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Perturbative QCD, a universal QCD scale, long-range spin-orbit potential, and the properties of heavy quarkonia

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A modified version of Richardson's potential is used to calculate the energies, fine-structure splittings, leptonic widths, and dipole transition rates of charmonium and the Υ system. The effects of the perturbative color-magnetic (spin-dependent) potentials are included to the full radiative one-loop level. The question of the consistency of the data with a universal QCD scale and its expression in the central and spin-dependent potentials is addressed.

I. INTRODUCTION

Since quantum chromodynamics (QCD) is one of the cornerstones of the standard model, it is imperative that its predictions receive careful experimental scrutiny in a wide variety of experimental contexts. Because of the special role played in the theory by the hypothesis of asymptotic freedom, it has been necessary to carry out precision tests of the theory at high-energy scales [1], where perturbative QCD is expected to be valid. Recently, Kwong *et al.* [2] have shown that it is feasible to extract precise information about the QCD coupling constant α_s at heavy-quark-mass scales by a careful examination of the gluonic and leptonic decays of heavy-quarkonium states. To implement this program it is necessary to assume that the hadronic and leptonic decay widths can be factorized into a nonperturbative part, whose effects are adequately considered by a potential model that includes the effects of a confining potential, and a perturbative part. However, the controversy [3] surrounding the recent results of the European Muon Collaboration for polarized deep-inelastic scattering has called into question the validity of perturbative QCD at low-energy scales. Thus, it is a pressing concern to explore the possibility of additional tests of perturbative QCD at low-energy scales.

Heavy-quarkonium systems afford an opportunity to develop further tests of QCD at energy scales somewhat lower than those appropriate for heavy-quarkonium annihilations. The nature of the spectra quite naturally suggests a division into a hierarchy of fine structure, hyperfine structure, and gross structure, in analogy with the spectrum of positronium [4,5]. To apply perturbative QCD to these systems, one needs a means of separating the perturbative effects from the nonperturbative effects. The hypothesis of electric confinement [6] provides one context for this separation. As initially formulated [6], this hypothesis held that color-electric effects alone were responsible for the confining potential of heavy-quark–antiquark systems. The effects of the color-magnetic degrees of freedom were assumed to be short ranged and thus amenable to treatment by perturbation theory. However, at least one of the spin-orbit potentials should have a nonperturbative component because, as Buchmüller pointed out [7], a rotating tube of color-electric flux in the center-of-mass frame would also have color-magnetic properties. In two recent papers [8,9], I have shown how one can use Gromes's consistency condition [10] to determine the nonperturbative spin-orbit potential in a manner that does not make any *ad hoc* assumptions about the Lorentz nature of the confining potential. A straightforward consequence of this application of Gromes's condition is that one expects destructive interference between the nonperturbative spin-orbit potential and the leading perturbative contribution to this potential. Thus, one expects a value of the fine-structure ratio r that is smaller than 0.8, a universal value for P states that can be derived from the one-gluon-exchange (OGE) potential. Observations of the fine-structure ratio yield $r \simeq 0.60-0.70$ for the $1P$ and $2P$ states of the Υ system [1,11,12] and $r \simeq 0.40-0.50$ for the $1P$ states of charmonium. All of these values are below the OGE value, which is experimental support for the destructive interference between perturbative and nonperturbative spin-orbit effects [8,9].

There has been a steady march of progress in the attempt to establish connections between both the spin-independent and spin-dependent portions [6] of the quarkonium potential with the underlying theory of QCD. An early calculation of hadron masses by De Rújula, Georgi, and Glashow [13] emphasized the role of OGE in the explanation of the fine structure and hyperfine structure. The potential of Eichten *et al.* [14] gave an increased precision in accounting for the locations of the heavy-quarkonium levels. Another important advance occurred when Gupta, Radford, and Repko [15] extended the QCD perturbative calculation of both the central potential and the spin-dependent potentials to the full radiative one-loop level. Of course, it was necessary to supplement their perturbative potentials with a linear confining potential to obtain agreement with the measured spectra. Pantaleone, Tye, and Ng [16] pointed out that using the radiative one-loop expressions for the perturbative spin-dependent potentials allowed one to determine the QCD scale parameter Λ, which should be a universal value, although the value of the coupling constant α_s might exhibit flavor dependence.

