Inflation in generalized Einstein theories

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We analyze the details of soft inflationary models, which have two scalar fields: one is the standard inflaton, whose potential is exponentially coupled to the other field. Such models are derived from both fundamental theories, and in the conformal frame of generalized Einstein theories. In the latter case, a nonstandard exponential coupling to the inflaton kinetic term also may arise. We list and discuss the various theories which give soft inflation, and then consider the satisfaction of the inflationary constraints in general models. We then specialize to new and chaotic inflation potentials, with both standard and nonstandard kinetic terms. The density perturbations are reduced sufficiently so that new inflation works well, with the coupling constant near the values allowed by grand unified theories. For chaotic inflation with a massive inflaton, we find successful inflation without any fine-tuning of the coupling constant or initial data.

I. INTRODUCTION: CLASSIFICATION OF INFLATIONARY MODELS

The inflationary universe model attempts to solve some long-standing problems of cosmology, and has been the subject of much investigation during the past decade [1]. However, as yet no fully viable model exists for the source of inflation. The original model, called old inflation [2], uses the vacuum energy created by the SU(5) Higgs field, when trapped in a metastable state, to act as an effective cosmological constant. As the Universe cools, this vacuum energy dominates, and the scale factor expands exponentially. However, tunneling from the metastable state proved too difficult, and a satisfactory termination of the inflationary phase was impossible [3].

The new inflationary scenario [4] avoids this problem by utilizing the Coleman-Weinberg effective potential [5], whose shape is very flat for small field values. A finite-temperature effective potential should localize the field to small expectation values at the start of inflation. The scalar field slowly rolls down this potential at first, with exponential expansion occurring. Inflation terminates when the field leaves the slow-rolling regime, quickly evolves to the true minimum, and reheats via oscillations about the bottom of the potential which couple to matter [6]. While this model predicted the spectrum of density fluctuations which would act as the seeds for galaxy formation, these fluctuations were unfortunately far too large for standard grand-unified-theory (GUT) parameters, and conflicted with the observed isotropy of the cosmic-microwave-background radiation [7]. To agree with this constraint, self-interactions and couplings to other fields would have to be excessively small. Thermal equilibrium could not be established by the onset of inflation, and hence the field would not be localized near zero expectation value [8,9]. Therefore, a sufficient amount of expansion will not be obtained.

The chaotic inflation scenario [10] showed that inflation need not occur only in very special field theories. For a broad range of general potentials, the field evolves slowly compared to the Hubble parameter, with inflation ensuing. Although an inflationary model which satisfies all constraints may be constructed, density fluctuations again force the couplings to be excessively small. While these models are not ruled out, they do suffer from a fine-tuning problem.

All models discussed so far have used field theories at very high energies to drive inflation. We shall call these models class I. However, inflation may also be generated by changing the gravitational sector, which we call class II. One such example is R^2 inflation, in which a term quadratic in the Ricci scalar is added to the standard Einstein action [11]. However, density fluctuations again force the coupling of the R^2 term to be inordinately large. Furthermore, even higher-order terms severely restrict the initial conditions which would lead to a successful transition out of the inflationary era [12]. Other examples of class II models utilize the induced gravity model [13] and nonminimal coupling terms [14].

Recently, much interest has focused on models which change both the matter and gravitational sectors, which we call class III. For example, extended inflation models [15] use the old-inflation-style potentials with their false vacuum in the Jordan-Brans-Dicke (JBD) theory of gravity. While this combination allows the phase transition to be completed, unfortunately homogeneity afterwards is achieved only for Brans-Dicke parameters which violate observations [16]. Hopefully, several variations on this theme have appeared which may bypass this problem [17].

While extended inflation uses the JBD theory and variants to allow successful old inflation, new and chaotic inflation models [18,19] can also be utilized in class III. In this paper we consider soft inflation [19], where a standard new or chaotic inflaton potential is coupled to an exponential potential, which is conformally related to class III. Heuristically, the success of soft inflation may be seen as follows: The exponential potential multiplied by any coupling constant of the theory acts as an effective coupling constant, with a value which is constantly de-

TABLE I. Inflationary theories and their difficulties. The models are classified according to whether their gravitational, matter, or both sectors are modified.

Class	L_{gravity}	L_{matter}	Models	Difficulties
I	Einstein gravity	Vacuum energy (GUT's)	Old inflation	No completion of phase transition
			New inflation ($V=V_0-\frac{1}{4}\lambda\psi^4$)	$\lambda \leq 10^{-12}$ while $\lambda \sim 0.5$ for SU(5)
			Chaotic inflation ($V=\frac{1}{4}\lambda\psi^4, \frac{1}{2}m^2\psi^2$)	$\lambda \leq 10^{-12}$, $m \leq 10^{-6} m_{\text{Pl}}$
			Power-law inflation ($V=e^{-\beta\kappa\psi}$)	$\beta < \sqrt{2}$ (no realistic model)
II	Generalized Einstein theories (GET's)	Usual matter	R^2 inflation [$L_{\text{gravity}}=(1/2\kappa^2)R+\alpha R^2$]	$\alpha \geq 10^{12}$
			Induced gravity inflation	Quantum gravity at GUT's scale?
			Nonminimal scalar inflation	$\lvert\xi\rvert > 10^4$ and $\xi < 0$
			Kaluza-Klein inflation	No successful inflationary model
III	Generalized Einstein theories (GET's)	Vacuum energy (GUT's)	Extended inflation	$\omega < 30$ while $\omega > 500$ from observation
			Hyperextended inflation	
			Soft inflation	

creasing. Thus, for quite standard values of parameters, density fluctuations can be suppressed, "softening" the constraints. Different theories which lead to inflation, as well as their dificulties, are given in Table I.

The exponential potential of this form arises from two different sources. One is in certain superstring and supergravity models [20], in which case the theory may be class I, where only the matter sector is changed. The other source is when generalized Einstein theories (GET's) containing a standard inflaton field are conformally transformed. The power of this approach is that a wide range of GET's may be modeled with the exponential potential [21]. Examples are the Jordan-Brans-Dicke theory [22], the induced gravity model [23], Kaluza-Klein theories [24], R^2 terms in the action [11], and models with nonminimal coupling [14]. Models using GET's plus an inflaton are of class III, where both the matter and gravitational sectors are changed.

The outline of this paper is as follows. In Sec. II we discuss the origin of the exponential potential, with careful attention given to conformal transformations in various GET's. In particiar, the kinetic term of the inflaton may pick up a nonstandard coupling to the exponential field, which will have important consequences. In Sec. III the constraints on the parameters for general kinetic coupling are given. These constraints arise from a sufficient amount of inflation, successful reheating, and suppression of density perturbations. In Sec. 4 we specify to the new inflation model, while Sec. V considers chaotic inflation. The models arising from GET's, which possess a nonstandard kinetic term, are given special detail. Section VI contains comments and plans for further work.

II. TRANSFORMATION OF CLASS III MODEL INTO SOFT INFLATION

In class III, we have the following theories, which may provide a natural inflationary model.

(1) Theories with a scalar field coupled to gravity, with the action

$$S=\int d^4x\sqrt{-g}\,[f(\Phi)R-h(\Phi)(\nabla\Phi)^2-V(\Phi)+L_{\text{inf}}]\ , \tag{2.1}$$

where f and h are arbitrary function of Φ and

$$L_{\text{inf}} \equiv -\tfrac{1}{2}(\nabla\psi)^2-V(\psi) \tag{2.2}$$

is the inflaton piece. These theories include the following subclasses.

(1a) Jordan-Brans-Dicke (JBD) theory [22] with an inflaton:

$$f=\frac{1}{16\pi}\Phi\ ,\quad h=\frac{\omega}{16\pi\Phi}\ ,\quad V(\Phi)=0 \tag{2.3}$$

where observations give the JBD parameter $\omega > 500$ [25].

(1b) Induced gravity model [23] with an inflaton:

$$f=\frac{\epsilon}{2}\Phi^2\ ,\quad h=\tfrac{1}{2}\ ,\quad V(\Phi)=\frac{\lambda}{8}(\Phi^2-\eta^2)^2\ , \tag{2.4}$$

where ϵ and η are a coupling constant and the present value of Φ, respectively.

(1c) Nonminimally coupled scalar field and an inflaton:

$$f=\frac{1}{2\kappa^2}-\frac{1}{2}\xi\Phi^2\ ,\quad h=\tfrac{1}{2}\ ,\quad V(\Phi)=0\ , \tag{2.5}$$

where $\kappa^2=8\pi G$. All three of these models could have additional potential terms for Φ.

