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Simplest Z' model

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The differences in family-lepton numbers are anomaly-free in the minimal standard model (MSM), and can therefore be gauged. For three generations of quarks and leptons, three models emerge depending on whether (i) L_e-L_μ, (ii) L_e-L_τ, or (iii) L_μ-L_τ are gauged. These are the simplest models to feature a Z' boson because no fermions beyond those already present in the MSM are required to cancel gauge anomalies. We analyze the phenomenology of models (i) and (ii) in detail, and present constraints derived from low-energy neutral-current data and CERN LEP data. We find that these Z' bosons may have a relatively low mass yet still evade present experimental bounds, while remaining detectable in current accelerators. The introduction of neutrino masses into the models is then considered. We discuss how one may incorporate both the reported 17-keV neutrino, and the Mikheyev-Smirnov-Wolfenstein effect solution of the solar-neutrino problem. We then describe how to embed the extra U(1) gauge group into a horizontal SU(2)-symmetry group acting on leptons.

I. INTRODUCTION

The minimal standard model (MSM) is constructed on the basis of a number of principles. The most fundamental of these principles is that the gauge group $G_{\rm MSM}$ is given by $G_{\rm MSM}$ $=$SU(3)$_c\otimes$SU(2)$_L\otimes$U(1)$_Y$. Of equal importance is the requirement that $G_{\rm MSM}$ be spontaneously broken to SU(3)$_c\otimes$U(1)$_Q$ where Q is the electric charge generator. This symmetry breaking is associated with two pieces of physics, which although related, may not be performed by exactly the same sector of the theory: gauge-boson masses and mixing, and fermion masses and mixing. The MSM has the additional assumption that both of these phenomena are generated in the minimal manner, namely, by the nonzero vacuum expectation value (VEV) of a fundamental Higgs doublet.

In the MSM, fermion masses and mixings arise from the Yukawa Lagrangian

$$\mathcal{L}_{\rm Yuk} = \lambda_1 \bar{\ell}_L \ell_R \phi + \lambda_2 \bar{Q}_L u_R \phi^c + \lambda_3 \bar{Q}_L d_R \phi + \text{H.c.} \qquad (1)$$

where the quark and lepton fields are classified under $G_{\rm MSM}$ as

$$\begin{aligned} &\ell_L \sim (1,2)(-1), \quad \ell_R \sim (1,1)(-2), \\ &Q_L \sim (3,2)(1/3), \quad u_R \sim (3,1)(4/3), \\ &d_R \sim (3,1)(-2/3), \end{aligned} \qquad (2)$$

and the Higgs field ϕ is given by

$$\phi \sim (1,2)(1). \qquad (3)$$

Note that we are considering the *minimal* standard model here, so there are no right-handed neutrinos.

The above Yukawa Lagrangian results from the minimal symmetry-breaking sector of the MSM. As well as generating fermion masses and quark mixing angles it leads to a number of exact global symmetries being present in the MSM. These global symmetries are generated by the family-lepton numbers (L_e, L_μ, and L_τ for the three-generation case) and baryon number B. There is no confirmed laboratory evidence for either family-lepton number or baryon-number-nonconserving processes, although there have been some claims recently of an admixture of a 17-keV neutrino state in the electron-neutrino state vector [1]. Other groups, however, claim to see no such effect [2]. If confirmed, the experiments of Ref. [1] would provide the first evidence for family-lepton–number symmetry breaking. The current situation therefore appears to support, on balance, the prediction by the MSM of family-lepton–number conservation. We will assume for the first part of this paper that family-lepton–number invariance is indeed exact, and comment on how the 17-keV neutrino may be incorporated into extensions of our models later.

In their pioneering paper on non-Abelian gauge theory, Yang and Mills [3] expressed the view that local symmetries are more in accord with the modern view of local interactions than are global symmetries. Global symmetries have the property that fields at widely separated spacetime points are transformed by the same amount. Local symmetries, on the other hand, allow the change in the field to vary with spacetime coordinate, and thus a subset of the possible local symmetry transformations will be in accord with intuitive notions of locality. In other words, according to Yang and Mills, local symmetries are defined in a more "physically acceptable" way than global symmetries, and thus may be more fundamental. Presumably this argumentation should be viewed as being of heuristic rather than rigorous intent, since there is also a "physically unacceptable" subset of local symmetry transformations (for instance, every local symmetry implies a global symmetry). At any rate, heuristic notions such as these have led people to postulate that every global symmetry should ultimately be embedded in a local symmetry, and thus all symmetry currents should couple to associated gauge fields.

In the context of the MSM the question therefore arises of whether or not the symmetry currents of family-lepton–number and baryon-number invariance are coupled to gauge fields. From a model-building point of view, this raises a serious issue. There are strong indications that gauge theories are viable only if they are free from gauge anomalies. This in turn has implications for the fermion spectrum of the theory.

The anomaly cancellation requirement allows one to elaborate on the heuristic idea that all symmetries should be local. Consider a gauge theory, such as the MSM, which has a given fermion spectrum and a number of global symmetries. *In general, one would expect to be able to gauge those linear combinations of the global symmetry generators that are anomaly-free with respect to the given fermion spectrum.* If one accepts that all global symmetries should be ultimately embedded in local symmetries, then one should be even *more* convinced that the subset of these global symmetries which is anomaly-free given a preexisting fermion spectrum should be gauged.

The remaining global symmetries, which are anomalous with respect to the given fermions, cannot be gauged without also extending the fermion spectrum. In general, there would be many possibilities *a priori* available for these exotic fermions. So without some indication from experiment as to the identity of these fermions the task of the model builder is quite ill defined. Thus it appears sensible to adopt the intermediate position that all global symmetries which are anomaly-free given the known fermions should be gauged, while retaining the anomalous global symmetries as just global. One would then have to wait and see whether any exotic fermions are discovered.

This situation actually pertains to the MSM, though this does not appear to be a widely appreciated fact [4, 5]. Of the four global symmetry generators of the MSM, the linear combinations

$$L_1 = L_e - L_\mu, \quad L_2 = L_e - L_\tau, \quad L_3 = L_\mu - L_\tau \tag{4}$$

are anomaly-free, although they are not simultaneously anomaly-free. Therefore the MSM gauge group can be extended to [5]

$$G_1 = G_{\rm MSM} \otimes \quad {\rm U}(1)_{L_1}, \tag{5}$$

or

$$G_2 = G_{\rm MSM} \otimes \quad {\rm U}(1)_{L_2}, \tag{6}$$

or

$$G_3 = G_{\rm MSM} \otimes \quad {\rm U}(1)_{L_3} \tag{7}$$

without the addition of any new fermions, not even right-handed neutrinos.

A purpose of this paper is to study the three theories defined by the gauge groups $G_{1,2,3}$. A preliminary analysis was performed in Ref. [5]. In the present work we give a more complete account of the phenomenological constraints, as well as discussing some theoretical developments.

Before commencing this analysis, more theoretical issues need to be addressed. First of all, there is another way of implementing the "all symmetries are local symmetries" hypothesis. Global symmetries can simply be explicitly broken by terms in the Lagrangian. For instance, in the MSM the family-lepton numbers and total lepton number can be explicitly broken by introducing right-handed Majorana neutrinos and a mixing matrix in the lepton sector. (In this case the baryon number remains as an exact though anomalous global symmetry, whose role is to be determined by future developments.) So there are three possibilities regarding anomaly-free global symmetries: (i) they are gauged, (ii) they are explicitly broken, or (iii) they are just exact global symmetries and no particular significance should be given to the circumstance that they are anomaly-free.

The MSM adheres to possibility (iii). In the MSM case, scenario (ii) has been exhaustively studied under the subject heading of "neutrino masses and mixing." We believe scenario (i) is also an interesting, and unorthodox, possibility.

Before discussing the details of gauging ${\rm U}(1)_{L_i}$ it is pertinent to discuss possibility (iii) a little further. This point of view actually leads to a certain predictivity problem. In fact in the case of the MSM it leads to hypercharge and electric-charge dequantization. The reason for this is quite simple, as was first pointed out by Foot [4, 6]. Let us focus on the anomaly-free symmetry generated by $L_1 = L_e - L_\mu$. There is one other anomaly-free U(1) symmetry in the theory, and that is of course standard hypercharge Y. If there are two anomaly-free symmetries of a theory, and one only chooses to gauge one of them, then no theoretical principle demanded by the theory can tell us which linear combination of the two should be the actual local symmetry, and thus the actual hypercharge in the case of the MSM. Therefore the MSM has a 1-parameter arbitrariness in the definition of the actual hypercharge:

$$Y_{\rm actual} = \cos\theta Y_{\rm standard} + \sin\theta L_1 . \tag{8}$$

Actually, it is also possible to replace L_1 by either L_2 or L_3 in Eq. (8). Thus the MSM has both a one-parameter continuous ambiguity and a trichotomic discrete ambiguity in the definition of actual hypercharge. Through symmetry breaking, electric charge is defined in terms of Y, and so an arbitrariness in actual hypercharge leads to an arbitrariness in electric charge. If ${\rm U}(1)_{L_i}$ were explicitly broken or anomalous this potential dequantization would not arise. Therefore the existence of anomaly-free global symmetries leads to a predictivity problem, which in the MSM manifests itself as the old problem of charge quantization.

What happens to charge quantization if we couple the anomaly-free global symmetries of the MSM to gauge fields? In order to answer this question it is useful to consider an arbitrary U(1)⊗U(1) gauge theory. Let the generators be denoted by Q_1 and Q_2. These can always be chosen to be Hermitian. The gauge-boson sector of this theory is specified by the gauge kinetic energy terms and the covariant derivative. The most general forms for these two objects consistent with gauge invariance are

$$D_\mu = \partial_\mu + ig_1(aQ_1 + bQ_2)A_\mu + ig_2(cQ_1 + dQ_2)B_\mu, \tag{9}$$

$$\mathcal{L}_{\rm KE} = k_1 F^{\mu\nu}F_{\mu\nu} + k_2 F^{\mu\nu}G_{\mu\nu} + k_3 G^{\mu\nu}G_{\mu\nu}, \tag{10}$$

where $g_{1,2}$ are real coupling constants, $F_{\mu\nu}$ and $G_{\mu\nu}$ are the field-strength tensors of the real gauge fields A_μ and B_μ, respectively, and a,b,c,d are real parameters. There exists a region of (k_1,k_2,k_3) parameter space for which the kinetic energy Lagrangian can be written in the canonical form for two properly normalized, independent, propagating U(1) gauge fields [7]. This corresponds to rewriting $\mathcal{L}_{\rm KE}$ in terms of gauge fields which are certain linear combinations of A_μ and B_μ. It is clear that reexpressing D_μ in terms of these new linear combinations can only redefine $g_{1,2}$ and a,b,c,d. So without loss of generality we may choose $k_1 = k_3 = -1/4$ and $k_2 = 0$. Which of $g_{1,2}$ and a,b,c,d are physical parameters? Suppose $a,d \neq 0$. Then we may rescale the coupling constants so that $a = d = 1$, and thus a and d are unphysical. We are also free to rotate the gauge fields by an arbitrary 2×2 orthogonal transformation (specified by an angle ϕ), since this preserves the canonical form of $\mathcal{L}_{\rm KE}$. This allows us to remove one more parameter. In fact a simple calculation shows that one can always choose ϕ so that the covariant derivative can be put in the form

$$D_\mu = \partial_\mu + ig_1' Q_1 A_\mu' + ig_2'(Q_2 + \epsilon Q_1)B_\mu', \tag{11}$$

where $g_{1,2}'$ and ϵ are appropriate combinations of $g_{1,2}$ and b,c. This is the most general form that the covariant derivative of U(1)⊗U(1) gauge theory can take when written in terms of physical parameters. Note that apart from the gauge coupling constants, there is an additional free parameter ϵ which cannot be transformed away by an orthogonal redefinition of the gauge fields or by rescaling. The cases where one of a or d or both are zero do not lead to a more general form than given by Eq. (11).

