J/ψ decays and pseudoscalar-meson mixing

Norikazu Morisita, Ichijiro Kitamura, and Tadayuki Teshima
Natural Sciences–Physics, College of Engineering, Chubu University, Kasugai, Aichi 487, Japan
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We have examined the branching ratios of the decays $J/\psi \to VP$ and $J/\psi \to P\gamma$ in order to determine the magnitudes of components other than $N=(u\bar{u}+d\bar{d})/\sqrt{2}$ and $S=s\bar{s}$ in η and η', using the data of the world average in 1990. Although the possibility that these magnitudes are very small is not excluded, there is a solution in which these magnitudes are not small, suggesting that η and η' have glueball origins. Then, one state of two or more states in the η (1440) region is considered to be a glueball. The mixing angle θ_1 corresponding to the mixing angle θ_P between η and η' is estimated to be about $-20°$. The estimated N component in this glueball is larger than S. Using the mixing angles obtained, we have calculated the decay widths for various $P\to\gamma\gamma$, $P\to V\gamma$, and $V\to P\gamma$ decays. The results are, in general, consistent with experiments.

I. INTRODUCTION

QCD predicts the existence of the glueball. The most probable candidate for the glueball is $\eta(1440)$ [which was named $\iota(1440)$ previously]. Since this is a pseudoscalar and isoscalar meson, it can mix with the $q\bar{q}$ pseudoscalar and isoscalar mesons η and η'. On the other hand, $\eta(1440)$ can also be a partner of $\eta(1295)$ which has been established recently as the radially excited pseudoscalar meson [1]. Experimentally, the mass region near $\eta(1440)$ has a complicated aspect as there may be two or more pseudoscalar states in this region [2–5]. Then, there also is the possibility that a glueball and a radially excited state exist in this region. Therefore, it is very important to study the mixing among η, η', and $\eta(1440)$.

The value of the η and η' mixing angle has been determined as $\theta_p \sim -10°$ by the quadratic Gell-Mann–Okubo mass formula and many experimental results have advocated this value [6]. However, in the past few years, new data of $J/\psi\to\eta(\eta')\gamma$ [7] and $\eta(\eta')\to\gamma\gamma$ decays [8] and $\pi^- p$ charge-exchange reactions [9] have accumulated, which favor a mixing angle of $\theta_p \sim -20°$. From these situations it is important to determine the mixing angle between them.

Several experimental groups [10] have studied the gluonic contents in η and η' by analyzing the J/ψ decays into a vector meson V $(\rho, K^*, \omega, \phi)$ plus a pseudoscalar meson P (π, K, η, η'), $J/\psi\to VP$, with the use of their experimental data. Their results are that the gluonic contents in η and η' are very small and the mixing angle θ_P is about $-20°$. In this paper, we examine not only the decay $J/\psi\to VP$ but also the radiative decay $J/\psi\to\gamma P$ in order to determine possible glueball contents or/and radially excited meson contents in η and η'. Since J/ψ is almost a $c\bar{c}$ quarkonium, the decays to hadrons composed of noncharmed quarks are believed to proceed mainly through a virtual photon, two gluons, and three gluons, as shown in Figs. 1 and 2. We here use the data of the world average in 1990 [11].

The obtained result is as follows: though the possibility that contents other than N and S in η and η' are very small is not excluded, there is a solution in which these contents are not very small, suggesting that these have glueball origins. Of course, these may have both a glueball origin and a radially excited state. Then, one state of two or more states in the $\eta(1440)$ region is considered to be a glueball. The estimated N component in this glueball is larger than S. The obtained mixing angle θ_1 corre-

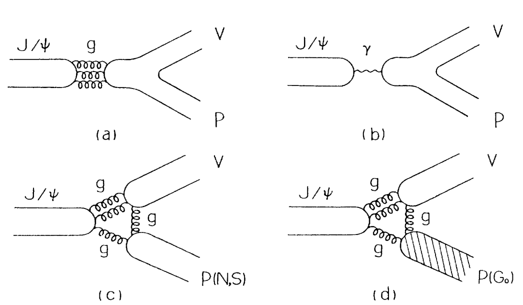

FIG. 1. Diagrams contributing to $J/\psi\to VP$ decays: (a) Three-gluon-annihilation diagram. (b) Electromagnetic diagram. (c) Double-OZI-suppression diagram connected to $q\bar{q}$ states. (d) Double-OZI-suppression diagram connected to a pure glueball state.

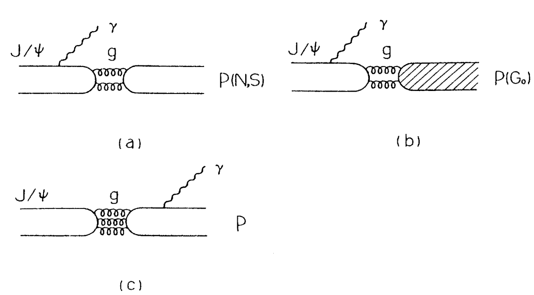

FIG. 2. Diagrams contributing to $J/\psi\to\gamma P$ decays: (a) Two-gluon-annihilation diagram connected to $q\bar{q}$ states. (b) Two-gluon-annihilation diagram connected to a pure glueball state. (c) Three-gluon-annihilation diagram.

sponding to the mixing angle θ_P between η and η' is about $-20°$. We calculated the decay widths for various $P\to V\gamma$, $V\to P\gamma$, and $P\to\gamma\gamma$ decays using the calculated mixing parameters. The results are, in general, consistent with experiment.