(2) $f(R)$ theories [26], including R^2 theory [11], with an inflaton, with the action

$$S=\int d^4x\sqrt{-g}\,[f(R)+L_{\text{inf}}]\ , \tag{2.6}$$

(3) $(4+D)$-dimensional Kaluza-Klein theories with an inflaton field, in which case the models have two subclasses.

(3a) The inflaton is introduced in the effective four-dimensional theory, in which the action is

$$S=\int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}\left[\Phi^2R+\frac{4(D-1)}{D}(\nabla\Phi)^2\right]-U_{\text{eff}}(\Phi)+L_{\text{inf}}\right]\ , \tag{2.7}$$

where $\Phi \equiv (b/b_0)^{D/2}$ with b an internal radius and b_0 its present value.

(3b) The inflation is defined in $(4+D)$-dimensions, with the action

$$S=\int d^{4+D}X\sqrt{-G}\left[\frac{1}{2\kappa_{(4+D)}^2}R+L_{\rm inf}\right], \qquad (2.8)$$

where G and $\kappa_{(4+D)}^2$ are the $(4+D)$-dimensional metric and gravitational constant, with D the dimension of the internal space.

(4) An effective action from a superstring model [27] with an inflaton:

$$S=\int d^{10}X\sqrt{-G}\,[e^{-2\Phi}R+4(\nabla\Phi)^2+L_{\rm inf}], \qquad (2.9)$$

where Φ is a dilaton field, with the ansatz of the rank-three antisymmetric tensor $H_{\mu\nu\lambda}=0$.

Most of the models of class III are conformally equivalent to those of an extra scalar field with a modified potential in the Einstein gravity theory [28]. Conveniently, many of these models in the conformal frame have the Lagrangian form

$$S=\int d^4x\sqrt{-g}\left[\frac{1}{2\kappa^2}R-\tfrac{1}{2}(\nabla\phi)^2-\tfrac{1}{2}e^{-\gamma\kappa\phi}(\nabla\psi)^2 - e^{-\beta\kappa\phi}V(\psi)-U(\phi)\right], \qquad (2.10)$$

where ϕ is the scalar field Φ redefined to obtain the canonical kinetic term, $U(\phi)$ is a modified potential of Φ or a newly appearing potential as in the $f(R)$ theory, and β and γ are dimensionless coupling constants.

In Table II, we list the relation between Φ and ϕ, and the values of β and γ for each model. In order to find power-law inflation due to a flat potential $V(\psi)$, β must be smaller than $\sqrt{2}$; this condition is also shown in Table II. The model based on superstring theory is reduced to four-dimensional Einstein gravity with two dilaton fields, $\kappa\phi_S\equiv(6\ln b-\Phi/2)/\sqrt{2}$ and $\kappa\phi_T\equiv\sqrt{6}(2\ln b+\Phi/2)/2$, via a conformal transformation. It does not give an inflationary solution; however, such a fundamental theory is still incomplete and future work may yield exponential potentials with the desired form.

If $U(\phi)=0$, the model may significantly ease the inflationary constraints, and hence acts as the soft inflation potential [19]. We will consider such models in this paper, first dealing with general β and γ, and then specializing to the cases of γ equal to 0 and $\beta/2$. We will comment on the effects of nonzero $U(\phi)$ in the conclusion.

III. SOFT INFLATION

In this section, we examine the constraints on soft inflation arising from sufficient inflation, reheating, and suppression of density perturbations in a general model. The action considered is (2.10), with $U(\phi)=0$. Variation of this action in a spatially flat Robertson-Walker universe yields the field equations

$$\ddot\phi+3H\dot\phi+\frac{\gamma\kappa}{2}e^{-\gamma\kappa\phi}\dot\psi^2-\beta\kappa e^{-\beta\kappa\phi}V(\psi)=0, \qquad (3.1)$$

$$\ddot\psi+3H\dot\psi-\gamma\kappa\dot\phi\dot\psi+e^{(\gamma-\beta)\kappa\phi}V'(\psi)=0, \qquad (3.2)$$

$$H^2=\frac{\kappa^2}{3}[\tfrac{1}{2}\dot\phi^2+\tfrac{1}{2}e^{-\gamma\kappa\phi}\dot\psi^2+e^{-\beta\kappa\phi}V(\psi)], \qquad (3.3)$$

where a is the scale factor of the Universe, $H=\dot a/a$ is the Hubble parameter, an overdot indicates a time derivative, and a prime denotes differentiation with respect to ψ.

Recall that in order for power-law inflation to occur in a potential-dominated stage, the condition $\beta^2<2$ is required. For large H, the second terms in (3.1) and (3.2) act to damp the motion of the scalar fields. Any initially high ϕ kinetic terms are damped rather quickly if we assume $\gamma^2\lesssim\beta^2\ll 6$, so that the potential contribution will indeed soon dominate (3.3) [29]. High ψ kinetic terms are similarly damped in chaotic inflation [30], while in new

TABLE II. Generalized Einstein theories, the definition of ϕ, and the values of β and γ in the conformal frame, as given by Eq. (2.10). The last column gives the condition for power-law inflation, $\beta<\sqrt{2}$.

Theory	Scalar field ϕ	β	γ	Power-law inflation
JBD theory	$\kappa\phi=\left[\frac{2\omega+3}{2}\right]^{1/2}\ln\left[\frac{\kappa^2}{2}\Phi\right]$	$\left[\frac{8}{2\omega+3}\right]^{1/2}$	$\beta/2$	$\omega>\frac{1}{2}$
Induced gravity	$\kappa\phi=\left[\frac{1+6\epsilon}{\epsilon}\right]^{1/2}\ln(\kappa\sqrt{\epsilon}\Phi)$	$\left[\frac{16\epsilon}{1+6\epsilon}\right]^{1/2}$	$\beta/2$	$\epsilon<\frac{1}{2}$
Higher-dimensional theories (a)	$\kappa\phi=\left[\frac{2(D+2)}{D}\right]^{1/2}\ln\Phi$	$\left[\frac{8D}{D+2}\right]^{1/2}$	$\beta/2$	No solution
Higher-dimensional theories (b)	$\kappa\phi=\left[\frac{2(D+2)}{D}\right]^{1/2}\ln\Phi$	$\left[\frac{2D}{D+2}\right]^{1/2}$	0	For all D
Superstring model	$\kappa\phi_S=\frac{1}{\sqrt{2}}(6\ln b-\Phi/2)$	$2\sqrt{2}$	$\beta/2$	No solution
	$\kappa\phi_T=\frac{\sqrt{6}}{2}(2\ln b+\Phi/2)$	$-\sqrt{6}$	β	No solution

inflation the high-temperature effective potential localizes the field and keeps the derivatives small at the onset of inflation. In this slow-rolling approximation, the second-derivative terms will be negligible, while the fact that the potential will dominate the kinetic contributions allows us to ignore the terms involving two first derivatives. One point to note is that when

$$\dot{\phi} \gg \frac{V'}{\beta\kappa V}\dot{\psi} , \tag{3.4}$$

the $\ddot{\phi}$ contribution to (3.1) and the $\frac{1}{2}\dot{\phi}^2$ term in (3.3) may be included to give power-law inflation [19,29,31]. However, this power-law solution breaks down when the two fields' kinetic contributions become comparable. Our approach of neglecting the ϕ derivative terms allows us to treat the full range of behavior without resorting to different sets of equations for different regimes. The deviation from the exact solution is minor, as will be illustrated later by numerical results.

With these factors taken into account, the equations we consider are

$$3H\dot{\phi} = \beta\kappa e^{-\beta\kappa\phi}V(\psi) , \tag{3.5}$$

$$3H\dot{\psi} = -e^{(\gamma-\beta)\kappa\phi}V'(\psi) , \tag{3.6}$$

$$H^2 = \frac{\kappa^2}{3}e^{-\beta\kappa\phi}V(\psi) . \tag{3.7}$$

Solving yields the general behavior

$$\phi - \phi_0 = \frac{\beta}{\kappa}\ln\frac{a}{a_0} , \tag{3.8}$$

$$f(\psi) = f(\psi_0) - \frac{e^{\gamma\kappa\phi_0}}{\beta\gamma}\left[\left[\frac{a}{a_0}\right]^{\beta\gamma} - 1\right] , \tag{3.9}$$

with the definition

$$f(\psi) \equiv \kappa^2 \int d\psi \frac{V}{V'} . \tag{3.10}$$

The first constraint to which any inflationary theory is subject comes from achieving sufficient inflation to solve the cosmological problems. This requires

$$\frac{a_f}{a_0} \gtrsim e^{65} , \tag{3.11}$$

so that scales just entering the horizon today came from a causally connected region [2]. The subscripts f and 0 indicate final and initial values of the inflationary epoch. Using Eq. (3.9) above, this condition may be written as

$$\frac{1}{\beta\gamma}\ln\{1 + \beta\gamma e^{-\gamma\kappa\phi_0}[f(\psi_0) - f(\psi_f)]\} \gtrsim 65 . \tag{3.12}$$