Let us now apply this result to the MSM with gauged $\mathrm{U}(1)_{L_i}$. The most general gauge group is therefore $G_{\epsilon i}$ where

$$G_{\epsilon i} = \mathrm{SU}(3)_c \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y \otimes \mathrm{U}(1)_{L_i+\epsilon Y}. \tag{12}$$

There are two observations one can make about this. First, there is a one-parameter class of Z' models one can study. In this paper we will study the $\epsilon = 0$ theory, because in this case the Z' boson does not couple to quarks, and so the experimental constraints are expected to be the least severe. Second, there is still a "hypercharge" quantization problem, because no theoretical principle demanded by the theory can be used to restrict ϵ. However, since one can always choose a symmetry-breaking scheme so that the ϵ-dependent generator does not contribute to electric charge, this arbitrariness will only manifest itself in the interactions of the Z' boson, rather than the standard Z or the photon.

Note that the parameter ϵ must be zero if the extra U(1) is embedded in the non-Abelian gauge group SU(2), whose diagonal generator is just L_i. We study such models in Sec. IV.

The plan of this paper is as follows. In Sec. II the neutral-current phenomenology associated with gauging $\mathrm{U}(1)_{L_i}$ where $i = 1,2,3$ is analyzed, and constraints are placed on the Z_i' coupling constant and mass from low-energy neutral current data and CERN e^+e^- collider LEP data. Section III discusses extensions of the models which introduce neutrino messes and solve a U(1) quantization problem. We show, in Sec. IV, how $\mathrm{U}(1)_{L_i}$ can be easily embedded in a non-Abelian horizontal SU(2) gauge symmetry acting on leptons. Section V is a conclusion.

II. Z' PHENOMENOLOGY

The major phenomenological consequence of gauging $\mathrm{U}(1)_{L_1}$ or $\mathrm{U}(1)_{L_2}$ or $\mathrm{U}(1)_{L_3}$ is the existence of a second neutral gauge boson, which we call Z_1', Z_2', and Z_3', respectively. [The reader should not forget that we are considering *three* different models here, because no two of these U(1)'s can be gauged simultaneously.] These three Z' bosons have different phenomenological implications. The first two are of great relevance to current collider experiments, because they couple to electrons. Therefore, data collected at e^+e^- colliders such as KEK TRISTAN [8], the SLAC Linear Collider [9] (SLC), and LEP [10] will place bounds on their masses and coupling constants. The obvious difference between Z_1' and Z_2' is that the former couples to muons and muon neutrinos, while the latter couples to τ's and τ neutrinos. Thus Z_1' boson effects would add a nonstandard contribution to $e^+e^- \to \mu^+\mu^-$ while leaving the corresponding process involving τ's in the final state to standard neutral-current effects. The Z_3' boson does not couple to first-generation particles, and thus its properties are less severely constrained than the other two. The constraints derived in Ref. [5] for Z_3' do not need to be updated. None of these gauge bosons couple to hadrons.

We first need to specify how $\mathrm{U}(1)_{L_i}$ (where $i = 1,2,3$) is spontaneously broken in these theories. This is easily achieved by the introduction of a Higgs field S which is neutral under $G_{\rm MSM}$ but has a nonzero L_i charge:

$$S \sim (1,1)(0,\eta), \quad \eta \neq 0. \tag{13}$$

So far there is no constraint in the theory to tell us what the L_i charge η of S should be. This represents a U(1) charge quantization problem for the theory, in addition to the problem of arbitrary ϵ. We will ignore this issue in the present section, because it is irrelevant for neutral-current phenomenology, but return to it in the following section.

Because S is neutral under the electroweak group, there is no tree-level mixing between the standard Z boson and Z_i'. This is phenomenologically significant because it removes a major constraint that usually reduces the acceptable parameter space in Z' models quite severely. In particular, there will be no observable shift in the value of the standard Z-boson mass. (One can easily check that higher-order processes which induce Z-Z_i'

mixing do so at a level which is experimentally inaccessible at present.)

Another important consequence of the quantum numbers of S is that it cannot couple to fermion bilinears in the Lagrangian. This means that the global symmetries associated with family-lepton–number conservation are actually *independent* of the local symmetry $\mathrm{U}(1)_{L_i}$. In other words, the symmetry associated with family-lepton number is actually

$$G_{\text{family lepton No.}} = [\mathrm{U}(1)_{L_i}]_{\text{local}} \otimes [\mathrm{U}(1)_{L_e} \otimes \mathrm{U}(1)_{L_\mu} \otimes \mathrm{U}(1)_{L_\tau}]_{\text{global}}. \quad (14)$$

When S develops a nonzero VEV, the local $\mathrm{U}(1)_{L_i}$ is broken, but the global family-lepton–number symmetries remain exact, because the latter symmetries *only act on fermions.* Consequently the existence of a massive Z'_i is not necessarily associated with family-lepton–number-violating processes. This is obviously very important phenomenologically, since the experimental constraints on such processes are severe.

In summary then, with our choice of symmetry-breaking scheme the Z'_i boson is flavor conserving, and its mixing with the standard Z boson is negligible.

In Ref. [5] a preliminary phenomenological analysis was performed to obtain constraints on the (M_{Z_i}, g'_i) parameter space, where these quantities are the mass and coupling constant of the Z'_i, respectively. We used experimental results for the total cross sections relative to QED and forward-backward asymmetries for the processes

$$e^+e^- \to \mu^+\mu^-, \quad \tau^+\tau^- \quad (15)$$

as measured at TRISTAN and at earlier machines [8, 11]. We denote these quantities by $R_{\mu,\tau}$ and $A_{FB}^{\mu,\tau}$. We also used cross-section data for [12]

$$\bar{\nu}_\mu e \to \bar{\nu}_\mu e, \quad \nu_\mu e \to \nu_\mu e, \quad \bar{\nu}_e e \to \bar{\nu}_e e, \quad \nu_e e \to \nu_e e. \quad (16)$$

We did not incorporate any LEP data into this initial study.

In this paper, we complete this analysis by (i) including some more recent data from TRISTAN, (ii) incorporating corrections where necessary for initial-state radiation near the Z'_i and Z resonances through a radiator function which is convoluted with the bare tree level cross-section formula, and (iii) including results from LEP on the standard Z resonance. The incorporation of constraints from Bhabha scattering is beyond the scope of this analysis, because t-channel contributions complicate the tree-level and especially the radiatively corrected cross sections enormously. Although it is interesting to look at Z'_i effects in Bhabha scattering, we expect that the incorporation of constraints from experimental data on this process will only change our results slightly.

The interaction Lagrangians we need are those for photon (γ), standard Z and $Z'_{1,2}$ coupling to fermions. They are given by

$$\mathcal{L}_A = eA^\mu(\bar{e}\gamma_\mu e + \bar{\mu}\gamma_\mu\mu + \bar{\tau}\gamma_\mu\tau), \quad (17)$$

$$\mathcal{L}_Z = \frac{g_2}{\sqrt{1-x}} Z^\mu[\,(x - \tfrac{1}{2})(\bar{e}_L\gamma_\mu e_L + \bar{\mu}_L\gamma_\mu\mu_L + \bar{\tau}_L\gamma_\mu\tau_L) + x(\bar{e}_R\gamma_\mu e_R + \bar{\mu}_R\gamma_\mu\mu_R + \bar{\tau}_R\gamma_\mu\tau_R)], \quad (18)$$

$$\mathcal{L}_{Z'_1} = g'_1 {Z'_1}^\mu(\bar{e}\gamma_\mu e - \bar{\mu}\gamma_\mu\mu), \quad (19)$$

$$\mathcal{L}_{Z'_2} = g'_2 {Z'_2}^\mu(\bar{e}\gamma_\mu e - \bar{\tau}\gamma_\mu\tau), \quad (20)$$

where $x \equiv \sin^2\theta_W$ and g_2 is the $\mathrm{SU}(2)_L$ coupling constant. Remember that we are considering two theories here; Z'_1 and Z'_2 cannot appear in a theory simultaneously. It should be clear from context in each of the following equations whether we are referring to one or the other or either.

Our theoretical treatment of neutrino-electron scattering and the experimental data we used in the fit are the same in this paper as in our previous study, so we refer the interested reader to this previous work [5].

The analysis of $e^+e^- \to \mu^+\mu^-, \tau^+\tau^-$ is, however, quite different, so we will now discuss it fully. The basic tree-level process is depicted in Fig. 1. Two quantities are of interest: the total cross section and the forward-backward asymmetry. We first present the cross-section formulas.

There are six terms in the formula for the cross section. Three of these correspond to direct contributions from γ, Z and Z'_i, while the other three are the $\gamma - Z, \gamma - Z'_i$ and $Z - Z'_i$ interference terms. The uncorrected total cross section is a function of the center-of-mass energy squared (s) and is the sum of all six of these terms:

$$\hat{\sigma}_{\text{tot}}(s) = \hat{\sigma}_\gamma(s) + \hat{\sigma}_Z(s) + \hat{\sigma}_{Z'_i}(s) + \hat{\sigma}_{\gamma Z}(s) + \hat{\sigma}_{\gamma Z'_i}(s) + \hat{\sigma}_{ZZ'_i}(s), \quad (21)$$

where "$\hat{\sigma}$" denotes an uncorrected quantity.

