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Shapes of solar-neutrino spectra: Unconventional tests of the standard electroweak model

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The shapes of the neutrino energy spectra produced by β decays or by nuclear reactions in the solar interior are compared to the corresponding spectral shapes that are produced in a terrestrial laboratory. The most important effects of the Sun, caused by thermal motion of the neutrino-emitting nuclei and by gravitational redshifts, are shown to be small. Any currently measurable difference between the observed shape of a solar-neutrino energy spectrum and the spectrum shape for a terrestrial neutrino source must be due to a departure from the standard electroweak model.

I. INTRODUCTION

In this paper, I show that the energy spectrum of neutrinos created by a specific nuclear fusion reaction or by a specific β-decay reaction is the same independent of whether the neutrinos are produced in the center of the Sun or in a terrestrial laboratory, provided the standard theory of electroweak interactions [1–3] is correct. The absolute number of neutrinos produced by each fusion reaction or β-decay can depend sensitively upon parameters of the solar interior. For example, the computed flux of ^{8}B solar neutrinos, $\phi(^8B)$, varies approximately like $\phi(^8\mathrm{B}) \propto T^{18}_{\mathrm{central}}$, where T_{central} is the central temperature of the solar model. Nevertheless, the shape of the neutrino energy spectrum from any specific reaction or decay will be shown to be essentially independent of all solar parameters. This result is valid to the accuracy achievable in currently practical experiments. Thus any measured departure from the standard calculated neutrino spectrum will be prima facie evidence for physics beyond the standard electroweak model, without regard to specific proposals for new physics such as the Mikheyev-Smirnov-Wolfenstein [4] (MSW) effect. At present, there is no direct experimental evidence that the spectrum of energies from a specific solar fusion reaction or decay is different from the spectrum predicted by the standard electroweak model.

Assuming some well-defined departure from the standard electroweak model, many authors have calculated changes in the energy spectrum of solar neutrinos. However, this is the first paper with which I am familiar that calculates the dependence on solar parameters of the energy spectrum from a given fusion reaction or β decay. There is also large and important literature on precision tests of electroweak theory [5]. The discussion in the present paper is unconventional in that it focuses on a generic prediction that applies to sources produced in a nonterrestrial laboratory.

In addition to improved calculations of the energy loss by solar neutrinos, this paper presents two main results, one for β decay of free nuclei (such as ^{8}B or ^{13}N) and one for nuclear fusion reactions [such as the pp or $^3\mathrm{He}+p$ (hep) reactions]. For β decays, I show that the spectrum $P_{\odot}(q)$ of neutrinos of energy q that are produced in the Sun in an interval dq is related to the spectrum of energies determined from laboratory experiments with terrestrial sources, $P_{\mathrm{lab}}(q)$, by the relation

$$P_{\odot}(q)=P_{\mathrm{lab}}(q)[1+O(10^{-5})] , \tag{1}$$

where the largest modification of the laboratory spectrum is caused by the general-relativistic redshift of the neutrino energies. For nuclear fusion reactions, the energy spectrum produced in the sun is related to the spectrum produced in the laboratory by the equation

$$P_{\odot}(q,Q)=P_{\mathrm{lab}}(q,Q+E_0) , \tag{2}$$

where Q is the difference between initial and final nuclear states that is measured in the laboratory and E_0 is the relative kinetic energy in the center of mass of the fusing particles. For the pp reaction, the relation given in Eq. (2) corresponds to a 1% change in the shape of the neutrino energy spectrum. The change in the shape of the hep neutrino spectrum is less than 0.1%. The invariance of the shape of the ^{8}B neutrino energy spectrum can be used to predict the rate of neutrino events in the ^{37}Cl experiment [6] (threshold 0.8 MeV) given the rate of neutrino events in the Kamiokande II experiment [7] (threshold 7.5 MeV). A comparison of the neutrino event rates in the two experiments shows that the measurements are not consistent if the standard version of electroweak physics is correct [8].

The results given in Eq. (1) and Eq. (2) constitute testable predictions of the standard electroweak model. For example, the deuterium solar-neutrino experiment [9] [Sudbury Neutrino Observatory (SNO)] will measure accurately the shape of the ^{8}B neutrino energy spectrum. The neutrino reaction that is most easily studied in the SNO detector is neutrino absorption by deuterium via the charged-current process

$$\nu_e + d \rightarrow p + p + e^- \ . \tag{3}$$

The energy spectrum of the neutrinos can be determined by measuring the energy of the electrons produced in the reaction shown in Eq. (3); the recoiling deuteron takes away a major fraction of the momentum but a negligible amount of energy. Moreover, the relatively large energy threshold in the deuterium experiment ensures that the observed neutrinos are from a single nuclear process, ^{8}B β decay, over nearly all the accessible energy range. The electron scattering reaction

$$\nu + e \rightarrow \nu' + e' \tag{4}$$

is studied, for example, in the Kamiokande II [7] and Super Kamiokande [10] experiments, which measure the energies of individual electrons produced by neutrino scattering. These experiments are sensitive almost entirely to ^{8}B neutrinos and provide useful information about the shape of the ^{8}B neutrino energy spectrum. However, there are nonadiabatic MSW solutions that predict for Eq. (4) an energy spectrum that is indistinguishable [8] from $P_{\text{lab}}(q)$. Radiochemical experiments, for example, with ^{37}Cl [6] or with ^{71}Ga [11], do not provide unique information on the shape of the energy spectrum from a specific neutrino source. Instead, the radiochemical experiments each determine one number, the total rate of neutrino events produced by all neutrino sources.