In the limit of $\gamma = 0$, the horizon constraint becomes simply

$$f(\psi_0) - f(\psi_f) \gtrsim 65 . \tag{3.13}$$

Another constraint comes from density perturbations [7]. In particular, limits from the microwave background radiation imply that $\delta\rho/\rho < (\delta\rho/\rho)_{\rm cr} \sim 10^{-4}$ on scales the size of the horizon, where $\delta\rho$ is the perturbation in the density ρ. The action (2.10) has two coupled scalar fields and a nonstandard kinetic term; the correct expression for density perturbations may be found by generalizing the results of Lyth [32]. From his Eq. (25), we have

$$\left[\left[1 + \frac{2}{3}\frac{1}{1+w}\right]\frac{\delta\rho}{\rho}\right]_1 = \left[\left[1 + \frac{2}{3}\frac{1}{1+w}\right]\frac{\delta\rho}{\rho}\right]_2 , \tag{3.14}$$

where $w \equiv P/\rho$ is the pressure over the energy density, and the subscripts 1 and 2 indicate the times when the perturbation first left and then reentered the horizon, respectively. Then at t_1,

$$\rho = \tfrac{1}{2}\dot{\phi}^2 + \tfrac{1}{2}\dot{\psi}^2 e^{-\gamma\kappa\phi} + V e^{-\beta\kappa\phi} , \tag{3.15}$$

$$P = \tfrac{1}{2}\dot{\phi}^2 + \tfrac{1}{2}\dot{\psi}^2 e^{-\gamma\kappa\phi} - V e^{-\beta\kappa\phi} , \tag{3.16}$$

while at t_2, $\sigma \equiv 1 + 2/[3(1+w)]$ is equal to $\frac{3}{2}$ during radiation domination and $\frac{5}{3}$ during matter domination. For the potential dominated inflationary phase, $P \approx -\rho$, and so

$$\left.\frac{\delta\rho}{\rho}\right|_2 = \left.\frac{2}{3\sigma(\rho+P)}\delta\rho\right|_1 . \tag{3.17}$$

Varying (3.15) and using the approximate equations of motion (3.5) and (3.6) gives

$$\delta\rho = \dot{\phi}\delta\dot{\phi} + \dot{\psi}\delta\dot{\psi}e^{-\gamma\kappa\phi} - 3H(\dot{\phi}\delta\phi + \dot{\psi}\delta\psi e^{-\gamma\kappa\phi}) . \tag{3.18}$$

Dimensional considerations give $\delta\dot{\phi} \approx H\delta\phi$, and similary for the inflaton. From the two-point correlation function [33], we find

$$\delta\phi \approx H , \quad \delta\psi \approx H e^{\gamma\kappa\phi/2} , \tag{3.19}$$

where the latter expression arises from proper normalization upon second quantization. With these considerations, we ultimately find

$$\frac{\delta\rho}{\rho} = C\frac{H^2(|\dot{\phi}| + |\dot{\psi}|e^{-\gamma\kappa\phi/2})}{\dot{\phi}^2 + \dot{\psi}^2 e^{-\gamma\kappa\phi}} , \tag{3.20}$$

where the left-hand side is evaluated at t_2 and the right-hand side at t_1, with C a numerical constant of order unity. This expression leads to two regimes; namely,

$$|\dot{\phi}| > |\dot{\psi}|e^{-\gamma\kappa\phi/2} \Longrightarrow \frac{\delta\rho}{\rho} \approx H^2/|\dot{\phi}| \quad \text{(region A)} , \tag{3.21}$$

$$|\dot{\phi}| < |\dot{\psi}|e^{-\gamma\kappa\phi/2} \Longrightarrow \frac{\delta\rho}{\rho} \approx H^2 e^{\gamma\kappa\phi/2}/|\dot{\psi}| \quad \text{(region B)} . \tag{3.22}$$

In these regions, the standard result for one dominant scalar field is recovered. For the microwave background constraint, all quantities above are to be evaluated at the time t_h when the perturbation originally left the horizon such that $\ln(a_f/a_h) \equiv \alpha_{f/h} \approx 65$ for scales currently entering the horizon. Using the approximate equations of motion, the density perturbation constraint may be written as

$$\frac{\delta\rho}{\rho} \sim \begin{cases} \frac{\kappa^2}{\sqrt{3}\beta} e^{-\beta\kappa\phi_h/2}\sqrt{V_h} & \text{(region A)} , \\ \frac{\kappa^3 V_h^{3/2}}{\sqrt{3}V_h'} e^{-(\beta+\gamma)\kappa\phi_h/2} & \text{(region B)} . \end{cases} \quad (3.23)$$

The last constraint we will consider is successful reheating. We take reheating to commence at the end of slow rolling of the inflaton, $\ddot{\psi} \approx 3H\dot{\psi}$, when oscillations in the inflaton field should couple to matter. This condition is equivalent to

$$(V^{1/2})'' = \frac{3\kappa^2}{2}\left[1+\frac{\beta}{3}\left[\gamma-\frac{\beta}{2}\right]\right]V^{1/2}e^{-\gamma\kappa\phi_f} . \quad (3.24)$$

However, for some potentials, the $\dot{\psi}^2$ terms in (3.1) and (3.3) become dominant before the above condition can be met. The approximations under which (3.5)–(3.7) are valid occurs for

$$e^{\gamma\kappa\phi} \leq \frac{6\kappa^2 V^2}{V'^2} . \quad (3.25)$$

Once the $\dot{\psi}$ terms become dominant, the slow-rolling condition is no longer valid. Solving the basic equations with large inflaton kinetic terms, we find that $\ddot{\psi}/3H\dot{\psi}|$ becomes order of unity for $\gamma=0$, as well as for $\gamma=\beta/2$ provided $\beta/\sqrt{6}$ is small compared to one. Therefore, when the $\dot{\psi}^2$ terms become important, the slow-rolling approximation of ψ quickly breaks down, and again reheating ensues. We thus take the end of inflation and the subsequent reheating to occur when either condition (3.24) or (3.25) is met.

For standard baryogenesis [34] through the decay of heavy Higgs bosons to account for the observed baryon asymmetry, the reheat temperature needs to be greater than 10^{10} GeV [35]. We assume efficient reheating, with all of the potential energy density at the end of inflation being converted to radiation, so that

$$T_{\rm RH} \approx \left[\frac{30e^{-\beta\kappa\phi_f}V(\psi_f)}{\pi^2 g_*}\right]^{1/4} > T_{\rm RH,min} \sim 10^{10}\ \text{GeV} , \quad (3.26)$$

where $g_*(T) \sim 100$ is the effective number of particle species. For nonstandard baryogenesis theories, reheat temperatures as low as 100 GeV are possible, and such theories would loosen the reheating constraint accordingly. Because of the presence of the negative exponential potential, there will be less potential energy at the end of inflation than in the corresponding model with just $V(\psi_f)$; however, the larger values of the self-coupling allowed by our model allow for much stronger coupling to radiation and thus make efficient reheating far more plausible. Also, in the original frame, if the present model Lagrangian is derived via a conformal transformation, $T_{\rm RH}$ in (3.26) should be replaced by that with $\beta=0$.

At this point, proceeding further is difficult without explicitly stating the model to be used for $V(\psi)$. In the next two sections we will consider new and then chaotic inflation potentials.

IV. SOFT INFLATION—NEW INFLATION TYPE

We will take the new inflation potential to be of the form

$$V(\psi) = V_0 - \frac{\lambda}{4}\psi^4 , \quad (4.1)$$

where V_0 is the GUT scale. This expression is a good approximation for the full, more complicated Coleman-Weinberg GUT potential [5]. We then find

$$f(\psi) = \kappa^2 V_0/2\lambda\psi^2 \quad \text{and} \quad \kappa\psi_f = [\kappa^4 V_0/\lambda(1-\beta^2/6)]^{1/2} , \quad (4.2)$$

where Eq. (3.24) determines the end of inflation. The case of general γ will be dealt with first, and then we will specialize to the values $\gamma=0$ and $\gamma=\beta/2$.