In order to properly compare the predictions of our models with LEP data on cross sections and forward-backward asymmetries near the Z peak, we have to incorporate initial-state radiation effects. This is done by convoluting the uncorrected cross section with a radiator function R [13]. The corrected cross section is then

$$\sigma_{\text{tot}}(s) = \int_0^{1-s/s_0} dz\; \hat{\sigma}_{\text{tot}}[s(1-z)]\; R(z), \quad (22)$$

where s_0 is a cutoff. The radiator function R is a com-

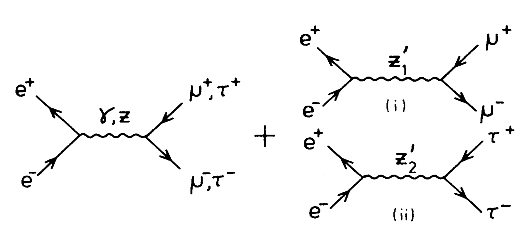

FIG. 1. Feynman diagrams showing the contributions of γ, Z, and $Z'_{1,2}$ to $e^+e^- \to$ (i) $\mu^+\mu^-$, (ii) $\tau^+\tau^-$.

plicated object, and the above integral cannot be done analytically. It is possible to perform this integration numerically, but for efficiency we choose in our analysis to instead adopt an approximate analytical form [14]. This form is used to change both the Z and Z_i' line shapes near their respective peaks. The accuracy claimed for these analytical approximations is about 0.4% for the interval $(M_G - 3\Gamma_G, M_G + 2\Gamma_G)$ where $G = Z, Z_i'$ and Γ_G is the total width of G.

The uncorrected direct photon cross section is

$$\hat{\sigma}_\gamma(s) = \frac{4\pi\alpha_{\rm em}(s)^2}{3s}, \tag{23}$$

where $\alpha_{\rm em}(s)$ is the running electromagnetic fine-structure constant. The analytic approximation for the corrected quantity is

$$\sigma_\gamma(s) = \hat{\sigma}_\gamma(s)\left[1 + \frac{\beta}{2}\ln\left(\frac{s}{s_0}\right) + O(\beta^2)\right], \tag{24}$$

where $\sqrt{s_0} \approx 1$ GeV and

$$\beta \approx \frac{2\alpha_{\rm em}(0)}{\pi}\left[\ln\left(\frac{s}{m_e^2}\right) - 1\right]. \tag{25}$$

The rest of the uncorrected cross section can be written in the generic form

$$\hat{\sigma}_{\rm tot}(s) - \hat{\sigma}_\gamma(s) = \frac{K}{|D_Z(s)|^2}(As+B) + \frac{K}{|D_{Z_i'}(s)|^2}(A's+B'), \tag{26}$$

where

$$D_G(s) = s - M_G^2 + is\gamma_G, \quad \gamma_G \equiv \frac{\Gamma_G}{M_G}. \tag{27}$$

The quantities A, B, A', and B' will be specified shortly. The analytic approximation to the corrected cross section is then

$$\sigma_{\rm tot}(s) - \sigma_\gamma(s)$$
$$\approx \frac{K}{|D_Z(s)|^2}\eta_Z^\beta \frac{\pi\beta}{\sin\pi\beta}\left((As+B)\frac{\sin[(1-\beta)\xi_Z]}{\sin\xi_Z}\Delta - (2As+B)\eta_Z\frac{\sin\beta\xi_Z}{\sin\xi_Z}\right) + (Z \to Z'), \tag{28}$$

where

$$\xi_Z = \frac{\pi}{2} + \arctan\left(\frac{s(1+\gamma_Z^2) - M_Z^2}{\gamma_Z M_Z^2}\right) \tag{29}$$

and

$$\Delta = 1 + \frac{3}{4}\beta + \frac{\alpha_{em}}{\pi}\left(\frac{\pi^2}{3} - \frac{1}{2}\right) - \frac{3}{4}\frac{\alpha_{em}}{\pi} + \frac{1}{4}\beta^2\left(\frac{9}{8} - \frac{\pi^2}{8}\right) + O(\beta^3). \tag{30}$$

The quantity K is given by

$$K = \frac{G_F^2 M_Z^4}{2\pi}, \tag{31}$$

where G_F is the Fermi constant. The coefficient A is

$$A = 2(\tfrac{1}{8} - \tfrac{1}{2}x + x^2)^2 + 4(1-x)x(\tfrac{1}{4} - x)^2 + y\left(\frac{M_{Z_i'}}{M_Z}\right)^2 (2\alpha_1\tilde{M}_Z^2 - \beta_1)(\tfrac{1}{4} - x), \tag{32}$$

where the first, second, and third terms are the direct Z, photon-Z interference, and part of the $Z - Z_i'$ interference contributions, respectively. The coefficient B is

$$B = (x - \tfrac{1}{4})M_Z^2\left[4x(1-x)(\tfrac{1}{4} - x) + y\left(\frac{M_{Z_i'}}{M_Z}\right)^2 \alpha_1\tilde{M}_Z^2\right], \tag{33}$$

where the first term is from photon-Z interference and the second is part of the Z-Z_i' interference contribution. The coefficients A' and B' are, respectively,

$$A' = \frac{y^2}{8}\left(\frac{M_{Z_i'}}{M_Z}\right)^4 - y\left(\frac{M_{Z_i'}}{M_Z}\right)^2 [x(1-x) + (\tfrac{1}{4} - x)(2\alpha_2\tilde{M}_{Z_i'}^2 - \beta_2)], \tag{34}$$

$$B' = yM_{Z_i'}^2\left(\frac{M_{Z_i'}}{M_Z}\right)^2 [x(1-x) + \alpha_2\tilde{M}_{Z_i'}^2(\tfrac{1}{4} - x)]. \tag{35}$$

The first, second and third terms of A' are the direct Z_i', γ-Z_i' interference and part of the Z-Z_i' interference contributions respectively. The two terms in B' refer to γ-Z_i' and part of the Z-Z_i' interference effects, in that order. The other quantities appearing in these expressions are

$$\eta_G = \frac{\sqrt{(s - M_G^2)^2 + s^2\gamma_G^2}}{s\sqrt{1+\gamma_G^2}}, \tag{36}$$

$$y = \frac{g_i'^2}{\sqrt{2}G_F M_{Z_i'}^2}, \tag{37}$$

$$\tilde{M}_G = \frac{M_G}{\sqrt{1+\gamma_G^2}}, \tag{38}$$

$$\alpha_1 = \frac{\Delta_1 + \Delta_2(\gamma_Z - \gamma_{Z_i'}) + \Delta_2\gamma_Z\gamma_{Z_i'}}{(\Delta_1^2 + \Delta_2^2)(1+\gamma_{Z_i'}^2)}, \tag{39}$$

$$\alpha_2 = \alpha_1\frac{1+\gamma_{Z_i'}^2}{1+\gamma_Z^2}, \tag{40}$$

$$\beta_1 = \frac{M_Z^2(\Delta_1 - \Delta_2\gamma_{Z_i'})}{(\Delta_1^2 + \Delta_2^2)(1+\gamma_Z^2)}, \tag{41}$$

$$\beta_2 = \frac{M_{Z_i'}^2(\Delta_1 + \Delta_2\gamma_Z)}{(\Delta_1^2 + \Delta_2^2)(1+\gamma_Z^2)}, \tag{42}$$

where

$$\Delta_1 = \tilde{M}_Z^2 - \tilde{M}_{Z'}^2, \tag{43}$$

$$\Delta_2 = \tilde{M}_Z^2\gamma_Z + \tilde{M}_{Z_i'}^2\gamma_{Z_i'}. \tag{44}$$

The uncorrected forward-backward asymmetry $\hat{A}_{\rm FB}$ is defined as

$$\hat{A}_{\rm FB}(s) \equiv \int_0^{\pi/2} \frac{d\hat{\sigma}(s,\theta) - d\hat{\sigma}(s,\pi-\theta)}{\hat{\sigma}(s)} \equiv \frac{\hat{\sigma}_{\rm FB}(s)}{\hat{\sigma}(s)}. \tag{45}$$

The corrected forward-backward asymmetry $A_{\rm FB}$ is simply obtained by inputting the corrected cross section in the above equation. The result of the approximate analytic integration for the numerator is

$$\sigma_{\rm FB}(s) \approx \frac{K_Z}{|D_Z(s)|^2}\eta_Z^\beta \frac{\pi\beta}{\sin\pi\beta} \times\left((Cs+D)\frac{\sin[(1-\beta)\xi_Z]}{\sin\xi_Z}\Delta - (2Cs+D)\eta_Z\frac{\sin\beta\xi_Z}{\sin\xi_Z}\right) + (Z\to Z'), \tag{46}$$

where

$$K_Z = K(\tfrac{1}{4}-x)^2, \tag{47}$$

$$C = 1 + \frac{1}{2(\frac{1}{4}-x)^2}\left[x(1-x) - \frac{y}{4}\left(\frac{M_{Z_i'}}{M_Z}\right)^2 (2\alpha_1\tilde{M}_Z^2 - \beta_1)\right], \tag{48}$$

$$D = \frac{M_Z^2}{2(\frac{1}{4}-x)^2}\left[\frac{y}{4}\left(\frac{M_{Z_i'}}{M_Z}\right)^2 \alpha_1\tilde{M}_Z^2 - x(1-x)\right], \tag{49}$$

$$K_{Z'} = \frac{G_F^2 M_Z^2 M_{Z_i'}^2}{16\pi} y(2\alpha_2\tilde{M}_{Z_i'}^2 - \beta_2), \tag{50}$$

$$C' = 1, \tag{51}$$

$$D' = \frac{\alpha_2 M_{Z_i'}^2 \tilde{M}_{Z_i'}^2}{\beta_2 - 2\alpha_2\tilde{M}_{Z_i'}^2}. \tag{52}$$

The data used in the fit were obtained from Refs. [8, 10–12]. A scan was made over a grid in $(r_i', M_{Z_i'})$ $(i=1,2)$ space for the region

$$40 \text{ GeV} < M_{Z_i'} < 1040 \text{ GeV}, \tag{53}$$

where $r_i \equiv (g_i'/g_2)^2$ (it turns out that the range $r_i < 0.01$ is the most interesting phenomenologically). The corrected cross sections and forward-backward asymmetries were used in the LEP region of 88–96 GeV, while the uncorrected quantities were sufficient for use in the TRISTAN region of 40–64 GeV.

An ideal analysis would consist of performing a global fit to both MSM and Z_i' parameters. We have chosen for simplicity to instead input the MSM parameters $\sin^2\theta_W, M_Z$, and Γ_Z from a fit performed within the MSM to experimental data. These results were taken from Ref. [15]. This approximate procedure should yield accurate results, because Z_i' physics only affects purely leptonic processes. Since the bulk of the neutral-current data concerns hadronic processes and our theories are identical with the MSM in this sector, a combined fit should yield standard parameters which are very close to those obtained from a fit purely in the MSM.