In this paper we will argue that recent improvements in the precision of the measured fine-structure splittings of the Υ system [1,11,12] and a better understanding of the implications of Gromes's consistency condition [10] provide a fresh opportunity to address questions of the expression of a universal QCD scale in the spectra and leptonic widths of charmonium and the Υ system. Input from recent lattice gauge calculations [17,18] of the long-range behavior of the spin-dependent parts of the quark–antiquark potential will also be important. In previous report [9], I have established that a modified form of Richardson's potential [19,20] leads to superb agreement with the measured values of the leptonic widths, level spacings, and fine-structure splittings of both charmonium and the Υ system. However, the results for the charmonium hyperfine splittings were 50–70 % too large. Since the perturbative magnetic potentials of the previous calculations were determined from OGE, it is natural to ask whether extending the magnetic part of the calculation to the next level of perturbation theory, the full radiative one-loop level, will remove this discrepancy. The contributions to the perturbative spin-dependent potentials from the eight Feynman graphs of this order of perturbation theory have been evaluated by Gupta and Radford [15].

Some of the goals of our present calculation can be summarized as follows: (1) to emphasize the importance of including the nonperturbative spin-orbit potential to obtain reasonable fine-structure ratios; (2) to show that extending the perturbative magnetic potentials to the full radiative one-loop level leads to better agreement with experiment than the OGE potentials; (3) to explore whether it is reasonable to view the scale parameter Λ of Richardson's potential as the QCD scale parameter of the perturbative magnetic potentials. Are the heavy-quarkonium data [16] consistent with a universal QCD scale Λ? Finally, I would like to note three other important calculations [21–23] of the properties of heavy-quarkonium systems.

II. RICHARDSON'S POTENTIAL

Most of the successful calculations of the properties of heavy quarkonium [8,9,15,16,20–24] have some mechanism for softening the QCD coupling constant at small distances, as required by the hypothesis of asymptotic freedom. A careful analysis of all the one-loop vacuum-polarization bubble graphs [25] leads to the usual logarithmic dependence of the running coupling constant on momentum transfer, that is,

$$\alpha_s(|q^2|)=\frac{12\pi}{(33-2n_f)\ln(|q^2|/\Lambda^2)} , \tag{2.1}$$

where n_f is the number of quark degrees of freedom. To apply Eq. (2.1) to heavy-quark spectroscopy one has to have some strategy for dealing with the singularity at $|q^2|=\Lambda^2$. Richardson [19] showed that the replacement

$$\frac{|q^2|}{\Lambda^2}\rightarrow 1+\frac{|q^2|}{\Lambda^2} \tag{2.2}$$

leads to a potential that does a satisfactory job of predicting the locations of the spin-averaged heavy-quarkonium levels. However, since Richardson's substitution prescribes a value of $8\pi\Lambda^2/(33-2n_f)$ for the string constant, it is very restrictive. Moreover, the relationship of Λ to the small-distance behavior of QCD is blurred because the string constant is mostly determined from the large-distance behavior of the confining potential.