Approximating the potential (4.1) as just V_0 during the slow-rolling phase, Eqs. (3.7) and (3.8) yield

$$\frac{a}{a_0} = \left[\frac{1}{2}\beta^2\kappa e^{-\beta\kappa\phi_0/2}\left[\frac{V_0}{3}\right]^{1/2}(t-t_*)\right]^{2/\beta^2} , \quad (4.3)$$

where t_* is a constant. As expected, the expansion is power law, with inflation occurring for $\beta<\sqrt{2}$, just as in the standard one-field exponential potential case [31]. The behavior of ϕ is then

$$\phi-\phi_0 = \frac{2}{\beta\kappa}\ln\left[\frac{1}{2}\beta^2\kappa e^{-\beta\kappa\phi_0/2}\left[\frac{V_0}{3}\right]^{1/2}(t-t_*)\right] , \quad (4.4)$$

and Eqs. (3.9) and (4.2) give

$$\frac{1}{\psi^2} = \frac{1}{\psi_0^2} - \frac{2\lambda e^{\gamma\kappa\phi_0}}{\beta\gamma\kappa^2 V_0}\left\{\left[\frac{1}{2}\beta^2\kappa e^{-\beta\kappa\phi_0/2}\left[\frac{V_0}{3}\right]^{1/2}(t-t_*)\right]^{2\gamma/\beta} - 1\right\} . \quad (4.5)$$

Now consider the satisfaction of the inflationary constraints. Written in terms of the value of the inflaton field at the onset of inflation, the horizon, reheating and density perturbation conditions become, respectively,

$$\psi_0 < \psi_H \equiv \left[\frac{\beta\gamma\kappa^2 V_0}{2\lambda}(e^{\beta\gamma\alpha_{f/h}}-1)^{-1}\right]^{1/2} e^{-\gamma\kappa\phi_0/2} , \quad (4.6)$$

$$\psi_0 > \psi_{\rm RH} \equiv \left[\frac{3\gamma\kappa^2 V_0}{2\lambda}\right]^{1/2} e^{-\gamma\kappa\phi_0/2}\left[\left[\frac{30V_0}{\pi^2 g_*}\right]^{\gamma/\beta} e^{-\gamma\kappa\phi_0} T_{\rm RH,min}^{-4\gamma/\beta} - 1\right]^{-1/2} , \quad (4.7)$$

$$\psi_0 < \psi_D \equiv \left[\frac{\beta\gamma\kappa^2 V_0}{2\lambda}\right]^{1/2} e^{-\gamma\kappa\phi_0/2}\left\{\left[\left[\frac{\delta\rho}{\rho}\right]_{\rm cr}^{-1}\frac{\kappa^2}{\beta}\left[\frac{V_0}{3}\right]^{1/2}\right]^{2\gamma/\beta} e^{-\gamma\kappa\phi_0+\beta\gamma\alpha_{f/h}}-1\right\}^{-1/2}, \tag{4.8}$$

where the last equation assumes that the Universe will be in region A, with $\delta\rho/\rho \approx H^2/\dot{\phi}$, at 65 e-foldings before the end of inflation. Because the new inflation potential is exceptionally flat near $\psi=0$, $\dot{\psi}$ will generally be much smaller than $\dot{\phi}$ until just before reheating, so this approximation is valid, except for very small β. Explicitly, the density fluctuations will be dominated by $\dot{\phi}$ as long as

$$\psi^3 < \frac{\beta\kappa V_0}{\lambda} e^{-\gamma\kappa\phi/2} \tag{4.9}$$

at the first horizon crossing, which is almost always true. For example, with $\gamma=0$ and using (3.13) and (4.2), this condition becomes

$$\frac{\kappa^2 V_0^{1/2}}{\beta\lambda^{1/2}(2\alpha_{f/h})^{3/2}} < 1 . \tag{4.10}$$

Next we must address the issue of initial conditions. In standard new inflation, the natural initial value of the inflaton field is considered to be the Hubble parameter, $\psi_0 = H_0$ (see Brandenberger in [1]). This value comes in part from dimensional considerations, as the Hubble parameter sets a natural scale which ψ_0 would be expected to be near. The more compelling argument comes from consideration of quantum fluctuations of ψ in a curved background. For new inflation to be successful, the inflaton field must start near zero expectation value. Finite-temperature effects lead to a potential which produces this localization. Indeed, when suppression of density perturbations was shown to imply such small values of λ that thermal equilibrium could not be achieved by the onset of inflation, the new inflation scenario was abandoned. However, we will soon see that λ need not be small in soft inflation; therefore the finite-temperature potential is well defined and ψ_0 should be localized near zero. However, quantum uncertainty exists, and ψ_0 may be calculated from the two-point correlation function $\langle\psi(0)\psi(x)\rangle$[33]. Because of the noncanonical kinetic term, the usual expression of $\psi_0 = H_0$ must be multiplied by an exponential factor of ϕ in order to maintain proper normalization. We therefore take, as previously argued for Eq. (3.19),

$$\psi_0 = H_0 e^{\gamma\kappa\phi_0/2} = \left[\frac{\kappa^2}{3} V_0 e^{(\gamma-\beta)\kappa\phi_0}\right]^{1/2} \tag{4.11}$$

as the natural initial value.

With the above choice of ψ_0, we are now left with three constraints in terms of λ, V_0, β, γ, and ϕ_0. Our approach will be to fix the latter three parameters, coming from the exponential potential sector, and determine what values of the GUT potential parameters λ and V_0 produce successful inflation. We then will discuss how the allowed regions change when the latter three parameters are varied. Hence, for a particular new-inflation-type potential, one may compare the parameters with the successful values of soft inflation to see if such a scenario is possible. We plot the SU(5) GUT value as an example.

Using (4.11), the constraints (4.6)–(4.8) become

$$\lambda < \frac{3\beta\gamma e^{(\beta-2\gamma)\kappa\phi_0}}{2(e^{\beta\gamma\alpha_{f/h}}-1)}, \tag{4.12}$$

$$\lambda > \frac{3\beta\gamma}{2} e^{(\beta-2\gamma)\kappa\phi_0}\left[\left[\frac{30V_0}{\pi^2 g_* T^2_{\rm RH,min}}\right]^{\gamma/\beta} e^{-\gamma\kappa\phi_0}-1\right]^{-1}, \tag{4.13}$$

$$\lambda < \frac{3\beta\gamma}{2} e^{(\beta-2\gamma)\kappa\phi_0}\left[e^{-\gamma\kappa\phi_0+\beta\gamma\alpha_{f/h}}\left[\left[\frac{\delta\rho}{\rho}\right]_{\rm cr}^{-1}\frac{\kappa^2}{\beta}\left[\frac{V_0}{3}\right]^{1/2}\right]^{2\gamma/\beta}-1\right]^{-1}, \tag{4.14}$$

respectively. When the quantity in square brackets is negative in (4.14), this condition will always be met. We next deal specifically with the two cases of interest, $\gamma=0$ and $\gamma=\beta/2$.

A. $\gamma=0$: Fundamental exponential potential

In the case where the exponential potential comes from some fundamental theory, then the kinetic terms may be standard. Taking the limit of $\gamma=0$ carefully in Eqs. (4.12)–(4.14), we obtain

$$\lambda < \frac{3}{2\alpha_{f/h}} e^{\beta\kappa\phi_0}, \tag{4.15}$$

$$\lambda > \frac{3\beta^2}{2} e^{\beta\kappa\phi_0}\left[\ln\left[\frac{30V_0}{\pi^2 g_* T^4_{\rm RH,min}}\right]-\beta\kappa\phi_0\right]^{-1}, \tag{4.16}$$

$$\lambda < \frac{3\beta^2}{2} e^{\beta\kappa\phi_0}\left[-\beta\kappa\phi_0+\beta^2\alpha_{f/h} -2\ln\left[\frac{\sqrt{3}\beta}{\kappa^2\sqrt{V_0}}\left[\frac{\delta\rho}{\rho}\right]_{\rm cr}\right]\right]^{-1} \tag{4.17}$$

for the sufficient inflation, reheating, and density perturbation constraints, respectively. These conditions on λ and V_0, for $\beta=0.1$ and $\phi_0=10m_{\rm Pl}$ are plotted in Fig. 1, with similar results to previous work [19]. A wide range

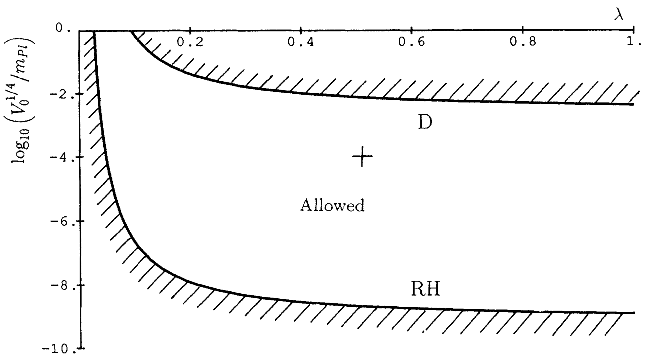

FIG. 1. The constraints from the horizon problem, reheating temperature ($T_{\rm RH} > 10^{10}$ GeV) and density perturbations ($\delta\rho/\rho < 10^{-4}$) for $\beta=0.1$, $\psi_0 = H_0$, and $\phi_0 = 10 m_{\rm Pl}$. The cross-hatched regions are not permitted, both here and in all other figures. The curves RH and D are the constraints from the reheating temperature and density perturbations, respectively. The point + corresponds to the SU(5) model with $V_0^{1/4} = 10^{15}$ GeV and $\lambda \sim \frac{1}{2}$.