The test of the electroweak model that is discussed in this paper is complimentary to the neutral-current tests using solar neutrinos that have been stressed by Chen [9], Weinberg [12], and Raghavan [13]. These authors have emphasized that the total number of neutrino events observed in a neutral-current reaction is a measure of the total number of neutrinos produced in the Sun. Two neutral-current reactions have been discussed relatively often in the literature; they are the neutrino disintegration of the deuteron [9],

$$\nu + d \rightarrow \nu + p + n \ , \tag{5}$$

and the neutrino excitation of ^{11}B [14],

$$\nu + {}^{11}\text{B} \rightarrow \nu + {}^{11}\text{B}^* \ . \tag{6}$$

Other experiments have been proposed [15] in which neutrinos interacting by the neutral current with electrons or with nucleons would be detected by bolometric means, by a time projection chamber, or by phonons. Measurements of the total number of neutral-current reactions test most directly the standard solar model rather than the standard electroweak interactions because the number of neutrino events observed in reactions such as Eq. (5) and Eq. (6) is compared with the number calculated using a solar model. Of course, one can combine neutral-current measurements with other solar-neutrino measurements to test in different ways the standard electroweak model.

I discuss in Sec. II the velocity distribution of nuclei that are responsible for neutrino production in the Sun. In Sec. III I calculate the neutrino energy spectrum produced by a nuclear β decay in the solar interior, and in Sec. IV I derive the neutrino energy spectrum for a nuclear fusion reaction. Thermal effects increase slightly the average steller energy loss via neutrinos. I calculate in Sec. V the increased energy loss. In Sec. VI, I discuss the effects of neutrino energy spectra of atomic binding energies, excited nuclear states, electron captures, and the Pauli exclusion principle. I summarize and discuss the main results in Sec. VII.

II. THERMAL EQUILIBRIUM

Neutrinos are produced in the Sun by nuclear fusion reactions, β decay, and electron capture. Are the nuclei responsible for neutrino production in thermal equilibrium with the ambient plasma? To answer this question, we compare the characteristic time τ_{Coulomb} for significant energy exchanges by Coulomb collisions with the characteristic time it takes to produce a neutrino in the solar interior, $\tau_{\text{production}}$. The typical time required to reach thermal equilibrium for ions of energy E in a hydrogenic plasma of density ρ is of order [16]

$$\tau_{\text{Coulomb}} \sim 10^{-12} \text{ s} \left[\frac{E}{20 \text{ keV}} \right]^{3/2} \left[\frac{150 \text{ g cm}^{-3}}{\rho} \right] . \tag{7}$$

The nuclei that are moving in the plasma of the solar interior have characteristic energies of order keV. However, the nuclei that β decay are produced by nuclear reactions that immediately precede the β decay. For example, the ^{7}Be$(p,\gamma)^8$B reaction produces ^{8}B nuclei with a significant recoil energy; the ^{8}B nuclei subsequently β decay. The typical nuclear recoil energy that results from a reaction in the solar interior is of order

$$E_{\text{recoil}} = \frac{(Q/c)^2}{2M_A} \sim \left[\frac{50}{A} \right] \left[\frac{Q}{10 \text{ MeV}} \right]^2 \text{keV} . \tag{8}$$

Here Q is the exothermic energy release in the nuclear reaction and A is the atomic number of the recoiling nucleus. For energies of order E_{recoil}, the characteristic Coulomb interaction time, Eq. (7), is increased by only a moderate factor with respect to the interaction times of typical solar ions.

Table I lists the reactions that produce solar neutrinos and shows in the second column the order of magnitude of the neutrino production times [16], which vary from seconds to many billions of years. In all cases,

$$\tau_{\text{Coulomb}} \ll \tau_{\text{production}} , \tag{9}$$

where τ_{Coulomb} is very much less than a second [see Eq. (7)]. I will assume, therefore, that solar neutrinos are produced by nuclei which are in thermal equilibrium with the ambient plasma.

The normalized distribution function for nuclei with masses M_A is

$$f(v_z) = (M_A/2\pi kT)^{1/2} \exp(-M_A v_z^2/2kT) , \tag{10}$$

where v_z is the component of the velocity in the z direction and T is the temperature. The characteristic ratio of v_z/c is

$$\frac{\langle v_z^2 \rangle^{1/2}}{c} = \left[\frac{kT}{M_A c^2} \right]^{1/2} \sim 3 \times 10^{-4} \left[\frac{8}{A} \right]^{1/2} . \tag{11}$$

TABLE I. Energy loss by neutrinos.