The results of the χ^2 fits for both relevant models are shown in Figs. 2 and 3, where the 90% C.L. allowed regions are plotted, for $M_{Z'}$ up to 100 GeV and r_i' up to 0.008. Several features of these graphs are worth noting. (i) The most striking aspect is that a $Z_{1,2}'$ boson is allowed to exist in the region between 64 GeV and 87 GeV, for values of r_i up to 0.003 for model L_1, and up to 0.007 for model L_2. Note also that the L_1 model is more severely constrained than the L_2 model in this region. This energy regime is as yet unexplored experimentally, because TRISTAN has only searched up to about 64 GeV, while LEP has concentrated on the standard Z resonance region. Our analysis shows that it is possible for a $Z_{1,2}'$ to exist in this "window" without affecting the successful predictions of the MSM at energies already explored. (ii) The allowed value of $g_{1,2}'$ rises rapidly with mass for the area above the standard Z resonance. Clearly this represents a large allowed area in parameter space, extending to relatively large values for the coupling constant. (iii) There are small pockets of parameter space allowed at 90% C.L. for both models in the regions already explored by LEP and TRISTAN. The reason for this is that $Z_{1,2}'$ is a very narrow resonance (see later), and so χ^2 decreases when $M_{Z_{1,2}}'$ is chosen to lie between data points.

The best fit values are

$$L_1 = L_e - L_\mu \text{ model}: \quad r_1' \to 0, \quad M_{Z_1'} \to \infty, \tag{54}$$

$$L_2 = L_e - L_\tau \text{ model}: \quad r_2' = 5\times 10^{-4}, \quad M_{Z_2'} = 58 \text{ GeV}. \tag{55}$$

The first result indicates that Z_1' interactions cannot be used to improve on the agreement between theory and experiment over the MSM. The second result shows that a slightly better fit to neutral-current data can be obtained in the gauged $L_e - L_\tau$ model than in the MSM. The best fit point (indicated by a cross in Fig. 3) actually occurs inside one of the small allowed regions in the

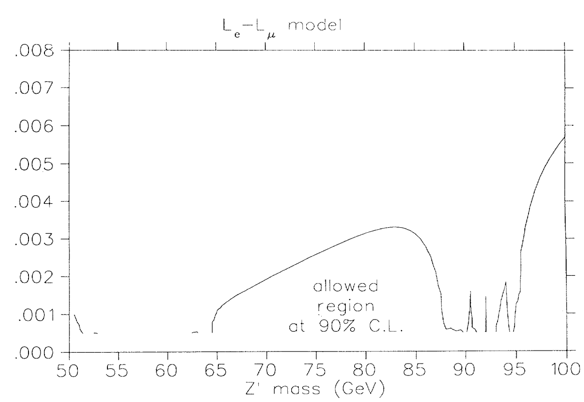

FIG. 2. The allowed region at 90% C.L. in $(g_1', M_{Z_1'})$ space for the gauged L_e-L_μ model.

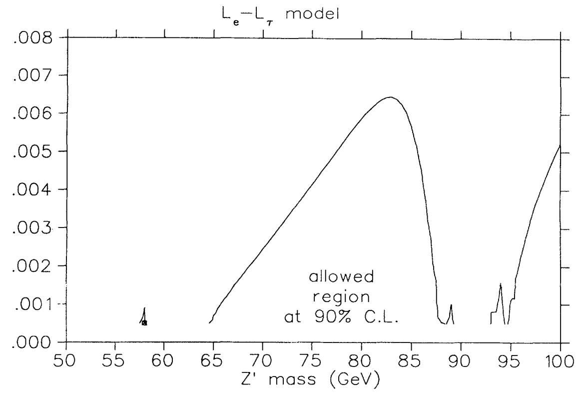

FIG. 3. The allowed region at 90% C.L. in $(g_2', M_{Z_2'})$ space for the gauged L_e-L_τ model.

TRISTAN regime.

Figs. 4(a) and 4(b) show R_τ and $A_{\rm FB}^\tau$ for this best fit point in the $\mathrm{U}(1)_{L_2}$ model, compared with the MSM best-fit curve. The experimental data fluctuates, and the measurement errors are quite significant. The Z_2' resonance shows up as a narrow peak for both quantities lying between two TRISTAN data points. The Z_2' line shape in R_τ indicates that $Z_2' - \gamma$ interference is important, as well as the direct Z_2' contribution. The statistical significance of this result should not be overestimated: there is considerable experimental uncertainty exhibited in the data, and it may be that the sharp spike produced by Z_2' effects happens to imitate a random fluctuation in the data for this particular point, rather than indicating the presence of an actual Z_2' contribution. Note also that the best-fit point in the present analysis is at a slightly different location from our previous analysis [5]. This reflects the presence of new data from TRISTAN. The effect of the best fit Z_2' in the LEP region is negligible.

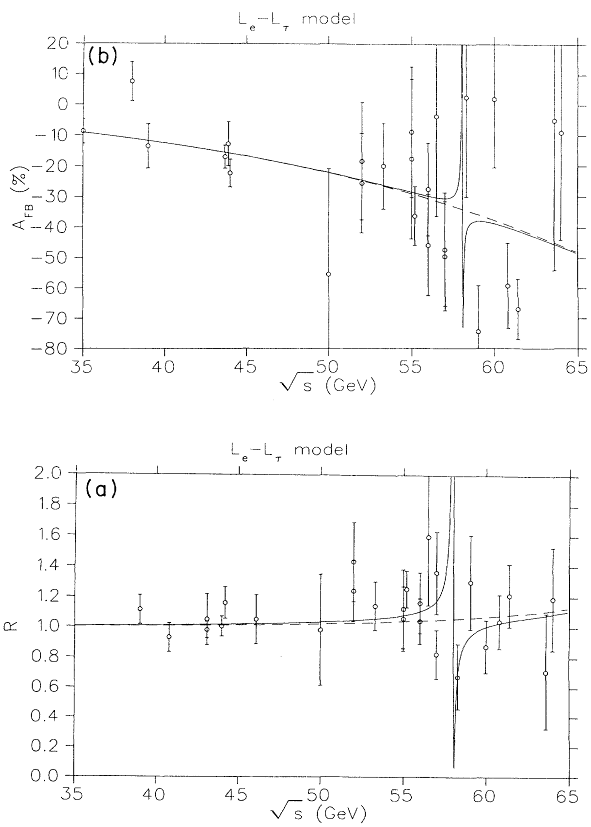

FIG. 4. (a) The best-fit point contribution to R_τ for the gauged L_e-L_τ model (solid line), compared with the MSM best fit (dashed line). (b) The best-fit point contribution to A_{FB}^τ in the TRISTAN region for the gauged L_e-L_τ model (solid line), compared with the MSM best fit (dashed line).

Figures 5(a)–5(d) show the comparison between the $\mathrm{U}(1)_{L_2}$ model and the MSM for cross-sections and forward-backward asymmetries in the TRISTAN and LEP regions corresponding to the point

$$r_2' = 0.006, \qquad M_{Z_2'} = 80.5 \text{ GeV}. \tag{56}$$

These values have been chosen as an illustration of the effect which a Z_2' lying in the window has on measurable physical quantities at experimentally accessible energies for TRISTAN, LEP, and the SLC. The value of the coupling constant corresponds to the maximum allowed at 90% C.L. in the window. Note first of all that there are negligible differences between the MSM predictions near the Z resonance and the predictions of the above example. The differences are greater in the TRISTAN region, particularly for R_τ, because the standard cross section here is relatively small. So, a decrease in the percentage error at TRISTAN to about 10% may be able to detect virtual effects from a Z_2' in the window, or significantly reduce the allowed parameter space. Note that the effects of a Z_2' would have the characteristic signature of increasing R_τ relative to the MSM for $s < M_{Z_2'}$, while leaving R_μ and hadronic processes unaffected.

Figures 6(a) and 6(b) display the corresponding graphs for the $\mathrm{U}(1)_{L_1}$ model with values

$$r_1' = 0.003, \quad M_{Z_1'} = 84 \text{ GeV} \tag{57}$$

which again corresponds to the maximum allowed value for the coupling constant in the window. Observations similar to those in the previous paragraph can be made here as well. We do not include graphs for the Z resonance region, because nonstandard effects are negligible there [as in Figs. 5(a) and 5(b) for the other model].

How may one understand these results qualitatively? First of all it should be noted that the way we have constructed the model allows *any* value of the Z_i' mass to be experimentally consistent, provided the coupling constant is made small enough. Since there is no theoretical constraint on the value of the coupling constant, it can be made arbitrarily small. However, the interesting observation to be made here is that *even with r_i values of the order of 0.005, the Z_i' boson can still provide a very striking experimental signature while simultaneously evading all current bounds.* This is illustrated in Figs. 4(a) and 4(b), where one sees a very narrow resonance with small off-resonance effects. One should note also that $r_i \approx 0.005$ is not so small when an appropriate comparison is made. One may see this by calculating the value of the coupling constant for the standard Z boson coupling to left-handed charged leptons, and comparing it with the same vertex involving the Z_i' boson. These two quantities are about 0.18 and 0.04, respectively, for $r_i \approx 0.005$, so the nonstandard number is only about a factor of 4 or 5 smaller than the standard one.

As is depicted in Fig. 4(a), the Z_i' boson has a very small width. It is given by

$$\Gamma_{Z_i'} = \frac{g_i'^2}{4\pi} M_{Z'} . \tag{58}$$

For $r_i = 0.006$ and $M_{Z_i'} = 80$ GeV, this yields

$$\Gamma_{Z_i'} = 16 \text{ MeV}. \tag{59}$$

This small width means that the energy interval affected by the Z_i' resonance is extremely short. Therefore a tall Z_i' peak can exist in the window between the TRISTAN and LEP regions, without disturbing standard-model predictions for those energy ranges which have already been explored.

In summary then, we have found that a Z_i' boson with a mass in the window between TRISTAN and LEP data can be consistent with known neutral-current results, while allowing large and interesting effects near its resonance. Both Z_1' and Z_2' are expected to be narrow resonances. Z_1' effects increase the cross section for $e^+e^- \to \mu^+\mu^-$ for $s < M_{Z_1'}$, decrease it for $s > M_{Z_1'}$, while leaving the $e^+e^- \to \tau^+\tau^-$ and hadronic final-state processes unaffected. The same observations apply for the Z_2' boson with the roles of μ and τ interchanged. An improvement by a factor of 5 or so in the accuracy of measurements of $R_{\mu,\tau}$ in the TRISTAN energy regime may either observe virtual effects from a Z_i' in the window, or rule out a considerable piece of the currently allowed parameter space. Of course the largest connected region of parameter space occurs for Z_i' masses greater than the standard Z mass, an area that could be explored by LEP2.