The paper is organized as follows. Section II is devoted to the interrelation between the branching ratios of J/ψ decays and the mixing angles of the pseudoscalar mesons P. Section III is devoted to the numerical analysis and the discussions. The Appendix is devoted to the discussions of radially excited states.

II. THE BRANCHING RATIOS AND THE MIXING ANGLES

A. $J/\psi\to VP$ decays

First, we consider the case in which there is a glueball which we call ι and there are no radially excited states. Since J/ψ meson is an almost pure $c\bar{c}$ state, its decays into V and P are the Okubo-Zweig-Iizuka- (OZI) rule-suppressed ones shown in Fig. 1(a) (a three-gluon-annihilation diagram) and Fig. 1(b) (an electromagnetic-interaction diagram). Aside from the OZI-suppression diagrams common to all hadronic J/ψ decays, the doubly disconnected diagrams shown in Figs. 1(c) and 1(d) are also expected to contribute to the J/ψ decays, where Fig. 1(c) represents the diagram connected to $q\bar{q}$ states N, S and Fig. 1(d) represents the diagram connected to a possible glueball state.

The amplitudes for the $J/\psi\to VP$ decays are expressed in terms of an SU(3)-symmetric coupling strength g [12] which is contributed from the three-gluon diagram with a small probable SU(3)-symmetry breaking s, an electromagnetic coupling strength e which is contributed from the electromagnetic interaction diagram with a small probable SU(3)-symmetry breaking s_e and another SU(3)-symmetric coupling strength which is written by g with suppression factors r and r' contributed from the doubly disconnected diagrams. The coupling strength e may have a relative phase to the strength g because these are produced from different origins. The SU(3)-symmetry-breaking term s is taken into account by pure octet-SU(3) breaking; then this is equivalent to reducing g for each strange quark in the final state. The s's of V and P are written as s_v and s_p, respectively. s_e can be determined as the ratio of the quark magnetic moment μ_s and μ_u for the strange and nonstrange quarks, as

$$1-s_e=\frac{\mu_s}{\mu_u}\simeq\frac{m_u}{m_s}\ , \tag{1}$$

where m_u and m_s are the nonstrange- and strange-quark masses.

The effective Lagrangian for the $J/\psi-V-P$ interaction without the kinematical quantities is expressed as [12]

$$L=\frac{g}{2}\mathrm{Tr}([P]\{[V],[S_v]\})+\frac{e}{2}\mathrm{Tr}([P]\{[V],[S_e]\}) +rg\,\mathrm{Tr}([P][S_p])\mathrm{Tr}([V][S_v])+r'gG_0\mathrm{Tr}([V][S_v])\ , \tag{2}$$

where $[P]$ and $[V]$ are the following matrices of fields P_i and V_i composed of $q\bar{q}$:

$$[P]=\frac{1}{\sqrt{2}}\lambda_i P_i\ ,\quad [V]=\frac{1}{\sqrt{2}}\lambda_i V_i\ . \tag{3}$$

G_0 is a pure glueball field. $[S_v]$, $[S_p]$, and $[S_e]$ are the matrices of spurious fields corresponding to the three-, two-gluon-annihilation interaction, and the electromagnetic interaction, respectively, expressed as

$$[S_v]=\frac{1}{\sqrt{3}}\begin{bmatrix}1&0&0\\0&1&0\\0&0&1-s_v\end{bmatrix}\ , \tag{4a}$$

$$[S_p]=\frac{1}{\sqrt{3}}\begin{bmatrix}1&0&0\\0&1&0\\0&0&1-s_p\end{bmatrix}\ , \tag{4b}$$

$$[S_e]=\frac{1}{3}\begin{bmatrix}2&0&0\\0&-1&0\\0&0&-(1-s_e)\end{bmatrix}\ . \tag{4c}$$

In the case where there are radially excited states, we consider the Lagrangian similar to Eq. (2) containing the interactions of the radially excited states. (See Appendix.)