Moxhay and Rosner's [20] proposal provides a way out of this dilemma. These authors suggested treating the coefficient of the linear term as a parameter, and thus their modified form of Richardson's potential is given by

$$V(r)=Ar-\frac{8\pi}{(33-2n_f)r}f(\Lambda r) , \tag{2.3}$$

where

$$f(t)=\frac{4}{\pi}\int_0^\infty \frac{\sin tx}{x}\left[\frac{1}{\ln(1+x^2)}-\frac{1}{x^2}\right]dx . \tag{2.4}$$

From the viewpoint of our calculation, the real advantage of Eq. (2.3) is that the scale parameter Λ is exclusively associated with the short-range behavior of the potential, and thus it is reasonable to preserve the interpretation of Λ as the scale parameter of perturbative QCD. The separation of long- and short-range behaviors embodied in Eq. (2.3) will allow us to ask questions about the relationship of the scale parameter Λ of the central potential to the coupling constant α_s and the renormalization scale μ associated with the spin-dependent potentials. In particular, do these three parameters satisfy the QCD relation

$$\Lambda=\mu\exp\left[\frac{-6\pi}{(33-2n_f)\alpha_s}\right] ? \tag{2.5}$$

To state the challenge faced by perturbative QCD in heavy-quarkonium systems more explicitly, can one reconcile the set of parameters α_s and μ that accounts for all the fine-structure and hyperfine splittings in the Υ system with the set that accounts for these splittings in charmonium, as demanded by Eq. (2.5)? After a value for Λ has been determined from the magnetic potentials underlying the fine structure and hyperfine splittings, one can ask if this value is consistent with the scale parameter Λ in the short-range part of Richardson's potential.

III. POTENTIAL MODEL

Our calculation is based on a Hamiltonian that describes the interaction of a heavy-quark–antiquark ($Q\bar{Q}$) pair in mutual orbit about its center of mass [9] through a central potential $E(r)$ and a spin-dependent potential $V_{\rm SD}(r)$, that is,

$$H=K+E(r)+V_{\rm SD}(r) . \tag{3.1}$$

In Eq. (3.1) the kinetic energy operator includes the leading relativistic correction, and the central potential includes spin-independent relativistic corrections ($V_{\rm SI}$) as well as the modified Richardson's potential of Eq. (2.3). The defining equations for the unperturbed problem,

$$H_0=2m+p^2/m+V(r), \quad H_0\Psi_0=E_0\Psi_0 , \tag{3.2}$$

are used to determine the unperturbed wave functions Ψ_0 and the unperturbed energies E_0 by numerically solving Schrödinger's equation [26] with the potential of Eq. (2.3).

Before comparing with experiment, we must consider the effects of the perturbation:

$$H' = -p^4/4m^3 + V_{\rm SI}(r) + V_{\rm SD}(r) \ . \tag{3.3}$$

The form of the spin-independent relativistic corrections [27] in Eq. (3.3) is the same as that of Ref. [9]. Thus, our energies and fine-structure and hyperfine-structure splittings are determined from the first-order perturbation theory expression

$$E(nLJ) = E_0(nL) + \langle JMLSn|H'|JMLSn\rangle \ . \tag{3.4}$$

The perturbation theory formalism affords a means of estimating how well we could reasonably expect our calculated results to agree with experiments. Surely, it is not reasonable to expect agreement from first-order perturbation theory that exceeds a bound established by the magnitudes of the corrections of second-order perturbation theory. Using the measured fine-structure splittings as a means of assessing the strength of the matrix elements and the spacing between levels as a means of estimating the energy denominators, we can make a rough estimate of the size of the second-order corrections. Hence,

$$\begin{aligned} \Delta E_b^{(2)} &\simeq (50\ \text{MeV})^2/500\ \text{MeV} = 5\ \text{MeV} \ , \\ \Delta E_c^{(2)} &\simeq (100\ \text{MeV})^2/500\ \text{MeV} = 20\ \text{MeV} \ , \end{aligned} \tag{3.5}$$

where the first result pertains to the Υ system and the second result to charmonium. Although the approximations used to obtain these two numbers are very crude, they are nevertheless useful as rough guides to the kind of accuracy we can expect.