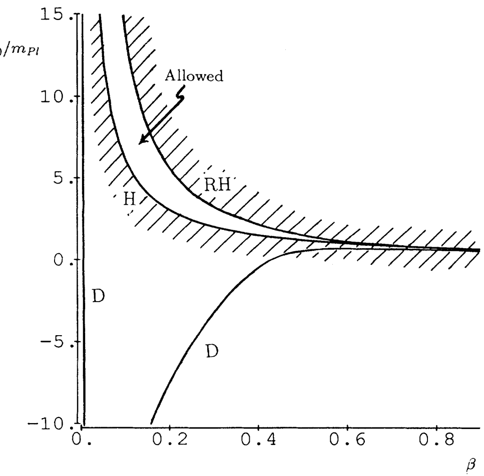

FIG. 2. The allowed region of (ϕ_0,β) parameter space for new inflation with $\gamma=0$, using the SU(5) GUT values. The curve H is from the horizon constraint.

of λ-V_0 phase space leads to successful inflation, including the standard SU(5) GUT values. Especially significant, these values of λ are large enough so that, as advertised previously, a finite-temperature potential has a well-defined meaning, and can be used to localize ψ_0 near the zero expectation value.

Next, consider the effects of changing ϕ_0 (see Fig. 2). As seen from the equations immediately above, an increase in ϕ_0 eases the sufficient inflation and density perturbation constraints, while tightening the reheating condition. Physically, the reasons are as follows. For the initial value of ψ_0 given by (4.11), an increase in ϕ_0 pushes the starting value of the inflaton field closer to zero by decreasing the initial energy density and hence the initial Hubble parameter. Therefore, a much longer period of inflation ensues, as ψ evolves very slowly for small expectation value. Consequently, there is a greater amount of expansion. This same suppression of H acts to decrease the density fluctuations: From Eq. (3.8), a larger ϕ_0 leads to a larger value ϕ_h, especially considering that a_h/a_0 will also increase. Therefore, the Hubble parameter will be smaller at the first horizon crossing. Since the density perturbation is given by $H^2/\dot{\phi}$ in the new inflation scenario, this smaller H_h leads to smaller density perturbations. In contrast, the smaller energy density makes the reheating constraint tighter. As ϕ_0 increases, the pattern of Fig. 1 is shifted so that the acceptable range of phase space is for higher λ; as ϕ_0 decreases, only lower λ can meet the constraints. For the values of Fig. 1, however, these effects on the latter two constraints are only noticeable when λ is small. Only when β is near unity will a change by a factor of 2 in ϕ_0 make an appreciable difference. The reason is that the horizontal asymptotes for the reheating and density perturbation constraints in Fig. 1 are determined by the quantities in square brackets in Eqs. (4.16) and (4.17), which are not strongly ϕ_0 dependent. The turning point where these curves begin to steeply increase is more sensitive to ϕ_0, but for a small β there will not be much change unless ϕ_0 becomes several orders above the Planck scale.

Similar to the reasoning concerning ϕ_0, a decrease in β lessens the importance of the exponential potential and hence makes the horizon and perturbation constraints more severe, while loosening the reheating condition. As β gets closer to zero, the standard new inflation model is recovered, and fully successful new inflation becomes impossible. As β becomes larger, the reheating and density constraints cannot both be met. Furthermore, as $\beta \to 1$, which is the limit of validity of our approximation, the expansion slows significantly, as seen in Eq. (4.3). The horizon constraint thus also becomes more difficult to satisfy, and indeed cannot be achieved for $\beta \geq \sqrt{2}$.

B. $\gamma=\beta/2$: Conformal exponential potential

As seen above in Sec. II, many generalized Einstein theories, when considered in the conformal frame, have $\gamma=\beta/2$. In this case, the constraints (4.6)–(4.8) become

$$\lambda < \frac{3\beta^2}{4(e^{\beta^2\alpha_{f/h}/2}-1)}\ , \tag{4.18}$$

$$\lambda > \frac{3\beta^2}{4}\left[\left[\frac{30V_0}{\pi^2 g_* T^4_{\rm RH,min}}\right]^{1/2} e^{-\beta\kappa\phi_0/2}-1\right]^{-1}\ , \tag{4.19}$$

$$\lambda < \frac{3\beta^2}{4}\left[\frac{\kappa^2}{\beta}\left[\frac{\delta\rho}{\rho}\right]^{-1}_{\rm cr}\left[\frac{V_0}{3}\right]^{1/2} e^{-\beta\kappa\phi_o/2+\beta^2\alpha_{f/h}/2}-1\right]^{-1}\ , \tag{4.20}$$

respectively. These constraints are plotted in Fig. 3 along with exact numerical results for the full system calculated using a fourth-order Runge-Kutta routine [36], and in Fig. 4. As can be seen, the approximations made are well justified by the agreement with exact results. The main difference from the $\gamma=0$ case is the significant restriction

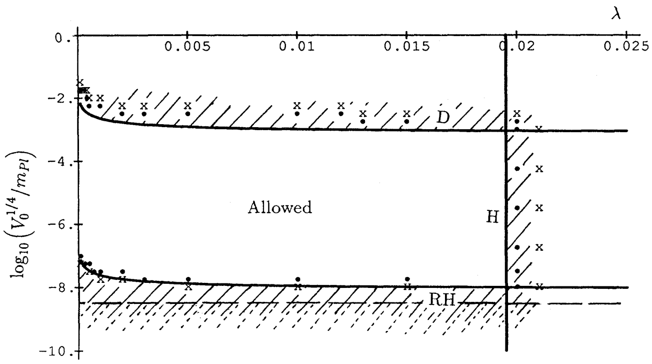

FIG. 3. The inflationary constraints for new inflation when $\gamma=\beta/2$, with $\beta=0.1$, $\psi_0=H_0$, and $\phi_0=10m_{\rm PL}$. The solid lines are in analogy with Fig. 1. The dotted line shows the reheating constraint for the GET frame. Numerical results are shown as dots if they meet all constraints, and as an X if they do not.

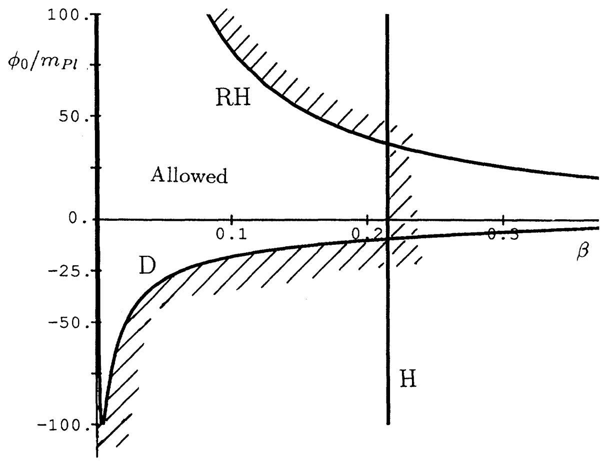

FIG. 4. The allowed region of (ϕ_0,β) phase space for new inflation with $\gamma=\beta/2$, $V_0^{1/4}=10^{15}$ GeV, and $\lambda=0.01$.

caused by the horizon constraint, corresponding to $\lambda \lesssim 0.02$ for $\beta=0.1$. As mentioned before, $\dot\psi$ will be greater in the $\gamma=\beta/2$ case, and hence inflation will terminate quicker. Therefore, the horizon constraint becomes important. The maximum that λ may be is $3/(2\alpha_{f/h})\approx 0.023$ for $\beta=0$, and changing ϕ_0 does not affect this constraint. While λ is thus an order of magnitude below typical GUT values, it is still many orders better than the standard one-field model.