Vector mesons V may be thought as the almost ideal nonet, but pseudoscalar mesons P are not ideal and the isoscalar mesons in P mix with each other. Furthermore, they can mix with the glueball or/and the radially excited contents, if they exist near the η and η' mass regions. First, we treat the case in which the glueball ι mixes with η and η' in P. In Rosner's notation, the meson states can be written as

$$\begin{aligned}\eta&=X_\eta N+Y_\eta S+Z_\eta G_0\ ,\\ \eta'&=X_{\eta'}N+Y_{\eta'}S+Z_{\eta'}G_0\ ,\\ \iota&=X_\iota N+Y_\iota S+Z_\iota G_0\ ,\end{aligned} \tag{5}$$

where the basis states are

$$N=(u\bar{u}+d\bar{d})/\sqrt{2},\quad S=s\bar{s},\quad G_0=\text{pure glueball state}\ .$$

We introduce the mixing angles θ_1, θ_2, and θ_3 on the basis η_8, η_0, and G_0 as

$$\begin{aligned}\eta&=\alpha_8\eta_8+\alpha_0\eta_0+\alpha_G G_0\ ,\\ \eta'&=\beta_8\eta_8+\beta_0\eta_0+\beta_G G_0\ ,\\ \iota&=\gamma_8\eta_8+\gamma_0\eta_0+\gamma_G G_0\ ,\end{aligned} \tag{6}$$

where

$$\alpha_8=c_1c_2\ ,\quad \alpha_0=-c_1s_2s_3-s_1c_3\ ,\quad \alpha_G=-c_1s_2c_3+s_1s_3\ ,$$

$$\beta_8=s_1c_2\ ,\quad \beta_0=-s_1s_2s_3+c_1c_3\ ,\quad \beta_G=-s_1s_2c_3-c_1s_3\ ,$$
$$\gamma_8=s_2\ ,\quad \gamma_0=c_2s_3\ ,\quad \gamma_G=c_2c_3\ , \tag{7}$$
$$c_i\equiv\cos\theta_i\ ,\quad s_i\equiv\sin\theta_i\quad (i=1,2,3)\ .$$

If θ_2 and θ_3 are 0, then ι is disconnected from (η,η') and θ_1 is reduced to an 8-0 mixing angle θ_p. However, it should be stressed that θ_1 is not the 8-0 mixing angle θ_p, if θ_2 and θ_3 are not 0. These are related to X_η, etc., as follows:

$$X_\eta=\alpha_8/\sqrt{3}+\sqrt{\tfrac{2}{3}}\alpha_0\ ,\quad Y_\eta=-\sqrt{\tfrac{2}{3}}\alpha_8+\alpha_0/\sqrt{3}\ ,\quad Z_\eta=\alpha_G\ ,$$
$$X_{\eta'}=\beta_8/\sqrt{3}+\sqrt{\tfrac{2}{3}}\beta_0\ ,\quad Y_{\eta'}=-\sqrt{\tfrac{2}{3}}\beta_8+\beta_0/\sqrt{3}\ ,\quad Z_{\eta'}=\beta_G\ , \tag{8}$$
$$X_\iota=\gamma_8/\sqrt{3}+\sqrt{\tfrac{2}{3}}\gamma_0\ ,\quad Y_\iota=-\sqrt{\tfrac{2}{3}}\gamma_8+\gamma_0/\sqrt{3}\ ,\quad Z_\iota=\gamma_G\ .$$

The decay width of $J/\psi\to VP$ is given by

$$\Gamma(J/\psi\to VP)=\frac{1}{3}\frac{g_{\psi VP}^2}{4\pi}\frac{q^3}{m_\psi^2}\ , \tag{9}$$

where $g_{\psi VP}$ is the coupling constant defined by the amplitude

$$H_{\psi VP}=\frac{g_{\psi VP}}{m_\psi}\varepsilon_{\mu\nu\rho\sigma}p^\mu\varepsilon_\psi^\nu(p)k^\rho\varepsilon_V^\sigma(k)\ , \tag{10}$$

and various $g_{\psi VP}$'s are tabulated in Table I. q is the center-of-mass momentum of P.

The case where there are radially excited states is ar-

TABLE I. Amplitudes for the decays of $J/\psi\to VP$ and $J/\psi\to\gamma P$.