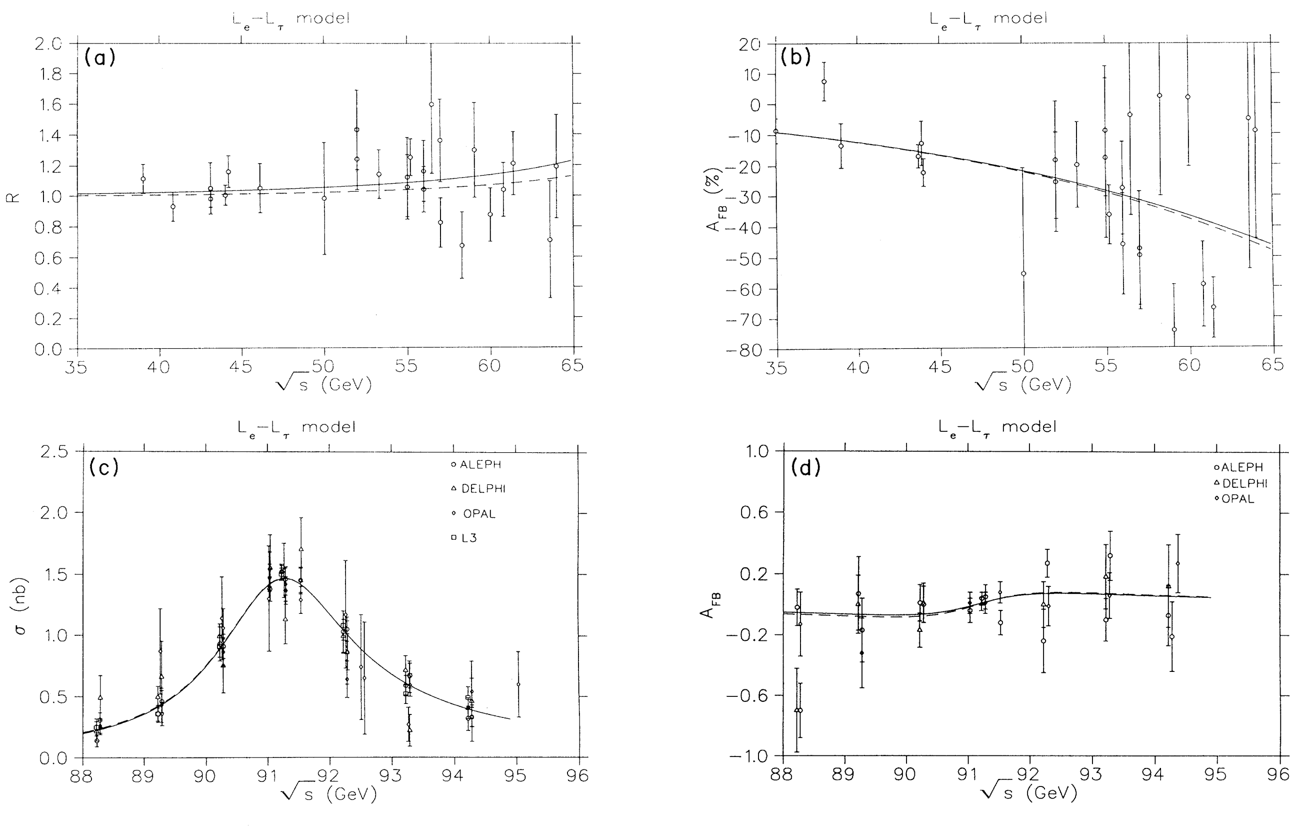

FIG. 5. (a) R_τ corresponding to the maximum allowed coupling constant case in the TRISTAN/LEP window for the gauged L_e-L_τ model (solid line), compared with the MSM best fit (dashed line). (b) A_{FB}^τ for the TRISTAN region corresponding to the maximum allowed coupling constant case in the TRISTAN/LEP window for the gauged L_e-L_τ model (solid line), compared with the MSM best fit (dashed line). (c) Z line shape corresponding to the maximum allowed coupling constant case in the TRISTAN/LEP window for the gauged L_e-L_τ model (solid line), compared with the MSM best fit (dashed line). (d) A_{FB}^τ in the LEP region corresponding to the maximum allowed coupling constant case in the TRISTAN/LEP window for the gauged L_e-L_τ model (solid line), compared with the MSM best fit (dashed line).

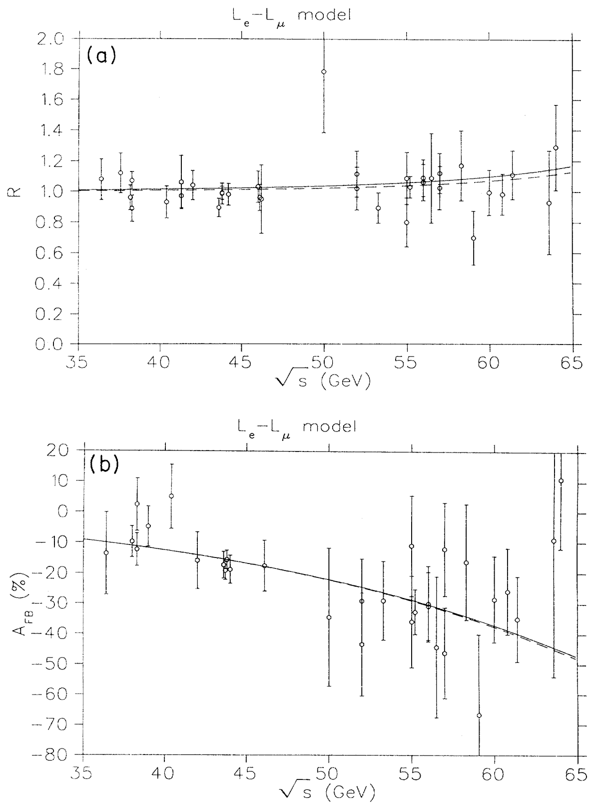

FIG. 6. (a) R_μ corresponding to the maximum allowed coupling constant case in the TRISTAN/LEP window for the gauged L_e-L_μ model (solid line), compared with the MSM best fit (dashed line). (b) A^{μ}_{FB} in the TRISTAN region corresponding to the maximum allowed coupling constant case in the TRISTAN/LEP window for the gauged L_e-L_μ model (solid line), compared with the MSM best fit (dashed line).

III. NEUTRINO MASSES

In this section we will discuss how neutrino masses may be introduced into our model. We will consider constraints from experimental results on oscillations [16], neutrinoless double-β decay [17], and direct neutrino mass measurements [18]. Implications for our theory of the recent results [1] indicating a 17-keV neutrino mass will be considered, and the MSW solution [19] to the solar-neutrino problem will be addressed.

In the previous sections, we pointed out that a gauged family-lepton–number difference was inherent in the MSM due to its automatic anomaly freedom. However when neutrino masses are introduced as an extension of the MSM, the family-lepton numbers are generally no longer conserved, and so we do not have the same motivation as before for gauging one of the L_i. It is nevertheless possible to *impose* gauged L_i on our massive neutrino theory and to study its consequences. One should note that although the motivation for doing this is different from our original motivation, it is still a respectable one.

The Higgs sector of the theory will have to be extended in order to generate neutrino masses. Symmetry-breaking effects due to these fields will then often also lead to Z-Z_i' mixing. These new VEV's, however, will be related to neutrino masses, and will thus have to be very small. Z-Z_i mixing will therefore also be small, and so we do not expect the phenomenological analysis of the previous section to be greatly altered.

We now systematically study neutrino masses in the case where only left-handed neutrinos exist, and then we consider the introduction of right-handed neutrinos also.

A. Left-handed neutrinos

In this case the only way of generating neutrino masses at the tree level is to introduce $SU(2)_L$-triplet Higgs multiplets χ with appropriate quantum numbers under the $U(1)_{L_i}$ group. We will consider the $U(1)_{L_1}$ model for definiteness; the other cases can be similarly treated.

There are different ways of generating neutrino masses depending on what Higgs triplet χ exists in the theory. The possibilities are

$$\begin{aligned}
&(A) \quad \frac{m_{\nu_e}}{v_1}\bar{\ell}^c_{eL}\ell_{eL}\chi_1 : \chi_1 \sim (1,3)(2,-2),\\
&(B) \quad \frac{m_{\nu_\mu}}{v_2}\bar{\ell}^c_{\mu L}\ell_{\mu L}\chi_2 : \chi_2 \sim (1,3)(2,2),\\
&(C) \quad \frac{m_{\nu_\tau}}{v_3}\bar{\ell}^c_{\tau L}\ell_{\tau L}\chi_3 : \chi_3 \sim (1,3)(2,0),\\
&(D) \quad \frac{m_{\nu_{e\mu}}}{v_4}\bar{\ell}^c_{eL}\ell_{\mu L}\chi_4 : \chi_4 \sim (1,3)(2,0),\\
&(E) \quad \frac{m_{\nu_{e\tau}}}{v_5}\bar{\ell}^c_{eL}\ell_{\tau L}\chi_5 : \chi_5 \sim (1,3)(2,-1),\\
&(F) \quad \frac{m_{\nu_{\mu\tau}}}{v_6}\bar{\ell}^c_{\mu L}\ell_{\tau L}\chi_6 : \chi_6 \sim (1,3)(2,1),
\end{aligned} \tag{60}$$

where v_i is the VEV of χ_i. Models C and D are different only by global family-lepton numbers.

If *one* of the above Higgs multiplets is introduced into the theory, some neutrinos will have Majorana masses, but there will be no neutrino oscillations. It is also interesting to note that of the above theories, only C and D have physical Majorons, provided the relevant global family-lepton numbers are imposed as exact symmetries of the Lagrangian. We in fact do choose to impose these symmetries because later on it will allow us to use the Majoron as a decay product of the 17-keV neutrino. (Alternatively, one may explicitly break the global symmetries through the Higgs potential terms $\phi^2\chi_3^\dagger$ and $\phi^2\chi_4^\dagger$.) Given that there are Majorons in C and D then, if there is no further extension of the Higgs sector, they are already ruled out (as is the similar Gelmini-Roncadelli model [20]) from the Z-width data at LEP. However, C and D can be rescued by introducing one Higgs-singlet field S', which is completely neutral under the gauge group, but transforms under the relevant global family-lepton–number symmetries. If $\langle S'\rangle \equiv v_{S'}$ is much larger than v_i, then the Majoron is mostly ImS'. Therefore the contribution of the Majoron to the Z width is suppressed by a mixing angle

Since there are no neutrino oscillations, the only relevant experimental constraints are from double-β decay and direct mass measurements. These yield the bounds: $m_{\nu_e} < 1.8$ eV [17], $m_{\nu_\mu} < 270$ keV and $m_{\nu_\tau} < 35$ MeV [18]. These can obviously be satisfied.

