frac{\partial L_{\rm lin}}{\partial(\partial_\mu\bar{\phi})}+J_\phi\right]_{\to\sim}+\text{H.c.}\right]+\text{terms in higher powers of quantum fluctuations}\ . \tag{3.6}$$

The symbol $\to\sim$ means that the evaluation is at $\phi=\tilde{\phi}$, $t_{L,R}=\tilde{t}_{L,R}$, $b_{L,R}=\tilde{b}_{L,R}$, $A_\mu^a=\tilde{A}_\mu^a$, and $B_\mu=\tilde{B}_\mu$. Here, we must be careful to take into account the expansions of the linear theory, but evaluate around solutions of the equations of motion of $L_{\rm nl}$. For A_μ^a and B_μ, there is no difference in the equations for the linear or the nonlinear theory. They both vanish, which also explains why there are no linear fluctuation terms in $A_\mu^{a\,\rm qf}$ and $B_\mu^{\rm qf}$ in Eq. (3.6). For the linear theory, we have

$$\left.\frac{\partial L_{\rm lin}}{\partial\bar{b}_R}\right|_{\to\sim}+\eta_R=-\frac{1}{i}\gamma_\mu D^\mu\tilde{b}_R-h(\tilde{\phi}^0\tilde{b}_L-\tilde{\phi}^-\tilde{t}_L)+\eta_R\ ,$$
$$\left.\frac{\partial L_{\rm lin}}{\partial\bar{b}_L}\right|_{\to\sim}+\eta_L=-\frac{1}{i}\gamma_\mu\left[D^\mu\begin{bmatrix}\bar{\tilde{t}}_L\\ \bar{\tilde{b}}_L\end{bmatrix}\right]_\downarrow-h\tilde{\phi}^{0\dagger}\tilde{b}_R+\eta_L\ ,$$
$$\left.\frac{\partial L_{\rm lin}}{\partial\bar{t}_L}\right|_{\to\sim}=-\frac{1}{i}\gamma_\mu\left[D^\mu\begin{bmatrix}\tilde{t}_L\\ \tilde{b}_L\end{bmatrix}\right]^\dagger+h\tilde{\phi}^+\tilde{b}_R\ ,\quad \left.\frac{\partial L_{\rm lin}}{\partial\bar{t}_R}\right|_{\to\sim}=0\ ,$$
$$\left.\frac{\partial L_{\rm lin}}{\partial\phi^0}\right|_{\to\sim}+J_\phi^{0\dagger}=D_\mu D^\mu\tilde{\phi}^0-\lambda(\tilde{\phi}^0\tilde{\phi}^{0\dagger}+\tilde{\phi}^+\tilde{\phi}^-)\tilde{\phi}^0-\mu^2\tilde{\phi}^0-h\bar{\tilde{b}}_L\tilde{b}_R+J_\phi^{0\dagger}\ ,$$
$$\left.\frac{\partial L}{\partial\phi^-}\right|_{\to\sim}+J_\phi^+=D_\mu D^\mu\tilde{\phi}^+-\lambda(\tilde{\phi}^0\tilde{\phi}^{0\dagger}+\tilde{\phi}^+\tilde{\phi}^-)\tilde{\phi}^+-\mu^2\tilde{\phi}^+-h\bar{\tilde{b}}_R\tilde{t}_L+J_\phi^+\ , \tag{3.7}$$

and similar equations for the conjugate fields. Use has been made of the constraints of Eq. (2.13). On the other hand, the equations of motion for the nonlinear Lagrangian are

$$\frac{\partial L_{\rm nl}}{\partial\bar{\tilde{b}}_R}+\eta_R=-\frac{1}{i}\gamma^\mu D_\mu\tilde{b}_R-h(\tilde{\phi}^0\tilde{b}_L-\tilde{\phi}^-\tilde{t}_L)+\eta_R=0\ ,$$
$$\frac{\partial L_{\rm nl}}{\partial\bar{\tilde{b}}_L}+\frac{\partial\bar{\tilde{t}}_L}{\partial\bar{\tilde{b}}_L}\frac{\partial L_{\rm nl}}{\partial\bar{\tilde{b}}_L}+\eta_L=-\frac{1}{i}\gamma^\mu\left[D_\mu\begin{bmatrix}\tilde{t}_L\\ \tilde{b}_L\end{bmatrix}\right]_\downarrow-h\tilde{\phi}^{0\dagger}\tilde{b}_R-\frac{1}{\tilde{\phi}^0}\tilde{\phi}^-\left\{-\frac{1}{i}\gamma^\mu\left[D_\mu\begin{bmatrix}\tilde{t}_L\\ \tilde{b}_L\end{bmatrix}\right]^\dagger+h\tilde{\phi}^+\tilde{b}_R\right\}+\eta_L=0\ ,$$
$$\frac{\partial L_{\rm nl}}{\partial\tilde{\phi}^0}+\frac{\partial\bar{\tilde{t}}_L}{\partial\tilde{\phi}^0}\frac{\partial L_{\rm nl}}{\partial\bar{\tilde{t}}_L}-\partial_\mu\frac{\partial L_{\rm nl}}{\partial\partial_\mu\tilde{\phi}^0}+J_\phi^{0\dagger}=D_\mu D^\mu\tilde{\phi}^0-\lambda(\tilde{\phi}^0\tilde{\phi}^{0\dagger}+\tilde{\phi}^+\tilde{\phi}^-)\tilde{\phi}^0-\mu^2\tilde{\phi}^0-h\bar{\tilde{b}}_L\tilde{b}_R$$
$$-\frac{1}{\tilde{\phi}^0}\bar{\tilde{t}}_L\left\{-\frac{1}{i}\gamma_\mu\left[D^\mu\begin{bmatrix}\bar{\tilde{t}}_L\\ \bar{\tilde{b}}_L\end{bmatrix}\right]^\dagger-h\tilde{\phi}^+\tilde{b}_R\right\}+J_\phi^{0\dagger}=0\ ,$$
$$\frac{\partial L_{\rm nl}}{\partial\tilde{\phi}^-}+\frac{\partial\bar{\tilde{t}}_L}{\partial\tilde{\phi}^-}\frac{\partial L_{\rm nl}}{\partial\bar{\tilde{t}}_L}-\partial_\mu\frac{\partial L_{\rm nl}}{\partial\partial_\mu\tilde{\phi}^-}+J_\phi^+=D_\mu D^\mu\tilde{\phi}^+-\lambda(\tilde{\phi}^0\tilde{\phi}^{0\dagger}+\tilde{\phi}^+\tilde{\phi}^-)\tilde{\phi}^0-\mu^2\tilde{\phi}^+-h\bar{\tilde{b}}_R\tilde{t}_L$$
$$-\frac{1}{\tilde{\phi}^0}\bar{\tilde{b}}_L\left\{-\frac{1}{i}\gamma_\mu\left[D^\mu\begin{bmatrix}\bar{\tilde{t}}_L\\ \bar{\tilde{b}}_L\end{bmatrix}\right]_\downarrow-h\tilde{\phi}^+\tilde{b}_R\right\}+J_\phi^+=0\ . \tag{3.8}$$

Combining Eq. (3.6) and Eq. (3.7), we have, for Eq. (3.5),

$$L=L_{\rm nl}+L'\ , \tag{3.9}$$

where

$$L'=[(\bar{B}_L\tilde{\phi}^- + \bar{T}_L\tilde{\phi}^0)\tilde{t}_R^{(1)} + (\Phi^0\bar{\tilde{t}}_L + \Phi^-\bar{\tilde{b}}_L)\tilde{t}_R^{(1)} + \text{H.c.}] + \text{terms in higher powers of quantum fluctuations .} \quad (3.10)$$

When this is substituted into Eq. (3.2), we find that loop corrections are contained behind the functional integration

$$\exp(iW)=\exp\left[i\int dx\,L_{\rm nl}\right]\int dB\,d\bar{B}\,dT\,d\bar{T}\,dA_\mu^{\rm qf}dB_\mu^{\rm qf}d\Phi\,d\bar{\Phi}\,\Delta\exp\left[i\int dxL'\right], \quad (3.11)$$

where Δ is the relevant Faddeev-Popov determinant. We already know that $(\tilde{\phi}^0,\tilde{\phi}^-)$ and $(\tilde{t},\tilde{b})_L$ transform as isodoublets. We can demand that the integration variables (Φ^0,Φ^-) and $(T,B)_L$ transform likewise. In this way, we can easily show that all the terms which are higher in powers of quantum fluctuations are isospin invariant. The only quantity which is not invariant is $\tilde{t}_R^{(1)}$ in Eq. (3.10), which is an on-shell singlet. Hence, quantum loop corrections to W will be made up of SU(2)⊗U(1) gauge-invariant quantities of $(\tilde{\phi}^0,\tilde{\phi}^-)$, $(\tilde{t},\tilde{b})_L$, D_μ, $G^a_{\mu\nu}$, and $F_{\mu\nu}$, and powers of $\tilde{t}_R^{(1)}$.

We now turn to a discussion of the meaning of an effective Lagrangian in the present context. Naively, this may mean that the top-quark fields have been *integrated out,* which is formally written as

$$\lim_{m_t\gg m,p_{\rm ext}}\int db\,d\bar{b}\,dt\,d\bar{t}\,dA_\mu dB_\mu d\phi\,d\bar{\phi}\,\Delta[A,B,\phi,\bar{\phi};\tilde{A},\tilde{B},\tilde{\phi},\bar{\tilde{\phi}}]\exp\left[i\int dxL\right]$$
$$=\int db\,d\bar{b}\,dA_\mu dB_\mu d\phi\,d\bar{\phi}\,\Delta[A,B,\phi,\bar{\phi};\tilde{A},\tilde{B},\tilde{\phi},\bar{\tilde{\phi}}]\exp\left[i\int dxL_{\rm eff}\right]. \quad (3.12)$$

There are two comments we want to make concerning this approach: (1) The large mass limit can be taken behind any integral sign, only if the ultraviolet behavior is sufficiently convergent to permit it. This is obviously not the case for a renormalizable theory. The grouping of terms to produce effective vertices and to give a meaning to the limiting procedure is the essence of the theory of effective Lagrangians and has been developed elsewhere for other theories. We do not intend to repeat this program here in its entirety. We will just use the implied technology in constructing the one-loop effective Lagrangian. (2) As we have repeated a few times, we must maintain SU(2)⊗U(1) invariance to make this otherwise rather formidable calculation possible. We shall obtain $L_{\rm eff}$ to one-loop order in two steps. First, we shall conduct a derivative expansion for $\Gamma_{\rm 1LPI}^{\rm loop}$, to a certain maximum power, to be determined by graphs with internal top-quark lines. This will be explained in more detail in the next section. Then we impose the symmetry requirement on $\Gamma_{\rm 1LPI}^{\rm loop}$, which has the consequence that even graphs with loops made up purely of light particles are also included and developed to the same powers in external momenta.

In operator language, we will obtain

$$\Gamma_{\rm 1LPI}^{\rm loop}=\sum C_i^{\rm loop}\mathcal{O}_i^{\rm loop}, \quad (3.13)$$

where $\mathcal{O}_i^{\rm loop}$ are local operators of the light fields $\tilde{A}^a_\mu$, $\tilde{B}_\mu$, $\tilde{\phi}$, and $\tilde{b}$. They are SU(2)⊗U(1) invariant up to $t_R^{(1)}$. C_i are coefficient functions which encompass all the $m_t^n\ln^m m_t(m,n\geq 0)$ effects.

Because we conduct derivative expansions also for graphs with purely light lines (for example, $\Gamma_{\rm 1LPI}^{\rm 1\,loop}$) strictly speaking the result is generally valid only for $m_t,m_{b,W,Z}\gg p_{\rm ext}$, although from it one can pick out those terms with explicit H dependence and isolate the dominant top-quark effects. The situation can be improved. If we subtract out one-loop contributions due to $L_{\rm eff}^{\rm tree}$ in the same powers in external momenta, as implied by Eq. (3.12), we obtain $L_{\rm eff}^{\rm 1\,loop}$. As we shall see, the amplitudes constructed through $L_{\rm eff}$ will have kinematic validity for $m_t>p_{\rm ext}$. Furthermore, it will respect all the analyticity requirements of the light-particle sector. They are renormalizable, in the sense that the only renormalization parameters are the conventional wave functions, couplings, and masses. At the one-loop level, for instance, the extra divergences coming from loops induced by $L_{\rm eff}^{\rm tree}$ will be canceled out by those from $L_{\rm eff}^{\rm 1\,loop}$, which are automatically generated by over-subtractions to go from $\Gamma_{\rm 1LPI}$ to $L_{\rm eff}$. Clearly, this is the most compact way to look for physically interesting processes which are sensitive to the heavy-top-quark mass effects.

IV. POWER COUNTING

Given a graph with at least one heavy-top-quark internal line, we want to carry out an expansion in inverse powers of m_t. The accompanying external parameters in the expansion are the light fields and their covariant derivatives. We want to know up to what powers in D_μ, $\tilde{F}_{\mu\nu}$, $\tilde{G}^a_{\mu\nu}$, $\tilde{\phi}$, $\tilde{b}$ we should include before contributions of graphs with heavy-top-quark internal lines become negligible in $\Gamma_{\rm 1LPI}^{\rm loop}$. As discussed before, we must also include graphs with only light b internal lines to the same derivative order to maintain SU(2)⊗U(1) invariance. We will see, however, that in actuality it works like this: we shall calculate processes which involve graphs with at least one heavy-top-quark line. Because of symmetry, they already provide enough information to determine $\Gamma_{\rm 1LPI}^{\rm loop}$. This means that if graphs with only light b-quark internal lines can exist in a process, they are automatically included to the required order in the momentum expansion.

Our present discussion will be carried out for a general-loop order, although we should remark that the actual extraction of $L_{\rm eff}$ can only be an iterative procedure. A formula, which gives us the maximum number of derivatives a local operator may have for a given loop order will be derived. After this is determined, in the next few sections we can combine it with the invariance requirement to write down all the possible local operators for $\Gamma_{\rm 1LPI}$ at the one-loop order.

Consider a fermion loop. The derivative expansion will produce integrals such as

$$\int d^4k \frac{(k^2)^m}{(k^2+m_b^2)^{n_1}(k^2+m_t^2)^{n_2}} . \tag{4.1}$$

In order that m_t be the scale of expansion in the problem, we must require that $m+2-n_1 \geq 0$. In other words, we must assume that there is no power infrared divergence with respect to m_b. It turns out that in constructing $\Gamma_{\rm 1LPI}^{\rm 1\,loop}$, there are graphs in which this condition is not satisfied, an example being the $W_\mu^+\phi^-\phi^0$ vertex. However, after we deduce $L_{\rm eff}^{\rm 1\,loop}$, out of which we can construct physical processes, this potential complication does not led to any effect.

A graph can potentially have the strongest power behavior in H if all its vertices carry H. Let us call the number of vertices in this graph V. Let n_F denote the number of external fermions ($\tilde{b}$) and n_B the number of external scalar bosons ($\tilde{\phi}$). (The number of external gauge bosons will eventually be decided by gauge-covariant derivatives.) The integral under consideration will be written as $H^V I$, where we have explicitly factored out the power of H from vertices. The mass dimension of I is $4-\frac{3}{2}n_F-n_B$. If we use p to denote a generic external momentum, the expansion takes the form

$$H^V I = \left[\frac{m_t}{v}\right]^V m_t^{4-3n_F/2-n_B} \times \left[a_0+a_1\frac{p}{m_t}+\cdots+a_N\left[\frac{p}{m_t}\right]^N\right] . \tag{4.2}$$

This series terminates at

$$4-\tfrac{3}{2}n_F-n_B+V=N . \tag{4.3}$$

The neglected terms in Eq. (4.2) are suppressed by at least $O(1/m_t)$.

Let i_F and i_B be the number of internal fermion and scalar boson lines, respectively, and let L be the loop order. We have

$$3i_F+2i_B+\tfrac{3}{2}n_F+n_B=4V ,$$

$$2(2i_B+n_B)=2i_F+n_F ,$$

and

$$i_F+i_B=L-1+V . \tag{4.4}$$

These three equations are used to eliminate i_F and i_B, which yield

$$2(L-1)+n_F+n_B=V . \tag{4.5}$$

We combine it with Eq. (4.3) to give the result we are after, namely

$$N=2(L+1)-\tfrac{1}{2}n_F . \tag{4.6}$$

Note that the number of external scalar boson lines does not appear in this formula. What it means is that we can have an arbitrary number of them. It is quite understandable how this has to be so: the relevant $SU(2)\otimes U(1)$-invariant scale we have is the scalar product $\bar{\tilde{\phi}}\tilde{\phi}$, which can occur in inverse powers or as an argument in the logarithm function. Each of these will give us an arbitrary number of scalar bosons when expanded around the vacuum expectation value v. In view of this, it is advisable that calculations be performed with the external field technique.

V. PROPAGATORS IN EXTERNAL FIELDS AND THE DERIVATIVE EXPANSION

For the rest of this article, we will be determining to one-loop order local vertices with external bosons only. Therefore, the necessary internal propagators are all fermionic.

We have explained that because there will be an arbitrary number of scalar bosons for each of the local vertices, a convenient tool to facilitate calculations in this situation is the external field technique, coupled with the derivative expansion [13]. Two remarks are now in order. First, the derivatives will naturally turn out to be covariant derivatives after all graphs are summed. On the other hand, if we use covariant derivatives throughout the whole calculation, then their proper ordering in a string should be closely monitored. We shall bypass this by using ordinary derivatives in the propagators, complemented by evaluating explicitly the noncommuting parts to the necessary orders, which will give rise to $F_{\mu\nu}$ and $G_{\mu\nu}^a$'s. This is justified, because after all we are performing a derivative expansion. Second, as we shall see, we need processes with at most two pairs of external charged scalar fields together with an arbitrary number of neutral scalars to determine all the coefficients C_i (to the extent that they can be uniquely determined; see later.) It will be sufficient for this purpose to have propagators in the presence only of external neutral scalar fields. The calculation will be done in the symmetric phase, which can be immediately carried over to the broken phase for applications.

Because the quarks will occur in the propagators only, they are quantum fluctuation fields, while the bosons will occur only externally and are thus external classical fields. There should then be no confusion if we just drop all ornamental symbols introduced before to differentiate between them.

