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Quark mixing and CP violation

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In view of the experimental indications that the top-quark mass may exceed the upper bound predicted by the Fritzsch ansatz for the quark mass matrices, we follow a phenomenological approach to find a modified form of the mass matrices to deal with quark mixing and CP violation in electroweak interactions. The essential feature of our mass matrices is that the diagonal elements corresponding to the light quarks u, d, and s are vanishingly small, while those corresponding to the heavy quarks c, b, and t are approximately equal to the observed values of their masses. By comparing our theoretical results for the Kobayashi-Maskawa mixing matrix elements with the experimental data, we obtain for the top-quark mass and the CP-violating phase the values $m_t^{\text{phys}} \leq 170$ GeV and $91° \leq \delta \leq 105°$.

I. INTRODUCTION

Various forms of mass matrices have been proposed [1,2] to deal with quark mixing in the electroweak interactions. Of all the suggested schemes, the scheme of Fritzsch is the most economical [1]. However, when the Kobayashi-Maskawa (KM) mixing matrix obtained from Fritzsch's form of mass matrices is compared with the experimental results [3], the top-quark mass is required to have an upper bound [4] of about 90 GeV. Since the existing data [5] from the colliders puts $m_t > 89$ GeV, the Fritzsch scheme appears to be in trouble. Indeed, it is generally believed that the top-quark mass is well above the upper bound in the Fritzsch scheme, which implies that either this approach should be abandoned or at least one of the zeros in the mass matrices should be replaced by a nonzero element. We shall follow a phenomenological approach for the modification of the Fritzsch matrices.

A possible modification of these mass matrices would be to introduce an additional nonzero off-diagonal element. This interesting possibility has been investigated by Albright and Lindner [6], and they have shown that it does not lead to an appreciable increase in the upper bound for the top-quark mass. We have reinvestigated this possibility, and arrived at the same conclusion.

Another possible modification would be to introduce an additional diagonal element in the mass matrices, and it is more natural to do so for the c quark than the u, d, or s quarks. We shall develop this scheme in this paper, compare our theoretical results for the KM matrix elements with the experimental data, and obtain the allowed values for the top quark mass and the CP-violating phase. As we shall see, the mass matrices

$$\mathbf{M}^u = \begin{bmatrix} 0 & A^u & 0 \\ A^u & D^u & B^u \\ 0 & B^u & C^u \end{bmatrix}, \quad \mathbf{M}^d = \begin{bmatrix} 0 & -iA^d & 0 \\ iA^d & 0 & B^d \\ 0 & B^d & C^d \end{bmatrix}, \tag{1.1}$$

with seven real parameters, are not only consistent with the available experimental data but also allow us to accommodate a top quark with a mass well above the upper bound in the Fritzsch scheme.

An essential feature of the mass matrices (1.1) is that the diagonal elements corresponding to only the light quarks are vanishingly small, and thus our scheme recognizes a fundamental difference between the light and the heavy quarks, whereas in the Fritzsch scheme the charmed quark is treated in the same manner as the light quarks.

II. DERIVATION OF THE KM MATRIX

For the three families of quarks, we shall explore mass matrices of the form

$$\mathbf{M} = \begin{bmatrix} 0 & Ae^{i\alpha} & 0 \\ Ae^{-i\alpha} & D & Be^{i\beta} \\ 0 & Be^{-i\beta} & C \end{bmatrix}, \tag{2.1}$$

and require that the diagonal elements corresponding to the light quarks are vanishingly small, while those corresponding to the heavy quarks are approximately equal to the observed values of their masses, so that

$$D^d = 0, \quad D^u \approx m_c, \quad C^d \approx m_b, \quad C^u \approx m_t \ . \tag{2.2}$$

It should be noted that for $D^d \ll m_b$ and $D^u \ll m_t$ the mass matrix (2.1) yields $C^d \approx m_b$ and $C^u \approx m_t$, and therefore the condition (2.2) can be satisfied simply by setting $D^d = 0$ and $D^u \approx m_c$.