It is impossible for the above theories to use the Mikheyev-Smirnov-Wolfenstein (MSW) effect to solve the solar-neutrino problem, because there are no neutrino oscillations. Also, they cannot accommodate a possible 17-keV neutrino, with properties as indicated by the experiments on β decay and electron capture in various nuclei [1]. In order to accommodate these two things a further extension of the Higgs sector is necessary. We find, however, that it is impossible to accommodate both at the same time. This is because the neutrinoless double-β-decay experiments have constrained the effective electron mass $\langle m_e \rangle = |U_{ee}^2 m_{\nu_e} + U_{e\mu}^2 m_{\nu_\mu} + U_{e\tau}^2 m_{\nu_\tau}|$ to be less than 1.8 eV. The claimed experimental properties of the 17-keV neutrino yield $m_{\nu_\tau} = 17$ keV and $U_{e\tau}^2 \approx 0.01$. (The 17-keV neutrino cannot be identified with the muon-neutrino because neutrino oscillation experiments already constrain $U_{e\mu}^2$ to be less than 0.001.) Incorporating this constraint on $U_{e\mu}$, the effective electron mass bound yields 170 keV $< |m_{\nu_\mu}| <$ 270 keV for the muon-neutrino mass. With this neutrino mass spectrum it is not possible to have neutrino mass-squared differences and mixings in the right range for the MSW solution of the solar-neutrino problem to be invoked.

It is, however, possible to accommodate one of them. For example, if we introduce *both* χ_2 and χ_4 it is possible to have a solution for the solar-neutrino problem via the MSW mechansim. Remember that this requires [21] $|m_{\nu_e}^2 - m_{\nu_\mu}^2| \sin^2 2\alpha \approx 3 \times 10^{-8}\text{eV}^2$ and $|\alpha| > 0.03$ where α is the ν_e-ν_μ mixing angle. Of course a Higgs singlet S' with a large global family-lepton–number-breaking VEV, and which couples to χ_4, is necessary in order for the theory to be consistent with the Z-width measurement.

It is also possible to accommodate the properties of the 17-keV neutrino. This can be achieved by introducing $\chi_2, \chi_3 = \chi_4$ and χ_5. Note that here we are identifying χ_3 and χ_4 by explicitly breaking some family-lepton–number global symmetries. The resulting mass matrix after the χ's develop VEV's can satisfy the data from neutrinoless double-β-decay, neutrino oscillation experiments and the 17-keV neutrino. Again a Higgs singlet S' is need to break global family-lepton number at a high scale so that the Majoron-Z coupling is suppressed. Imχ_5 has a Majoron (J) component of order $v_5/v_{S'}$. Since the coupling of J to ν_τ does not conserve flavor, the 17-keV neutrino can decay through $\nu_{17} \to \nu_e + J$ with lifetime $< 10^7$ sec. This makes the theory consistent with cosmological and large-scale–structure formation constraints [22]. The ratio $v_5/v_{S'}$ can also be chosen so that the theory is consistent with big-bang nucleosynthesis and astrophysical constraints [22]. It is possible to have a viable theory with χ_4 and χ_6 instead of the above; this is similar to a model studied in Ref. [23]. In this case there are two 17-keV neutrinos.

Finally, note that one of the predictivity problems of the original theory can be solved in extensions A, B, E, and F. The L_i charge η of the original singlet-Higgs boson S can be fixed by demanding that the Higgs potential term $\chi^\dagger \phi^2 S$ exists.

B. Left- and right-handed neutrinos

Unfortunately, none of the theories considered above can yield a neutrino spectrum that is consistent with the 17-keV neutrino data and can also solve the solar-neutrino problem via the MSW effect. However, with further extensions to the fermion sector this is possible. The simplest extension is to introduce right-handed neutrinos, a scenario we now study. There are two distinguishable classes of models: (a) pure Dirac neutrinos, and (b) Dirac and Majorana neutrinos.

We consider pure Dirac neutrinos first. We assign the right-handed neutrinos to the same quantum numbers of $L_e - L_\mu$ and the global symmetry generator L_τ as the other leptons. We also introduce two Higgs doublets

$$\phi' \sim (1,2)(+1,+1)_{+1}, \quad \phi'' \sim (1,2)(+1,+1)_{-1}, \tag{61}$$

where the subscript is L_τ. The allowed Yukawa couplings are

$$\mathcal{L}_{\text{Yuk}} = \lambda_i^e \bar{\ell}_{iL} \ell_{iR} \phi + \lambda_i^\nu \bar{\ell}_{iL} \nu_{iR} \phi^c + \lambda_{e\tau}^e \bar{\ell}_{eL} \tau_R \phi'' + \lambda_{\tau\mu}^e \bar{\ell}_{\tau L} \mu_R \phi' + \lambda_{\mu\tau}^\nu \bar{\ell}_{\mu L} \nu_{\tau R} \phi^{c\prime} + \lambda_{\tau e}^\nu \bar{\ell}_{\tau L} \nu_{eR} \phi^{c\prime\prime}, \tag{62}$$

where $i = e, \mu, \tau$.

We require ϕ' and ϕ'' to acquire *small* VEV's; the global τ number is thus broken and very small Z-Z_i' mixing is induced. There is a massless pseudoscalar which is a linear combination of ϕ' and ϕ''. This leads to a possible dangerous $Z \to J+$light Higgs contribution to the Z width. To avoid this, and also to avoid a large coupling of the Majoron to charged leptons, we introduce a Higgs singlet S' which again is a gauge singlet but has nonzero τ number. So again the massless pseudoscalar is mostly ImS'.

In this model, both the charged lepton and neutrino mass matrices are nondiagonal. After diagonalizing these mass matrices it is possible to produce ν_e-ν_τ mixing at the 0.1 level, and to have the ν_e-ν_μ mixing and squared-mass difference in the correct range to invoke the MSW solution to the solar-neutrino problem. The 17-keV neutrino can again decay through $\nu_{17} \to J + \nu_e$ because both ϕ' and ϕ'' have Majoron components of order $v_{\phi',\phi''}/v_{S'}$. Note, of course, that the Majoron coupling to neutrinos is nondiagonal.

We now treat the case where there are both Dirac and Majorana neutrinos. Introducing right-handed neutrinos opens up the possibility of having small neutrino masses through the seesaw mechansim. One can induce Dirac neutrino masses through $SU(2)_L$ doublets and Majorana masses for the right-handed neutrinos through $SU(2)_L$ singlets. In order that the theory be consistent with the properties of the 17-keV neutrino, the τ neutrino needs to be almost a pure Dirac particle. This implies that the Majorana mass matrix for the right-handed neutrinos must be rank 2 rather than 3. This issue within the context of gauged $U(1)_{L_e - L_\mu}$ has recently been studied by Ng [24]. In addition to the standard doublet ϕ, and the singlet S which breaks $L_e - L_\mu$, there is one more Higgs doublet $\phi' \sim (1,2)(+1,-1)_1$ and another Higgs singlet $S'' \sim (1,1)(-1/2,1/2)_{-1/2}$. Both ϕ' and S'' are required to develop VEV's. The resulting mass matrix can satisfy constraints from experiments. There is a Majoron in this theory which has nondiagonal couplings to the neutrinos. Depending on the VEV of S'', the 17-keV neutrino can decay through Majoron emission with a lifetime as short as 10^{-3} sec, to satisfy cosmological bounds. The model as considered cannot utilize the MSW solution to the solar-neutrino problem. This can, however, be remedied by introducing more Higgs multiplets, e.g., $\phi''' \sim (1,2)(+1,-2)_0$.

In the above we have discussed possible ways of generating neutrino masses, including some discussion of the 17-keV neutrino and the MSW effect. Our model shares similar difficulties of yielding a 17-keV neutrino with the correct properties with most other theories: quite complicated and somewhat artificial schemes result. Clearly our model has the most aesthetic appeal in its pure form, where only massless left-handed neutrinos exist. The confirmation of the 17-keV neutrino or the MSW solution would obviously rule out the simplest version of our class of theories, and indeed the simplest versions of many other theories.

IV. NON-ABELIAN EXTENSIONS

A. Introduction

We have seen how the possibility of gauging the differences in family-lepton numbers is inherent in the MSM, in the sense that these symmetries are anomaly-free. We have also seen that the phenomenology of the resulting Z' boson is rather interesting. A question then naturally arises: if gauged $U(1)_{L_i}$ exists, what are its theoretical implications? For instance, is such a theory unifiable? If it is, then the resulting grand unified theory (GUT) would certainly have to possess rather nonstandard features. We are not too concerned at this stage with obtaining answers to questions such as these. Our motivation for gauging $U(1)_{L_i}$ was from a low-energy perspective: the MSM allows this possibility, the resulting theory is extremely simple and it has low-energy phenomenological implications. Nevertheless, we will in this section speculate about a stage of development that is possible beyond the gauging of $U(1)_{L_i}$, because these models can be embedded in a non-Abelian gauge theory based on a horizontal SU(2) symmetry acting on leptons. As pointed out earlier, this also requires ϵ to be zero.

The reason SU(2) is relevant is rather obvious. Its diagonal generator is proportional to diag$(1,-1)$ in generation space for the doublet representation, while for the triplet representation it is proportional to diag$(1,0,-1)$. Thus three generations of leptons can be assigned either to a $\mathbf{2}\oplus\mathbf{1}$ or a $\mathbf{3}$ of a horizontal SU(2) gauge group [denoted hereinafter by $SU(2)_H$] with the diagonal generator identified as the family-lepton–number differences operator. The gauge group of the non-Abelian extensions is thus G_{ext} where

$$G_{\text{ext}} = SU(2)_H \otimes SU(3)_c \otimes SU(2)_L \otimes U(1)_Y . \quad (63)$$

In addition to Z'_i coupling to the diagonal generator, $SU(2)_H$ contains two other neutral gauge bosons (call them $Z_\pm$) which couple to the raising and lowering generators. We will comment on the phenomenology of these extra neutral bosons later [25].