Decay mode	Amplitude
$\rho^+\pi^-$, $\rho^0\pi^0$, $\rho^-\pi^+$	$\frac{g}{\sqrt{3}}+\frac{e}{6}$
$K^{*+}K^-$, $K^{*-}K^+$	$\frac{g}{\sqrt{3}}\left[1-\frac{s_v}{2}\right]+\frac{e}{6}(1+s_e)$
$K^{*0}\bar{K}^0$, $\bar{K}^{*0}K^0$	$\frac{g}{\sqrt{3}}\left[1-\frac{s_v}{2}\right]-\frac{e}{3}\left[1-\frac{s_e}{2}\right]$
$\omega\eta$	$\left[\frac{g}{\sqrt{3}}+\frac{e}{6}+\frac{2}{3}rg\right]X_\eta+\frac{\sqrt{2}}{3}rg(1-s_p)Y_n+\left[\frac{2}{3}\right]^{1/2}r'gZ_\eta$
$\omega\eta'$	$\left[\frac{g}{\sqrt{3}}+\frac{e}{6}+\frac{2}{3}rg\right]X_{\eta'}+\frac{\sqrt{2}}{3}rg(1-s_p)Y_{\eta'}+\left[\frac{2}{3}\right]^{1/2}r'gZ_{\eta'}$
$\phi\eta$	$\frac{\sqrt{2}}{3}rg(1-s_v)X_\eta+\left[\frac{g}{\sqrt{3}}(1-s_v)-\frac{e}{3}(1-s_e)+\frac{rg}{3}(1-s_p)(1-s_v)\right]Y_\eta+\frac{r'g}{\sqrt{3}}(1-s_v)Z_\eta$
$\phi\eta'$	$\frac{\sqrt{2}}{3}rg(1-s_v)X_{\eta'}+\left[\frac{g}{\sqrt{3}}(1-s_v)-\frac{e}{3}(1-s_e)+\frac{rg}{3}(1-s_p)(1-s_v)\right]Y_{\eta'}+\frac{r'g}{\sqrt{3}}(1-s_v)Z_{\eta'}$
$\rho^0\eta$	$\frac{e}{2}X_\eta$
$\rho^0\eta'$	$\frac{e}{2}X_{\eta'}$
$\omega\pi^0$	$\frac{e}{2}$
$\gamma\eta'$	$\frac{2}{\sqrt{6}}\left[d+\frac{f}{3}\right]X_{\eta'}+\frac{1}{\sqrt{3}}\left[(1-s_p)d-\frac{f}{3}(1-s_v)(1-s_e)\right]Y_{\eta'}+\frac{r'}{r}dZ_{\eta'}$
$\gamma\eta$	$\frac{2}{\sqrt{6}}\left[d+\frac{f}{3}\right]X_\eta+\frac{1}{\sqrt{3}}\left[(1-s_p)d-\frac{f}{3}(1-s_v)(1-s_e)\right]Y_\eta+\frac{r'}{r}dZ_\eta$
$\gamma\pi^0$	$\frac{f}{\sqrt{6}}$

gued in the Appendix. We write the radially excited mesons as $\bar{P}$. We consider a simple model in which the mixing angles between π and $\bar{\pi}$, between K and $\bar{K}$ (the radially excited state of K), and between (η,η') and $(\bar{\eta},\bar{\eta}')$ are the same. The most probable candidates for $\bar{\eta}'$ and $\bar{\eta}$ are ones of two or more states in the $\eta(1440)$ and the $\eta(1295)$ regions, respectively. Making an appropriate replacement of the coupling constants, the amplitudes for the decays containing π and K which mix with $\bar{\pi}$ and $\bar{K}$ are written by the same expression as the ones for the case containing the glueball. The amplitudes for the decays containing η and η' that mix with $\bar{\eta}$ and $\bar{\eta}'$ differ from the ones of the case containing the glueball only in the terms for the mixing of glueball, i.e., the terms containing Z_η and $Z_{\eta'}$.

B. $J/\psi\to\gamma P$ decays

First, we consider the case in which there is a glueball. The radiative decays proceed through the virtual two-gluon and three-gluon annihilation diagrams as shown in Figs. 2(a), 2(b) and 2(c). The amplitudes for these radiative decays are expressed in terms of an SU(3)-symmetric electromagnetic coupling strength d with small SU(3) breaking s_p [contributed from Fig. 2(a)] and another electromagnetic coupling strength f with small SU(3) breaking s_e and s_v [contributed from Fig. 2(c)]. The amplitude produced from the pure glueball G_0 in P is expressed by the electromagnetic coupling strength d altered by r'/r contributed from the diagram Fig. 2(b). The effective Lagrangian L for these interactions without the kinematical quantities is expressed as

$$L=d\,\mathrm{Tr}([S_p][P])+\frac{f}{2}\mathrm{Tr}([S_v]\{[S_e],[P]\})+\frac{r'}{r}dG_0\ . \tag{11}$$

The decay width of $J/\psi\to\gamma P$ decay is given by

$$\Gamma(J/\psi\to\gamma P)=\frac{1}{3}\frac{g^2_{\psi\gamma P}}{4\pi}\frac{q^3}{m^2_\psi}\ , \tag{12}$$

where $g_{\psi\gamma P}$ is the coupling constant defined by

$$H_{\psi\gamma P}=\frac{g_{\psi\gamma P}}{m_\psi}\varepsilon_{\mu\nu\rho\sigma}p^\mu\varepsilon^\nu_\psi(p)k^\rho\varepsilon^\sigma_\gamma(k)\ , \tag{13}$$

which is proportional to the amplitude. Various $g_{\psi\gamma P}$'s are tabulated in Table I.

For the case in which there are radially excited states, see Appendix.