The equations of motion lead to the following set of equations for the Green's functions $\langle T[t_R(x)\bar{t}_R(y)]\rangle$ and $\langle T[t_L(x)\bar{t}_R(y)]\rangle$:

$$[-\partial_x^2+H^2\phi^0(x)\phi^{0\dagger}(x)]\langle T[t_L(x)\bar{t}_R(y)]\rangle = H\phi^0(x)\frac{1}{i}\delta(x-y)_L+\left[\frac{1}{i}\gamma\partial_x H\phi^0(x)\right]\langle T[t_R(x)\bar{t}_R(y)]\rangle , \tag{5.1}$$

and

$$[-\partial_x^2+H^2\phi^0(x)\phi^{0\dagger}(x)]\langle T[t_R(x)\bar{t}_R(y)]\rangle=-\frac{1}{i}\gamma\partial_x\frac{1}{i}\delta(x-y)_L+\left[\frac{1}{i}\gamma\partial_x H\phi^{0\dagger}(x)\right]\langle T[t_L(x)\bar{t}_R(y)]\rangle \; . \tag{5.2}$$

To solve these equations, we write formally ($u\equiv\phi^0\phi^{0\dagger}$)

$$\langle T[t_L(x)\bar{t}_R(y)]\rangle=\int d^4z\left\langle x\left|\frac{1}{-\partial^2+H^2u}\right|z\right\rangle\left[H\phi^0(z)\frac{1}{i}\delta(z-y)_L+\left[\frac{1}{i}\gamma\partial H\phi^0(z)\right]\langle T[t_R(z)\bar{t}_R(y)]\rangle\right] , \tag{5.3}$$

and

$$\langle T[t_R(x)\bar{t}_R(y)]\rangle=\int d^4z\left\langle x\left|\frac{1}{-\partial^2+H^2u}\right|z\right\rangle\left[-\frac{1}{i}\gamma\partial_z\frac{1}{i}\delta(z-y)_L+\left[\frac{1}{i}\gamma H\phi^{0\dagger}(z)\right]\langle T[t_L(z)\bar{t}_R(y)]\rangle\right] . \tag{5.4}$$

In the case of pure bosonic vertices, from Eq. (4.5), one has to iterate up to four derivatives on the right-hand sides of Eqs. (5.3) and (5.4). Note that because u has coordinate dependence, we also have to carry out a derivative expansion for it. In particular, the inverse operator $1/-\partial^2+H^2u$ is written as

$$\left\langle x\left|\frac{1}{-\partial^2+H^2u}\right|z\right\rangle=\int\frac{d^4p}{(2\pi)^2}e^{ipx}\left\langle p\left|\frac{1}{-\partial^2+H^2u}\right|z\right\rangle=\int\frac{d^4p}{(2\pi)^4}\frac{1}{p^2+H^2u[x-i(\overleftarrow{\partial}/\partial p)]}e^{ip(x-z)} , \tag{5.5}$$

which will be used with, e.g.,

$$\langle T[t_L(x)\bar{t}_R(y)]\rangle\equiv\langle x|t_L\bar{t}_R|y\rangle=\int\frac{d^4p}{(2\pi)^2}e^{ipx}\langle p|t_L\bar{t}_R|y\rangle=\int\frac{d^4p}{(2\pi)^4}\left\langle t_L\bar{t}_R\left[x-i\frac{\overleftarrow{\partial}}{\partial p}\right]\right\rangle e^{ip(x-y)} . \tag{5.6}$$

Here, the derivative within the argument acts on every p-dependent quantity to the left. We can write two equivalent equations for these Green's functions:

$$\langle T[t_R(x)\bar{t}_R(y)]\rangle=\int d^4z\left[\frac{1}{i}\gamma\partial_z\frac{1}{i}\delta(x-z)_L-\langle T[t_R(x)\bar{t}_L(z)]\rangle\left[\frac{1}{i}\gamma\partial H\phi^0(z)\right]\right]\left\langle z\left|\frac{1}{-\partial^2+H^2u}\right|y\right\rangle , \tag{5.7}$$

and

$$\langle T[t_R(x)\bar{t}_L(y)]\rangle=\int d^4z\left[-H\phi^{0\dagger}(z)\delta(x-z)_R+\langle T[t_R(x)\bar{t}_R(z)]\rangle\left[\frac{1}{i}\gamma\partial H\phi^{0\dagger}(z)\right]\right]\left\langle z\left|\frac{1}{-\partial^2+H^2u}\right|y\right\rangle . \tag{5.8}$$

For these equations, it will be more appropriate to write, e.g.,

$$\langle T[t_L(x)\bar{t}_R(y)]\rangle=\int\frac{d^4p}{(2\pi)^4}e^{ip(x-y)}\left\langle t_L\bar{t}_R\left[y+i\frac{\vec{\partial}}{\partial p}\right]\right\rangle , \tag{5.9}$$

with

$$\begin{aligned}\left\langle x\left|\frac{1}{-\partial^2+H^2u}\right|z\right\rangle&=\int\frac{d^4p}{(2\pi)^4}e^{ip(x-z)}\frac{1}{p^2+H^2u[z+i(\vec{\partial}/\partial p)]}\\&=\int\frac{d^4p}{(2\pi)^4}e^{ip(x-z)}\frac{1}{p^2+H^2u(z)}\\&\quad\times\sum_{m=0}^{\infty}\left[-\sum_{n=1}^{\infty}\frac{1}{n!}[\partial^z_{\mu_1}\cdots\partial^z_{\mu_n}H^2u(z)]i\frac{\partial}{\partial p_{\mu_1}}\cdots i\frac{\partial}{\partial p_{\mu_n}}\left[\frac{1}{p^2+H^2u(z)}\right]\right]^m .\end{aligned} \tag{5.10}$$

In the last line, we have made a series expansion. Of course, here we just have to keep up to four derivatives in z. One can check the consistency of the resulting expressions after left or right differentiations by using commutation relations of the following kind:

$$\left[\frac{1}{p^2+H^2u[x+i(\vec{\partial}/\partial p)]},\gamma p\right]=\frac{1}{p^2+H^2u[x+i(\vec{\partial}/\partial p)]}\left[\frac{1}{i}\gamma\partial^x H^2u\left[x+i\frac{\vec{\partial}}{\partial p}\right]\right]\frac{1}{p^2+H^2u[x+i(\vec{\partial}/\partial p)]} ,$$

and

$$\left[H\phi^0, \frac{1}{p^2+H^2u}\right] = -\frac{1}{p^2+H^2u}\frac{1}{i}\gamma p(\gamma\partial^x H\phi^0)\frac{1}{p^2+H^2u} - \frac{1}{p^2+H^2u}(\gamma\partial^x H\phi^0)\frac{1}{i}\gamma p\frac{1}{p^2+H^2u}\ , \tag{5.11}$$

where we omitted the argument $x+i(\vec{\partial}/\partial p)$ in u and ϕ^0 of the second equation.

To illustrate, we give the result for $\langle T(t_R\bar{t}_R)\rangle$, where all momentum derivatives are to act to the right and the argument in u, ϕ^0, and $\phi^{0\dagger}$ is understood to be $x+i(\vec{\partial}/\partial p)$:

$$\begin{aligned}\langle T[t_R(y)\bar{t}_R(x)]\rangle^{4\partial} = \int \frac{d^{4p}}{(2\pi)^4}e^{ip(y-x)}\Bigg[& -\frac{1}{i}\gamma p\frac{1}{p^2+H^2u} - \frac{1}{i}H\phi^{0\dagger}\frac{1}{p^2+H^2u}\left[\frac{1}{i}\gamma\partial H\phi^0\right]\frac{1}{p^2+H^2u} \\ & -\frac{1}{i}\gamma p\frac{1}{p^2+H^2u}\left[\frac{1}{i}\gamma\partial H\phi^{0\dagger}\right]\frac{1}{p^2+H^2u}\left[\frac{1}{i}\gamma\partial H\phi^0\right]\frac{1}{p^2+H^2u} \\ & -\frac{1}{i}H\phi^{0\dagger}\frac{1}{p^2+H^2u}\left[\frac{1}{i}\gamma\partial H\phi^0\right]\frac{1}{p^2+H^2u}\left[\frac{1}{i}\gamma\partial H\phi^{0\dagger}\right]\frac{1}{p^2+H^2u} \\ & \times\left[\frac{1}{i}\gamma\partial H\phi^0\right]\frac{1}{p^2+H^2u} \\ & -\frac{1}{i}\gamma p\frac{1}{p^2+H^2u}\left[\frac{1}{i}\gamma\partial H\phi^{0\dagger}\right]\frac{1}{p^2+H^2u}\left[\frac{1}{i}\gamma\partial H\phi^0\right]\frac{1}{p^2+H^2u} \\ & \times\left[\frac{1}{i}\gamma\partial H\phi^{0\dagger}\right]\frac{1}{p^2+H^2u}\left[\frac{1}{i}\gamma\partial H\phi^0\right]\frac{1}{p^2+H^2u}\Bigg]\ .\end{aligned} \tag{5.12}$$

This, in conjunction with Eq. (5.10), will generate a large number of terms. We use SCHOONSCHIP [14] to carry out this chore, and some others.

In an analogous way, we can construct $\langle T(t_R\bar{t}_L)\rangle$, $\langle T(t_L\bar{t}_R)\rangle$, $\langle T(t_L\bar{t}_L)\rangle$, and similar Green's functions for the b propagators, either with the derivatives acting on the right or on the left. With these ingredients, we are now in a position to calculate the diagrams necessary to determine the coefficient multiplying the invariants of $\Gamma_{\rm 1LPI}$.

VI. ONE-LOOP 1LPI GENERATING FUNCTIONAL

In this section, we shall determine the one loop $\Gamma_{\rm 1LPI}$ with techniques developed in previous sections. By power-counting arguments in Sec. IV, one-loop effective bosonic vertices are described by SU(2)⊗U(1)-invariant operators carrying zero, two, or four derivatives. The complete set of operators is listed as follows.

A. Potential terms

$$A_{11}(\bar{\phi}\phi)^2,\quad A_{12}(\bar{\phi}\phi)^2\ln\left[\frac{\bar{\phi}\phi}{\mu^2}\right]\ . \tag{6.1}$$

B. Two-derivative terms

$$\begin{aligned}& A_{20}(D_\mu\phi)^\dagger(D^\mu\phi)\ln\left[\frac{\bar{\phi}\phi}{\mu^2}\right]\ , \\ & A_{21}(D_\mu\phi)^\dagger(D^\mu\phi)\ , \\ & A_{22}[(D_\mu\phi)^\dagger\phi][(D^\mu\phi)^\dagger\phi]/(\bar{\phi}\phi)\ , \\ & A_{23}[\bar{\phi}(D_\mu\phi)][\bar{\phi}(D^\mu\phi)]/(\bar{\phi}\phi)\ , \\ & A_{24}[\bar{\phi}(D_\mu\phi)][(D^\mu\phi)^\dagger\phi]/(\bar{\phi}\phi)\ .\end{aligned} \tag{6.2}$$

C. Four-derivative terms

To compactify, we introduce the following notation ${}^{(\lambda_4,\kappa_4)}_{(\lambda_3,\kappa_3)}I^{(\lambda_1,\kappa_1)}_{(\lambda_2,\kappa_2)}$, where each λ or κ stands for a set of Lorentz indices. Each pair (λ_i,κ_i) operates on a bilinear $\bar{\phi}\phi$ in the following fashion: $(D^{(\lambda_i)}\phi)^\dagger(D^{(\kappa_i)}\phi)$. Note that the ordering of Lorentz indices in each λ and κ must be strictly observed. After all the covariant differentiations have been applied, we scale the resulting expression with appropriate powers of $\bar{\phi}\phi$ so that each ${}^{(\lambda_4,\kappa_4)}_{(\lambda_3,\kappa_3)}I^{(\lambda_1,\kappa_1)}_{(\lambda_2,\kappa_2)}$ has mass dimension four. For example

$$I^{\mu\nu,\nu}_{\mu,0} = [(D^\mu D^\nu\phi)^\dagger D_\nu\phi][(D_\mu\phi)^\dagger\phi]/(\bar{\phi}\phi)^2\ .$$

Then, the possible terms are

$$\begin{aligned}& C_1 I^{\mu\nu,\mu\nu}\ ,\quad C_2 I^{\mu\nu,0}_{0,\mu\nu}\ ,\quad C_3(I^{\mu\nu,0}_{\mu\nu,0}+\text{H.c.})\ , \\ & C_5(I^{\mu\nu,\nu}_{\mu,0}+\text{H.c.})\ ,\quad C_6(I^{\mu\nu,\nu}_{0,\mu}+\text{H.c.})\ , \\ & C_9(I^{\mu\nu,0}_{\nu,\mu}+\text{H.c.})\ ,\quad C_{11}I^{\nu,\mu}_{\nu,\mu}\ ,\quad C_{12}I^{\nu,\mu}_{\mu,\nu} \\ & C_{13}I^{\mu,\mu}_{\nu,\nu}\ ,\quad C_{14}({}_{\mu,0}I^{\nu\mu,0}_{\nu,0}+\text{H.c.})\ ,\quad C_{15}({}_{0,\mu}I^{\nu\mu,0}_{\nu,0}+\text{H.c.})\ , \\ & C_{16}({}_{0,\mu}I^{\nu\mu,0}_{0,\nu}+\text{H.c.})\ ,\quad C_{20}({}_{\mu,0}I^{\nu,\nu}_{\mu,0}+\text{H.c.})\ , \\ & C_{21}({}_{0,\mu}I^{\nu,\nu}_{\mu,0})\ ,\quad C_{23}({}_{\nu,0}I^{\nu,\mu}_{\mu,0}+\text{H.c.})\ ,\quad C_{24}({}_{0,\nu}I^{\nu,\mu}_{\mu,0})\ , \\ & C_{25}({}_{\nu,0}I^{\nu,\mu}_{0,\mu})\ ,\quad C_{27}({}^{\nu,0}_{\nu,0}I^{\mu,0}_{\mu,0}+\text{H.c.})\ ,\quad C_{28}({}^{0,\nu}_{\nu,0}I^{\mu,0}_{\mu,0}+\text{H.c.})\ , \\ & C_{29}({}^{0,\nu}_{0,\nu}I^{\mu,0}_{\mu,0})\ ,\quad C_{30}({}^{0,\nu}_{\nu,0}I^{\mu,0}_{0,\mu})\ .\end{aligned} \tag{6.3}$$

D. Terms with field strengths

$$
\begin{aligned}
&B_1 G^a_{\mu\nu}G^{a\mu\nu},\quad B_2 G^a_{\mu\nu}G^{a\mu\nu}\ln\left[\frac{\bar\phi\phi}{\mu^2}\right],\\
&B_3 F_{\mu\nu}F^{\mu\nu},\quad B_4 F_{\mu\nu}F^{\mu\nu}\ln\left[\frac{\bar\phi\phi}{\mu^2}\right],\\
&B_5(\bar\phi G_{\mu\nu}\phi)F^{\mu\nu}/(\bar\phi\phi)\ ,\quad B_6(\bar\phi G_{\mu\nu}\phi)(\bar\phi G^{\mu\nu}\phi)/(\bar\phi\phi)^2\ ,\\
&B_7[(D_\mu\phi)^\dagger(D_\nu\phi)-\text{H.c.}]F^{\mu\nu}/(\bar\phi\phi)\ ,\\
&B_8[(D_\mu\phi)^\dagger(D_\nu\phi)-\text{H.c.}](\bar\phi G^{\mu\nu}\phi)/(\bar\phi\phi)^2\ ,\\
&B_9(D_\mu\phi)^\dagger G^{\mu\nu}(D_\nu\phi)/(\bar\phi\phi)\ ,\\
&B_{10}\{[\bar\phi(D_\nu\phi)][(D_\mu\phi)^\dagger\phi]-\text{H.c.}\}F^{\mu\nu}/(\bar\phi\phi)^2\ ,\\
&B_{11}\{[\bar\phi G_{\mu\nu}(D^\nu\phi)][(D^\mu\phi)^\dagger\phi]-\text{H.c.}\}/(\bar\phi\phi)^2\ ,\\
&B_{12}\{[\bar\phi G_{\mu\nu}(D^\nu\phi)][\bar\phi(D^\mu\phi)]-\text{H.c.}\}/(\bar\phi\phi)^2\ ,\\
&B_{13}(\bar\phi G_{\mu\nu}\phi)\{[(D^\mu\phi)^\dagger\phi][\bar\phi(D^\nu\phi)]-\text{H.c.}\}/(\bar\phi\phi)^3\ .
\end{aligned}
\tag{6.4}
$$

Here $G_{\mu\nu}=G^a_{\mu\nu}(\tau^a/2)$. One need not use the charge-conjugated Higgs doublet; because of charge conservation it has to appear in pairs. The identity $\epsilon^{ij}\epsilon^{kl}=\delta^{ik}\delta^{jl}-\delta^{il}\delta^{jk}$ will convert them back into terms made with the ordinary Higgs doublet.

We now describe the processes we have chosen to determine these coefficients. We shall omit the color factor $N_c=3$ associated with a fermion loop for all the coefficients to be determined.

1. A_{11} to A_{24}

To compute A_{11} to A_{24} for the potential and vertices with two derivatives, it is enough to compute $\phi^+\phi^-$ self-energy with arbitrary numbers of neutral scalars emitted by fermion propagators. The coupling constants g and g' can be switched off, so that gauge fields do not play any role in this part of the calculation. The results will be compared with the same amplitude given by direct differentiation on relevant operators, where each invariant in general will contribute to several different terms. A_{11} to A_{24} will then be extracted.