The four models implied by the above discussion have the fermion spectra

$$\text{(A)}\quad \ell_L \sim (3,1,2)(-1),\quad \ell_R \sim (3,1,1)(-2); \quad (64)$$

$$\begin{aligned}\text{(B)}\quad &\ell_{1L} \sim (2,1,2)(-1),\quad \ell_{1R} \sim (2,1,1)(-2),\quad \nu_{1R} \sim (2,1,1)(0),\\ &\ell_{2L} \sim (1,1,2)(-1),\quad \ell_{2R} \sim (1,1,1)(-2),\quad \nu_{2R} \sim (1,1,1)(0);\end{aligned} \quad (65)$$

$$\begin{aligned}\text{(C)}\quad &\ell_{1L} \sim (2,1,2)(-1),\quad \ell_R \sim (3,1,1)(-2),\\ &\ell_{2L} \sim (1,1,2)(-1);\end{aligned} \quad (66)$$

$$\text{(D)}\quad \ell_L \sim (3,1,2)(-1),\quad \ell_{1R} \sim (2,1,1)(-2),\quad \ell_{2R} \sim (1,1,1)(-2). \quad (67)$$

Note that we have included right-handed neutrino multiplets in model B. This is in order to cancel the $SU(2)_H$ global anomaly that would otherwise result. Note also that ν_{2R} need not be introduced, but we choose to include it to maintain the three-generation structure for all types of leptons. As we have mentioned earlier, the introduction of right-handed neutrinos seems to be at odds with our original motivation for gauging family-lepton–number differences. Observe, however, that *all* of the non-Abelian extensions go against our original motivation because the MSM does not in general have an $SU(2)_H$ symmetry in its Yukawa Lagrangian. Therefore we do not think the presence of right-handed neutrinos in this theory is a great concern. We simply have to accept that the non-Abelian extensions cannot be motivated in the same way as their precursor. We will comment on anomaly cancellation for models C and D later.

We will now describe these four models in detail. The reader should remember that there are three precursor theories and thus three interpretations of each possible non-Abelian extension. These cases will be treated generically rather than specifically in what follows.

B. Model A

The standard Yukawa coupling Lagrangian for this model takes the form

$$\mathcal{L}_{\text{Yuk}}^{(1)} = \lambda_1 \bar{\ell}_L \ell_R \phi + \text{H.c.}, \tag{68}$$

where $\phi \sim (1,1,2)(1)$. If this Lagrangian was the only source of lepton masses, then the phenomenologically unacceptable mass relations

$$m_e = m_\mu = m_\tau \tag{69}$$

would result, due to the $\mathrm{SU}(2)_H$ symmetry and the choice of this simple Higgs sector. One way of avoiding this is to introduce the Higgs fields Δ and Ω where

$$\Delta \sim (3,1,2)(1), \quad \Omega \sim (5,1,2)(1) \tag{70}$$

which couple to leptons through Yukawa Lagrangians given by

$$\begin{aligned} \mathcal{L}_{\text{Yuk}}^{(2)} &= \lambda_2 \bar{\ell}_L \ell_R \Delta + \text{H.c.}, \\ \mathcal{L}_{\text{Yuk}}^{(3)} &= \lambda_3 \bar{\ell}_L \ell_R \Omega + \text{H.c.} \end{aligned} \tag{71}$$

Note that Δ and Ω act nontrivially in generation space. We require their $L_i = 0, I_{3L} = -1/2$ components to get nonzero VEV's, where L_i is of course the diagonal generator of $\mathrm{SU}(2)_H$. This performs the symmetry breaking

$$\mathrm{SU}(2)_H \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y \to \mathrm{U}(1)_{L_i} \otimes \mathrm{U}(1)_Q . \tag{72}$$

This breaks the degeneracy between e, μ, and τ and an acceptable (though not predictive) lepton mass spectrum can be arranged. Alternatively, one may attempt to use this horizontal symmetry as a basis for constructing some type of *predictive* lepton mass scheme. This program is beyond the scope of this paper, although we will note that since our theories have a horizontal symmetry in the lepton sector only, it would be nontrivial to also construct a predictive scheme for quark masses and mixing angles [26].

Nonzero VEV's for Δ and Ω are not sufficient to completely break $\mathrm{SU}(2)_H$ since the $\mathrm{U}(1)_{L_i}$ subgroup remains exact. In order to break this subgroup, we need to find a generalization of S. This is provided by another Higgs field Σ where

$$\Sigma \sim (2,1,1)(0). \tag{73}$$

Note that Σ is a natural generalization of S because it is neutral under the electroweak group, but not under $\mathrm{SU}(2)_H$, and therefore will not induce mixing between Z_i' and the standard Z at the tree level.

The one remaining issue in symmetry breaking in this model is that, as it stands, there cannot be a great mass difference between the gauge field Z_i' coupling to the diagonal generator of $\mathrm{SU}(2)_H$ and the gauge fields $Z_\pm$ which couple to the remaining two generators. Since these latter neutral gauge bosons contribute to $e^+e^- \to \mu^+\mu^-$, $\tau^+\tau^-$ in addition to Z_i', and since they also contribute to μ decay (see later), we would like the freedom to make them heavy, while keeping the Z_i' fairly light. Remember that the Abelian case was phenomenologically interesting because an obervable Z_i' could be fairly light, and we wish to preserve this feature in the non-Abelian extension. This requires us to introduce a Higgs boson σ where

$$\sigma \sim (3,1,1)(0). \tag{74}$$

The quantum numbers have been chosen so that a nonzero VEV for the $L_i = 0$ component of σ will perform the desired breaking

$$\mathrm{SU}(2)_H \to \mathrm{U}(1)_{L_i} \tag{75}$$

at an arbitrarily high scale. Note, of course, that $\langle\sigma\rangle$ does not induce any mixing between the standard Z and any of the Z' bosons.

This completes the specification of the fields in the theory. We now summarize the basic features of the model: The VEV hierarchy

$$\langle\sigma\rangle \gg \langle\Sigma\rangle \gg \langle\phi\rangle, \langle\Delta\rangle, \langle\Omega\rangle \tag{76}$$

is imposed so that the resulting symmetry-breaking cascade is

$$\begin{array}{c} \mathrm{SU}(2)_H \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y \\ \downarrow \langle\sigma\rangle \\ \mathrm{U}(1)_{L_i} \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y \\ \downarrow \langle\Sigma\rangle \\ \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y \\ \langle\phi\rangle \downarrow \langle\Delta\rangle, \langle\Omega\rangle \\ \mathrm{U}(1)_Q \end{array} \tag{77}$$

After the first stage of symmetry breaking, the effective theory has the same gauge group as the original model. The $Z_\pm$ gauge bosons pick up mass at this stage. Note that the presence of σ is essential for generating this intermediate scale. The Z_i' boson gains mass from the VEV of Σ, while electroweak symmetry breaking and fermion mass generation proceed via VEV's for ϕ, Δ, and Ω. Note that as in the Abelian case, no tree-level Z-Z_i', Z-$Z_\pm$, or Z_i'-$Z_\pm$ mixing is induced by the Higgs bosons. This is phenomenologically important.

We will discuss the neutral-gauge-boson sector in more detail after we have introduced all of the models, because of some common features.

C. Model B

The Yukawa Lagrangian for this model is

$$\begin{aligned} \mathcal{L}_{\text{Yuk}} = {} & \lambda_1 \bar{\ell}_{1L} \ell_{1R} \phi + \lambda_2 \bar{\ell}_{1L} \ell_{1R} \Delta + \lambda_3 \bar{\ell}_{1L} \nu_{1R} \phi^c \\ & + \lambda_4 \bar{\ell}_{1L} \nu_{1R} \Delta^c + \lambda_5 \bar{\ell}_{2L} \ell_{2R} \phi + \lambda_6 \bar{\ell}_{2R} \nu_{2R} \phi^c \\ & + h_1 \bar{\nu}_{1R}^c \nu_{2R} \Sigma + h_2 \bar{\nu}_{1R}^c \nu_{1R} \sigma + M \bar{\nu}_{2R}^c \nu_{2R} + \text{H.c.} \end{aligned} \tag{78}$$

The Higgs multiplet Δ is needed to break a potential degeneracy between e and μ in this case, while Ω is not necessary and thus not introduced. ϕ and Δ also give Dirac masses to the neutrinos. The fields Σ and σ give Majorana masses to the right-handed neutrinos in this model, and the singlet field ν_{2R} can have a bare Majorana mass. Note that these Majorana masses (apart from M) are naturally large, because they are correlated with the high symmetry-breaking scales represented by $\langle\Sigma\rangle$ and $\langle\sigma\rangle$. A natural seesaw mechanism can therefore be induced. An attractive feature of this model as compared with model A is that all of the Higgs bosons couple to fermion bilinears. This provides a deeper motivation for introducing Σ and σ, which were included in model A simply to obtain a phenomenologically interesting symmetry-breaking pattern. Also, Higgs bosons which couple to fermion bilinears may ultimately be interpreted as dynamically generated bound states from dynamical symmetry breakdown induced by four-Fermi interactions.

Since the Higgs spectrum in model B is essentially the same as in model A, the same symmetry-breaking cascade may be induced, and the same observations about neutral-gauge-boson masses and mixings will remain.

D. Models C and D

The Yukawa Lagrangian in model C is

$$\mathcal{L}_{\rm Yuk} = \lambda_1 \bar{\ell}_{1L}\ell_R\Theta + \lambda_2\bar{\ell}_{1L}\ell_R\rho + \lambda_3\bar{\ell}_{2L}\ell_R\Delta + \text{H.c.}, \tag{79}$$

where the Higgs multiplets Θ and ρ are given by

$$\Theta \sim (2,1,2)(1), \quad \rho \sim (4,1,2)(1). \tag{80}$$

It is immediately obvious that this model has significantly different properties to models A and B, because the fields Θ and ρ do not contain any $L_i = 0$ components. Therefore nonzero VEV's for Θ and ρ, which are necessary for lepton mass generation, would also induce tree-level mixing between the standard Z and Z'_i. Since the phenomenological constraints on any admixture in Z are quite stringent, we do not consider this model in any greater detail. [Note also that gauge anomalies do not cancel within the leptons. This requires either the introduction of exotic fermions, or the assignment of nontrivial $SU(2)_H$ properties to quarks.]

Finally, it is clear model D has the same qualitative features with regard to symmetry breaking and neutral-gauge-boson mixing as does model C, so we also consider it no further. (It also possesses gauge and global anomalies if no new fermions are introduced.)

E. Discussion

We now examine the neutral-gauge-boson sector of models A and B. Let us use the following matrix notation for the various VEV's:

$$\langle\phi\rangle = \begin{pmatrix} 0 \\ v \end{pmatrix}, \quad \langle\Delta\rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & w & 0 \end{pmatrix}, \quad \langle\Omega\rangle = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & w' & 0 & 0 \end{pmatrix}, \tag{81}$$

$$\langle\Sigma\rangle = \begin{pmatrix} u_1 \\ u_2 \end{pmatrix}, \quad \langle\sigma\rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & s & 0 \end{pmatrix}. \tag{82}$$

In Eqs. (81) and (82), $SU(2)_L$ acts vertically and $SU(2)_H$ acts horizontally. I_{3L} eigenvalues decrease down the page and L_i eigenvalues increase from left to right.