Using an expansion in inverse powers of the mass to evaluate relativistic corrections, Eichten and Feinberg [6] derived a general expression for the spin-dependent potential

$$\begin{aligned} V_{\rm SD} = {} & \frac{\mathbf{L}\cdot\mathbf{S}}{m^2 r}\left[\frac{1}{2}\frac{dV}{dr} + \frac{dV_1}{dr} + \frac{dV_2}{dr}\right] + \frac{\mathbf{S}_1\cdot\mathbf{S}_2}{3m^2}V_4 \\ & + \frac{1}{m^2}\left[\hat{\mathbf{r}}\cdot\mathbf{S}_1\hat{\mathbf{r}}\cdot\mathbf{S}_2 - \tfrac{1}{3}\mathbf{S}_1\cdot\mathbf{S}_2\right]V_3 \ , \end{aligned} \tag{3.6}$$

where $\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2$, V_1 and V_2 are the spin-orbit potentials, and V_3 and V_4 carry the radial dependence of the spin-spin and tensor potentials. By evaluating the expectation values of the bilinear combinations of color-electric and -magnetic fields derived by Eichten and Feinberg for the potentials V_i on a lattice, Michael and Campostrini *et al.* [17,18] have found that only the potential V_1 exhibits long-range, or nonperturbative behavior. Thus, it is *consistent with the results of these lattice gauge calculations to assume that perturbative QCD suffices to determine the remaining potentials* $V_2 - V_4$. Furthermore, we can simultaneously satisfy the requirements of Gromes's consistency condition [10] if we require

$$V_1(r) = V_2(r) - V(r) \ . \tag{3.7}$$

Substituting Eq. (3.7) into the first term of Eq. (3.6) yields an important result for the spin-orbit potential, that is,

$$V_{\rm SO} = \frac{\mathbf{L}\cdot\mathbf{S}}{m^2 r}\left[2\frac{dV_2}{dr} - \frac{1}{2}\frac{dV}{dr}\right] \ . \tag{3.8}$$

I have already discussed in some detail [8,9] how the minus sign of Eq. (3.8) is responsible for the destructive interference that reduces the fine-structure ratio r below 0.8, the OGE value.

Keeping only terms through the second order in α_s in the perturbative spin-dependent potentials [15], we have for the spin-spin, the tensor, and the spin-orbit potentials:

$$V_{\rm SS}(r) = \frac{32\pi\alpha_s}{9m^2}\mathbf{S}_1\cdot\mathbf{S}_2\left[\left[1 - \frac{\alpha_s}{12\pi}(26 + 9\ln 2)\right]\delta(\mathbf{r}) - \frac{\alpha_s}{24\pi^2}(33 - 2n_f)\nabla^2\left[\frac{\ln\mu r + \gamma_E}{r}\right] + \frac{21\alpha_s}{16\pi^2}\nabla^2\left[\frac{\ln mr + \gamma_E}{r}\right]\right] \ , \tag{3.9}$$

$$V_{\rm T}(r) = \frac{4\alpha_S^2}{m^2}\left[\frac{\mathbf{S}_1\cdot\hat{\mathbf{r}}\mathbf{S}_2\cdot\hat{\mathbf{r}} - \frac{1}{3}\mathbf{S}_1\cdot\mathbf{S}_2}{r^3}\right]\left[1 + \frac{4\alpha_s}{3\pi} + \frac{\alpha_s(33 - 2n_f)}{6\pi}(\gamma_E + \ln\mu r - \tfrac{4}{3}) - \frac{3\alpha_s}{\pi}(\gamma_E + \ln mr - \tfrac{4}{3})\right] \ , \tag{3.10}$$

$$V_{\rm SO}(r) = \frac{\mathbf{L}\cdot\mathbf{S}}{m^2}\left\{\frac{8\alpha_s}{3r^3}\left[1 + \frac{\alpha_s}{\pi}\left[\frac{33 - 2n_f}{6}(\ln\mu r + \gamma_E - 1) - \tfrac{5}{6} - \tfrac{3}{2}(\ln mr + \gamma_E - 1)\right]\right] - \frac{1}{2r}\frac{dV}{dr}\right\} \ , \tag{3.11}$$

where $\gamma_E = 0.5772\cdots$ is Euler's constant and μ is the renormalization scale. The coupling constant of Eqs. (3.9)–(3.11) is defined in the momentum-space subtraction scheme of Gupta and Radford [15]. It is related to the coupling constant defined in the modified-minimal-substraction scheme used by Pantaleone, Tye, and Ng [16] by the relation

$$(\alpha_s)_{\rm GRR} = \bar{\alpha}_s\left[1 + \frac{\bar{\alpha}_s}{4\pi}\left[6 + \frac{31}{3} - \frac{10}{9}n_f\right]\right] \ . \tag{3.12}$$