First of all, the $\phi^+\phi^-$ self-energy is written as

$$A_{\phi^+\phi^-}=I_1+I_2+I_3+I_4\ , \tag{6.5}$$

with

$$I_1=-i^2H^2\,\mathrm{Tr}\int d^4x\int d^4y\langle T[b_L(x)\bar b_L(y)]\rangle\phi^-(y)\langle T[t_R(y)\bar t_R(x)]\rangle\phi^+(x)\ , \tag{6.6}$$

$$I_2=-i^2h^2\,\mathrm{Tr}\int d^4x\int d^4y\langle T[b_R(x)\bar b_R(y)]\rangle\phi^-(y)\langle T[t_L(y)\bar t_L(x)]\rangle\phi^+(x)\ , \tag{6.7}$$

$$I_3=i^2Hh\,\mathrm{Tr}\int d^4x\int d^4y\langle T[b_L(x)\bar b_R(y)]\rangle\phi^-(y)\langle T[t_L(y)\bar t_R(x)]\rangle\phi^+(x)\ , \tag{6.8}$$

and

$$I_4=i^2Hh\,\mathrm{Tr}\int d^4x\int d^4y\langle T[b_R(x)\bar b_L(y)]\rangle\phi^-(y)\langle T[t_R(y)\bar t_L(X)]\rangle\phi^+(x)\ . \tag{6.9}$$

We use I_1 as an example to illustrate how the derivative expansion is applied to each individual integral. Following the notation introduced in Sec. V, we recast I_1 into

$$I_1=-i^2H^2\,\mathrm{Tr}\int d^4x\int d^4y\langle x|b_L\bar b_L|y\rangle\langle y|\phi^-t_R\bar t_R\phi^+|x\rangle\ . \tag{6.10}$$

A compact formula well suited for iteration can be derived by inserting a complete set of four-momentum eigenstates $|p\rangle$ into Eq. (6.10) and performing a few integrations by parts. Explicitly, we have

$$
\begin{aligned}
I_1&=-i^2H^2\,\mathrm{Tr}\int a^4x\int d^4y\int d^4p_1d^4p_2d^4p_3d^4p_4\langle x|p_1\rangle\langle p_1|b_L\bar b_L|p_2\rangle\langle p_2|y\rangle\langle y|p_3\rangle\langle p_3|\phi^-t_R\bar t_R\phi^+|p_4\rangle\langle p_4|x\rangle\\
&=-i^2H^2\,\mathrm{Tr}\left\{\int d^4x\int d^4y\int d^4p_1d^4p_2d^4p_3d^4p_4\frac{e^{ip_1x}}{(2\pi)^2}\left[\left\langle b_L\bar b_L\left[i\frac{\vec\partial}{\partial p_1}\right]\right\rangle\delta(p_1-p_2)\right]\right.\\
&\qquad\left.\times\frac{e^{-ip_2y}}{(2\pi)^2}\frac{e^{ip_3y}}{(2\pi)^2}\phi^-\left[i\frac{\vec\partial}{\partial p_3}\right]\left[\delta(p_3-p_4)\left\langle t_R\bar t_R\left[-i\frac{\overleftarrow\partial}{\partial p_4}\right]\right\rangle\right]\phi^+\left[-i\frac{\overleftarrow\partial}{\partial p_4}\right]\frac{e^{-ip_4x}}{(2\pi)^2}\right\}\\
&=-i^2H^2\,\mathrm{Tr}\left\{\int d^4x\int d^4y\int d^4p_1d^4p_3d^4p_4\left\langle b_L\bar b_L\left[x-i\frac{\overleftarrow\partial}{\partial p_1}\right]\right\rangle\frac{e^{ip_1(x-y)}}{(2\pi)^4}\frac{e^{ip_3y}}{(2\pi)^2}\right.\\
&\qquad\left.\times\phi^-\left[i\frac{\vec\partial}{\partial p_3}\right]\left[\delta(p_3-p_4)\left\langle t_R\bar t_R\left[-i\frac{\overleftarrow\partial}{\partial p_4}\right]\right\rangle\right]\phi^+(x)\frac{e^{-ip_4x}}{(2\pi)^2}\right\}\\
&=-i^2H^2\,\mathrm{Tr}\left\{\int d^4x\int d^4y\int d^4p_1d^4p_3d^4p_4\left\langle b_L\bar b_L\left[x-i\frac{\overleftarrow\partial}{\partial p_1}\right]\right\rangle\frac{e^{ip_1(x-y)}}{(2\pi)^4}\frac{e^{ip_3y}}{(2\pi)^2}\right.
\end{aligned}
$$

$$\times\left[\phi^-\left[-i\frac{\vec{\partial}}{\partial p_4}\right]\delta(p_3-p_4)\frac{e^{-ip_4x}}{(2\pi)^2}\right]\left\langle t_R\bar{t}_R\left[x+i\frac{\vec{\partial}}{\partial p_4}\right]\right\rangle\phi^+(x)\Bigg\}$$

$$=-i^2H^2\,\mathrm{Tr}\left[\int d^4x\int d^4y\int d^4p_1d^4p_3\left\langle b_L\bar{b}_L\left[x-i\frac{\overleftarrow{\partial}}{\partial p_1}\right]\right\rangle\frac{e^{ip_1(x-y)}}{(2\pi)^4}\frac{e^{-ip_3(x-y)}}{(2\pi)^4}\right.$$

$$\left.\times\phi^-\left[x+i\frac{\vec{\partial}}{\partial p_3}\right]\left\langle t_R\bar{t}_R\left[x+i\frac{\vec{\partial}}{\partial p_3}\right]\right\rangle\phi^+(x)\right]$$

$$=-i^2H^2\,\mathrm{Tr}\left[\int d^4x\int\frac{d^4p}{(2\pi)^4}\left\langle b_L\bar{b}_L\left[x-i\frac{\overleftarrow{\partial}}{\partial p}\right]\right\rangle\phi^-\left[x+i\frac{\vec{\partial}}{\partial p}\right]\langle t_R\bar{t}_R\left[x+i\frac{\vec{\partial}}{\partial p}\right]\right\rangle\phi^+(x)\right], \tag{6.11}$$

where use has been made of relations $F[i(\vec{\partial}/\partial p)]e^{-ipx}=e^{-ipx}F[x+i(\vec{\partial}/\partial p)]$, and $e^{ipx}F[-i(\overleftarrow{\partial}/\partial p)]=F[x-i(\overleftarrow{\partial}/\partial p)]e^{ipx}$. Equation (6.11), as it is written, in a basic equation in this calculation. Later on, a similar technique will be applied to four-point functions which are needed to determine the coefficients of four-derivative terms.

A derivative expansion of the last section is then applied to each term in Eq. (6.11). The trace over γ matrices and the momentum average are performed. In order to regulate the loop integration, we use dimensional continuation [15]. Since all external momenta enter only via x derivatives, the integrands all have the form $[1/p^2+H^2u(x)]^{n_1}[1/p^2+h^2u(x)]^{n_2}(p^2)^{n_3}$. Thus, we effectively shrink an n-point function into a set of local vertices. The numerical results can only be rational functions of H^2 and h^2 and/or logarithms $\ln(H^2)$ and $\ln(h^2)$, multiplied by binomial coefficients and fractional numbers. For the example, we find

$$A_{\phi^+\phi^-}=\int d^4x\,L_{\phi^+\phi^-}\,, \tag{6.12}$$

where

$$L_{\phi^+\phi^-}=\frac{1}{16\pi^2}\Bigg\{\left[2(H^4+h^4)\left[-\frac{1}{\epsilon}+\gamma_E+\ln(\pi)\right]-(H^4+h^4)+2H^4\ln(H^2)+2h^4\ln(h^2)\right](\phi^{0\dagger}\phi^0)(\phi^+\phi^-)$$

$$+2(H^4+h^4)(\phi^{0\dagger}\phi^0)(\phi^+\phi^-)\ln\left[\frac{\phi^{0\dagger}\phi^0}{\mu^2}\right]+\left[\frac{1}{6}H^2+\frac{1}{6}h^2+h^2\ln\left[\frac{H^2}{h^2}\right]\right](\partial_\mu\phi^{0\dagger})(\partial^\mu\phi^0)\phi^+\phi^-/(\phi^{0\dagger}\phi^0)$$

$$+(H^2+h^2)(\partial_\mu\phi^+)(\partial^\mu\phi^-)\ln\left[\frac{\phi^{0\dagger}\phi^0}{\mu^2}\right]+\left[(H^2+h^2)\ln(H^2)+(H^2+h^2)\right.$$

$$\left.\times\left[-\frac{1}{\epsilon}+\gamma_E+\ln(\pi)\right]\right](\partial_\mu\phi^+)(\partial^\mu\phi^-)$$

$$+\frac{1}{3}(H^2+h^2)[(\partial_\mu\phi^{0\dagger}\phi^0)(\partial^\mu\phi^+\phi^-)/(\phi^{0\dagger}\phi^0)+\mathrm{H.c.}]$$

$$+\left[\frac{5}{6}(H^2+h^2)-h^2\ln\left[\frac{H^2}{h^2}\right]\right][(\phi^+\partial_\mu\phi^-)(\partial^\mu\phi^{0\dagger}\phi^0)/(\phi^{0\dagger}\phi^0)+\mathrm{H.c.}]$$

$$-\tfrac{1}{6}(H^2+h^2)[(\partial_\mu\phi^{0\dagger}\phi^0)(\partial^\mu\phi^{0\dagger}\phi^0)\phi^+\phi^-/(\phi^{0\dagger}\phi^0)^2+\mathrm{H.c.}]\Bigg\}\,. \tag{6.13}$$

As mentioned previously, the same process can be generated by differentiating relevant operators, which leads to

$$L_{\phi^+\phi^-}=\frac{1}{16\pi^2}\Bigg[(2A_{11}+A_{12})(\phi^{0\dagger}\phi^0)(\phi^+\phi^-)+2A_{12}(\phi^{0\dagger}\phi^0)(\phi^+\phi^-)\ln\left[\frac{\phi^{0\dagger}\phi^0}{\mu^2}\right]$$

$$+(A_{20}-A_{24})(\partial_\mu\phi^{0\dagger})(\partial^\mu\phi^0)\phi^+\phi^-/(\phi^{0\dagger}\phi^0)+A_{20}(\partial_\mu\phi^+)(\partial^\mu\phi^-)\ln\left[\frac{\phi^{0\dagger}\phi^0}{\mu^2}\right]+A_{21}(\partial_\mu\phi^+)(\partial^\mu\phi^-)$$

$$+2A_{22}(\partial_\mu\phi^{0\dagger}\phi^0)(\partial^\mu\phi^+\phi^-)/(\phi^{0\dagger}\phi^0)+2A_{23}(\phi^{0\dagger}\partial_\mu\phi^0)(\phi^+\partial^\mu\phi^-)/(\phi^{0\dagger}\phi^0)$$

$$+A_{24}[(\partial_\mu\phi^{0\dagger}\phi^0)(\phi^+\partial^\mu\phi^-)/(\phi^{0\dagger}\phi^0)+\mathrm{H.c.}]-A_{22}(\partial_\mu\phi^{0\dagger}\phi^0)(\partial^\mu\phi^{0\dagger}\phi^0)\phi^+\phi^-/(\phi^{0\dagger}\phi^0)^2$$

$$-A_{23}(\phi^{0\dagger}\partial_\mu\phi^0)(\phi^{0\dagger}\partial^\mu\phi^0)\phi^+\phi^-/(\phi^{0\dagger}\phi^0)^2\Bigg]\,. \tag{6.14}$$

By matching Eq. (6.13) and Eq. (6.14), we obtain $[\mathrm{Tr}(I)=n, \epsilon=2-n/2]$

$$
\begin{aligned}
A_{11}&=\frac{1}{16\pi^2}\left[(H^4+h^4)\left[-\frac{1}{\epsilon}+\gamma_E+\ln(\pi)-1\right]+H^4\ln(H^2)+h^4\ln(h^2)\right] ,\\
A_{12}&=\frac{1}{16\pi^2}(H^4+h^4) ,\\
A_{20}&=\frac{1}{16\pi^2}(H^2+h^2) ,\\
A_{21}&=\frac{1}{16\pi^2}\left[(H^2+h^2)\left[-\frac{1}{\epsilon}+\gamma_E+\ln(\pi)\right]+(H^2+h^2)\ln(H^2)\right] ,\\
A_{22}&=A_{23}=\frac{1}{16\pi^2}[\tfrac{1}{6}(H^2+h^2)] ,\\
A_{24}&=\frac{1}{16\pi^2}\left[\frac{5}{6}(H^2+h^2)-h^2\ln\left[\frac{H^2}{h^2}\right]\right] .
\end{aligned}
\tag{6.15}
$$

2. C_1 to C_{30}

The determination of the four-derivative terms is much more involved, because there are overall 21 different invariants. Again, by the same strategy, we temporarily switch off the coupling constants g and g' to avoid complications due to gauge fields. The appropriate process sufficient to determine coefficients C_1 to C_{30} is the scattering of ϕ^+ and ϕ^- with the emission of arbitrary numbers of neutral scalars by fermion propagators.

Recall that we are to include the neutral scalar fields in the fermion propagators. Therefore, one easy way to generate the scattering amplitude $A_{\phi^+\phi^-\phi^+\phi^-}$ is through the interaction Lagrangian

$$L_{\mathrm{int}}=-H(\bar{b}_L\phi^- t_R+\bar{t}_R\phi^+ b_L)+h(\bar{t}_L\phi^+ b_R+\bar{b}_R\phi^- t_L) . \tag{6.16}$$

Obviously, only the fourth-order term in the expansion series is relevant. It reads

$$\frac{i^4}{4!}\int d^4x\, d^4y\, d^4z\, d^4w T[L_{\mathrm{int}}(x)L_{\mathrm{int}}(y)L_{\mathrm{int}}(z)L_{\mathrm{int}}(w)] . \tag{6.17}$$

This gives

$$A_{\phi^+\phi^-\phi^+\phi^-}=\sum_1^{10} I_i , \tag{6.18}$$

where I_1 to I_{10} come out as a result of performing Wick contractions. Explicitly,

$$
\begin{aligned}
I_1=-H^2h^2\,\mathrm{Tr}\int d^4x\, d^4y\, d^4z\, d^4w &\langle T[b_L(y)\bar{b}_L(x)]\rangle\phi^-(x)\langle T[t_R(x)\bar{t}_L(z)]\rangle\phi^+(z)\langle T[b_R(z)\bar{b}_R(w)]\rangle\\
&\times\phi^-(w)\langle T[t_L(w)\bar{t}_R(y)]\rangle\phi^+(y) ,
\end{aligned}
\tag{6.19}
$$

$$
\begin{aligned}
I_2=-H^2h^2\,\mathrm{Tr}\int d^4x\, d^4y\, d^4z\, d^4w &\langle T[b_R(y)\bar{b}_L(x)]\rangle\phi^-(x)\langle T[t_R(x)\bar{t}_R(z)]\rangle\phi^+(z)\langle T[b_L(z)\bar{b}_R(w)]\rangle\\
&\times\phi^-(w)\langle T[t_L(w)\bar{t}_L(y)]\rangle\phi^+(y) ,
\end{aligned}
\tag{6.20}
$$

$$
\begin{aligned}
I_3=H^3h\,\mathrm{Tr}\int d^4x\, d^4y\, d^4z\, d^4w &\langle T[b_R(y)\bar{b}_L(x)]\rangle\phi^-(x)\langle T[t_R(x)\bar{t}_R(z)]\rangle\phi^+(z)\langle T[b_L(z)\bar{b}_L(w)]\rangle\\
&\times\phi^-(w)\langle T[t_R(w)\bar{t}_L(y)]\rangle\phi^+(y) ,
\end{aligned}
\tag{6.21}
$$

$$
\begin{aligned}
I_4=H^3h\,\mathrm{Tr}\int d^4x\, d^4y\, d^4z\, d^4w &\langle T[b_L(y)\bar{b}_L(x)]\rangle\phi^-(x)\langle T[t_R(x)\bar{t}_R(z)]\rangle\phi^+(z)\langle T[b_L(z)\bar{b}_R(w)]\rangle\\
&\times\phi^-(w)\langle T[t_L(w)\bar{t}_R(y)]\rangle\phi^+(y) ,
\end{aligned}
\tag{6.22}
$$

$$
\begin{aligned}
I_5=Hh^3\,\mathrm{Tr}\int d^4x\, d^4y\, d^4z\, d^4w &\langle T[b_R(y)\bar{b}_L(x)]\rangle\phi^-(x)\langle T[t_R(x)\bar{t}_L(z)]\rangle\phi^+(z)\langle T[b_R(z)\bar{b}_R(w)]\rangle\\
&\times\phi^-(w)\langle T[t_L(w)\bar{t}_L(y)]\rangle\phi^+(y) ,
\end{aligned}
\tag{6.23}
$$

$$
\begin{aligned}
I_6=-\tfrac{1}{2}H^4\,\mathrm{Tr}\int d^4x\, d^4y\, d^4z\, d^4w &\langle T[b_L(y)\bar{b}_L(x)]\rangle\phi^-(x)\langle T[t_R(x)\bar{t}_R(z)]\rangle\phi^+(z)\langle T[b_L(z)\bar{b}_L(w)]\rangle\\
&\times\phi^-(w)\langle T[t_R(w)\bar{t}_R(y)]\rangle\phi^+(y) ,
\end{aligned}
\tag{6.24}
$$

$$I_7=-\tfrac{1}{2}H^2h^2\,\mathrm{Tr}\int d^4x\,d^4y\,d^4z\,d^4w\langle T[b_R(y)\bar{b}_L(x)]\rangle\phi^-(x)\langle T[t_R(x)\bar{t}_L(z)]\rangle\phi^+(z)\langle T[b_R(z)\bar{b}_L(w)]\rangle \times\phi^-(w)\langle T[t_R(w)\bar{t}_L(y)]\rangle\phi^+(y)\;, \tag{6.25}$$

$$I_8=-\tfrac{1}{2}H^2h^2\,\mathrm{Tr}\int d^4x\,d^4y\,d^4z\,d^4w\langle T[b_L(y)\bar{b}_R(x)]\rangle\phi^-(x)\langle T[t_L(x)\bar{t}_R(z)]\rangle\phi^+(z)\langle T[b_L(z)\bar{b}_R(w)]\rangle \times\phi^-(w)\langle T[t_L(w)\bar{t}_R(y)]\rangle\phi^+(y)\;, \tag{6.26}$$

$$I_9=-\tfrac{1}{2}h^4\,\mathrm{Tr}\int d^4x\,d^4\,d^4z\,d^4w\langle T[b_R(y)\bar{b}_R(x)]\rangle\phi^-(x)\langle T[t_L(x)\bar{t}_L(z)]\rangle\phi^+(z)\langle T[b_R(z)\bar{b}_R(w)]\rangle \times\phi^-(w)\langle T[t_L(w)\bar{t}_L(y)]\rangle\phi^+(y)\;, \tag{6.27}$$

$$I_{10}=Hh^3\,\mathrm{Tr}\int d^4x\,d^4y\,d^4z\,d^4w\langle T[b_R(y)\bar{b}_R(x)]\rangle\phi^-(x)\langle T[t_L(x)\bar{t}_R(z)]\rangle\phi^+(z)\langle T[b_L(z)\bar{b}_R(w)]\rangle \times\phi^-(w)\langle T[t_L(w)\bar{t}_L(y)]\rangle\phi^+(y)\;. \tag{6.28}$$