The lack of adequate expansion was previously seen in the new inflation scenario with Einstein gravity, and several possibilities exist to raise the allowable value of λ. One way is to postulate a fine-tuning in the initial condition (4.11) and place ψ_0 closer to the top of the potential. Another possibility is that either a different field theory gives a flatter potential or a smaller λ. Certainly λ on the order of 0.01 still allows the new inflation scenario from thermal equilibrium considerations, a dramatic improvement over the standard one-field model. Lastly, when stochastic effects are considered, the period of slow rolling is lengthened, and therefore a larger λ could still yield a sufficient amount of expansion. The allowed region of (ϕ_0,β) parameter space is shown for the GUT energy scale but with $\lambda=0.01$ in Fig. 4. In contrast to the $\gamma=0$ case, a wide range of parameters lead to successful inflation.

The above constraints are valid in the frame where $\gamma=\beta/2$. However, this frame is derived using a conformal transformation, and is usually not considered to be the physical one. The above conditions must be transformed back into the frame with GET's present and canonical kinetic coupling for the inflaton field. As discussed in Ref. [19], the reheat constraint is the only one to change significantly; it becomes

$$V_0 > \frac{\pi^2 g_*}{30} T^4_{\rm RH,min} \tag{4.21}$$

and is plotted as the dashed line in Fig. 3.

V. SOFT INFLATION—CHAOTIC INFLATION TYPE

We will investigate chaotic inflation [10] arising from potentials of the form

$$V(\psi)=\frac{\lambda_n}{n}\psi^n \,, \tag{5.1}$$

where n is an even integer. Correspondingly,

$$f(\psi)=\kappa^2\psi^2/2n \quad \text{and} \quad \kappa^2\psi_f^2=\frac{n^2}{6}e^{\gamma\kappa\phi_f} \,, \tag{5.2}$$

where Eq. (3.25) now determines the end of inflation. As with new inflationary potentials, we first derive expressions valid for general γ, and then specialize to the cases of 0 and $\beta/2$.

When written as conditions on ψ_0, the constraints become

$$\psi_0>\psi_H\equiv\left[\frac{2n}{\beta\gamma\kappa^2}(e^{\beta\gamma\alpha_{f/h}}-1)\right]^{1/2}e^{\gamma\kappa\phi_0/2} \,, \tag{5.3}$$

$$\psi_0<\psi_{\rm RH}\equiv\left[\frac{2n}{\beta\gamma\kappa^2}\right]^{1/2}\left[\left[\frac{\pi^2 g_* n T^4_{\rm RH,min}}{30\lambda_n}\right]^{2\gamma/(n\gamma-2\beta)}\left[\frac{6\kappa^2}{n^2}\right]^{n\gamma/(n\gamma-2\beta)}e^{-\gamma\kappa\phi_0}-1\right]^{1/2}e^{\gamma\kappa\phi_0/2} \,, \tag{5.4}$$

$$\psi_0 > \psi_D \equiv \left[\frac{2ne^{\gamma\kappa\phi_0}}{\beta\gamma\kappa^2} \right]^{1/2} \times \left\{ \begin{array}{l} \left[\left[\left[\frac{\delta\rho}{\rho} \right]_{\rm cr}^{-1} \frac{\kappa^2}{\beta} \left[\frac{\lambda_n}{3n} \right]^{1/2} \left[\frac{2n}{\beta\gamma\kappa^2} \right]^{n/4} e^{\beta^2\alpha_{f/h}/2} (1-e^{\beta\gamma\alpha_{f/h}})^{n/4} \right]^{4\gamma/(2\beta-n\gamma)} e^{-\gamma\kappa\phi_0} - 1 \right]^{1/2}, \\ \left[\left[\left[\frac{\delta\rho}{\rho} \right]_{\rm cr}^{-1} \frac{\kappa^3}{n^{3/2}} \left[\frac{\lambda_n}{3} \right]^{1/2} \left[\frac{2n}{\beta\gamma\kappa^2} \right]^{(n+2)/4} e^{\beta(\beta/2+\gamma)\alpha_{f/h}} (1-e^{-\beta\gamma\alpha_{f/h}})^{(n+2)/4} \right]^{4\gamma/(2\beta-n\gamma)} e^{-\gamma\kappa\phi_0} - 1 \right]^{1/2} \end{array} \right. \tag{5.5}$$

where the density perturbation conditions are for regions A and B, respectively. The reheating and density perturbation constraints are valid for $\beta > n\gamma/2$ and flip inequality signs when the above condition on β is not met. If a quantity in square brackets becomes negative, then that constraint cannot be met for those particular parameter values. The Universe will be in regime A at t_h when

$$\frac{1}{\beta\gamma} \ln \left[1 + \frac{n\gamma}{2\beta} \right] < \alpha_{f/h} \ . \tag{5.6}$$

The ratio of scale factors is given by

$$\frac{a_h}{a_0} = e^{-\alpha_{f/h}} \left[1 + \frac{\beta\gamma\kappa^2\psi_0^2}{4n} e^{-\gamma\kappa\phi_0} \right]^{2/\beta^2} , \tag{5.7}$$

an expression which is useful in deriving the density constraints above.

With standard chaotic inflation, the usual procedure is to start considering the classical evolution of the Universe at the first possible instant when this behavior should be valid, namely at the Planck scale. The heavy damping of the scalar field rapidly eliminates any initial kinetic energy, and is usually ignored [30]. We follow the same procedure here, with our initial conditions therefore taken as

$$\frac{\lambda_n}{n} \psi_0^n e^{-\beta\kappa\phi_0} = m_{\rm Pl}^4 \ . \tag{5.8}$$

This initial condition on ψ_0 results in some simplification, as the condition for sufficient inflation may now be written as

$$\lambda_n < n m_{\rm Pl}^4 e^{(\beta - n\gamma/2)\kappa\phi_0} \left[\frac{2n}{\beta\gamma\kappa^2} (e^{\beta\gamma\alpha_{f/h}} - 1) \right]^{-n/2} . \tag{5.9}$$

Again, we now specialize to the two values of γ of physical interest, as well as to specific n.

A. $\gamma = 0$: Fundamental exponential potential

As discussed in [19], when $\dot{\psi}$ is small, $V(\psi)$ will change slowly compared to the evolution in the ϕ direction. We therefore find a power-law expansion [31], just as in the constant-V_0 case of new inflation. However, as $\dot{\psi}$ increases, the expansion deviates from the power law, and the expressions without explicit time dependence become more convenient. From Eqs. (5.9), (5.4), and (5.5), the constraints for sufficient inflation, reheating, and density perturbations, respectively, become

$$\lambda_n < n m_{\rm Pl}^4 e^{\beta\kappa\phi_0} \left[\frac{\kappa^2}{2n\alpha_{f/h}} \right]^{n/2} , \tag{5.10}$$

$$\lambda_n > n m_{\rm Pl}^4 e^{\beta\kappa\phi_0} (\beta\kappa)^n \left[-2n\beta\kappa\phi_0 + 2n \ln \left[\frac{30\lambda_n}{\pi^2 g_* n T_{\rm RH,min}^4} \right] + n^2 \ln \left[\frac{n^2}{6\kappa^2} \right] \right]^{-n/2} , \tag{5.11}$$

$$\lambda_n < n m_{\rm Pl}^4 e^{\beta\kappa\phi_0} \left\{ \begin{array}{l} \left\{ \frac{4n}{\beta^2\kappa^2} \left[\ln \left[\left[\frac{\delta\rho}{\rho} \right]_{\rm cr}^{-1} \frac{\kappa^2}{\beta} \left[\frac{\lambda_n}{3n} \right]^{1/2} \left[\frac{2n\alpha_{f/h}}{\kappa^2} \right]^{n/4} \right] - \beta\kappa\phi_0/2 + \beta^2\alpha_{f/h}/2 \right] \right\}^{-n/2} , \\ \left\{ \frac{4n}{\beta^2\kappa^2} \left[\ln \left[\left[\frac{\delta\rho}{\rho} \right]_{\rm cr}^{-1} \frac{\kappa^3}{n^{3/2}} \left[\frac{\lambda_n}{3} \right]^{1/2} \left[\frac{2n\alpha_{f/h}}{\kappa^2} \right]^{(n+2)/4} \right] - \beta\kappa\phi_0/2 + \beta^2\alpha_{f/h}/2 \right] \right\}^{-n/2} , \end{array} \right. \tag{5.12}$$

with the density conditions for regions A and B, in that order, and $\beta^2\alpha_{f/h} > n/2$ the condition to be in region A at the first horizon crossing. Both the reheating and density constraints also have a λ_n term on the right-hand side; however, as the logarithm is taken, its effects may easily be accounted for by an iterative procedure. The allowable region of λ_n-β space is plotted in Figs. 5 and 6, for the cases of $n=2$ and $n=4$, with $\phi_0 = 10 m_{\rm Pl}$. Unfortunately, only a small region of parameter space is allowed. While the permitted region does occur for the desirable values of the coupling constant being near unity, the narrowness of this region makes this scenario of chaotic inflation with canonical coupling highly unlikely.