We have verified that the potentials of Refs. [15] and [16] agree with each other when allowance is made for Eq. (3.12).

IV. RESULTS

Our calculation for the energies, leptonic widths, and dipole transition rates of the Υ system requires values for the parameters Λ, A, m, α_s, and μ. First, we select values of α_s and μ that give fine-structure splittings reasonably close to those of the OGE calculation [9]. Then we determine the values of Λ and A by fitting the differences between the centers of gravity of the $1P$ and

TABLE I. Energies of the low-lying states of the Υ system (MeV). The parameters used for the present calculation are $\Lambda=0.388$ GeV, $m=4.8855$ GeV, $\mu=3.25$ GeV, $\alpha_s=0.33$, $A=0.164$ GeV^2.

State	GRS (Ref. [24])	Fulcher (Ref. [9])	Fulcher (present work)	Expt.
$1\,^3S_1(\Upsilon)$	9460	9461	9459	9460.3 ± 0.2[a]
$1\,^1S_0(\eta_b)$	9412	9369	9413	
$2\,^3S_1$	10016	10019	10015	10023.3 ± 0.3
$2\,^1S_0$	9993	9975	9992	
$3\,^3S_1$	10358	10357	10356	10355.3 ± 0.5
$3\,^1S_0$	10340	10324	10338	
$1\,^3P_2(\chi_b)$	9914	9912	9911	9913.2 ± 0.6
$1\,^3P_1$	9893	9893	9893	9891.9 ± 0.7
$1\,^3P_0$	9862	9865	9865	9859.8 ± 1.3
$1\,^1P_1(h_b)$	9900	9900	9900	
r_{1P}	0.65	0.67	0.65	0.66
$2\,^3P_2$	10270	10270	10269	10269.0 ± 0.7
$2\,^3P_1$	10254	10254	10256	10255.2 ± 0.4
$2\,^3P_0$	10229	10232	10234	10235.3 ± 1.1
$2\,^1P_1$	10259	10261	10261	
r_{2P}	0.65	0.70	0.63	0.69
$1\,^3D_3$	10163	10172	10172	
$1\,^3D_2$	10153	10169	10166	
$1\,^3D_1$	10141	10163	10158	
$1\,^1D_2$	10154	10169	10167	
r_{1D}	0.96	0.55	0.77	

[a] Particle Data Group (Ref. [1]).

$2P$ states and the $1\,^3S_1$ state [28]. Then the values of α_s and μ are fine-tuned to give the best fit to the fine-structure splittings. Finally the constituent mass m is adjusted to fit the measured $1\,^3S_1$ mass at 9460 MeV. Our results for the energies of the Υ system are listed in Table I, where they are compared with experiment, with my earlier calculation of Ref. [9], and with the recent calculation of Gupta, Repko, and Suchyta [24]. It is worth noting that the results of Table I did not require the addition of an arbitrary constant to the central potential. Agreement with experiment is excellent.