Following the procedure used before, we write I_1, for example, as

$$I_1=-H^2h^2\,\mathrm{Tr}\left[\int d^4y\int\frac{d^4p}{(2\pi)^4}\left\langle b_L\bar{b}_L\phi^-t_R\bar{t}_L\left[y-i\frac{\overleftarrow{\partial}}{\partial p}\right]\right\rangle\phi^+\left[y+i\frac{\overrightarrow{\partial}}{\partial p}\right]\left\langle b_R\bar{b}_R\phi^-t_L\bar{t}_R\left[y+i\frac{\overrightarrow{\partial}}{\partial p}\right]\right\rangle\phi^+(x)\right]\;. \tag{6.29}$$

With similar formulae for other integrals, the amplitude $A_{\phi^+\phi^-\phi^+\phi^-}$ can be computed efficiently with the help of SCHOONSCHIP (Ref. [14]). The same amplitude, which we denote as $A'_{\phi^+\phi^-\phi^+\phi^-}$, is also obtained by differentiating the four-derivative vertices listed at the beginning of this section. By requiring

$$A_{\phi^+\phi^-\phi^+\phi^-}-A'_{\phi^+\phi^-\phi^+\phi^-}=0\;, \tag{6.30}$$

we obtain 52 equations to solve for the coefficients:

$$\begin{aligned}
&C_1=\frac{1}{16\pi^2}\left[\frac{1}{3}\right],\quad C_2=\frac{1}{16\pi^2}\left[-\frac{1}{15}\right],\quad C_3=\frac{1}{16\pi^2}\left[-\frac{1}{30}\right],\\
&C_5=\frac{1}{16\pi^2}\left[-\frac{5}{18}\right],\quad C_6=\frac{1}{16\pi^2}\left[-\frac{4}{9}\right],\quad C_9=\frac{1}{16\pi^2}\left[-\frac{7}{90}\right],\\
&C_{11}=\frac{1}{16\pi^2}\left[-\frac{17}{30}+\frac{1}{3}\ln\left[\frac{H^2}{h^2}\right]\right],\quad C_{14}=\frac{1}{16\pi^2}\left[\frac{1}{9}\right],\quad C_{15}=\frac{1}{16\pi^2}\left[\frac{1}{9}\right],\\
&C_{16}=\frac{1}{16\pi^2}\left[\frac{2}{9}\right],\quad C_{20}=\frac{1}{16\pi^2}\left[\frac{2}{9}\right],\quad C_{23}=\frac{1}{16\pi^2}\left[\frac{1}{3}\right],\\
&C_{25}=\frac{1}{16\pi^2}\left[\frac{10}{9}-\frac{2}{3}\ln\left[\frac{H^2}{h^2}\right]\right],\quad C_{27}=\frac{1}{16\pi^2}\left[-\frac{1}{12}\right],\quad C_{28}=\frac{1}{16\pi^2}\left[-\frac{1}{3}\right],\\
&C_{29}=\frac{1}{16\pi^2}\left[-\frac{7}{18}+\frac{1}{3}\ln\left[\frac{H^2}{h^2}\right]\right],\quad C_{30}=\frac{1}{16\pi^2}\left[-\frac{1}{9}-\frac{1}{3}\ln\left[\frac{H^2}{h^2}\right]\right],
\end{aligned} \tag{6.31}$$

and

$$\begin{aligned}
&2C_{12}+C_{24}=\frac{1}{16\pi^2}\left[\frac{8}{15}\right],\\
&2C_{13}+C_{21}=\frac{1}{16\pi^2}\left[\frac{1}{9}\right],\\
&C_{12}+C_{13}=\frac{1}{16\pi^2}\left[\frac{17}{45}-\frac{1}{3}\ln\left[\frac{H^2}{h^2}\right]\right].
\end{aligned} \tag{6.32}$$

Evidently, Eqs. (6.32) are not enough to uniquely determine the four coefficients there. In fact, it can be shown that $C_{12,13,21,24}$ always appear in these combinations in any physical process; they cannot be resolved any further. These results agree completely with those in our earlier publication.

3. B_1 to B_{13}

With a major part of Γ_{1LPI} determined, we now turn our attention to terms with explicit dependence on field strengths. Even though we have taken much advantage of the derivative expansion and external field technique before, we shall also rely on conventional methods of calculation in some cases here.

To determine B_1 to B_6, it is sufficient to compute $B_\mu B_\nu$, $A_{3,\mu}A_{3,\nu}$, and $W_\mu^+ W_\nu^-$ self-energies and the $B_\mu A_{3,\nu}$ mixing. Here we have defined $W_\mu^\pm = (1/\sqrt{2})(A_\mu^1 \mp i A_\mu^2)$. For this part, our calculation is entirely based on Feynman rules derived from the tree-level linear Lagrangian where the spontaneous symmetry breaking has already taken place. Feynman integrals arising from internal fermion loops are expanded in inverse powers of m_t. Since we are only interested in the heavy-top-quark limit, terms proportional to positive powers of $1/m_t$ are set to zero. Comparing this calculation with the method of operator differentiation, we have

$$\begin{aligned}
B_1 &= \frac{g^2}{16\pi^2}\left[\frac{1}{12}\left[-\frac{1}{\epsilon}+\ln(\pi)+\gamma_E-\frac{1}{3}+\ln(H^2)\right]\right],\\
B_2 &= \frac{g^2}{16\pi^2}\left[\frac{1}{12}\right],\\
B_3 &= \frac{g'^2}{16\pi^2}\left[\frac{11}{108}\left[-\frac{1}{\epsilon}+\ln(\pi)+\gamma_E\right]+\frac{11}{216}\right.\\
&\qquad\left.+\frac{17}{216}\ln(H^2)+\frac{5}{216}\ln(h^2)\right],\\
B_4 &= \frac{g'^2}{16\pi^2}\left[\frac{11}{108}\right],\\
B_5 &= \frac{gg'}{16\pi^2}\left[\frac{1}{18}\ln\left[\frac{H^2}{h^2}\right]\right],\\
B_6 &= \frac{g^2}{16\pi^2}\left[\frac{5}{18}-\frac{1}{6}\ln\left[\frac{H^2}{h^2}\right]\right].
\end{aligned} \tag{6.33}$$

To come up with the rest of the B_i's, we compute $B_\mu W_\nu^+ W_\sigma^-$ and $A_{3,\mu}W_\nu^+ W_\sigma^-$ triangle diagrams by the conventional method and obtain

$$\begin{aligned}
B_7 &= \frac{ig'}{16\pi^2}\left[-\frac{49}{180}+\frac{1}{18}\ln\left[\frac{H^2}{h^2}\right]\right],\\
B_9+B_{11}-B_{12} &= \frac{ig}{16\pi^2}\left[\frac{2}{3}\right],\\
2B_8-B_9 &= \frac{ig}{16\pi^2}\left[\frac{1}{5}-\frac{2}{3}\ln\left[\frac{H^2}{h^2}\right]\right].
\end{aligned} \tag{6.34}$$

To provide further information on the unknown coefficients as well as more consistency checks on existing results, we once again resort to the technique of derivative expansions operating on the configuration space. In particular, we compute B_μ and $A_{3,\mu}$ tadpoles with the emission of arbitrary numbers of neutral scalars by fermion propagators. For the computation of the B_μ tadpole, let us begin with an interaction Lagrangian which reads

$$\begin{aligned}
L_{\rm int} &= \frac{g'}{2}(Y_L\bar{t}_L\gamma_\mu t_L + Y_{tR}\bar{t}_R\gamma_\mu t_R)B^\mu\\
&\quad+\frac{g'}{2}(Y_L\bar{b}_L\gamma_\mu b_L + Y_{bR}\bar{b}_R\gamma_\mu b_R)B^\mu .
\end{aligned} \tag{6.35}$$

Only first-order terms in the S-matrix expansion can contribute to the tadpole diagram. They are

$$\begin{aligned}
S = \frac{ig'}{2}\int d^4x\,\{ & Y_L T[\bar{t}_L(x)\gamma_\mu t_L(x)B^\mu(x)]\\
&+Y_{tR}T[\bar{t}_R(x)\gamma_\mu t_R(x)B^\mu(x)]\\
&+Y_L T[\bar{b}_L(x)\gamma_\mu b_L(x)B^\mu(x)]\\
&+Y_{bR}T[\bar{b}_R(x)\gamma_\mu b_R(x)B^\mu(x)]\}\\
+\cdots .
\end{aligned} \tag{6.36}$$

The application of Wick contractions converts the S matrix into

$$\begin{aligned}
S = \frac{-ig'}{2}\int d^4x\,{\rm Tr}(& Y_L\gamma_\mu\langle x|t_L\bar{t}_L B^\mu|x\rangle\\
&+Y_{tR}\gamma_\mu\langle x|t_R\bar{t}_R B^\mu|x\rangle\\
&+Y_L\gamma_\mu\langle x|b_L\bar{b}_L B^\mu|x\rangle\\
&+Y_{bR}\gamma_\mu\langle x|b_R\bar{b}_R B^\mu|x\rangle)\\
+\cdots .
\end{aligned} \tag{6.37}$$

The first term in the parentheses, for example, is now transformed into

$$\begin{aligned}
I_1 = -i\frac{g'}{2}Y_L\,{\rm Tr}\left[\int d^4x\int\frac{d^4p}{(2\pi)^4}\gamma_\mu\right.\\
\left.\times\left\langle t_L\bar{t}_L\left[x+i\frac{\vec{\partial}}{\partial p}\right]\right\rangle B^\mu(x)\right].
\end{aligned} \tag{6.38}$$

With similar expressions for the other terms in Eq. (6.37), the B_μ tadpole can be easily calculated. Here only terms with three derivatives are kept to compare with the results given by differentiation of Eq. (6.4). The calculation of the $A_{3,\mu}$ tadpole is exactly parallel to this. Furnished with the results of these two calculations, we acquire two more relations:

$$\begin{aligned}
B_{10} &= \frac{ig'}{16\pi^2}\left[\frac{1}{12}-\frac{1}{18}\ln\left[\frac{H^2}{h^2}\right]\right],\\
2(B_8+B_{11}+B_{13})+B_9 &= \frac{ig'}{16\pi^2}\left[\frac{34}{45}\right],
\end{aligned} \tag{6.39}$$

plus others, which further check some coefficients already obtained. Finally, we compute the $W_\mu^+\phi^-$ mixing, again with arbitrary numbers of neutral scalars emitted by internal fermion propagators. Here we adopt the convention that all electric charges flow into the two-point function. This calculation is similar to computing the $\phi^+\phi^-$ self-energy. Instead of repeating the details, we simply list all the new relations:

$$2B_8-B_{11}+2B_{13}=\frac{ig}{16\pi^2}\left[-\frac{7}{90}\right],$$
$$B_9+B_{11}=\frac{ig}{16\pi^2}\left[\frac{5}{6}\right], \qquad (6.40)$$
$$B_{12}=\frac{ig}{16\pi^2}\left[\frac{1}{6}\right].$$

Solving Eqs. (6.34), (6.39), and (6.40), we arrive at

$$B_7=\frac{ig'}{16\pi^2}\left[-\frac{49}{180}+\frac{1}{18}\ln\left[\frac{H^2}{h^2}\right]\right],$$
$$B_8=\frac{ig}{16\pi^2}\left[\frac{31}{60}-\frac{1}{3}\ln\left[\frac{H^2}{h^2}\right]\right],$$
$$B_9=\frac{ig}{16\pi^2}\left[\frac{5}{6}\right],$$
$$B_{10}=\frac{ig'}{16\pi^2}\left[\frac{1}{12}-\frac{1}{18}\ln\left[\frac{H^2}{h^2}\right]\right], \qquad (6.41)$$
$$B_{11}=\frac{ig}{16\pi^2}(0),$$
$$B_{12}=\frac{ig}{16\pi^2}\left[\frac{1}{6}\right],$$
$$B_{13}=\frac{ig}{16\pi^2}\left[-\frac{5}{9}+\frac{1}{3}\ln\left[\frac{H^2}{h^2}\right]\right].$$

This completes our determination of the one-loop bosonic effective vertices. Before we leave this section, let us remark that the divergences $1/\epsilon$ in A_{11}, A_{21}, B_1, and B_3 can be removed by coupling renormalization of λ and wave-function renormalizations of ϕ, A^a_μ, and B_μ.

VII. ONE-LOOP EFFECTIVE LAGRANGIAN

In Sec. VI, we completed the construction of Γ_{1LPI} to one-loop order for bosonic vertices in the standard model. One may recall that the approach there was to perform a derivative expansion with respect to both bottom- and top-quark internal lines, irrespective of the compositions of the graphs. Generally, the validity of this effective functional is only in a region where the external momenta are less than both of the top- and the bottom-quark masses. In view of this, additional work is needed to construct an effective Lagrangian, capable of describing any low-energy light-particle process with external momenta less than m_t. One can formally infer the procedure to arrive at such an effective Lagrangian $L_{\rm eff}^{\rm light\ theory}$ from the operator equation:

$$T\left[\exp\left[i\int L^{\rm full\ theory}\right]\right]=T\left[\exp\left[i\int L_{\rm eff}^{\rm light\ theory}\right]\right]. \qquad (7.1)$$

The $L_{\rm eff}^{\rm light\ theory}$ at the tree level, which we shall denote as $L_{\rm eff}^{\rm tree}$, is just the nonlinear Lagrangian. To facilitate the discussion of one-loop corrections, let us write $\Gamma_{1LPI}^{1\ \rm loop}=\int d^4x\ \Omega_{1LPI}^{1\ \rm loop}$. The one-loop contributions from $L_{\rm eff}^{\rm tree}$ will be denoted by $\Omega_{\rm ind}^{1\ \rm loop}$. Then, as implied by Eq. (7.1), $L_{\rm eff}^{1\ \rm loop}\equiv L_{\rm eff}^{1\ \rm loop\ light\ theory}$ is given by

$$L_{\rm eff}^{1\ loop}=\Omega_{1LPI}^{1\ \rm loop}-\Omega_{\rm ind}^{1\ \rm loop}. \qquad (7.2)$$

We shall illustrate in the following that $L_{\rm eff}^{1\ \rm loop}$, given by Eq. (7.2) along with $L_{\rm eff}^{\rm tree}$, gives correctly all the light-particle processes with external momenta less than m_t. Parenthetically, $L_{\rm eff}^{\rm light\ theory}$ can be constructed to any loop order with the procedure given above.

As we have pointed out at the beginning, Ω_{1LPI} (or Γ_{1LPI}) is obtained by performing derivative expansions with respect to both bottom- and top-quark propagators, which implies that Ω_{1LPI} is valid only when all the external momenta are less than both the bottom- and the top-quark masses. In the case that some of the external momenta are actually greater than m_b, Ω_{1LPI} strictly speaking cannot be used. The simplest example is its failure to describe a process involving diagrams with only bottom-quark internal lines. Another example is to consider a diagram which has at least one top-quark internal line and several bottom-quark lines, where the external momenta are large enough to reach the threshold of bottom-quark pair productions. This is the case for the process $W_\mu^+\to\phi^+\phi'^{0\dagger}$. There we expect to encounter terms behaving as $\ln[p^2+x(1-x)m_b^2]$, where x is a Feynman parameter and p is a generic external momentum. These terms are nonlocal and cannot be described properly by Ω_{1LPI}. In fact, under our approach for constructing the Ω_{1LPI}, such nonlocal logarithmic corrections are made local in Ω_{1LPI} at the price of assuming $p_{\rm ext}\ll m_b$, which permits a further expansion. If this were the best we could do, the kinematic validity of our result would have been highly restrictive. Fortunately, Zimmermann's oversubtraction identity ensures that, if we just follow Eq. (7.2), these threshold logarithms, which are required by analyticity, can be reproduced. At the one-loop level, the leftover piece $L_{\rm eff}^{1\ \rm loop}$ will no longer contain any term which originates from nonlocal logarithmic functions. These nonlocal logarithmic corrections will now come out directly from exact calculations of the relevant one-loop diagrams based on Feynman rules given by $L_{\rm eff}^{\rm tree}$. In summary, one will see that $L_{\rm eff}^{\rm tree}$ and $L_{\rm eff}^{1\ \rm loop}$ constitute an effective Lagrangian, which respects unitarity and analyticity in the extraction of the heavy-top-quark effects for any low-energy process.

It is our task now to construct $\Omega_{\rm ind}^{1\ \rm loop}$ with which one can determine $L_{\rm eff}^{1\ \rm loop}$. As we have mentioned before, $\Omega_{\rm ind}^{1\ \rm loop}$ is induced by $L_{\rm eff}^{\rm tree}\equiv L_{\rm nl}$. We again exploit the $SU(2)\times U(1)$ symmetry which remains valid for the nonlinear model. Following the same procedure as in the previous section, we first write down all possible gauge-invariant operators carrying zero, two, or four derivatives. They are listed as follows.