The neutral-vector-boson mass Lagrangian which results from using these VEV's in the Higgs-boson kinetic terms is

$$\begin{aligned}\mathcal{L}_{\rm gauge\ mass} = {} & \tfrac{1}{4}g_Z^2(v^2 + w^2 + w'^2)Z_\mu Z^\mu \\ & + \tfrac{1}{2}g_H^2(w^2 + w'^2 + u_1^2 + u_2^2 + s^2)Z_{+\mu}Z_-^\mu \\ & + \tfrac{1}{4}g_H^2(u_1^2 + u_2^2)Z'_{i\mu}Z_i'^\mu .\end{aligned} \tag{83}$$

This equation summarizes the observations made about neutral-gauge-boson masses and mixings in the above text. First, the standard Z boson, the Z'_i and $Z_\pm$ do not mix with each other at the tree level. This means that any nonstandard admixture in Z is very small. The fact that Z'_i and $Z_\pm$ do not mix at the tree level is also significant, because it means that the mass eigenstate Z'_i has almost identical couplings to fermions in both the original Abelian theory and the two non-Abelian extensions A and B. Second, we can see clearly from Eq. (83) that the VEV hierarchy $s \gg u_1, u_2 \gg v, w, w'$ leads to $M_{Z_\pm} \gg M_{Z'_i}$. Note that the last VEV hierarchy does not necessarily translate into a significant hierarchy between the standard Z mass and $M_{Z'_i}$ because a small coupling constant g'_i can compensate.

The neutral gauge bosons $Z_\pm$ contribute to μ decay because second-generation leptons are changed into first-generation leptons at the vertex. This has been discussed by Babu and Mohapatra in Ref. [25]. Using their result we obtain the bound

$$M_{Z_\pm} > 3200\sqrt{r_i}\ \text{GeV}. \tag{84}$$

For $r_i = 0.005$ this yields $M_{Z_\pm} > 230$ GeV. Therefore a significant mass splitting between Z'_i and $Z_\pm$ is required if we want to have a light Z'_i.

Finally, we remark that the Higgs potentials in models A and B are very complicated. The Higgs potential of model B is similar (though not identical) to that analyzed in Ref. [25]. They showed that a VEV spectrum similar to that in Eqs. (81) and (82) was possible in a region of parameter space. We expect the same to apply in our models.

V. CONCLUSION

The status of the global symmetries of the MSM is a subject of great importance and much discussion. Although the conservation laws of family-lepton number and baryon number are obeyed very closely by nature, there has been considerable speculation about exotic processes which violate these laws. Such effects are seen as a way to probe physics beyond the MSM.

On the other hand, the important theoretical observation that differences in family-lepton numbers are anomaly-free in the MSM allows an obvious and hitherto unexplored phenomenon to occur: one of them may be coupled to a gauge field. This has very interesting phenomenological consequences, because it defines a class of models which are the simplest possible extensions of the MSM to feature a second neutral massive gauge boson. Our phenomenological analysis of two of these theories, where a gauge field is coupled to $L_e - L_\mu$ or $L_e - L_\tau$, shows that their effects can be made consistent with all present data, while allowing the possibility of their detection in the next few years. Such a gauge boson may, for instance, show up in e^+e^- colliders as a tall, thin resonance in the unexplored region between 64 GeV and 87 GeV. The narrowness of the peak presents a challenge to our experimentalist colleagues.

We also explored some other implications of these theories. We discussed how neutrino masses may be introduced as a further extension of the models, and showed how the solar-neutrino deficit and the possible existence of a 17-keV neutrino admixture in β decay may be addressed. Some extensions to the non-Abelian gauge group SU(2) were also proposed.

If family-lepton–number symmetries are exact, then the fact that anomaly-free linear combinations of them exist is very significant theoretically. As well as having implications for charge quantization in the MSM, it also allows their status to be upgraded to local symmetries coupled to gauge fields. The discovery of such a neutral gauge boson would represent an exciting direction for nature to take.

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[1] J. J. Simpson, Phys. Rev. Lett. **54**, 1891 (1985); J. J. Simpson and A. Hime, Phys. Rev. D **39**, 1825 (1989); **39**, 1837 (1989); A. Hime and N. Jelley, Phys. Lett. B **257**, 441 (1991); B. Sur *et al.*, Phys. Rev. Lett. **66**, 2444 (1991); I. Žlimen *et al.*, *ibid.* **67**, 560 (1991).

[2] T. Altzitzoglou *et al.*, Phys. Rev. Lett. **55**, 807 (1985); T. Ohi *et al.*, Phys. Lett. **160B**, 322 (1985); V. Datar *et al.*, Nature (London) **318**, 547 (1985); A. Apalikov *et al.*, Pis'ma Zh. Eksp. Teor. Fiz. **42**, 233 (1985) [JETP Lett. **42**, 289 (1985)]; J. Markey and F. Boehm, Phys. Rev. C **32**, 2215 (1985); D. Hetherington *et al.*, *ibid.* **36**, 1504 (1987); M. Borge *et al.*, Phys. Scr. **34**, 591 (1988); I. Zlimen *et al.*, *ibid.* **34**, 529 (1989); H.-W. Becker *et al.*, presented at Moriond, Caltech report, 1991 (unpublished).

[3] C. N. Yang and R. L. Mills, Phys. Rev. **96**, 191 (1954).

[4] R. Foot, Mod. Phys. Lett. A **6**, 527 (1991).

[5] X.-G. He, G. C. Joshi, H. Lew, and R. R. Volkas, Phys. Rev. D **43**, 22 (1991); in *Proceedings of the 25th International Conference on High Energy Physics*, Singapore, 1990, edited by K. K. Phua and Y. Yamaguchi (World Scientific, Singapore, 1991).

[6] For a review, see R. Foot, G. C. Joshi, H. Lew, and R. R. Volkas, Mod. Phys. Lett. A **5**, 2721 (1990).

[7] For a discussion of nondiagonal kinetic-energy terms, see R. Foot and X.-G. He, Phys. Lett. B (to be published).

[8] AMY Collaboration, A. Bacala *et al.*, Phys. Lett. B **218**, 112 (1989); TOPAZ Collaboration, I. Adachi *et al.*, *ibid.* **208**, 319 (1988); VENUS Collaboration, K. Abe *et al.*, *ibid.* **246**, 297 (1990); Z. Phys. C **48**, 13 (1990).

[9] G. S. Abrams *et al.*, Phys. Rev. Lett. **63**, 2173 (1989).

[10] OPAL Collaboration, M. Z. Akrawy *et al.*, Phys. Lett. B **235**, 379 (1990); **247**, 458 (1990); **240**, 497 (1990); L3 Collaboration, B. Adeva *et al.*, *ibid.* **247**, 473 (1990); **250**, 183 (1990); ALEPH Collaboration, D. Decamp *et al.*, Z. Phys. C **48**, 365 (1990); DELPHI Collaboration, P. Abreu *et al.*, Report No. CERN-PPE/90-119 (unpublished).

[11] U. Amaldi *et al.*, Phys. Rev. D **36**, 1385 (1987).

[12] J. Panman, in *Proceedings of the XIVth International Symposium on Lepton and Photon Interactions*, Stanford, California, 1989, edited by M. Riordan (World Scientific, Singapore, 1990); K. Abe *et al.*, *ibid.*; CHARM-II Collaboration, D. Geiregat *et al.*, *ibid.*; G. S. Vidyakin *et al.*, *ibid.*

[13] F. A. Berends, R. Kleiss, and S. Jadach, Nucl. Phys. **B202**, 63 (1982); M. Greco, G. Pancheri-Srivastava, and Y. Srivastava, *ibid.* **B171**, 118 (1980); **B197**, 543(E) (1982); E. A. Kuraev and V. S. Fadin, Yad. Fiz. **41**, 733 (1985) [Sov. J. Nucl. Phys. **41**, 466 (1985)]; F. A. Berends, G. Burgers, and W. L. van Neervan, Nucl. Phys. **B297**, 429 (1988); **B304**, 921(E) (1988); Phys. Lett. B **177**, 191 (1986); G. Alterelli and G. Martinelli, in *Physics at LEP*, LEP Jamboree, Geneva, Switzerland, 1985, edited by J. Ellis and R. Peccei (CERN Yellow Report No. 86-02, Geneva, 1986), p. 47; O. Nicrosini and L. Trentadue, Phys. Lett. B **196**, 551 (1987); V. S. Fadin and V. S. Khoze, Yad. Fiz. **47**, 1693 (1988) [Sov. J. Nucl. Phys. **47**, 1073 (1988)].

[14] The analytic approximation we use does not correspond precisely with any so far published in the literature, because we include the effects of s-channel Z_i' exchange. References to the Z line shape in the MSM are A. Borrelli *et al.*, Nucl. Phys. **B333**, 357 (1990); F. A. Berends and R. Kleiss, *ibid.* **B177**, 237 (1981); R. N. Cahn, Phys. Rev. D **36**, 2666 (1987); P. Wang and Y.-S. Zhu, *ibid.* **43**, 1459 (1991).

[15] F. Dydak, in *Proceedings of the 25th International Conference on High Energy Physics* (Ref. [5]).

[16] For a recent review, see F. Pierre, in *Proceedings of the Workshop on Tau-Lepton Physics*, Orsay, France, 1990 (CEN Report No. Saclay-DPHPE-90-19, Saclay, 1990).

[17] D. Caldwell *et al.*, Phys. Rev. Lett. **59**, 419 (1987).

[18] Particle Data Group, J. J. Hernández *et al.*, Phys. Lett. B **239**, 1 (1990).

[19] S. Mikheyev and A. Smirnov, Yad. Fiz. **42**, 1441 (1986) [Sov. J. Nucl. Phys. **42**, 913 (1986)]; L. Wolfenstein, Phys. Rev. D **17**, 2369 (1979).

[20] G. B. Gelmini and M. Roncadelli, Phys. Lett. **99B**, 411 (1981).

[21] A. Smirnov, in Proceedings of the 26th Rencontres de Moriond, Les Arcs, France, 1991 (unpublished).

[22] J. M. Cline and T. P. Walker, Ohio State University Report No. OSU-TA.2/91, 1991 (unpublished).
[23] M. J. Dugan *et al.*, Phys. Rev. Lett. **54**, 2302 (1985).
[24] D. Ng, North Carolina Report No. IFP-403-UNC, 1991 (unpublished).
[25] K. S. Babu and R. N. Mohapatra, Phys. Rev. D **42**, 3778 (1990). This paper discusses the phenomenology of $Z_{\pm}$ bosons in a similar model.
[26] For an attempt along these lines, see K. S. Babu and R. N. Mohapatra, Phys. Rev. D **43**, 2278 (1991).