III. NUMERICAL ANALYSIS AND DISCUSSION

In this section, we determine the coupling strengths and the mixing angles using the experimental data for the $J/\psi\to VP$ and $J/\psi\to\gamma P$ decays. We use the data [11] listed in Table II, which are the world average in 1990. We do not use the experimental data concerning $\eta(1440)$, because the mass region near $\eta(1440)$ has a complicated aspect as there may be two or more pseudoscalar states in this region. In fact, in the hadronic reaction $\pi^-p\to K\bar{K}\pi n$, the $K\bar{K}\pi$ system has two narrow η resonances in the 1410–1480 MeV region [2], and in the reaction $\pi^-p\to\eta\pi\pi n$, the $\eta\pi\pi$ system [3] has the state of mass 1388 MeV. In the J/ψ radiative decay, the $K\bar{K}\pi$ channel in the reaction $J/\psi\to\eta(1440)\gamma\to K\bar{K}\pi\gamma$ is described by the two-Breit-Wigner fit [4], and the $\eta\pi\pi$ channel in the reaction $J/\psi\to\eta(1440)\gamma\to\eta\pi\pi\gamma$ peaks at 1390 MeV [5].

As calculated in the last section and the Appendix, the amplitudes for $J/\psi\to PV$ and $P\gamma$ where P's mix with a glueball and radially excited states are represented by two parts. One part does not contain the mixing parameters and another part is proportional to the mixing parameters. Although the P's mix with the glueball or radially excited states, the part not containing the mixing parameters has the same expression for either case. If there are both the glueball and radially excited states, the part containing the mixing parameters is a superposition of the term for the glueball mixing and the term for radially excited states mixing. However, the part containing the radially excited states mixing is not so large, and the mixing angle is considered to be smaller than about 10°. In fact, the mixing angle between ρ and the radially excited state ρ' is calculated to be less than 9.5° from the following analysis. We adopt $\rho(1450)$ for ρ'. A part of $\rho'\to\pi\pi$ decay is caused through the $\rho\to\pi\pi$ decay of the ρ in the ρ' meson. Using the experimental data $\Gamma(\rho'\to\pi\pi)\times\Gamma(\rho'\to e^+e^-)/\Gamma(\rho'\to\mathrm{all})=0.12$ keV [11] and the assumption

$$\Gamma(\rho'\to e^+e^-)/\Gamma(\rho\to e^+e^-)=\Gamma(J/\psi(3685)\to e^+e^-)/\Gamma(J/\psi(3097)\to e^+e^-)\ ,$$

the above result is obtained. Then, we analyze a case in which there is only glueball mixing. If the glueball mixing angle obtained is not so small and larger than about 10°, then part of the radially excited states mixing is hidden by part of the glueball, although there exist radially

TABLE II. Branching ratios for the decays of $J/\psi\to VP$ and $J/\psi\to\gamma P$. The experimental data are the world average cited by the Particle Data Group [11]. The calculated values are given by using the results (15) and (16) obtained in the least-squares method.

Decay mode	Branching ratios (10^{-3}) Expt.	Calc.
$\rho\pi$	12.8±1.0	11.5
$K^{*+}K^-$+c.c.	3.8±0.7	4.7
$K^{*0}\bar{K}^0$+c.c.	3.7±0.8	4.8
$\omega\eta$	1.71±0.22	1.4
$\omega\eta'$	0.166±0.025	0.17
$\phi\eta$	0.714±0.030	0.67
$\phi\eta'$	0.38±0.04	0.36
$\rho\eta$	0.193±0.032	0.22
$\rho\eta'$	0.096±0.018	0.089
$\omega\pi^0$	0.48±0.07	0.43
$\gamma\eta'$	4.2±0.4	4.2
$\gamma\eta$	0.86±0.08	0.85
$\gamma\pi^0$	0.039±0.013	0.04

excited states. If the glueball mixing angle obtained is small, then both glueball mixing and radially excited states mixing can exist and we cannot estimate these mixing angles explicitly in the present analysis.

The s_e defined in (1) is given as

$$s_e=0.30\ , \tag{14}$$

for the quark masses $m_u=350$ MeV and $m_s=500$ MeV. In order to determine the coupling strengths and mixing angles, we use the method of least squares. The results obtained are

$$\begin{aligned}&|g|=9.58\times10^{-3}\ ,\ |e/g|=0.389\ ,\\ &\phi(\text{relative phase between } e \text{ and } g)=91.0^\circ\ ,\\ &r=-0.416\ ,\quad r'=-0.353\ ,\quad s=0.319\ ,\\ &\theta_1=-19.0^\circ\ ,\quad \theta_2=7.06^\circ\ ,\quad \theta_3=20.7^\circ\ ,\\ &|d|=1.22\times10^{-2}\ ,\quad f/d=0.102\ ,\end{aligned} \tag{15}$$

with a χ^2 of 4.4 for 2 degrees of freedom. The mixing parameters for the obtained angles are given as

$$\begin{aligned}&\eta=0.757N-0.613S-0.224G_0\ ,\\ &\eta'=0.547N+0.783S-0.296G_0\ ,\\ &\iota=0.357N+0.102S+0.929G_0\ .\end{aligned} \tag{16}$$

A change of 30% of s_e affects the above results within 5%. The characteristics of the obtained results are as follows. (a) The mixing angle θ_1 corresponding to θ_P is rather large, about $-20°$. (b) The relative phase ϕ between e and g is about $\pi/2$. (c) Mixing angles θ_2 and θ_3 are not so small; i.e., the glueball components in η and η' are appreciably large; then there is a glueball which mixes with η and η' appreciably, and one state in the $\eta(1440)$ region is considered to be the glueball. There can be also a radially excited state in this region, for which we cannot say anything. (d) The estimated N component in ι is larger than the S component; then it has not only a $KK^*(892)$ intermediate-state contribution but also an $a_0(980)\pi$ one, and it should have a character different from the radially excited state which has an almost $KK^*(892)$ intermediate-state contribution because of $s\bar{s}$ dominance. (e) r' and r have the same order values; then the pure glueball creation process and the $q\bar{q}$ pair creation process proceeding through the intermediate 2-gluons state as shown in Figs. 2(b) and 2(a) are the same order. We have listed the branching ratios calculated for the above best-fit parameters in Table II.