A. Potential terms

$$a_{11}(\bar{\phi}\phi)^2,\quad a_{12}(\bar{\phi}\phi)^2\ln\left[\frac{\bar{\phi}\phi}{\mu^2}\right]. \tag{7.3}$$

B. Two-derivative terms

$$\begin{aligned}
&a_{20}(D_\mu\phi)^\dagger(D^\mu\phi)\ln\left[\frac{\bar{\phi}\phi}{\mu^2}\right],\\
&a_{21}(D_\mu\phi)^\dagger(D^\mu\phi),\\
&a_{22}[(D_\mu\phi)^\dagger\phi][(D^\mu\phi)^\dagger\phi]/(\bar{\phi}\phi),\\
&a'_{22}[(D_\mu\phi)^\dagger\phi][(D^\mu\phi)^\dagger\phi]\ln\left[\frac{\bar{\phi}\phi}{\mu^2}\right]\Big/(\bar{\phi}\phi),\\
&a_{23}[\bar{\phi}(D_\mu\phi)][\bar{\phi}(D^\mu\phi)]/(\bar{\phi}\phi),\\
&a'_{23}[\bar{\phi}(D_\mu\phi)][\bar{\phi}(D^\mu\phi)]\ln\left[\frac{\bar{\phi}\phi}{\mu^2}\right]\Big/(\bar{\phi}\phi),\\
&a_{24}[\bar{\phi}(D_\mu\phi)][(D^\mu\phi)^\dagger\phi]/(\bar{\phi}\phi),\\
&a'_{24}[\bar{\phi}(D_\mu\phi)][(D^\mu\phi)^\dagger\phi]\ln\left[\frac{\bar{\phi}\phi}{\mu^2}\right]\Big/(\bar{\phi}\phi).
\end{aligned} \tag{7.4}$$

C. Four-derivative terms

In addition to the invariants in Sec. VI, for example,

$$I^{\mu\nu,\nu}_{\mu,0}=[(D^\mu D^\nu\phi)^\dagger D_\nu\phi][(D_\mu\phi)^\dagger\phi]/(\bar{\phi}\phi)^2,$$

we introduce

$$J^{\mu\nu,\nu}_{\mu,0}=[(D^\mu D^\nu\phi)^\dagger D_\nu\phi][(D_\mu\phi)^\dagger\phi]\ln\left[\frac{\bar{\phi}\phi}{\mu^2}\right]\Big/(\bar{\phi}\phi)^2.$$

In general, the invariants denoted by J's carry an additional factor $\ln(\bar{\phi}\phi/\mu^2)$ to the corresponding I's. They are introduced, because we expect to encounter new divergences, as the nonlinear Lagrangian $L_{\rm nl}$ is "nonrenormalizable." Now the possible four-derivative terms are

$$\begin{aligned}
&e_1I^{\mu\nu,\mu\nu},\quad e_2(I^{\mu\nu,0}_{0,\mu\nu}+\text{H.c.}),\quad e_3I^{\mu\nu,0}_{\mu\nu,0},\\
&e_5(I^{\mu\nu,\nu}_{\mu,0}+\text{H.c.}),\quad e_6(I^{\mu\nu,\nu}_{0,\mu}+\text{H.c.}),\\
&e_9(I^{\mu\nu,0}_{\nu,\mu}+\text{H.c.}),\quad e_{11}I^{\nu,\mu}_{\nu,\mu},\quad e_{12}I^{\nu,\mu}_{\mu,\nu},\\
&e_{13}I^{\mu,\mu}_{\nu,\nu},\quad e_{14}({}_{\mu,0}I^{\nu\mu,0}_{\nu,0}+\text{H.c.}),\quad e_{15}({}_{0,\mu}I^{\nu\mu,0}_{\nu,0}+\text{H.c.}),\\
&e_{16}({}_{0,\mu}I^{\nu\mu,0}_{0,\nu}+\text{H.c.}),\quad e_{20}({}_{\mu,0}I^{\nu,\nu}_{\mu,0}+\text{H.c.}),\\
&e_{21}({}_{0,\mu}I^{\nu,\nu}_{\mu,0}),\quad e_{23}({}_{\nu,0}I^{\nu,\mu}_{\mu,0}+\text{H.c.}),\quad e_{24}({}_{0,\nu}I^{\nu,\mu}_{\mu,0}),\\
&e_{25}({}_{\nu,0}I^{\nu,\mu}_{0,\mu}),\quad e_{27}({}^{\nu,0}_{\nu,0}I^{\mu,0}_{\mu,0}+\text{H.c.}),\quad e_{28}({}^{0,\nu}_{\nu,0}I^{\mu,0}_{\mu,0}+\text{H.c.}),\\
&e_{29}({}^{0,\nu}_{0,\nu}I^{\mu,0}_{\mu,0}),\quad e_{30}({}^{0,\nu}_{\nu,0}I^{\mu,0}_{0,\mu}),
\end{aligned} \tag{7.5}$$

and

$$\begin{aligned}
&d_1J^{\mu\nu,\mu\nu},\quad d_2(J^{\mu\nu,0}_{0,\mu\nu}+\text{H.c.}),\quad d_3J^{\mu\nu,0}_{\mu\nu,0},\\
&d_5(J^{\mu\nu,\nu}_{\mu,0}+\text{H.c.}),\quad d_6(J^{\mu\nu,\nu}_{0,\mu}+\text{H.c.}),\\
&d_9(J^{\mu\nu,0}_{\nu,\mu}+\text{H.c.}),\quad d_{11}J^{\nu,\mu}_{\nu,\mu},\quad d_{12}J^{\nu,\mu}_{\mu,\nu},\\
&d_{13}J^{\mu,\mu}_{\nu,\nu},\quad d_{14}({}_{\mu,0}J^{\nu\mu,0}_{\nu,0}+\text{H.c.}),\quad d_{15}({}_{0,\mu}J^{\nu\mu,0}_{\nu,0}+\text{H.c.}),\\
&d_{16}({}_{0,\mu}J^{\nu\mu,0}_{0,\nu}+\text{H.c.}),\quad d_{20}({}_{\mu,0}J^{\nu,\nu}_{\mu,0}+\text{H.c.}),\\
&d_{21}({}_{0,\mu}J^{\nu,\nu}_{\mu,0}),\quad d_{23}({}_{\nu,0}J^{\nu,\mu}_{\mu,0}+\text{H.c.}),\quad d_{24}({}_{0,\nu}J^{\nu,\mu}_{\mu,0}),\\
&d_{25}({}_{\nu,0}J^{\nu,\mu}_{0,\mu}),\quad d_{27}({}^{\nu,0}_{\nu,0}J^{\mu,0}_{\mu,0}+\text{H.c.}),\quad d_{28}({}^{0,\nu}_{\nu,0}J^{\mu,0}_{\mu,0}+\text{H.c.}),\\
&d_{29}({}^{0,\nu}_{0,\nu}J^{\mu,0}_{\mu,0}),\quad d_{30}({}^{0,\nu}_{\nu,0}J^{\mu,0}_{0,\mu}).
\end{aligned} \tag{7.6}$$

D. Terms with field strengths

$$\begin{aligned}
&b_1G^a_{\mu\nu}G^{a\mu\nu},\quad b_2G^a_{\mu\nu}G^{a\mu\nu}\ln\left[\frac{\bar{\phi}\phi}{\mu^2}\right],\\
&b_3F_{\mu\nu}F^{\mu\nu},\quad b_4F_{\mu\nu}F^{\mu\nu}\ln\left[\frac{\bar{\phi}\phi}{\mu^2}\right],\\
&b_5(\bar{\phi}G_{\mu\nu}\phi)F^{\mu\nu}/(\bar{\phi}\phi),\quad b_6(\bar{\phi}G_{\mu\nu}\phi)(\bar{\phi}G^{\mu\nu}\phi)/(\bar{\phi}\phi)^2,\\
&b'_5(\bar{\phi}G_{\mu\nu}\phi)\ln\left[\frac{\bar{\phi}\phi}{\mu^2}\right]F^{\mu\nu}/(\bar{\phi}\phi),\\
&b'_6(\bar{\phi}G_{\mu\nu}\phi)(\bar{\phi}G^{\mu\nu}\phi)\ln\left[\frac{\bar{\phi}\phi}{\mu^2}\right]\Big/(\bar{\phi}\phi)^2,\\
&b_7[(D_\mu\phi)^\dagger(D_\nu\phi)-\text{H.c.}]F^{\mu\nu}/(\bar{\phi}\phi),\\
&b'_7[(D_\mu\phi)^\dagger(D_\nu\phi)-\text{H.c.}]F^{\mu\nu}\ln\left[\frac{\bar{\phi}\phi}{\mu^2}\right]\Big/(\bar{\phi}\phi),\\
&b_8[(D_\mu\phi)^\dagger(D_\nu\phi)-\text{H.c.}](\bar{\phi}G^{\mu\nu}\phi)/(\bar{\phi}\phi)^2,\\
&b'_8[(D_\mu\phi)^\dagger(D_\nu\phi)-\text{H.c.}](\bar{\phi}G^{\mu\nu}\phi)\ln\left[\frac{\bar{\phi}\phi}{\mu^2}\right]\Big/(\bar{\phi}\phi)^2,\\
&b_9(D_\mu\phi)^\dagger G^{\mu\nu}(D_\nu\phi)/(\bar{\phi}\phi),\\
&b'_9(D_\mu\phi)^\dagger G^{\mu\nu}(D_\nu\phi)\ln\left[\frac{\bar{\phi}\phi}{\mu^2}\right]\Big/(\bar{\phi}\phi),\\
&b_{10}\{[\bar{\phi}(D_\nu\phi)][(D_\mu\phi)^\dagger\phi]-\text{H.c.}\}F^{\mu\nu}/(\bar{\phi}\phi)^2,\\
&b'_{10}\{[\bar{\phi}(D_\nu\phi)][(D_\mu\phi)^\dagger\phi]-\text{H.c.}\}F^{\mu\nu}\ln\left[\frac{\bar{\phi}\phi}{\mu^2}\right]\Big/(\bar{\phi}\phi)^2,\\
&b_{11}\{[\bar{\phi}G^{\mu\nu}(D_\nu\phi)][(D_\mu\phi)^\dagger\phi]-\text{H.c.}\}/(\bar{\phi}\phi)^2,\\
&b'_{11}\{[\bar{\phi}G^{\mu\nu}(D_\nu\phi)][(D_\mu\phi)^\dagger\phi]-\text{H.c.}\}\ln\left[\frac{\bar{\phi}\phi}{\mu^2}\right]\Big/(\bar{\phi}\phi)^2,\\
&b_{12}\{[\bar{\phi}G^{\mu\nu}(D_\nu\phi)][\bar{\phi}(D_\mu\phi)]-\text{H.c.}\}/(\bar{\phi}\phi)^2,\\
&b'_{12}\{[\bar{\phi}G^{\mu\nu}(D_\nu\phi)][\bar{\phi}(D_\mu\phi)]-\text{H.c.}\}\ln\left[\frac{\bar{\phi}\phi}{\mu^2}\right]\Big/(\bar{\phi}\phi)^2,\\
&b_{13}(\bar{\phi}G^{\mu\nu}\phi)\{[(D_\mu\phi)^\dagger\phi][\bar{\phi}(D_\nu\phi)]-\text{H.c.}\}/(\bar{\phi}\phi)^3,\\
&b'_{13}(\bar{\phi}G^{\mu\nu}\phi)\{[(D_\mu\phi)^\dagger\phi][\bar{\phi}(D_\nu\phi)]\\
&\qquad-\text{H.c.}\}\ln\left[\frac{\bar{\phi}\phi}{\mu^2}\right]\Big/(\bar{\phi}\phi)^3.
\end{aligned} \tag{7.7}$$

To determine the above coefficients, it is sufficient to compute the same set of processes as in the last section. The only difference is the interaction Lagrangian on which our calculations are based. For determining the coefficients in the potential, two-derivative and four-derivative terms, one can again set the coupling constants g and g' to zero. The interaction Lagrangian between fermions and scalar bosons reads

$$L_{\rm int}=\left[\frac{\phi^+\phi^-\partial_\mu\phi^{0\dagger}}{\phi^{0\dagger}\phi^0\phi^{0\dagger}}-\frac{\phi^-\partial_\mu\phi^+}{\phi^{0\dagger}\phi^0}\right]\bar{b}_L\frac{1}{i}\gamma^\mu b_L -\frac{\phi^+\phi^-}{\phi^{0\dagger}\phi^0}\bar{b}_L\frac{1}{i}\gamma_\mu\partial^\mu b_L -h\frac{\phi^+\phi^-}{\phi^0}\bar{b}_L b_R-h\frac{\phi^+\phi^-}{\phi^{0\dagger}}\bar{b}_R b_L \;. \tag{7.8}$$

1. a_{11} to a'_{24}

As we have shown in the last section, these coefficients can be completely determined by computing the $\phi^+\phi^-$ self-energy with arbitrary numbers of neutral scalars emitted by fermion propagators. The $\phi^+\phi^-$ self-energy according to the interaction Lagrangian in Eq. (7.8) can be written as

$$A_{\phi^+\phi^-}=I_a+I_b+I_c+I_d \;, \tag{7.9}$$

with

$$I_a=-{\rm Tr}\int d^4x\,\gamma_\mu\langle T[b_L(x)\bar{b}_L(x)]\rangle \times\left[\frac{\phi^+(x)\phi^-(x)\partial^\mu\phi^{0\dagger}(x)}{\phi^{0\dagger}(x)\phi^0(x)\phi^{0\dagger}(x)}-\frac{\phi^-(x)\partial^\mu\phi^+(x)}{\phi^{0\dagger}(x)\phi^0(x)}\right], \tag{7.10}$$

$$I_b={\rm Tr}\int d^4x\langle T[\partial\!\!\!/ b_L(x)\bar{b}_L(x)]\rangle\frac{\phi^+(x)\phi^-(x)}{\phi^{0\dagger}(x)\phi^0(x)}\;, \tag{7.11}$$

$$I_c=ih\,{\rm Tr}\int d^4x\langle T[b_R(x)\bar{b}_L(x)]\rangle\frac{\phi^+(x)\phi^-(x)}{\phi^0(x)}\;, \tag{7.12}$$

$$I_d=ih\,{\rm Tr}\int d^4x\langle T[b_L(x)\bar{b}_R(x)]\rangle\frac{\phi^+(x)\phi^-(x)}{\phi^{0\dagger}(x)}\;. \tag{7.13}$$

Applying the derivative expansion technique on the above integrals and comparing the result with that given by differentiating the relevant operators in Eqs. (7.3) and (7.4), we end up with

$$\begin{aligned}
a_{11}&=\frac{1}{16\pi^2}\left[h^4\left[\frac{-1}{\epsilon}+\gamma_E+\ln(\pi)\right]-h^4+h^4\ln(h^2)\right],\\
a_{12}&=\frac{1}{16\pi^2}(h^4)\;,\\
a_{20}&=\frac{1}{16\pi^2}(0)\;,\\
a_{21}&=\frac{1}{16\pi^2}(0)\;,\\
a_{22}&=\frac{1}{16\pi^2}\left[\frac{1}{6}h^2\right],\\
a'_{22}&=\frac{1}{16\pi^2}(0)\;,\\
a_{23}&=\frac{1}{16\pi^2}\left[\frac{1}{6}h^2\right],\\
a'_{23}&=\frac{1}{16\pi^2}(0)\;,\\
a_{24}&=\frac{1}{16\pi^2}\left[h^2\left[-\frac{1}{\epsilon}+\gamma_E+\ln(\pi)\right]+\frac{5}{6}h^2+h^2\ln(h^2)\right],\\
a'_{24}&=\frac{1}{16\pi^2}(h^2)\;.
\end{aligned} \tag{7.14}$$

2. e_1 to d_{30}

As in the last section, the amplitude for $\phi^+\phi^-$ scattering is used to determine the coefficients of this sector. For convenience, we define

$$S_\mu=\left[\frac{\phi^+\phi^-\partial_\mu\phi^{0\dagger}}{\phi^{0\dagger}\phi^0\phi^{0\dagger}}-\frac{\phi^-\partial_\mu\phi^+}{\phi^{0\dagger}\phi^0}\right].$$

Performing Wick contraction on the given interaction Lagrangian, we have the scattering amplitude written as

$$A_{\phi^+\phi^-\phi^+\phi^-}=\sum_1^{10}I_i\;, \tag{7.15}$$

with

$$I_1=\frac{-1}{2}{\rm Tr}\int d^4x\,d^4y\langle T[b_L(x)\bar{b}_L(y)]\rangle\gamma_\mu S^\mu(y)\langle T[b_L(y)\bar{b}_L(x)]\rangle\gamma_\nu S^\nu(x)\;, \tag{7.16}$$

$$I_2={\rm Tr}\int d^4x\,d^4y\langle T[b_L(x)\bar{b}_L(y)]\rangle\frac{\phi^+\phi^-}{\phi^{0\dagger}\phi^0}(y)\langle T[\partial\!\!\!/ b_L(y)\bar{b}_L(x)]\rangle\gamma_\mu S^\mu(x)\;, \tag{7.17}$$

$$I_3=ih\,{\rm Tr}\int d^4x\,d^4y\langle T[b_L(x)\bar{b}_L(y)]\rangle\frac{\phi^+\phi^-}{\phi^0}(y)\langle T[b_R(y)\bar{b}_L(x)]\rangle\gamma_\mu S^\mu(x)\;, \tag{7.18}$$

$$I_4=ih\,{\rm Tr}\int d^4x\,d^4y\langle T[b_L(x)\bar{b}_R(y)]\rangle\frac{\phi^+\phi^-}{\phi^{0\dagger}}(y)\langle T[b_L(y)\bar{b}_L(x)]\rangle\gamma_\mu S^\mu(x)\;, \tag{7.19}$$

$$I_5=-\tfrac{1}{2}{\rm Tr}\int d^4x\,d^4y\langle T[\partial\!\!\!/ b_L(x)\bar{b}_L(y)]\rangle\frac{\phi^+\phi^-}{\phi^{0\dagger}\phi^0}(y)\langle T[\partial\!\!\!/ b_L(y)\bar{b}_L(x)]\rangle\frac{\phi^+\phi^-}{\phi^{0\dagger}\phi^0}(x)\;, \tag{7.20}$$

$$I_6=-ih\,\mathrm{Tr}\int d^4x\, d^4y\langle T[b_R(x)\bar{b}_L(y)]\rangle\frac{\phi^+\phi^-}{\phi^{0\dagger}\phi^0}(y)\langle T[\partial\!\!\!/ b_L(y)\bar{b}_L(x)]\rangle\frac{\phi^+\phi^-}{\phi^0}(x)\,, \tag{7.21}$$

$$I_7=-ih\,\mathrm{Tr}\int d^4x\, d^4y\langle T[b_L(x)\bar{b}_L(y)]\rangle\frac{\phi^+\phi^-}{\phi^{0\dagger}\phi^0}(y)\langle T[\partial\!\!\!/ b_L(y)\bar{b}_R(x)]\rangle\frac{\phi^+\phi^-}{\phi^{0\dagger}}(x)\,, \tag{7.22}$$

$$I_8=\frac{h^2}{2}\,\mathrm{Tr}\int d^4x\, d^4y\langle T[b_R(x)\bar{b}_L(y)]\rangle\frac{\phi^+\phi^-}{\phi^0}(y)\langle T[b_R(y)\bar{b}_L(x)]\rangle\frac{\phi^+\phi^-}{\phi^0}(x)\,, \tag{7.23}$$

$$I_9=h^2\,\mathrm{Tr}\int d^4x\, d^4y\langle T[b_R(x)\bar{b}_L(y)]\rangle\frac{\phi^+\phi^-}{\phi^{0\dagger}}(y)\langle T[b_L(y)\bar{b}_L(x)]\rangle\frac{\phi^+\phi^-}{\phi^0}(x)\,, \tag{7.24}$$

$$I_{10}=\frac{h^2}{2}\,\mathrm{Tr}\int d^4x\, d^4y\langle T[b_L(x)\bar{b}_R(y)]\rangle\frac{\phi^+\phi^-}{\phi^{0\dagger}}(y)\langle T[b_L(y)\bar{b}_R(x)]\rangle\frac{\phi^+\phi^-}{\phi^{0\dagger}}(x)\,. \tag{7.25}$$