Changing the initial value ϕ_0 is of little avail, as the pattern of the figures is merely shifted, with a slender region of phase space remaining. When ϕ_0 is increased, the

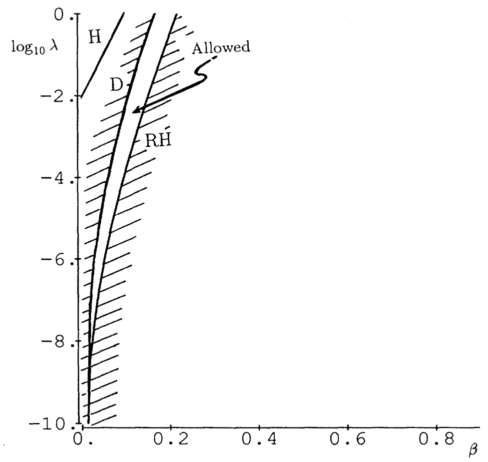

FIG. 5. The inflationary constraints for $n=4$ chaotic inflation with $\gamma=0$ and $\phi_0=10m_{\rm Pl}$.

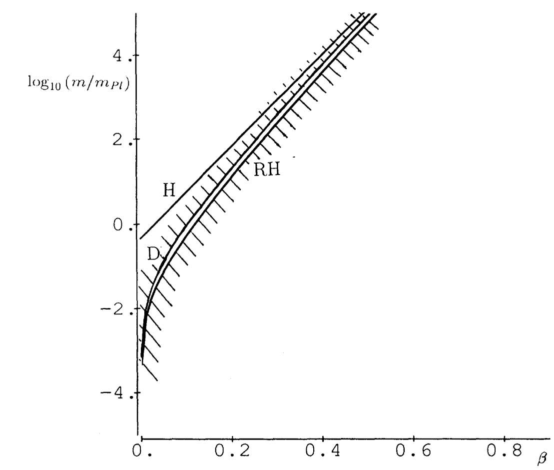

FIG. 6. The inflationary constraints for $n=2$ (massive) chaotic inflation with $\gamma=0$ and $\phi_0=10m_{\rm Pl}$.

density constraint is loosened, while the reheating condition becomes more severe. As mentioned earlier, a larger ϕ_0 produces a smaller effective coupling, and hence a much larger λ_n can still suppress density perturbations. Concurrently, the smaller exponential potential at the end of the inflation results in less energy density and hence greater difficulty in reheating. Furthermore, even if a much lower reheating temperature is allowed by a different mechanism of baryogenesis, there is little change in the reheating constraint due to the logarithmic dependence in Eq. (5.11).

B. $\gamma=\beta/2$: Conformal exponential potential

From Eqs. (3.5)–(3.7), the time dependence is found to be

$$\psi=\begin{cases}\left[\psi_0^{(4-n)/2}+\dfrac{n-4}{2\kappa}\left[\dfrac{n\lambda_n}{3}\right]^{1/2}(t-t_0)\right]^{2/(4-n)} & (n\neq4)\,,\\[2ex] \psi_0\exp\left[-\dfrac{2}{\kappa}\left[\dfrac{\lambda_4}{3}\right]^{1/2}(t-t_0)\right] & (n=4)\,,\end{cases}\tag{5.13}$$

for the inflaton and hence the scale factor is given by

$$\left[\frac{a}{a_0}\right]^{\beta^2/2}-1=\begin{cases}\dfrac{\beta^2\kappa^2}{4n}e^{-\beta\kappa\phi_0/2}\psi_0^2\left[1-\left[1+\dfrac{n-4}{2\kappa}\left[\dfrac{n\lambda_n}{3}\right]^{1/2}\psi_0^{(n-4)/2}(t-t_0)\right]^{4/(4-n)}\right] & (n\neq4)\,,\\[2ex] \left[\dfrac{\beta\kappa\psi_0}{4}\right]^2 e^{-\beta\kappa\phi_0/2}\left\{1-\exp\left[-\dfrac{4}{\kappa}\left[\dfrac{\lambda_4}{3}\right]^{1/2}(t-t_0)\right]\right\} & (n=4)\,.\end{cases}\tag{5.14}$$

By fixing, say ϕ_0 and n, we may plot the allowable range of λ_n-β parameter space as given by the implicit Eqs. (5.4) and (5.5). Below we give special attention to the renormalizable cases of n equal to 2 and 4. When $n=4$, a more careful treatment must be made. The constraints then become

$$\lambda_4<\pi^2\beta^4(e^{\beta^2\alpha_{f/h}/2}-1)^{-2}\,,\tag{5.15}$$

$$\lambda_4\gtrsim100g_*\left[\frac{T_{\rm RH,min}}{m_{\rm Pl}}\right]^4,\tag{5.16}$$

$$\lambda_4<\begin{cases}\dfrac{3\beta^6}{64}\left[\dfrac{\delta\rho}{\rho}\right]_{\rm cr}^2(e^{\beta^2\alpha_{f/h}/2}-1)^{-2} & (\text{region A: } \beta^2\alpha_{f/h}>2\ln2)\,,\\[2ex] \dfrac{3\beta^6}{64}\left[\dfrac{\delta\rho}{\rho}\right]_{\rm cr}^2(e^{\beta^2\alpha_{f/h}/2}-1)^{-3} & (\text{region B: } \beta^2\alpha_{f/h}<2\ln2)\,,\end{cases}\tag{5.17}$$

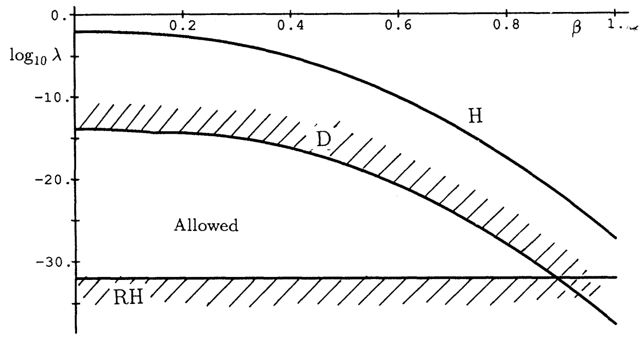

FIG. 7. The inflationary constraints for $n=4$ chaotic inflation with $\gamma=\beta/2$. The results are ϕ_0 independent, due to conformal invariance.

for the horizon, reheating, and density perturbation constraints, respectively. That the reheating condition has no dependence on any parameters is not surprising, resulting from the fact that the $n=4$ case is conformally invariant and hence independent of the exponential sector. For the other conditions, the appearance of a dependence on β results from the initial condition (5.8), whereas the amount of reheating only depends on the final values.

These results are plotted in Fig. 7. Disappointingly, the density perturbations force λ_4 to be excessively small, around 10^{-12}, in contrast with the natural values possible in the $\gamma=0$ case. While this scenario certainly is not ruled out, there is little benefit compared to standard $n=4$ chaotic inflation. Because the inflaton evolves faster compared to the $\gamma=0$ case, inflation will terminate sooner, and thus ϕ will evolve less, resulting in little change to the effective λ_n. Changing ϕ_0 is of no avail; because of conformal invariance, there is no ϕ_0 dependence, as Eqs. (5.15)–(5.17) show.

The case of $n=2$, corresponding to a massive scalar field, is also of physical interest. Along with the $n=4$ case discussed above, these are the only powers of ψ which both lead to chaotic inflation and are renormalizable in standard field theory. Writing $\lambda_2=m^2$, we find

$$\frac{m}{m_{\rm Pl}} < \sqrt{2\pi}\beta e^{\beta\kappa\phi_0/4}(e^{\beta^2\alpha_{f/h}/2}-1)^{-1/2}\,, \tag{5.18}$$

$$\frac{m}{m_{\rm Pl}} > \left[\frac{8g_*}{5}\right]^{1/4}\frac{T_{\rm RH,min}}{m_{\rm Pl}}\pi\beta e^{\beta\kappa\phi_0/4}\,, \tag{5.19}$$

$$\frac{m}{m_{\rm Pl}} < \left[\frac{\delta\rho}{\rho}\right]_{\rm cr}^{1/2}\beta^{3/2}e^{\beta\kappa\phi_0/4}\begin{cases}(\frac{3}{16})^{1/4}(1-e^{-\beta^2\alpha_{f/h}/2})^{-1/4}e^{-\beta^2\alpha_{f/h}/4} & (\beta^2\alpha_{f/h}>2\ln\frac{3}{2})\,,\\ (\frac{3}{32})^{1/4}(1-e^{-\beta^2\alpha_{f/h}/2})^{-1/2}e^{-3\beta^2\alpha_{f/h}/8} & (\beta^2\alpha_{f/h}<2\ln\frac{3}{2}a\end{cases}\,, \tag{5.20}$$

for the horizon, reheating, and density perturbation constraints. These conditions are plotted in Fig. 8, along with numerical computations, for $\phi_0=10m_{\rm Pl}$. There is an improvement of several orders of magnitude over the standard chaotic scenario, with the mass being as high as 0.01 times the Planck scale. Thus, when soft inflation arises from conformal transformations of GET's, quite reasonable values of the mass are possible.