In the analysis of the $J/\psi\to PV$ decay by the Mark III and DM2 Collaborations [10], the u-, d-, and s-quark contents of the η and η' saturate the wave function, ruling out gluonia in the η and η', and the mixing angle θ_P is calculated as about $-19°$. In order to clarify the difference between the Mark III and DM2 results and our result, we analyze only the $J/\psi\to PV$ decay using the DM2 experimental data and our amplitudes in Table I. Two solutions are obtained.

Solution I (χ^2 for 1 degree of freedom is 1.15):

$$\begin{aligned}&|g|=9.67\times10^{-3},\quad |e/g|=0.351\ ,\\ &\phi(\text{relative phase between } e \text{ and } g)=77.3^\circ\ ,\\ &r=-0.196\ ,\quad r'=0.197\ ,\quad s=0.331\ ,\\ &\theta_1=-21.9^\circ\ ,\quad \theta_2=8.12^\circ\ ,\quad \theta_3=15.9^\circ\ ,\\ &\eta=0.793N-0.564S-0.228G_0\ ,\\ &\eta'=0.528N+0.825S-0.203G_0\ ,\\ &\iota=0.303N+0.041S+0.952G_0\ .\end{aligned} \tag{17}$$

Solution II (χ^2 for 1 degree of freedom is 1.48):

$$\begin{aligned}&|g|=9.58\times10^{-3},\quad |e/g|=0.351\ ,\\ &\phi(\text{relative phase between } e \text{ and } g)=80.3^\circ\ ,\\ &r=-0.277\ ,\quad r'=0.0\ ,\quad s=0.323\ ,\\ &\theta_1=-20.2^\circ\ ,\quad \theta_2=0.0^\circ\ ,\quad \theta_3=0.0^\circ\ ,\\ &\eta=0.824N-0.567S-0.000G_0\ ,\\ &\eta'=0.567N+0.824S-0.000G_0\ ,\\ &\iota=0.000N+0.000S+1.000G_0\ .\end{aligned} \tag{18}$$

The result of solution II is very close to the one of DM2. Then, the difference between the result of DM2 and Mark III and our result (15) and (16) can be considered to arise from the containing of the $J/\psi\to P\gamma$ decay in the analysis.

In the above analysis, we have assumed $s_v=s_p$. However, if we relax the restriction $s_v=s_p$, then we can get two solutions with χ^2 of 4.4 for 1 degree of freedom (DF); one of which is very close to the Eq. (15) solution and the other is that

$$\begin{aligned}&|g|=9.58\times10^{-3}\ ,\quad |e/g|=0.388\ ,\\ &\phi=90.9^\circ\ ,\quad r=-0.199\ ,\quad r'=0.113\ ,\\ &s_v=0.318\ ,\quad s_p=0.004\ ,\\ &\theta_1=-19.1^\circ\ ,\quad \theta_2=7.28^\circ\ ,\quad \theta_3=20.0^\circ\ ,\\ &|d|=0.580\times10^{-2}\ ,\quad f/d=0.215\ .\end{aligned} \tag{15$'$}$$

It is remarkable that the mixing angles θ_1, θ_2, and θ_3 for both solutions are very close. Furthermore, we can get a solution with χ^2 of 4.8 fixing $\theta_2=\theta_3=r'=0$;

$$\begin{aligned}&|g|=9.47\times10^{-3}\ ,\quad |e/g|=0.373\ ,\\ &\phi=91.7^\circ\ ,\quad r=-0.264\ ,\quad r'=0.0\ (\text{fixed})\ ,\\ &s_v=0.301\ ,\quad s_p=0.078\ ,\\ &\theta_1=-18.3^\circ\ ,\quad \theta_2=\theta_3=0.0\ (\text{fixed})\ ,\\ &|d|=0.672\times10^{-2}\ ,\quad f/d=0.186\ .\end{aligned} \tag{15$''$}$$

This solution is very similar to (15′), aside from θ_2, θ_3, and r'. The χ^2 in this case is not so large compared with the one of case (15); then a possibility that the contents other than N and S in η and η' are very small is not excluded.

TABLE III. The widths for the decays of $V \to \gamma P$ and $P \to \gamma V$ and $P \to \gamma\gamma$. The experimental data are taken from Ref. [11]. The datum with superscript a is given by assuming $\Gamma(K^{*\pm} \to \text{all}) = \Gamma(K^{*0} \to \text{all})$. The calculated values are taken by using the results (15) and (16).