The values of the parameters μ (3.25 GeV) and α_s (0.33) require further comment. If perturbative QCD is valid, then these parameters should be related to the QCD scale parameter by Eq. (2.5). However, the appropriate value of n_f to use there is uncertain. As the derivation in Griffiths's text [25] makes clear, this quantity measures the number of different kinds of quark–antiquark pairs that contribute to vacuum-polarization effects underlying the running coupling constant. The question is how many of these contribute to a given process, which in this case is the magnetic scattering of a heavy-quark–antiquark pair in a mutual orbit. One viewpoint is that the value of n_f appropriate for magnetic scattering should be the same as that determined from the short-range behavior of the running coupling constant in the central potential. Under this assumption, $n_f=3$ and Eq. (2.5) yields $(\Lambda)_{\text{fine structure}}=0.392$ GeV, which is essentially the same value, $\Lambda=0.388$ GeV, used in the central potential. The attractive feature of this viewpoint is that it allows us to *fit all of the Υ data with a single QCD scale parameter*, which is significant progress towards establishing the kind of behavior expected from perturbative QCD. An alternative viewpoint will be discussed below.

The results of Table I suggest a good experimental test of the universal scale hypothesis. The hyperfine splittings are very sensitive to the value of the renormalization scale μ after the value of α_s is fixed from the fine-structure splittings. Thus, the measured values of the hyperfine splittings will allow an additional determination of the QCD scale parameter. Will this value prove to be near 0.390 GeV? Hence, the measurements of the Υ hyperfine splittings should lead to a crucial test of the applicability of perturbative QCD in the Υ system.

Our results for the Υ leptonic widths are presented in Table II, where they are compared with experiment and the results of Gupta, Repko, and Suchyta [24]. These transition rates were obtained from the leptonic width formula [2,5]

$$\Gamma_{ee}=\frac{4\alpha^2 e_Q^2}{M^2(Q\bar{Q})}|R(0)|^2\left[1-\frac{16\alpha_s}{3\pi}\right], \tag{4.1}$$

where α is the fine-structure constant, e_Q is the quark charge, and M denotes the mass of the initial state. The factor in large parentheses in Eq. (4.1) arises because of radiative QCD corrections. Since Eq. (4.1) describes an

TABLE II. Leptonic widths of the low-lying S states of the Υ system (keV).

State	GRS (Ref. [24])	Fulcher (present work)	Expt.
$\Upsilon(1S)$	1.21	1.31	1.34 ± 0.05[a]
$\Upsilon(2S)$	0.55	0.57	0.60 ± 0.04
$\Upsilon(3S)$	0.41	0.41	0.44 ± 0.03

[a] Particle Data Group (Ref. [1]).

annihilation process, the value of momentum transfer in the argument of α_s there should be substantially larger than that appropriate for the magnetic scattering potentials of Eqs. (3.9)–(3.11). Thus, we expect the value of α_s in Eq. (4.1) to be somewhat smaller than that used in Table I. Thus we choose $\alpha_s=0.18$ from Ref. [2], a value obtained from heavy-quarkonium decays. The good agreement of our leptonic widths with experiment is solid support for the values of our radial wave functions at the origin $R(0)$. The agreement of our results for the $2\,^3S\rightarrow1\,^3P$, $3\,^3S\rightarrow1\,^3P$, and the $3\,^3S\rightarrow2\,^3P$ dipole transition rates with experiment is also very good. These results do not differ substantially from those presented in Table IV of Ref. [9] and will not be repeated here.

Our results for the energies and leptonic widths of charmonium are presented in Tables III and IV. These results are based on a value of $\Lambda=0.388$ GeV, the same value used for the Υ system. However, it was necessary to introduce some flavor dependence into the central potential to achieve high-quality agreement with the data. For example, the charmonium value of A was about 20% larger than the Υ value, and it was necessary to add an arbitrary constant to the central potential. A substantial increase of α_s was necessary to give good results for the charmonium fine-structure splittings. From Table III, it is clear that the agreement of the hyperfine splittings of the present calculation with experiment is much better than those of the OGE calculation of Ref. [9]. Thus, we conclude that *the full radiative one-loop expressions* of Eqs. (3.9)–(3.11), where the coupling constant softens at short distances in accord with asymptotic freedom, *offer a significant advantage over the lowest-order perturbative expressions of OGE* in simultaneously accounting for the fine-structure splittings and the hyperfine-structure splittings of both heavy-quarkonium systems.