The $n=2$ case is further successful under a wide range of ϕ_0, as illustrated in Fig. 9. Thus, chaotic inflation in GET's can be realized for a massive potential with quite natural values of all parameters, including the mass itself.

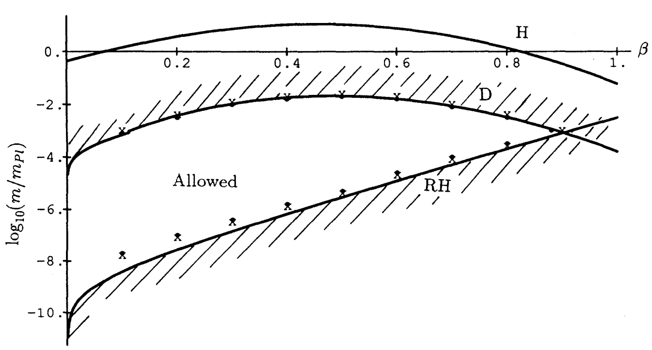

FIG. 8. The inflationary constraints for $n=2$ (massive) chaotic inflation with $\gamma=\beta/2$ and $\phi_0=10m_{\rm Pl}$. Exact numerical results are given by dots and X's as in Fig. 3.

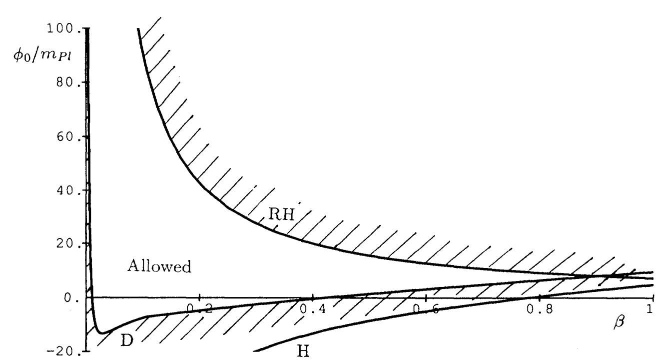

FIG. 9. Inflationary constraints on (ϕ_0,β) space for $n=2$ chaotic inflation with $\gamma=\beta/2$ and $m=10^{-4}m_{\rm Pl}$.

Just as with the new inflation scenario, the main change is the softening of density perturbations due to the simultaneous rolling of two fields.

VI. CONCLUSION AND REMARKS

We have investigated the effects of an exponential potential multiplicatively coupled to the standard new and chaotic inflation potentials. When the kinetic terms are standard, the inflaton coupling constant can be near unity [19]. If the coupling is found in the conformally transformed frame of generalized Einstein theories, or GET's, then the inflaton kinetic term also contains exponential coupling. For new inflation, this coupling tightens the horizon problem constraint, although not to a severe degree. With either kinetic term, a successful new inflation scenario is achieved. For $n=4$ chaotic inflation, the above nonstandard kinetic term forces the self-coupling to near the same excessively small values of the standard one-field case, removing the advantages found with canonical coupling. One may find comfort that GET's, which arise in many string theories and renormalization of quantum field theories in curved spacetime, are still compatible with $\lambda_4\psi^4$ inflation; nonetheless, the continued need for fine tuning is a disappointment. In the $n=2$ (massive) chaotic case with this kinetic term, all constraints can be met for a mass as large as $10^{-2}m_{\rm Pl}$. Furthermore, such natural values are found in a wide range of parameter space, as seen in Fig. 9. Table III gives a summary of the results found in this paper. The successes of soft inflation seem to merit further inquiry into this theory.

As noted above, our results are all derived in the "Einstein" frame, with standard gravity. However, if our Universe has nonstandard gravity, then the "GET" frame is the physical one. Upon transforming back to the GET frame, only the reheating condition is expected to change significantly [37], and this constraint should be loosened. In addition, the initial condition for chaotic inflation will also change, as the effective Planck scale in the GET frame should be used to set the initial energy scale. Since the reheating and density perturbation constraints are determined by values at the end of inflation, and the horizon condition is usually not a problem in chaotic inflation, we expect little change in our results. A full discussion of the relations between quantities in the two frames will be presented in a later paper.

One promising avenue for future research with soft inflation is the formation of large-scale structure. While we have considered the constraint from suppression of density perturbations at current horizon scales, inflation produces these fluctuations at all scales. The standard new and chaotic models predict an almost scale-free spectrum [7]; however, models with exponential potentials are known to give a spectrum with increasing power at larger scales [31]. Observational evidence is starting to indicate more structure at large scales than previously thought. Furthermore, as soft inflation proceeds, the dominant contribution to $\delta\rho/\rho$ will change [see Eqs. (3.21) and (3.22)], setting natural scales at which the behavior of large-scale structure will change. Thus soft inflation contains a greater richness of structure formation than standard one-field inflation models.

An examination of the R^2 theory, which we expect not to soften the inflationary constraints [38], is instructive, as it illuminates what is required in a successful model. In order for the soft inflation scenario to ease the constraints arising from density perturbations, both fields must be slowly rolling at horizon crossing. The potential in the conformal frame contains two pieces [11,28]. The $U(\phi)$ term fixes the value of the exponential field, corresponding to $R=0$, in the $f(R)$ theory, in contrast with the continuously rolling ϕ of the JBD theory. Once ϕ becomes fixed, the softening of the constraints will cease, and behavior similar to the conventional inflation scenarios will ensue. If the $f(R)$ sector is initially such that ϕ rolls for a sufficiently long time, then the inflation ψ may become fixed first. With only ϕ evolving, the fine-tuning problem of $f(R)$ gravity [11] still exists. Both fields must end their evolution at roughly the same time, or else excessive density fluctuations will exist, and such an occurrence seems difficult to imagine except by contrived means.

The difference with the models considered in this paper is the absence of a $U(\phi)$ piece. Hence ϕ rolls throughout inflation, which ends when the inflaton nears its potential minimum. There is no separate $U(\phi)$ potential to continue the rapid expansion after the inflaton stops its evolution. In some string models or extensions of GET's potentials are added to fix the value of ϕ. These potentials must not end the evolution of ϕ until inflation has terminated, or else the softening effect will be mitigated.

If the minimum of $V(\psi)$ is zero, then after the termination of inflation, (3.1) implies that ϕ behaves like a damped oscillator without a driving term. Its kinetic en-

TABLE III. Summary of results, for new and chaotic inflation potentials with both usual and nonstandard kinetic coupling. Both compatibility and naturalness are considered.

Potential	$\gamma=0$	$\gamma=\beta/2$
New inflation ($V=V_0-\frac{1}{4}\lambda\psi^4$)	GUT's: compatible fine tuning of initial data	$\lambda\leq 0.02$ (not extremely small), but simple SU(5): excluded no fine tuning
Chaotic inflation ($V=\frac{1}{4}\lambda\psi^4$)	$\lambda\sim 1$: compatible fine tuning of λ for given β	$\lambda\leq 10^{-12}$
Chaotic inflation ($V=\frac{1}{2}m^2\psi^2$)	$m\leq m_{\rm Pl}$: compatible fine tuning of m for given β	$m\leq 10^{-(2\sim 3)}m_{\rm Pl}$ no fine tuning

ergy decreases as a^{-6}, and quickly becomes negligible. However, if the minimum is not zero, then the potential term acts as an effective decaying cosmological "constant." The effects of such a term on the subsequent evolution of the Universe and the possibility of it acting as dark matter was examined in detail for a negative-power-law potential in [39], with the exponentially decreasing case also discussed. Our preliminary work indicates that the energy contained in the scaler field will grow faster than radiation until it dominates. For the matter and scalar contributions to be of the same order today, the minimum of $V(\psi)$ must be fine tuned to a very small value. However, if this value is related to some other energy scale, for example the schizon mass [40], then the scenario of a scalar field rolling on an exponential potential today becomes more attractive. Such work is currently in progress.

Finally, we note that while we took ϕ_0 as being near unity for "naturalness," there is in fact no real argument for determining this value. Perhaps the transition from quantum to classical gravity in a more fundamental theory such as superstrings will be able to predict plausible values of ϕ_0. Another possibility is to utilize quantum cosmology.

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