Decay mode	Γ^{expt} (keV)	Γ^{calc} (keV)
$\rho^\pm \to \pi^\pm\gamma$	67.1±7.6	74.7
$\omega \to \pi^0\gamma$	717±43	717(input)
$K^{*\pm} \to K^\pm\gamma$	50.3±4.6	62.1
$K^{*0} \to K^0\gamma$	115±10[a]	141
$\rho \to \eta\gamma$	56.7±10.5	50.2
$\omega \to \eta\gamma$	3.96±1.69	6.51
$\phi \to \eta\gamma$	56.4±2.8	66.8
$\eta' \to \rho\gamma$	62.4±7.0	59.0
$\eta' \to \omega\gamma$	6.24±0.90	5.27
$\phi \to \eta'\gamma$	<1.81	1.49
$\pi^0 \to \gamma\gamma$	7.84×10^{-3}	7.84×10^{-3} (input)
$\eta' \to \gamma\gamma$	4.49±0.58	4.70
$\eta \to \gamma\gamma$	0.463±0.047	0.508

Aizawa *et al.* [13] suggested a large value of the mixing angle θ_1 ($\sim 30°$) from the analysis of the radiative decays of J/ψ, using the chiral-U(3)×U(3) algebraic approach, but their conclusion is that the width of $\iota \to \gamma\gamma$ decay is very large compared with the experimental one. One of us (T.T.) studied the mixing problem of the $\eta - \eta' - \iota$ in various approaches, the chiral U(3)×U(3) or U(4)×U(4) QCD algebraic approach [14] and asymptotic flavor-SU(3)-symmetry approach [15]. In these approaches, he has discussed the problems of the masses of P and branching ratios of the radiative decays of J/ψ and ι, and obtained the results that the mixing angle θ_1 has been about $-10°$ and the glueball component in η' has been significantly large.

Finally we calculate the decay widths for various $P \to V\gamma$, $V \to P\gamma$, and $P \to \gamma\gamma$ decays using the values of the parameters (15) and (16) obtained, and tabulate the results in Table III. In the calculations, we have used relation (12) for the $V \to P\gamma$ decay. For the $P \to V\gamma$ decay, we have used a similar relation

$$\Gamma(P \to \gamma V) = \frac{g_{V\gamma P}^2}{4\pi} \frac{q^3}{m_P^2} , \tag{19}$$

where $g_{V\gamma P}$ is defined by

$$g_{V\gamma P} = \frac{g}{2} \text{Tr}([V]\{[S_e],[P]\}) , \tag{20}$$

and the datum $\Gamma(\omega \to \pi^\circ\gamma) = 717$ keV was used as input. The electromagnetic symmetry-breaking parameter s_e in S_e has been estimated to be 0.2 by fitting the ratio $\Gamma(K^{*\pm} \to K^\pm\gamma)/\Gamma(K^{*0} \to K^0\gamma)$ to the experimental result, which is comparable with the value estimated in (14). We have thus used the value 0.2 in the calculation of the $P \to V\gamma$ and $V \to P\gamma$ decay widths. For the two γ decays, we have used the relation

$$\Gamma(p^A \to \gamma\gamma) = \left[\frac{m_A}{m_\pi}\right]^3 \left|\frac{F^A}{F^{\pi 0}}\right|^2 \Gamma(\pi^0 \to \gamma\gamma) , \tag{21}$$

where F^A for a pseudoscalar meson A is defined by

$$F^A = F\,\text{Tr}([P][Q][Q]) ,$$
$$[Q] = \frac{1}{3} \begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} . \tag{22}$$

F is the coupling constant common to all coupling $P\gamma\gamma$. The datum $\Gamma(\pi^\circ \to \gamma\gamma) = 7.84 \times 10^{-3}$ keV [11] was used as input. Almost all the calculated results are consistent with the experimental data.

IV. CONCLUSION

The branching ratios of the $J/\psi \to VP$ and $P\gamma$ decays have been examined in order to determine the magnitudes of components other than N and S in η and η'. The data used are the world average in 1990. Although a possibility that the magnitudes are very small is not excluded, there is a solution that these magnitudes are found to be not very small. Therefore, the η and the η' have glueball origins. A state in the $\eta(1440)$ region is considered to be the glueball. Radially excited states are not excluded from existing in this region. The mixing angle θ_1 corresponding to the angle θ_P between η and η' has been obtained as about $-20°$. The estimated N component in this glueball is larger than the component S. Using the mixing parameters obtained, we have calculated the decay widths for various $P \to \gamma\gamma$, $P \to V\gamma$, and $V \to P\gamma$ decays. Almost all the results obtained are consistent with experiments.