The mass matrix (2.1) is expressible as

$$\mathbf{M} = \mathbf{P}\overline{\mathbf{M}}\mathbf{P}^\dagger \ , \tag{2.3}$$

where

$$\overline{\mathbf{M}} = \begin{bmatrix} 0 & A & 0 \\ A & D & B \\ 0 & B & C \end{bmatrix}, \quad \mathbf{P} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{-i\alpha} & 0 \\ 0 & 0 & e^{-i(\alpha+\beta)} \end{bmatrix}, \tag{2.4}$$

and $\overline{\mathbf{M}}$ can be diagonalized by means of the transformation

$$\overline{\mathbf{M}}=\mathbf{O}\mathbf{M}_{\mathrm{diag}}\mathbf{O}^{\dagger} \tag{2.5}$$

with

$$\mathbf{M}_{\mathrm{diag}}=\begin{bmatrix} m_1 & 0 & 0 \\ 0 & -m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} . \tag{2.6}$$

It then follows that

$$\begin{aligned} &m_1-m_2+m_3=C+D , \\ &m_1m_3-m_1m_2-m_2m_3=CD-A^2-B^2 , \\ &m_1m_2m_3=A^2C , \end{aligned} \tag{2.7}$$

or

$$\begin{aligned} A&=\left[\frac{m_1m_2m_3}{m_1-m_2+m_3-D}\right]^{1/2} , \\ B&=\left[-\frac{m_1m_2m_3}{m_1-m_2+m_3-D}+CD+m_1m_2+m_2m_3-m_1m_3\right]^{1/2} , \\ C&=m_1-m_2+m_3-D . \end{aligned} \tag{2.8}$$

Furthermore, upon retaining only the leading powers of m_1/m_2, m_1/m_3, m_2/m_3, and D/m_3, in view of our knowledge of the quark masses [7], $\mathbf{O}$ is found to be

$$\mathbf{O}=\begin{bmatrix} 1 & -\left[\frac{m_1}{m_2}\right]^{1/2} & \left[\frac{m_1}{m_3}\right]^{1/2}\frac{[m_2(m_2+D)]^{1/2}}{m_3} \\ \left[\frac{m_1}{m_2}\right]^{1/2} & 1 & \left[\frac{m_2+D}{m_3}\right]^{1/2} \\ -\left[\frac{m_1(m_2+D)}{m_2m_3}\right]^{1/2} & -\left[\frac{m_2+D}{m_3}\right]^{1/2} & 1 \end{bmatrix} . \tag{2.9}$$

According to (2.3) and (2.5),

$$\mathbf{M}=\mathbf{R}\mathbf{M}_{\mathrm{diag}}\mathbf{R}^{\dagger} \text{ with } \mathbf{R}=\mathbf{P}\mathbf{O} , \tag{2.10}$$

which yields the KM matrix

$$\mathbf{V}=\mathbf{R}^{u\dagger}\mathbf{R}^{d}=\mathbf{O}^{u\dagger}\mathbf{P}^{ud}\mathbf{O}^{d} , \tag{2.11}$$

where

$$\mathbf{P}^{ud}=\mathbf{P}^{u\dagger}\mathbf{P}^{d}=\begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{i\phi_1} & 0 \\ 0 & 0 & e^{i\phi_2} \end{bmatrix} , \tag{2.12}$$

and

$$\phi_1=\alpha^u-\alpha^d , \quad \phi_2-\phi_1=\beta^u-\beta^d . \tag{2.13}$$

The matrix $\mathbf{V}$, obtained by the substitution of (2.9) and (2.12) into (2.11), is

$$V=\begin{bmatrix} 1 & -b_0+a_0e^{i\phi_1} & b_0c_0^3+a_0c_0e^{i\phi_1}-a_0\lambda d_0e^{i\phi_2} \\ -a_0+b_0e^{i\phi_1} & e^{i\phi_1} & c_0e^{i\phi_1}-\lambda d_0e^{i\phi_2} \\ a_0\lambda d_0^3+b_0\lambda d_0e^{i\phi_1}-b_0c_0e^{i\phi_2} & \lambda d_0e^{i\phi_1}-c_0e^{i\phi_2} & e^{i\phi_2} \end{bmatrix} , \tag{2.14}$$