The hyperfine splittings of the P states listed in Tables I and III are very small. The reason for this is easy to determine from the spin-spin potential of Eq. (3.9). There the contribution of the δ function vanishes and one can evaluate explicitly the action of the ∇^2 operators on the terms in square brackets. This yields

$$\langle V_{\mathrm{SS}}\rangle_{1\neq0}=\frac{\alpha_s^2}{\pi m^2}\left[\frac{2}{9}-\frac{8n_f}{27}\right]\langle r^{-3}\rangle \mathbf{S}_1\cdot\mathbf{S}_2\;, \tag{4.2}$$

which gives a small positive shift of about 0.4–0.6 MeV

TABLE III. Energies of the low-lying states of the charmonium system (MeV). The parameters used in the present calculation are $\Lambda=0.388$ GeV, $m=1.30$ GeV, $\mu=1.22$ GeV, $\alpha_s=0.54$, $A=0.195$ GeV^2.

State	GRS (Ref. [24])	Fulcher (Ref. [9])	Fulcher (present work)	Expt.
$1\,^3S_1(J/\psi)$	3097	3125	3104	3096.9 ± 0.1[a]
$1\,^1S_0(\eta_c)$	2981	2921	2987	2979.6 ± 1.7
$2\,^3S_1$	3690	3685	3670	3686.0 ± 0.4
$2\,^1S_0$	3619	3546	3584	3594.0[b]
$1\,^3P_2(\chi_c)$	3554	3561	3557	3556.3 ± 0.4
$1\,^3P_1$	3507	3506	3513	3510.6 ± 0.5
$1\,^3P_0$	3412	3407	3404	3415.1 ± 1.0
$1\,^1P_1(h_c)$	3518	3525	3529	
r_{1P}	0.49	0.56	0.40	0.48
$1\,^3D_3$		3867	3884	
$1\,^3D_2$		3872	3871	
$1\,^3D_1$		3860	3840	3770(?)
$1\,^1D_2$		3867	3872	
r_{1D}			0.41	

[a] Particle Data Group (Ref. [1]).
[b] Particle Data Group (Ref. [29]).

TABLE IV. Leptonic widths of the low-lying S states of charmonium (keV).

State	GRS (Ref. [24])	Fulcher (present work)	Expt.
$J/\psi(1S)$	5.57	5.23	4.72 ± 0.35^{a}
$\psi(2S)$	2.87	2.56	2.15 ± 0.21

[a] Particle Data Group (Ref. [1]).

for location of the single P state relative to the center of gravity of the triplet P states of the Υ system. This splitting is about 3 MeV for charmonium. The signs of these corrections and the magnitudes are in agreement with the approximate calculation of Pantaleone and Tye [30]. However, a recent determination of the hyperfine splitting by Dixit *et al.* [31] came to the conclusion that this splitting should have the opposite sign. Their work was based on a phenomenological short-range potential. Thus, the sign of the hyperfine P-state splitting may afford a good test of perturbative QCD.

It is also of interest to investigate whether the values of α_s and μ used in Tables III and IV lead to the same value of the QCD scale parameter Λ as obtained from the Υ system. Substituting the values of α_s and μ from Table III into Eq. (2.5) yields $(\Lambda)_{\text{charm mag}}=0.335$ GeV, about 14% lower than the value derived from the Υ fine structure. It is not possible to decide if this value is close enough to 0.390 GeV to support the hypothesis of a universal QCD scale without further refined study. As an example of the kind of uncertainty that enters this determination, we note that one could adopt a different viewpoint from that used above to determine $\Lambda_{\text{fine structure}}$. Setting $n_f=4$ in Eq. (2.5), which is suggested by a consideration of Υ annihilation processes [2], yields $(\Lambda')_{\Upsilon\ \text{mag}}=0.331$ GeV. In this case, the two determinations of Λ from magnetic scattering in the Υ system and charmonium are consistent but differ somewhat from the value in the central potential.

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