We remark that $\langle T[\partial\!\!\!/ b_L(y)\bar{b}_L(x)]\rangle$, which appears in Eqs. (7.17), (7.20), and (7.21), should be taken as

$$\langle T[\partial\!\!\!/ b_L(y)\bar{b}_L(x)]\rangle=\gamma_\mu\partial^\mu\langle T[b_L(y)\bar{b}_L(x)]\rangle\,. \tag{7.26}$$

Similarly, $\langle T[\partial\!\!\!/ b_L(y)\bar{b}_R(x)]\rangle$ in Eqs. (7.11) and (7.22) is to be substituted for by $\gamma_\mu\partial^\mu\langle T[b_L(y)\bar{b}_R(x)]\rangle$. This is due to the fact that we have performed Wick contractions based on the interaction Lagrangian in Eq. (7.8), where there are derivative couplings, rather than the interaction Hamiltonian. One can show that contributions due to the extra "seagull" terms in the interaction Hamiltonian will be compensated completely by this treatment. Computing four-derivative terms of each integral and comparing them with those obtained by operator differentiation, we acquire

$$\begin{aligned}
&e_1=\frac{1}{16\pi^2}(0),\quad e_2=\frac{1}{16\pi^2}\left[\frac{2}{15}\right],\quad e_3=\frac{1}{16\pi^2}\left[-\frac{1}{60}\right],\\
&e_5=\frac{1}{16\pi^2}(0),\quad e_6=\frac{1}{16\pi^2}(0),\quad e_9=\frac{1}{16\pi^2}\left[\frac{1}{10}\right],\\
&e_{11}=\frac{1}{16\pi^2}\left[-\frac{1}{5}-\frac{1}{3}\ln(h^2)-\frac{1}{3}\left[-\frac{1}{\epsilon}+\gamma_E+\ln(\pi)\right]\right],\\
&e_{14}=\frac{1}{16\pi^2}\left[\frac{1}{18}\right],\quad e_{15}=\frac{1}{16\pi^2}\left[-\frac{2}{9}\right],\\
&e_{16}=\frac{1}{16\pi^2}\left[-\frac{1}{9}\right],e_{20}=\frac{1}{16\pi^2}(0),\quad e_{23}=\frac{1}{16\pi^2}\left[-\frac{1}{18}\right],\\
&e_{25}=\frac{1}{16\pi^2}\left[-\frac{1}{18}+\frac{2}{3}\ln(h^2)+\frac{2}{3}\left[-\frac{1}{\epsilon}+\gamma_E+\ln(\pi)\right]\right],\\
&e_{27}=\frac{1}{16\pi^2}\left[-\frac{1}{24}\right],\quad e_{28}=\frac{1}{16\pi^2}\left[\frac{1}{6}\right],\\
&e_{29}=\frac{1}{16\pi^2}\left[\frac{1}{3}-\frac{1}{3}\ln(h^2)-\frac{1}{3}\left[-\frac{1}{\epsilon}+\gamma_E+\ln(\pi)\right]\right],\\
&e_{30}=\frac{1}{16\pi^2}\left[\frac{1}{6}+\frac{1}{3}\ln(h^2)+\frac{1}{3}\left[-\frac{1}{\epsilon}+\gamma_E+\ln(\pi)\right]\right];
\end{aligned}\tag{7.27}$$

$$\begin{aligned}
&d_1=\frac{1}{16\pi^2}(0),\quad d_2=\frac{1}{16\pi^2}(0),\quad d_3=\frac{1}{16\pi^2}(0)\,,\\
&d_5=\frac{1}{16\pi^2}(0),\quad d_6=\frac{1}{16\pi^2}(0),\quad d_9=\frac{1}{16\pi^2}(0)\,,\\
&d_{11}=\frac{1}{16\pi^2}\left[-\frac{1}{3}\right],\quad d_{14}=\frac{1}{16\pi^2}(0),\quad d_{15}=\frac{1}{16\pi^2}(0)\,,\\
&d_{16}=\frac{1}{16\pi^2}(0),\quad d_{20}=\frac{1}{16\pi^2}(0),\quad d_{23}=\frac{1}{16\pi^2}(0)\,,\\
&d_{25}=\frac{1}{16\pi^2}\left[\frac{2}{3}\right],\quad d_{27}=\frac{1}{16\pi^2}(0),\quad d_{28}=\frac{1}{16\pi^2}(0)\,,\\
&d_{29}=\frac{1}{16\pi^2}\left[-\frac{1}{3}\right],\quad d_{30}=\frac{1}{16\pi^2}\left[\frac{1}{3}\right];
\end{aligned}\tag{7.28}$$

$$\begin{aligned}
&2e_{12}+e_{24}=\frac{1}{16\pi^2}\left[\frac{19}{90}\right],\\
&2e_{13}+e_{21}=\frac{1}{16\pi^2}(0)\,,\\
&e_{12}+e_{13}=\frac{1}{16\pi^2}\left[\frac{3}{10}+\frac{1}{3}ln(h^2)\right.\\
&\qquad\left.+\frac{1}{3}\left[-\frac{1}{\epsilon}+\gamma_E+\ln(\pi)\right]\right],\\
&2d_{12}+d_{24}=\frac{1}{16\pi^2}(0)\,,\\
&2d_{13}+d_{21}=\frac{1}{16\pi^2}(0)\,,\\
&d_{12}+d_{13}=\frac{1}{16\pi^2}\left[\frac{1}{3}\right].
\end{aligned}\tag{7.29}$$

Once again, we see that $e_{12,13,21,24}$ and $d_{12,13,21,24}$ cannot be further resolved.

3. b_1 to b'_{13}

With the conventional method, we compute the $B_\mu B_\nu$, $A_{3,\mu}A_{3,\nu}$, and $W_\mu^+W_\nu^-$ self-energies and the $B_\mu A_{3,\nu}$ mixing. It is easy to see that the neutral-boson diagrams would receive contributions only from internal b-quark loops in the nonlinear theory. Furthermore, the $W_\mu^+W_\nu^-$

self-energy receives no contributions from internal fermion loops. Knowing these two-point functions, we are ready to determine coefficients b_1 to b'_6. They are

$$
\begin{aligned}
b_1 &= \frac{g^2}{16\pi^2}(0) , \\
b_2 &= \frac{g^2}{16\pi^2}(0) , \\
b_3 &= \frac{g'^2}{16\pi^2}\left[\frac{5}{216}\left[-\frac{1}{\epsilon}+\ln(\pi)+\gamma_E\right] + \frac{5}{432}+\frac{5}{216}\ln(h^2)\right] , \\
b_4 &= \frac{g'^2}{16\pi^2}\left[\frac{5}{216}\right] , \\
b_5 &= \frac{gg'}{16\pi^2}\left[-\frac{1}{36}-\frac{1}{18}\ln(h^2) - \frac{1}{18}\left[-\frac{1}{\epsilon}+\ln(\pi)+\gamma_E\right]\right] , \\
b'_5 &= \frac{gg'}{16\pi^2}\left[-\frac{1}{18}\right] , \\
b_6 &= \frac{g^2}{16\pi^2}\left[\frac{1}{12}+\frac{1}{6}\ln(h^2)+\frac{1}{6}\left[-\frac{1}{\epsilon}+\ln(\pi)+\gamma_E\right]\right] , \\
b'_6 &= \frac{g^2}{16\pi^2}\left[\frac{1}{6}\right] .
\end{aligned}
\tag{7.30}
$$

To determine the rest of the coefficients, we observe that both the $B_\mu W^+_\nu W^-_\sigma$ and $A_{3,\mu} W^+_\nu W^-_\sigma$ triangle diagrams vanish in the nonlinear model. Then, with the external field technique, we compute B_μ and $A_{3,\mu}$ tadpoles as well as $W^+_\mu \phi^-$ mixing with the emission of arbitrary numbers of neutral scalars by fermion propagators. One can easily see that both tadpole diagrams are contributed to only by internal b-quark loops. For $W^+_\mu \phi^-$ mixing, one starts with the interaction Lagrangian

$$
L_{\rm int} = -\frac{g}{\sqrt{2}}\bar{b}_L\gamma_\mu b_L \frac{W^{+\mu}\phi^-}{\phi^0} . \tag{7.31}
$$

The S-matrix element is hence given by

$$
S = i\frac{g}{\sqrt{2}}{\rm Tr}\int d^4x\,\gamma_\mu\langle T[b_L(x)\bar{b}_L(x)]\rangle\frac{W^{+\mu}\phi^-}{\phi^0}(x) . \tag{7.32}
$$

This expression can be transformed in the same way as that shown in Eq. (6.38). In summary, we obtain

$$
\begin{aligned}
b_7 &= \frac{ig'}{16\pi^2}\left[-\frac{17}{180}-\frac{1}{18}\left[-\frac{1}{\epsilon}+\ln(\pi)+\gamma_E\right] - \frac{1}{18}\ln(h^2)\right] , \\
b_8 &= \frac{ig}{16\pi^2}\left[\frac{3}{10}+\frac{1}{3}\left[-\frac{1}{\epsilon}+\ln(\pi)+\gamma_E\right]+\frac{1}{3}\ln(h^2)\right] , \\
b_9 &= \frac{ig}{16\pi^2}(0) , \\
b_{10} &= \frac{ig'}{16\pi^2}\left[-\frac{101}{180}+\frac{1}{18}\left[-\frac{1}{\epsilon}+\ln(\pi)+\gamma_E\right] + \frac{1}{18}\ln(h^2)\right] , \\
b_{11} &= \frac{ig}{16\pi^2}(0) , \\
b_{12} &= \frac{ig}{16\pi^2}(0) , \\
b_{13} &= \frac{ig}{16\pi^2}\left[\frac{1}{36}-\frac{1}{3}\left[-\frac{1}{\epsilon}+\ln(\pi)+\gamma_E\right]-\frac{1}{3}\ln(h^2)\right] ,
\end{aligned}
\tag{7.33}
$$

and

$$
\begin{aligned}
b'_7 &= \frac{ig'}{16\pi^2}\left[-\frac{1}{18}\right] , \\
b'_8 &= \frac{ig}{16\pi^2}\left[\frac{1}{3}\right] , \\
b'_9 &= \frac{ig}{16\pi^2}(0) , \\
b'_{10} &= \frac{ig'}{16\pi^2}\left[\frac{1}{18}\right] , \\
b'_{11} &= \frac{ig}{16\pi^2}(0) , \\
b'_{12} &= \frac{ig}{16\pi^2}(0) , \\
b'_{13} &= \frac{ig}{16\pi^2}\left[-\frac{1}{3}\right] .
\end{aligned}
\tag{7.34}
$$

Now that we have completed the construction of $\Omega^{1\,\rm loop}_{\rm ind}$, the one-loop effective Lagrangian $L^{1\,\rm loop}_{\rm eff}$ is simply given by Eq. (7.2). For convenience, we write $L^{1\,\rm loop}_{\rm eff}$ as

$$
L^{1\,\rm loop}_{\rm eff} = -V + L^{(2)}_{\rm eff} + L^{(4)}_{\rm eff} + L^{\rm gf}_{\rm eff} , \tag{7.35}
$$

where

$$
V = \frac{1}{16\pi^2}\left\{H^4\left[-\left[-\frac{1}{\epsilon}+\gamma_E+\ln\pi\right]+1-\ln\left[\frac{H^2\bar{\phi}\phi}{\mu^2}\right]\right]\right\}(\bar{\phi}\phi)^2 , \tag{7.36}
$$

$$L_{\text{eff}}^{(2)}=\frac{1}{16\pi^2}\left[\left\{(H^2+h^2)\left[-\frac{1}{\epsilon}+\gamma_E+\ln\pi+\ln\left[\frac{H^2\bar{\phi}\phi}{\mu^2}\right]\right]\right\}(D_\mu\phi)^\dagger(D^\mu\phi)\right.$$
$$+\tfrac{1}{6}H^2\{[(D_\mu\phi)^\dagger\phi][(D^\mu\phi)^\dagger\phi]+[\bar{\phi}(D_\mu\phi)][\bar{\phi}(D^\mu\phi)]\}/(\bar{\phi}\phi)$$
$$\left.+\left[-h^2\left[-\frac{1}{\epsilon}+\gamma_E+\ln\pi\right]+\frac{5}{6}H^2-h^2\ln\left[\frac{H^2\bar{\phi}\phi}{\mu^2}\right]\right][\bar{\phi}(D_\mu\phi)][(D^\mu\phi)^\dagger\phi]/(\bar{\phi}\phi)\right], \tag{7.37}$$

$$L_{\text{eff}}^{(4)}=\frac{1}{16\pi^2}\left[\frac{1}{6}I^{\mu\nu,\mu\nu}-\frac{1}{10}I^{\mu\nu,0}_{0,\mu\nu}-\frac{1}{60}I^{\mu\nu,0}_{\mu\nu,0}-\frac{5}{18}I^{\mu\nu,\nu}_{\mu,0}-\frac{4}{9}I^{\mu\nu,\nu}_{0,\mu}-\frac{8}{45}I^{\mu\nu,0}_{\nu,\mu}+\left[-\frac{1}{6\epsilon'}-\frac{11}{60}+\frac{1}{6}\ln H^2\right]I^{\nu,\mu}_{\nu,\mu}+\frac{1}{2}F_{12}I^{\nu,\mu}_{\mu,\nu}\right.$$
$$+\frac{1}{2}F_{13}I^{\mu,\mu}_{\nu,\nu}+\frac{1}{18}\,{}_{\mu,0}I^{\nu\mu,0}_{\nu,0}+\frac{1}{3}\,{}_{0,\mu}I^{\nu\mu,0}_{\nu,0}+\frac{1}{3}\,{}_{0,\mu}I^{\nu\mu,0}_{0,\nu}+\frac{2}{9}\,{}_{\mu,0}I^{\nu,\nu}_{\mu,0}+\frac{1}{2}F_{21}\,{}_{0,\mu}I^{\nu,\nu}_{\mu,0}+\frac{7}{18}\,{}_{\nu,0}I^{\nu,\mu}_{\mu,0}+\frac{1}{2}F_{24}\,{}_{0,\nu}I^{\nu,\mu}_{\mu,0}$$
$$+\left[\frac{1}{3\epsilon'}-\frac{1}{3}\ln H^2+\frac{7}{12}\right]{}_{\nu,0}I^{\nu,\mu}_{0,\mu}-\frac{1}{24}\,{}^{\nu,0}_{\nu,0}I^{\mu,0}_{\mu,0}-\frac{1}{2}\,{}^{0,\nu}_{\nu,0}I^{\mu,0}_{\mu,0}$$
$$+\left[-\frac{1}{6\epsilon'}+\frac{1}{6}\ln H^2-\frac{13}{36}\right]{}^{0,\nu}_{0,\nu}I^{\mu,0}_{\mu,0}+\left[\frac{1}{6\epsilon'}-\frac{1}{6}\ln H^2-\frac{5}{36}\right]{}^{0,\nu}_{\nu,0}I^{\mu,0}_{0,\mu}+\frac{1}{6}J^{\nu,\mu}_{\nu,\mu}$$
$$\left.+\frac{1}{2}f_{12}J^{\nu,\mu}_{\mu,\nu}+\frac{1}{2}f_{13}J^{\mu,\mu}_{\nu,\nu}+\frac{1}{2}f_{21\,0,\mu}J^{\nu,\nu}_{\mu,0}+\frac{1}{2}f_{24\,0,\nu}J^{\nu,\mu}_{\mu,0}-\frac{1}{3}\,{}_{\nu,0}J^{\nu,\mu}_{0,\mu}+\frac{1}{6}\,{}^{0,\nu}_{0,\nu}J^{\mu,0}_{\mu,0}-\frac{1}{6}\,{}^{0,\nu}_{\nu,0}J^{\mu,0}_{0,\mu}+\text{H.c.}\right], \tag{7.38}$$

$$L_{\text{eff}}^{\text{gf}}=\frac{1}{16\pi^2}\left\{g^2\left[-\frac{1}{24\epsilon'}-\frac{1}{72}+\frac{1}{24}\ln\left[\frac{H^2\bar{\phi}\phi}{\mu^2}\right]\right]G^a_{\mu\nu}G^{a\mu\nu}+g'^2\left[-\frac{17}{432\epsilon'}+\frac{17}{864}+\frac{17}{432}\ln\left[\frac{H^2\bar{\phi}\phi}{\mu^2}\right]\right]F_{\mu\nu}F^{\mu\nu}\right.$$
$$+gg'\left[-\frac{1}{36\epsilon'}+\frac{1}{72}+\frac{1}{36}\ln\left[\frac{H^2\bar{\phi}\phi}{\mu^2}\right]\right](\bar{\phi}G_{\mu\nu}\phi)F^{\mu\nu}/(\bar{\phi}\phi)$$
$$+g^2\left[\frac{1}{12\epsilon'}+\frac{7}{72}-\frac{1}{12}\ln\left[\frac{H^2\bar{\phi}\phi}{\mu^2}\right]\right](\bar{\phi}G_{\mu\nu}\phi)(\bar{\phi}G^{\mu\nu}\phi)/(\bar{\phi}\phi)^2$$
$$+ig'\left[-\frac{1}{18\epsilon'}-\frac{8}{45}+\frac{1}{18}\ln\left[\frac{H^2\bar{\phi}\phi}{\mu^2}\right]\right][(D_\mu\phi)^\dagger(D_\nu\phi)]F^{\mu\nu}/(\bar{\phi}\phi)$$
$$+ig\left[\frac{1}{3\epsilon'}+\frac{13}{60}-\frac{1}{3}\ln\left[\frac{H^2\bar{\phi}\phi}{\mu^2}\right]\right][(D_\mu\phi)^\dagger(D_\nu\phi)](\bar{\phi}G^{\mu\nu}\phi)/(\bar{\phi}\phi)^2$$
$$+\frac{5ig}{12}(D_\mu\phi)^\dagger G^{\mu\nu}(D_\nu\phi)/(\bar{\phi}\phi)+ig'\left[\frac{1}{18\epsilon'}+\frac{29}{45}-\frac{1}{18}\ln\left[\frac{H^2\bar{\phi}\phi}{\mu^2}\right]\right][\bar{\phi}(D_\nu\phi)][(D_\mu\phi)^\dagger\phi]F^{\mu\nu}/(\bar{\phi}\phi)^2$$
$$+\frac{ig}{6}[\bar{\phi}G^{\mu\nu}(D_\nu\phi)][\bar{\phi}(D_\mu\phi)]/(\bar{\phi}\phi)^2$$
$$\left.+ig\left[-\frac{1}{3\epsilon'}-\frac{7}{12}+\frac{1}{3}\ln\left[\frac{H^2\bar{\phi}\phi}{\mu^2}\right]\right](\bar{\phi}G^{\mu\nu}\phi)[(D_\mu\phi)^\dagger\phi][\bar{\phi}(D_\nu\phi)]/(\bar{\phi}\phi)^3+\text{H.c.}\right\}. \tag{7.39}$$

There are some remarks concerned with the one-loop effective Lagrangian $L_{\text{eff}}^{1\,\text{loop}}$ given above. First of all, $1/\epsilon'\equiv 1/\epsilon-\gamma_E-\ln\pi$ in expressions of $L_{\text{eff}}^{(4)}$ an $L_{\text{eff}}^{\text{gf}}$. Second, the operation of Hermitian conjugation in Eqs. (7.38) and (7.39) is understood to be performed even for those operators which are self-conjugate, e.g., $I^{\mu\nu,\mu\nu}$. In other words, it should be done with respect to every term in Eqs. (7.38) and (7.39). Finally, because of reasons given before, coefficients $F_{12,13,21,24}$ as well as $f_{12,13,21,24}$ cannot be further resolved. There are insufficient relations among them:

$$2F_{12}+F_{24}=\tfrac{29}{90}\,,$$
$$2F_{13}+F_{21}=\tfrac{1}{9}\,, \tag{7.40}$$
$$F_{12}+F_{13}=\frac{1}{3\epsilon'}+\frac{2}{45}-\frac{1}{3}\ln H^2\,,$$

and

$$2f_{12}+f_{24}=0\ ,$$
$$2f_{13}+f_{21}=0\ , \tag{7.41}$$
$$f_{12}+f_{13}=-\tfrac{1}{3}\ .$$

Armed with $L_{\text{eff}}^{\text{tree}}$ and $L_{\text{eff}}^{1\,\text{loop}}$, we are ready to extract heavy-top-quark effects for any low-energy process with $p_{\text{ext}} \ll m_t$ to one-loop order. The strategy works in the following way: if one is only interested in tree-level results, $L_{\text{eff}}^{\text{tree}}$ should provide the answers, with corrections of order $O(H^{-1})$. For one-loop corrections, one should add up contributions resulting directly from $L_{\text{eff}}^{1\,\text{loop}}$ and those coming from one-loop diagrams based on applying Feynman rules to $L_{\text{eff}}^{\text{tree}}$. Note the appearance of extra $1/\epsilon$'s in some of the coefficients in $L_{\text{eff}}^{1\,\text{loop}}$. They will disappear when these two one-loop contributions are added up. The effective theory in our approach is self-regulating; there are no new renormalization constants. In Sec. IX, we shall provide a few examples to demonstrate this procedure.