where

$$a_0=(m_u/m_c)^{1/2} , \quad b_0=(m_d/m_s)^{1/2} , \quad c_0=(m_s/m_b)^{1/2} , \quad d_0=(m_c/m_t)^{1/2} , \tag{2.15}$$

and, in view of (2.2), λ is approximately given by

$$\lambda=\left[\frac{m_c+D^u}{m_c}\right]^{1/2}=\sqrt{2}\ . \tag{2.16}$$

In order to relate the elements of (2.14) to experiments, it is customary to take the standard parametrization of the KM matrix [3]:

$$V=\begin{bmatrix} c_{12}c_{13} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12}-c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23}-s_{12}s_{23}s_{13}e^{i\delta} & c_{13}s_{23} \\ s_{12}s_{23}-c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23}-s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}\end{bmatrix}, \tag{2.17}$$

where $c_{ij}=\cos\theta_{ij}$ and $s_{ij}=\sin\theta_{ij}$, and θ_{12} is essentially the Cabibbo angle. Since θ_{13} and θ_{23} are known to be quite small, (2.17) can be approximated as

$$\mathbf{V}=\begin{bmatrix} c_{12} & s_{12} & s_{13}e^{-i\delta} \\ -s_{12} & c_{12} & s_{23} \\ s_{12}s_{23}-s_{13}e^{i\delta} & -s_{23} & 1\end{bmatrix}, \tag{2.18}$$

in which only two elements, containing the *CP*-violating phase, are complex.

It is possible to transform (2.14) into a similar form by the successive rotations of the quark fields

$$\begin{aligned} &c\to ce^{i\phi_1}\ ,\quad t\to te^{i\phi_2}\ ,\\ &u\to ue^{i\theta}\ ,\quad d\to de^{i\theta}\ ,\\ &b\to be^{i\chi}\ ,\quad t\to te^{i\chi}\ ,\end{aligned} \tag{2.19}$$

where

$$\begin{aligned}\theta&=\arctan\left[\frac{\sin\phi_1}{\cos\phi_1-b_0/a_0}\right],\\ \chi&=\arctan\left[\frac{\sin(\phi_2-\phi_1)}{\cos(\phi_2-\phi_1)-c_0/\lambda d_0}\right].\end{aligned} \tag{2.20}$$

We thus obtain

$$\mathbf{V}=\begin{bmatrix} 1-\frac{1}{2}\eta_1^2 & \eta_1 & \eta_3 \\ -\eta_1 & 1-\frac{1}{2}\eta_1^2 & \eta_2 \\ \eta_4 & -\eta_2 & 1\end{bmatrix}, \tag{2.21}$$

where

$$V_{us}=-V_{cd}=\eta_1=\sqrt{a_0^2+b_0^2-2a_0b_0\cos\phi_1}\ , \tag{2.22}$$

$$V_{cb}=-V_{ts}=\eta_2=\sqrt{c_0^2+\lambda^2d_0^2-2c_0\lambda d_0\cos(\phi_2-\phi_1)}\ , \tag{2.23}$$

$$V_{ub}=\eta_3=(b_0c_0^3+a_0c_0e^{i\phi_1}-a_0\lambda d_0e^{i\phi_2})e^{i(\chi-\theta)}\ , \tag{2.24}$$

$$V_{td}=\eta_4=(-b_0c_0+a_0\lambda d_0^3e^{i\phi_2}+b_0\lambda d_0e^{i(\phi_1-\phi_2)})e^{-i(\chi-\theta)}\ , \tag{2.25}$$

and we have added the next-to-leading terms in the diagonal elements by using the unitary condition.