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APPENDIX: THE MIXING BETWEEN THE GROUND STATES AND THE RADIALLY EXCITED STATES

We represent the radially excited states of pseudoscalar mesons by $\bar{P}$ and assume the interaction Lagrangian for $J/\psi - V - \bar{P}$ and $J/\psi - \bar{P} - \gamma$ to be

$$L = \frac{\bar{g}}{2} \text{Tr}([\bar{P}]\{[V],[S_v]\}) + \frac{\bar{e}}{2} \text{Tr}([\bar{P}]\{[V],[S_e]\}) + r\bar{g}\,\text{Tr}([\bar{P}][S_p])\text{Tr}([V][S_v]) , \tag{A1}$$

$$L = \bar{d}\,\text{Tr}([S_p][\bar{P}]) + \frac{\bar{f}}{2} \text{Tr}([S_v]\{[S_e],[\bar{P}]\}) . \tag{A2}$$

The mixing between the ground states and radially excited states are written as

$$\begin{bmatrix} \pi \\ \bar{\pi} \end{bmatrix} = \begin{bmatrix} \cos\theta_\pi & -\sin\theta_\pi \\ \sin\theta_\pi & \cos\theta_\pi \end{bmatrix} \begin{bmatrix} \pi_0 \\ \bar{\pi}_0 \end{bmatrix} , \tag{A3}$$

$$\begin{bmatrix} K \\ \bar{K} \end{bmatrix} = \begin{bmatrix} \cos\theta_K & -\sin\theta_K \\ \sin\theta_K & \cos\theta_K \end{bmatrix} \begin{bmatrix} K_0 \\ \bar{K}_0 \end{bmatrix} , \tag{A4}$$

where $(\pi_0,\bar{\pi}_0)$ and $(K_0,\bar{K}_0)$ are the pure ground and radially excited states of π and K. We assume a simple model for the mixing between (η,η') and $(\bar{\eta},\bar{\eta}')$:

$$\begin{bmatrix} \eta \\ \eta' \\ \bar{\eta} \\ \bar{\eta}' \end{bmatrix} = \begin{bmatrix} \cos\theta_\eta & 0 & -\sin\theta_\eta & 0 \\ 0 & \cos\theta_\eta & 0 & -\sin\theta_\eta \\ \sin\theta_\eta & 0 & \cos\theta_\eta & 0 \\ 0 & \sin\theta_n & 0 & \cos\theta_\eta \end{bmatrix} \begin{bmatrix} X_\eta & Y_\eta & 0 & 0 \\ X_{\eta'} & Y_{\eta'} & 0 & 0 \\ 0 & 0 & X_{\bar{\eta}} & Y_{\bar{\eta}} \\ 0 & 0 & X_{\bar{\eta}'} & Y_{\bar{\eta}'} \end{bmatrix} \begin{bmatrix} N \\ S \\ \bar{N} \\ \bar{S} \end{bmatrix} . \tag{A5}$$

Furthermore, we use a simplification: $\theta=\theta_\pi=\theta_K=\theta_\eta$.

If we make the replacements

$$\begin{aligned} & g\cos\theta-\bar{g}\sin\theta\rightarrow g \ , \\ & e\cos\theta-\bar{e}\sin\theta\rightarrow e \ , \\ & d\cos\theta-\bar{d}\sin\theta\rightarrow d \ , \\ & f\cos\theta-\bar{f}\sin\theta\rightarrow f \ , \end{aligned} \tag{A6}$$

then we can write the amplitudes for the decays $J/\psi\rightarrow\rho\pi$, K^*K, $\omega\pi^0$, and $\gamma\pi^0$ by the same expressions as in Table I. In other amplitudes, the terms involving Z_η and $Z_{\eta'}$ are replaced by the terms proportional to $\sin\theta$: for example,

$$\begin{aligned} A(J/\psi\rightarrow\omega\eta)= & \left[\frac{g}{\sqrt{3}}+\frac{e}{6}+\frac{2}{3}rg\right]X_\eta+\frac{\sqrt{2}}{3}rg(1-s_p)Y_\eta \\ & +\left[\left[\frac{\bar{g}}{\sqrt{3}}+\frac{\bar{e}}{6}+\frac{2}{3}r\bar{g}\right](X_\eta-X_{\bar{\eta}})+\frac{\sqrt{2}}{3}r\bar{g}(1-s_p)(Y_\eta-Y_{\bar{\eta}})\right]\sin\theta \ , \end{aligned} \tag{A7}$$

$$\begin{aligned} A(J/\psi\rightarrow\omega\eta')= & \left[\frac{g}{\sqrt{3}}+\frac{e}{6}+\frac{2}{3}rg\right]X_{\eta'}+\frac{\sqrt{2}}{3}rg(1-s_p)Y_{\eta'} \\ & +\left[\left[\frac{\bar{g}}{\sqrt{3}}+\frac{\bar{e}}{6}+\frac{2}{3}r\bar{g}\right](X_{\eta'}-X_{\bar{\eta}'})+\frac{\sqrt{2}}{3}r\bar{g}(1-s_p)(Y_{\eta'}-Y_{\bar{\eta}'})\right]\sin\theta \ . \end{aligned} \tag{A8}$$

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