VIII. WESS-ZUMINO TERMS

There is yet another residual signature of a heavy top quark after we remove it from the low-energy sector. It gives rise to Wess-Zumino terms (Ref. [8]). In this section, we shall determine this part of the effective Lagrangian, which consists of terms involving an $\epsilon^{\mu\nu\alpha\beta}$ symbol. Note that we have restricted ourselves to only one quark doublet in our model Lagrangian to simplify our analysis. Consequently, the theory is perturbatively *CP* invariant. As one can show that any operator involving an $\epsilon^{\mu\nu\alpha\beta}$ symbol cannot be both *CP* and gauge invariant, the operators we shall obtain will be gauge noninvariant. They will be a part of the Wess-Zumino Lagrangian induced by chiral anomalies [16]. In a bigger context, because the whole standard model is anomaly-free, the Wess-Zumino terms due to internal quark loops will be canceled by those given by lepton loops.

Wess-Zumino terms needed to cancel the axial anomalies in a general chiral theory have been derived in various different approaches [17]. Here we shall focus on perturbative anomalies [18]. Let us note that there have been some discussions in the literature on the Wess-Zumino terms induced by removing both fermions in a doublet [19,20]. As pointed out by D'Hoker and Farhi, they can be obtained directly by functional integration over fermion fields, to which masses are given through Yukawa couplings. Thus, it is not necessary to consider the determination of the anomalies and the computation of the Wess-Zumino terms as separate problems. In this spirit, we shall obtain the Wess-Zumino terms with the techniques of external fields and derivative expansions [21]. Since these terms contain the $\epsilon^{\mu\nu\alpha\beta}$ tensor, it is sufficient to compute all the one-loop diagrams with, at most, four external gauge fields. This is consistent with our power-counting rule of Sec. IV, because each gauge field is equivalent to a derivative in mass dimension. Note also that our calculational procedure will take care of the scalar fields automatically.

Now, let us suppose that we have already obtained the Wess-Zumino terms due to the removal of both quarks in a doublet, which we shall denote by $\Gamma_{\text{WZ}}^{\text{SU}(2)\otimes \text{U}(1)}$. Following the discussion in the last section, the same subtraction procedure should be applied to it to extract $\Gamma_{\text{WZ}}^{1\,\text{loop}}$. In other words, the Wess-Zumino terms would contain two parts, of which one is induced by forming one loop with $L_{\text{eff}}^{\text{tree}}$, whereas the other is a portion of $L_{\text{eff}}^{1\,\text{loop}}$. In practice, this subtraction is not necessary because the Wess-Zumino terms stay the same regardless of the relative magnitude of external momenta and masses of internal particles. We may as well for expediency assume that both the top- and the bottom-quark masses are much larger than external momenta and construct the Wess-Zumino terms via derivative expansions.

$\Gamma_{\text{WZ}}^{\text{SU}(2)\otimes \text{U}(1)}$ here is just a special case of the Wess-Zumino terms arising from general chiral theories. To utilize some known results, it is convenient to rewrite the standard-model lagrangian in an artificially more symmetric way [22]. For the standard model with top and bottom quarks, we write the Lagrangian of the fermion sector as

$$L_{\text{fermion}}=-\bar{\psi}_L\frac{1}{i}\gamma^\mu D_\mu\psi_L-\bar{\psi}_R\frac{1}{i}\gamma^\mu D_\mu\psi_R+\cdots\ , \tag{8.1}$$

where

$$\psi_L=\begin{bmatrix} t\\ b\end{bmatrix}_L,\quad \psi_R=\begin{bmatrix} t\\ b\end{bmatrix}_R\ ; \tag{8.2}$$
$$D_\mu\psi_L=(\partial_\mu-iA_\mu^L)\psi_L,\quad D_\mu\psi_R=(\partial_\mu-iA_\mu^R)\psi_R\ ,$$

with

$$A_\mu^L=\frac{g}{2}\tau^a A_\mu^a+\frac{g'}{2}Y_L B_\mu I,\quad A_\mu^R=\frac{g'}{2}(Y_L I+\tau_3)B_\mu\ , \tag{8.3}$$

where I is just the unit matrix in weak isospin space. As one can easily verify, L_{fermion} is invariant under the transformations

$$\psi_L\to e^{-i\omega_L}\psi_L,\quad \psi_R\to e^{-i\omega_R}\psi_R\ ,$$
$$A_\mu^L\to A_\mu^L-i[\omega_L,A_\mu^L]-\partial_\mu\omega_L\ , \tag{8.4}$$
$$A_\mu^R\to A_\mu^R-i[\omega_R,A_\mu^R]-\partial_\mu\omega_R\ ,$$

and the corresponding transformations of scalar fields, which will be written down later. In Eq. (8.4), $\omega_L=\omega_L^a(\tau^a/2)+\omega_L^0(I/2)$ and similarly $\omega_R=\omega_R^a(\tau^a/2)+\omega_L^0(I/2)$. Among the eight group parameters, we now set

$$\omega_R^1=\omega_R^2=0\ ,$$
$$\omega_R^0=\omega_L^0=Y_L\omega_R^3\ , \tag{8.5}$$

to reflect the fact there are only four independent ones for the SU(2)⊗U(1) gauge symmetry.

Now, with a color factor N_c understood, $\Gamma_{\text{WZ}}^{\text{SU}(2)\otimes \text{U}(1)}$ is given by (Ref. [17])

$$\Gamma_{\text{WZ}}^{\text{SU}(2)\otimes \text{U}(1)}=\frac{i}{48\pi^2}\int d^4x\ \epsilon^{\mu\nu\alpha\beta}W_{\mu\nu\alpha\beta}\ , \tag{8.6}$$

where

$$W_{\mu\nu\alpha\beta}=\mathrm{Tr}[i\Phi_\mu^L\Phi_\nu^L\Phi_\alpha^L A_\beta^L+\tfrac{1}{2}\Phi_\mu^L A_\nu^L\Phi_\alpha^L A_\beta^L-\Phi_\mu^L\Phi_\nu^L\Phi A_\alpha^R\Phi^{-1}A_\beta^L-\Phi_\mu^L(A_\nu^L\partial_\alpha A_\beta^L+\partial_\nu A_\alpha^L A_\beta^L)$$
$$-\Phi_\mu^L\Phi\partial_\nu A_\alpha^R\Phi^{-1}A_\beta^L+i\Phi_\mu^L A_\nu^L A_\alpha^L A_\beta^L+i\Phi_\mu^L A_\nu^L\Phi A_\alpha^R\Phi^{-1}A_\beta^L+i\partial_\mu A_\nu^R\Phi^{-1}A_\alpha^L\Phi A_\beta^R$$
$$+i\Phi^{-1}\partial_\mu A_\nu^L A_\alpha^L\Phi A_\beta^R+\Phi^{-1}A_\mu^L A_\nu^L A_\alpha^L\Phi A_\beta^R-(A_L\leftrightarrow A_R,\Phi^L\leftrightarrow\Phi^R,\Phi\leftrightarrow\Phi^{-1})]$$
$$+\tfrac{1}{2}\mathrm{Tr}(\Phi^{-1}A_\mu^L\Phi A_\nu^R\Phi^{-1}A_\alpha^L\Phi A_\beta^R)\,, \tag{8.7}$$

where

$$\Phi=\begin{bmatrix}-\phi^+ & \phi^0\\ \phi^{0\dagger} & \phi^-\end{bmatrix}\Big/\sqrt{\bar\phi\phi}\,, \tag{8.8}$$

which transforms according to

$$\Phi\to e^{i\omega_L}\Phi e^{-i\omega_R}\,. \tag{8.9}$$

Furthermore, we have defined

$$\Phi_\mu^L=\partial_\mu\Phi\Phi^{-1},\quad \Phi_\mu^R=\partial_\mu\Phi^{-1}\Phi\,. \tag{8.10}$$

We should mention that the nonperturbative SU(2) anomaly is not included in Eq. (8.7).

In the following, we shall present our method in computing the Wess-Zumino terms. For simplicity, we shall illustrate this with a simplified Abelian theory which amounts to discarding all the charged-boson fields and the bottom-quark field in the standard-model Lagrangian. The Lagrangian for this simplified theory is hence given by

$$L_{\mathrm{sim}}=-\bar t_L\frac{1}{i}\gamma^\mu(\partial_\mu-iC_\mu^L)t_L$$
$$-\bar t_R\frac{1}{i}\gamma^\mu(\partial_\mu-iC_\mu^R)t_R+\cdots\,, \tag{8.11}$$

where $C_\mu^L=-(g/2)A_\mu^3+(g'/2)Y_L B_\mu$; $C_\mu^R=(g'/2)\times Y_{tR}B_\mu$. It is easy to see that L_{sim} is invariant under two independent U(1) symmetries:

$$t_L\to e^{-i\Lambda_L(x)}t_L,\quad t_R\to e^{-i\Lambda_R(x)}t_R\,,$$
$$C_\mu^L\to C_\mu^L-\partial_\mu\Lambda_L(x),\quad C_\mu^R\to C_\mu^R-\partial_\mu\Lambda_R(x)\,. \tag{8.12}$$

Let us specify the Wess-Zumino terms caused by the top quark in this model as $\Gamma_{\mathrm{WZ}}^{\mathrm{U}(1)\otimes\mathrm{U}(1)}$. Applying the result from general chiral theories, we have

$$\Gamma_{\mathrm{WZ}}^{\mathrm{U}(1)\otimes\mathrm{U}(1)}=\frac{i}{48\pi^2}\int d^4x\,\epsilon^{\mu\nu\alpha\beta}Z_{\mu\nu\alpha\beta}\,, \tag{8.13}$$

with

$$Z_{\mu\nu\alpha\beta}=C_\mu^L U_\nu^R\partial_\alpha C_\beta^R-C_\mu^R U_\nu^L\partial_\alpha C_\beta^L$$
$$-2C_\mu^R\partial_\nu C_\alpha^R U_\beta^R+2C_\mu^L\partial_\nu C_\alpha^L U_\beta^L\,. \tag{8.14}$$

Similar to the non-Abelian theory, we have defined $U_\mu^L=\partial_\mu\varphi\varphi^{-1}$, and $U_\mu^R=\partial_\mu\varphi^{-1}\varphi$, where $\varphi=\phi^0/\sqrt{\phi^{0\dagger}\phi^0}$. Under the $\mathrm{U}(1)_L\otimes\mathrm{U}(1)_R$ symmetry, φ transforms according to

$$\varphi\to e^{i\Lambda_L(x)}\varphi e^{-i\Lambda_R(x)}\,. \tag{8.15}$$

Now we shall demonstrate that Eqs. (8.13) and (8.14) can be reproduced by the derivative expansion technique. To compute all possible terms with the antisymmetric tensor $\epsilon^{\mu\nu\alpha\beta}$, it is necessary to consider all the one-loop diagrams with two or three external gauge fields, together with scalars. We can rule out diagrams with only one gauge field and diagrams with four external gauge fields due to the antisymmetric property of $\epsilon^{\mu\nu\alpha\beta}$. By explicit calculations, we also find that diagrams with three gauge fields do not give rise to $\epsilon^{\mu\nu\alpha\beta}$ terms. Therefore, we need only to compute $C_\mu^L C_\nu^L$ and $C_\mu^R C_\nu^R$ self energies and $C_\mu^L C_\nu^R$ mixing, which we proceed to do momentarily.

First of all, the relevant interaction Lagrangian reads

$$L_{\mathrm{int}}=\bar t_L\gamma^\mu C_\mu^L t_L+\bar t_L\gamma^\mu C_\mu^L t_L\,. \tag{8.16}$$

Performing Wick contractions, we obtain three different amplitudes denoted as I_{LL}, I_{RR}, and I_{LR}, respectively. Explicitly, they are given by

$$I_{LL}=\tfrac{1}{2}\mathrm{Tr}\left[\int d^4x\int\frac{d^4p}{(2\pi)^4}\left\langle t_L\bar t_L\left[x-i\frac{\overleftarrow{\partial}}{\partial p}\right]\right\rangle\gamma^\nu C_\nu^L\left[x+i\frac{\vec\partial}{\partial p}\right]\left\langle t_L\bar t_L\left[x+i\frac{\vec\partial}{\partial p}\right]\right\rangle\gamma^\mu C_\mu^L(x)\right]\,, \tag{8.17}$$

$$I_R=\tfrac{1}{2}\mathrm{Tr}\left[\int d^4x\int\frac{d^4p}{(2\pi)^4}\left\langle t_R\bar t_R\left[x-i\frac{\overleftarrow{\partial}}{\partial p}\right]\right\rangle\gamma^\nu C_\nu^R\left[x+i\frac{\vec\partial}{\partial p}\right]\left\langle t_R\bar t_R\left[x+i\frac{\vec\partial}{\partial p}\right]\right\rangle\gamma^\mu C_\mu^R(x)\right]\,, \tag{8.18}$$

$$I_{LR}=\mathrm{Tr}\left[\int d^4x\int\frac{d^4p}{(2\pi)^4}\left\langle t_R\bar t_L\left[x-i\frac{\overleftarrow{\partial}}{\partial p}\right]\right\rangle\gamma^\nu C_\nu^L\left[x+i\frac{\vec\partial}{\partial p}\right]\left\langle t_L\bar t_R\left[x+i\frac{\vec\partial}{\partial p}\right]\right\rangle\gamma^\mu C_\mu^R(x)\right]\,. \tag{8.19}$$

To obtain the Wess-Zumino action, we can just look into all the two-derivative terms with the $\epsilon^{\mu\nu\alpha\beta}$ tensor in Eqs. (8.17), (8.18), and (8.19). This calculation has been done and the result does agree with that presented in Eq. (8.14).

To apply this method to obtain $\Gamma_{\mathrm{WZ}}^{\mathrm{SU}(2)\otimes\mathrm{U}(1)}$, one needs to compute all the one-loop diagrams up to four external gauge fields. The previous argument which rules out diagrams with one or four external gauge fields no longer holds because the generators of internal symmetry are not commutable. Although we have not actually performed this calculation explicitly to verify that Eq. (8.7) indeed follows by our method, our success in obtaining $\Gamma_{WZ}^{U(1)\otimes U(1)}$ seems compelling.

IX. EXAMPLES

In the last two sections we have completely determined the pure bosonic parts of $L_{\rm eff}^{\rm light\ theory}$ for the standard model in the heavy-top-quark limit. In this section we shall give three examples to illustrate different applications of this effective Lagrangian.

First of all, a heavy fermion is known to have prominent effects on one-loop corrections to the ρ parameter [23]. As will become clear later, these corrections are closely related to the self energies of the charged and the neutral vector bosons evaluated at zero external momentum. Although the effective Lagrangian $L_{\rm eff}^{\rm light\ theory}$ should be used to compute both self energies, $\Gamma_{\rm 1LPI}$ (or $\Omega_{\rm 1LPI}$) should also give the correct answer since the external momenta in this case are always less than both m_t and m_b.

The second example has to do with $H\to\gamma\gamma$ (Ref. [11]), which takes place through internal fermion loops. Since all the external states are neutral, the internal fermions could either be entirely top or bottom quarks. It is clear that $L_{\rm eff}^{\rm tree}$ will reproduce the result of the diagram with internal bottom quarks whereas $L_{\rm eff}^{\rm 1\,loop}$ will generate the result given by the diagram with internal top quarks. We shall demonstrate, in particular, that $L_{\rm eff}^{\rm 1\,loop}$ indeed gives rise to a correct answer for $H\to\gamma\gamma$ through internal top quarks.

Finally, we shall turn our attention to the virtual process $W_\mu^+\to\phi^+\phi'^{0\dagger}$ which has been mentioned in Sec. VII. This is an example where the entire effective Lagrangian including $L_{\rm eff}^{\rm tree}$ and $L_{\rm eff}^{\rm 1\,loop}$ is needed for obtaining the amplitude of the process in question.