III. TOP-QUARK MASS AND *CP*-VIOLATING PHASE

We have compared our theoretical results with the experimental data [3] for various values of the top quark mass at and above its experimental lower bound, and we have taken for the masses of the other quarks the commonly used values [7]

$$\begin{aligned} &m_u/m_c=0.0038\pm0.0012\ ,\\ &m_d/m_s=0.051\pm0.004\ ,\\ &m_s/m_b=0.033\pm0.011\ ,\\ &m_c(1\ \text{GeV})=1.35\pm0.05\ \text{GeV}\ ,\end{aligned} \tag{3.1}$$

Our results are shown in Tables I and II.

In Table I, the phases ϕ_1 and ϕ_2 were determined by allowing them to vary so as to achieve the closest agree-

TABLE I. Comparison of theoretical and experimental results for the KM matrix elements. The values of m_t^{phys} given in the table correspond to $m_t(1\ \text{GeV})=150$, 200, 250, and 300 GeV.

	m_t^{phys} (GeV) 90	117	143	170	Expt.
$\lvert V_{us}\rvert$, $\lvert V_{cd}\rvert$	0.2210	0.2210	0.2210	0.2215	0.218–0.224
$\lvert V_{cb}\rvert$, $\lvert V_{ts}\rvert$	0.0447	0.0454	0.0481	0.0571	0.030–0.058
$\lvert V_{ub}\rvert$	0.0035	0.0033	0.0033	0.0032	0.001–0.007
$\lvert V_{td}\rvert$	0.0101	0.0103	0.0107	0.0126	0.003–0.019
δ	105°	102°	97°	96°	
ϕ_1	77°	78°	80°	84°	
ϕ_2	82°	80°	81°	84°	

TABLE II. Comparison of theoretical and experimental results for the KM matrix elements with $\phi_1=\phi_2=90°$.

	$m_t^{\rm phys}$ (GeV) 90	117	143	170	Expt.
$\lvert V_{us}\rvert$, $\lvert V_{cd}\rvert$	0.2231	0.2231	0.2231	0.2231	0.218–0.224
$\lvert V_{cb}\rvert$, $\lvert V_{ts}\rvert$	0.0452	0.0459	0.0481	0.0571	0.030–0.058
$\lvert V_{ub}\rvert$	0.0027	0.0026	0.0026	0.0031	0.001–0.007
$\lvert V_{td}\rvert$	0.0098	0.0100	0.0104	0.0124	0.003–0.019
δ	105°	98°	94°	91°	

ment between the theoretical results and the experimentally determined central values of the magnitudes of the KM matrix elements. For this purpose, a nonlinear regression computer program was used. We found, however, that the theoretical results are not very sensitive to small variations in the values of ϕ_1 and ϕ_2 and they remain within experimental bounds of the matrix elements when ϕ_1 and ϕ_2 are increased to 90°. Since we would expect the basic parameters ϕ_1 and ϕ_2 to have simple values, we have put $\phi_1=\phi_2=90°$ in Table II.

Our theoretical upper bound for the top quark mass is

$$m_t^{\rm phys} \leq 170 \text{ GeV} , \tag{3.2}$$

which is obtained from $m_t(1 \text{ GeV})=300\text{GeV}$ by the same transformations as used by Albright *et al.*[8] in the Fritzsch scheme. The *CP*-violating phase δ, obtained from the matrix element η_3 in (2.21), is in the range

$$91° \leq \delta \leq 105° , \tag{3.3}$$

the smaller values corresponding to the larger values of the top quark mass. Our results also strongly suggest that $\phi_1=\phi_2=90°$, which implies that we can put

$$\alpha^u=0 , \quad \alpha^d=-\pi/2 , \quad \beta^u=\beta^d=0 , \tag{3.4}$$

and the mass matrices (2.1) reduce to the simpler form (1.1) with seven real parameters. They appear to be the simplest mass matrices consistent with the available experimental data.

Our phenomenological results should prove helpful in constructing models of the Higgs-boson couplings of quarks, which are currently of much experimental and theoretical interest.

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