A. The ρ parameter

The ρ parameter is defined as the ratio of strengths of the neutral-current process to the charged one. Operationally, one compares the strength of $\nu_\mu e$ scattering to muon decay. At the tree level, the effective four-fermion interaction for the $\nu_\mu e$ scattering is

$$L_{\rm eff}^{\rm NC}=\frac{g^2}{16c^2M_Z^2}[\bar\nu_\mu\gamma^\alpha(1+i\gamma_5)\nu_\mu][\bar e\gamma_\alpha(4s^2-1-i\gamma_5)e]\ . \tag{9.1}$$

For muon decay, the effective Lagrangian reads

$$L_{\rm eff}^{\rm CC}=\frac{g^2}{8M_W^2}[\bar\nu_\mu\gamma^\alpha(1+i\gamma_5)\mu][\bar e\gamma_\alpha(1+i\gamma_5)\nu_e]\ . \tag{9.2}$$

The parameter ρ is defined as the ratio of twice the overall coefficient of the effective neutral interaction to that of the charged one. At the tree level, this is

$$\rho=\frac{M_W^2}{c^2M_Z^2}\ , \tag{9.3}$$

where c stands for $\cos\theta$, which leads to $\rho_{\rm tree}=1$. At the one-loop level, the $L_{\rm eff}^{\rm NC}$ and $L_{\rm eff}^{\rm CC}$ are modified, respectively as [24]

$$L_{\rm eff}^{\rm NC}=\frac{g^2}{16c^2M_Z^2}\left[1+\frac{T_{ZZ}(0)}{(2\pi)^4iM_Z^2}\right][\bar\nu_\mu\gamma^\alpha(1+i\gamma_5)\nu_\mu]\times\left[\bar e\gamma_\alpha\left[4s^2-1-4sc\left.\frac{T_{AZ}(k^2)}{(2\pi)^4ik^2}\right|_{k^2\to0}-i\gamma_5\right]e\right]\ , \tag{9.4}$$

$$L_{\rm eff}^{\rm CC}=\frac{g^2}{8M_W^2}\left[1+\frac{T_{WW}(0)}{(2\pi)^4iM_W^2}\right][\bar\nu_\mu\gamma^\alpha(1+i\gamma_5)\mu]\times[\bar e\gamma_\alpha(1+i\gamma_5)\nu_e]\ .$$

Here $T_{ZZ}(k^2)$, $T_{AZ}(k^2)$, and $T_{WW}(k^2)$ are defined as

$$\Sigma_{\mu\nu}^{ZZ}(k^2)=T_{ZZ}(k^2)\left[g_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right]+L_{ZZ}(k^2)\frac{k_\mu k_\nu}{k^2}\ ,$$
$$\Sigma_{\mu\nu}^{W^+W^-}(k^2)=T_{W^+W^-}(k^2)\left[g_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right]+L_{W^+W^-}(k^2)\frac{k_\mu k_\nu}{k^2}\ , \tag{9.5}$$
$$\Sigma_{\mu\nu}^{AZ}(k^2)=T_{AZ}(k^2)\left[g_{\mu\nu}-\frac{k_\mu k_\nu}{k^2}\right]+L_{AZ}(k^2)\frac{k_\mu k_\nu}{k^2}\ ,$$

where $\Sigma_{\mu\nu}^{ZZ}$, $\Sigma_{\mu\nu}^{W^+W^-}$, and $\Sigma_{\mu\nu}^{AZ}$ [multiplied by $1/(2\pi)^4i$] are vacuum polarizations of Z and W bosons and the one-loop mixing between the photon and Z boson, respectively. According to Eq. (8.4), the ρ parameter becomes

$$\rho=\frac{(1/c^2M_Z^2)\{1+[T_{ZZ}(0)/(2\pi)^4iM_Z^2]\}}{1/M_W^2\{1+[T_{WW}(0)/(2\pi)^4iM_W^2]\}}=1+\Delta\rho\ , \tag{9.6}$$

where

$$\Delta\rho=\frac{c^2T_{ZZ}(0)}{(2\pi)^4iM_W^2}-\frac{T_{WW}(0)}{(2\pi)^4iM_W^2} \tag{9.7}$$

is the one-loop correction to the ρ parameter. It is our intent to obtain $\Delta\rho$ through our effective Lagrangian in what follows. For the reason given before, $\Omega_{\rm 1LPI}$ determined in Sec. VI is sufficient for our purpose here.

As for $T_{ZZ}(0)$, it receives contributions from operators with coefficients A_{20}, A_{21}, A_{22}, A_{23}, and A_{24}. In configuration space, we obtain the expression

$$A_{20}\left[\frac{g^2}{4c^2}Z_\mu Z^\mu \phi^{0\dagger}\phi^0 \ln\left[\frac{\phi^{0\dagger}\phi^0}{\mu^2}\right]\right]$$
$$+A_{21}\left[\frac{g^2}{4c^2}Z_\mu Z^\mu \phi^{0\dagger}\phi^0\right]$$
$$+(-A_{22}-A_{23}+A_{24})\left[\frac{g^2}{4c^2}Z_\mu Z^\mu \phi^{0\dagger}\phi^0\right] . \quad (9.8)$$

Since our aim is just to compute the vector-boson self-energy, we set $\phi^{0\dagger}\phi^0=v^2/2$, where v is the vacuum expectation value responsible for symmetry breaking in the electroweak theory. The masses of the top and the bottom quarks are then given by

$$m_t=\frac{Hv}{\sqrt{2}}, \quad m_b=\frac{hv}{\sqrt{2}} . \quad (9.9)$$

Substituting the values of the relevant A coefficients, we have, in momentum space,

$$\frac{T_{ZZ}(0)}{(2\pi)^4 i}=\frac{g^2}{32\pi^2c^2}\left[(m_t^2+m_b^2)\ln\left[\frac{m_t^2}{\mu^2}\right]-m_b^2\ln\left[\frac{m_t^2}{m_b^2}\right]\right.$$
$$+(m_t^2+m_b^2)\left[-\frac{1}{\epsilon}+\gamma_E\right.$$
$$\left.\left.+\ln(\pi)+\frac{1}{2}\right]\right] . \quad (9.10)$$

For $T_{WW}(0)$, only two operators contribute. In configuration space, one has

$$A_{20}\left[\frac{g^2}{2}W_\mu^+ W^{-\mu}\phi^{0\dagger}\phi^0\ln\left[\frac{\phi^{0\dagger}\phi^0}{\mu^2}\right]\right]$$
$$+A_{21}\left[\frac{g^2}{2}W_\mu^+ W^{-\mu}\phi^{0\dagger}\phi^0\right] . \quad (9.11)$$

Following the same procedure, we have

$$\frac{T_{WW}(0)}{(2\pi)^4 i}=\frac{g^2}{32\pi^2}\left[(m_t^2+m_b^2)\ln\left[\frac{m_t^2}{\mu^2}\right]\right.$$
$$\left.+(m_t^2+m_b^2)\left[-\frac{1}{\epsilon}+\gamma_E+\ln(\pi)\right]\right] . \quad (9.12)$$

From Eqs. (9.7), (9.10), and (9.12), we arrive at

$$\Delta\rho=\frac{g^2}{64\pi^2M_W^2}\left[(m_t^2+m_b^2)-2m_b^2\ln\left[\frac{m_t^2}{m_b^2}\right]\right] , \quad (9.13)$$

a result which agrees with that in the literature (Ref. [23]) for the limit $m_t \gg m_b$.

B. $H\rightarrow\gamma\gamma$

To extract effects of the top quark in this process, we simply look into the effective Lagrangian $L_{\rm eff}^{1\,\rm loop}$. It is not difficult to see that only the first four terms in Eqs. (7.39) contribute to $H\rightarrow\gamma\gamma$. The physical Higgs boson σ arises from the function $\ln(\bar{\phi}\phi/\mu^2)$. The two photons in the final state are given by field strength tenors $F_{\mu\nu}$. In configuration space, we summarize the result with an effective Higgs-photon coupling

$$L_{H\gamma\gamma}=\frac{2e^2}{108\pi^2 v}\sigma F_{\mu\nu}^{\rm em}F^{{\rm em},\mu\nu}$$
$$=\frac{g(eQ_t)^2}{48\pi^2M_W}\sigma F_{\mu\nu}^{\rm em}F^{{\rm em},\mu\nu} , \quad (9.14)$$

where eQ_t is the electric charge of the top quark. A color factor of 3 should be multiplied. This result agrees with that given by others (Ref. [11]).

C. $W_\mu^+\rightarrow\phi^+\phi'^{0\dagger}$

This process is equivalent to a $W_\mu^+\phi^-\phi'^0$ vertex (with $\phi^0=\phi'^0+v/\sqrt{2}$) with the relevant particles represented by their corresponding fields. Since this is a process which has not been calculated before, we shall present results given by direct calculations and by the effective Lagrangian method and show that they do agree.

For direct calculations, all the relevant Feynman rules are derived from the tree-level linear Lagrangian in the symmetry-broken phase. As shown in Fig. 1, there are two Feynman diagrams which contribute to this particular process. We expand each Feynman integral in inverse powers of m_t and discard terms proportional to positive powers of $1/m_t$. The results are

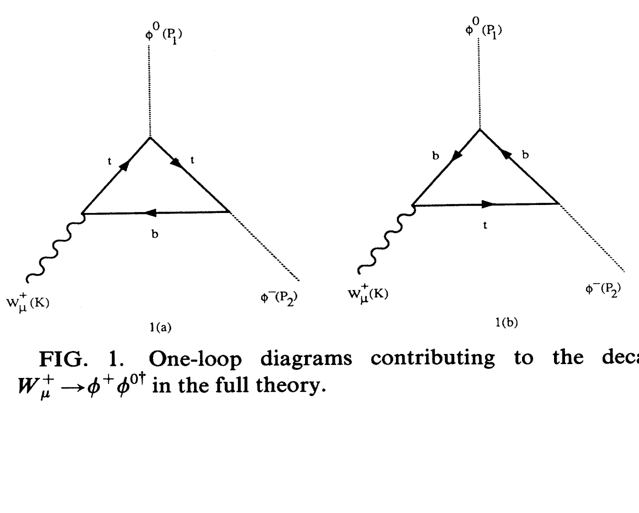

FIG. 1. One-loop diagrams contributing to the decay $W_\mu^+\rightarrow\phi^+\phi^{0\dagger}$ in the full theory.

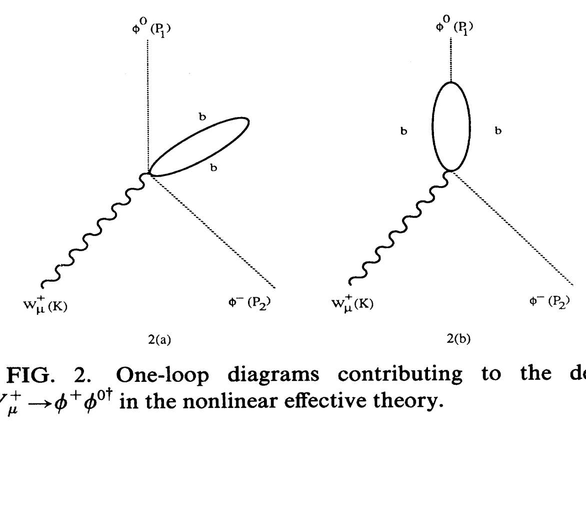

FIG. 2. One-loop diagrams contributing to the decay $W_\mu^+\rightarrow\phi^+\phi^{0\dagger}$ in the nonlinear effective theory.

$$\text{Fig. 1(a):}\quad \frac{-\sqrt{2}i\pi^2 g}{v^2}[(\tfrac{1}{9}P_1\cdot P_2+\tfrac{1}{6}P_2\cdot K+\tfrac{1}{6}P_1\cdot K)P_{1\mu}+(\tfrac{5}{18}P_1\cdot P_2+\tfrac{5}{18}P_2\cdot K+\tfrac{1}{3}P_1\cdot K)P_{2\mu}$$
$$+(\tfrac{1}{4}H^2v^2-\tfrac{1}{2}h^2v^2)P_{1\mu}+\tfrac{1}{2}H^2v^2P_{2\mu}]\,, \tag{9.15}$$

$$\text{Fig. 1(b):}\quad \frac{-\sqrt{2}i\pi^2 g}{v^2}\left\{\tfrac{3}{4}h^2v^2P_{1\mu}+\tfrac{1}{2}h^2v^2P_{2\mu}-\left[\int_0^1 dx\, h^2x\ln\left[\frac{H^2v^2}{h^2v^2+2P^2x(1-x)}\right]\right]v^2P_{1\mu}\right\}, \tag{9.16}$$

where P_1, P_2, and K are the incoming momenta of ϕ'^0, ϕ^-, and W_μ^+, respectively. To reproduce these results via the effective Lagrangian, we first consider the portion which is induced by $L_{\text{eff}}^{\text{tree}}$. In $L_{\text{eff}}^{\text{tree}}$, W_μ^+ interacts according to

$$L_{\text{int}}=-\frac{g}{\sqrt{2}}\bar{b}_L\gamma^\mu b_L\frac{W_\mu^+\phi^-}{\phi^0}\,. \tag{9.17}$$

One should notice that this interaction term is written in the symmetric phase. In the symmetry-broken phase, one has to perform a shift on the scalar field $\phi^0\to\phi'^0+v/\sqrt{2}$. In this way, L_{int} becomes

$$L_{\text{int}}=-g\bar{b}_L\gamma^\mu b_L\frac{W_\mu^+\phi^-}{v}\left[1-\frac{\sqrt{2}\phi'^0}{v}+\cdots\right]. \tag{9.18}$$

From Eq. (9.18) and other parts of $L_{\text{eff}}^{\text{tree}}$, one can easily see that there are two diagrams to be evaluated. The first diagram, as shown in Fig. 2(a), actually vanishes because it contains a fermionic tadpole. For Fig. 2(b), one has

$$\text{Fig. 2(b):}\quad \frac{-\sqrt{2}i\pi^2 g}{v^2}\left[\frac{h^2}{2}\left[-\frac{1}{\epsilon}+\gamma_E+\ln\pi+\frac{1}{2}\right]v^2P_{1\mu}+\frac{h^2}{2}\int_0^1 dx\ln\left[\frac{h^2v^2}{2}+P^2x(1-x)\right]v^2P_{1\mu}\right]. \tag{9.19}$$

In addition to $L_{\text{eff}}^{\text{tree}}$, $L_{\text{eff}}^{1\,\text{loop}}$ also contributes to this process. The contribution by $L_{\text{eff}}^{(2)}$ is found to be

$$L_{\text{eff}}^{(2)}:\quad \frac{-\sqrt{2}i\pi^2 g}{v^2}\left[-\frac{h^2}{2}\left[-\frac{1}{\epsilon}+\gamma_E+\ln\pi\right]v^2P_{1\mu}+\frac{1}{4}H^2v^2P_{1\mu}-\frac{h^2}{2}\ln\left[\frac{H^2v^2/2}{\mu^2}\right]v^2P_{1\mu}+\frac{1}{2}(H^2+h^2)v^2P_{2\mu}\right]. \tag{9.20}$$

There are also operators in $L_{\text{eff}}^{(4)}$ and $L_{\text{eff}}^{\text{gf}}$ contributing to this process. These operators are $I^{\mu\nu,\mu\nu}$, $I_{0,\mu\nu}^{\mu\nu,0}$, $I_{0,\mu\nu}^{0,\mu\nu}$, $I_{0,\mu}^{\nu,\mu\nu}$, $I_{0,\mu}^{\mu\nu,\nu}$, and $I_{\mu,\nu}^{0,\mu\nu}$ in $L_{\text{eff}}^{(4)}$ and $[\bar{\phi}G_{\mu\nu}(D_\nu\phi)][\bar{\phi}(D_\mu\phi)]/(\bar{\phi}\phi)^2$ in $L_{\text{eff}}^{\text{gf}}$. Altogether, they give

$$\frac{-\sqrt{2}i\pi^2 g}{v^2}[(\tfrac{1}{9}P_1\cdot P_2+\tfrac{1}{6}P_2\cdot K+\tfrac{1}{6}P_1\cdot K)P_{1\mu}+(\tfrac{5}{18}P_1\cdot P_2+\tfrac{5}{18}P_2\cdot K+\tfrac{1}{3}P_1\cdot K)P_{2\mu}]\,. \tag{9.21}$$

From Eqs. (9.19), (9.20), and (9.21), it is not difficult to see that $L_{\text{eff}}^{\text{tree}}$ and $L_{\text{eff}}^{1\,\text{loop}}$ indeed reproduce the result given by Eqs. (9.15) and (9.16) via direct calculation. The only relation needed for this comparison is

$$\int_0^1 dx\ln\left[\frac{h^2v^2}{2}+P^2x(1-x)\right]=2\int_0^1 x\,dx\ln\left[\frac{h^2v^2}{2}+P^2x(1-x)\right], \tag{9.22}$$

which can be easily proven by a change of variable $x=\frac{1}{2}(u+1)$ on each integral above. Both calculations give the same correct unitarity cut.

Other applications of the effective Lagrangian are under study and will be reported elsewhere.

X. CONCLUDING REMARKS

We have shown that when the top-quark mass becomes very heavy, constraints develop in field quantities so that the top-quark field becomes nonlinearly realized. The underlying symmetry SU(2)⊗U(1) is still preserved at the S-matrix level. We have also shown how to use the derivative expansion and the external field technique to construct $\Gamma_{1\text{LPI}}$ and $L_{\text{eff}}^{\text{light theory}}$, explicitly for the bosonic sector to one-loop order. We have given several examples to demonstrate how our results recover what are already known.

We have stressed that $\Gamma_{1\text{LPI}}$ is different from the effective Lagrangian $L_{\text{eff}}^{\text{light theory}}$. The latter is obtained, so that after wave-function and parameter renormalizations:

$$T\left[\exp\left[i\int L^{\text{full theory}}\right]\right]=T\left[\exp\left[i\int L_{\text{eff}}^{\text{light theory}}\right]\right]$$

for all the low-energy light-particle processes. The reason that we are interested in the construction of L_{eff} is because through it we can validate the effective theory for $m_t\gg p_{\text{ext}},m_{W,Z,b}$, without making any restriction between p_{ext} and $m_{W,Z,b}$. In addition, this is the most compact way to enforce some basic properties of quantum field theory, such as analyticity, unitarity, and *CPT*.

The construction of L_{eff} follows closely Zimmermann's oversubtraction identity [25] to define heavy vertices. As we have seen, there are self-generated prescriptions to renormalize some seemingly nonrenormalizable operators.

ACKNOWLEDGMENTS

This work was partially supported by the U.S. Department of Energy. H.S. would like to thank the Particle Theory Group at Ann Arbor for facilitating a series of visits, during which a part of this work was performed.

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