Neutrino source	$\tau_{\rm prod}$	$\langle q \rangle_0$ (keV)	Δq (keV; $T_6=15$)	$\langle q \rangle_{\rm total}$ (keV; $T_6=15$)
pp	10^{10} yr	268.5	3.6	272.1
pep	10^{12} yr	1,442.2	3.6+1.6	1,447.4
hcp	10^{12} yr	9,624.9	5.4	9,630.3
^{7}Be	10^{-1} yr	812.6	1.4	814.0
^{8}B	1 s	6,710.0	-	6,710.0
^{13}N	10^3 s	706.7	-	706.7
^{15}O	10^2 s	996.5	-	996.5
^{17}F	10^2 s	999.4	-	999.4

The small value of v_z/c in the solar interior prevents thermal effects from significantly influencing the shape of the neutrino energy spectrum from a given reaction.

III. β DECAY: ^{8}B, ^{13}N, ^{15}O, ^{17}F

The β decay of an unstable nucleus provides the simplest example of how neutrino energy spectra are affected by solar interior conditions. The β decay of ^{8}B produces the neutrinos that are expected [16] to dominate the ^{37}Cl [6], Kamiokande II [7], SNO [9], Super Kamiokande [10], and Borexino [14] experiments. The process of interest is

$$^8\mathrm{B} \rightarrow {}^8\mathrm{Be}^* + e^+ + \nu_e \ . \tag{12}$$

Solar conditions also affect in the same way the production of neutrinos via ^{13}N, ^{15}O, and ^{17}F β decays; however, these CNO β decays are much more difficult to detect experimentally [16]. I first discuss effects due to the thermal motion of the decaying nuclei and then evaluate the effect of the gravitational potential of the solar interior.

A. Thermal corrections

The probability $P_\odot dq$ that a neutrino is produced in the solar interior with an energy q can be calculated using the laboratory neutrino energy spectrum $P_{\rm lab}$. The differential contribution $\Delta P_\odot(q)$ to the observed solar neutrino spectrum can be expressed in terms of the laboratory spectrum $P_{\rm lab}(q_0)$ with the aid of $f(v_z)$, the one-dimensional distribution function [Eq. (10)]. Here $v_z(q,q_0)$ can be thought of as the velocity that causes a neutrino energy q_0 in the rest frame of the decaying nucleus to appear as an energy q in the observed frame. A fraction $f(v_z)dv_z$ of the β decays of nuclei moving with velocities near v_z and emitting neutrinos with energies near q_0 will produce solar neutrinos with energies near q. Thus

$$\Delta P_\odot(q)dq = P_{\rm lab}(q_0)dq_0 f(v_z(q,q_0))dv_z(q,q_0) \ . \tag{13}$$

While it is waiting to decay, a nucleus collides with surrounding ions and changes its velocity. I make the usual assumption of ergodicity; namely, that the average over time is equivalent to an ensemble average over nuclei. Taking account of the Doppler shift

$$q = (1 - v_z/c)q_0 \tag{14a}$$

and its differential relation

$$dq_0 = (1 + v_z/c)dq \ , \tag{14b}$$

the observed spectrum can be written

$$P_\odot(q) = \int_{-\infty}^{\alpha(q)c} dv_z (1 + v_z/c) P_{\rm lab}(q(1 + v_z/c)) f(v_z) \ . \tag{15}$$

The upper velocity limit in Eq. (15)

$$\alpha(q)c \equiv \left[1 - \frac{q}{q_{0,\rm max}} \right] c \tag{16}$$

corresponds to the maximum laboratory neutrino energy $q_{0,\rm max}$ being shifted to the energy q. For the β decays of interest, the values of $q_{0,\rm max}$ are 1.2 MeV (^{13}N), 1.7 MeV (^{15}O), 1.7 MeV (^{17}F), and ~ 15 MeV (^{8}B).

Using the fact that $v/c \ll 1$, one can write

$$P_{\rm lab}(q_0) \approx P_{\rm lab}(q) + \frac{dP_{\rm lab}(q)}{dq} \left[\frac{q v_z}{c} \right] \ . \tag{17}$$

Therefore,

$$P_\odot(q) = P_{\rm lab}(q)[1 - I(q, \langle v_z^2 \rangle)] + O(\langle v_z^2 \rangle / c^2) \ , \tag{18}$$

where

$$I \equiv \int_{\alpha(q)c}^{\infty} dv_z f(v_z) \tag{19}$$

and $\langle v_z^2 \rangle$ is the one-dimensional velocity dispersion. Since

$$\int_y^\infty dx \exp(-x^2) < (2y)^{-1} \exp(-y^2) \ , \tag{20}$$

the extra term in Eq. (18) is

$$I \le \left[\frac{1}{2\pi} \left[\frac{\langle v_z^2 \rangle}{\alpha(q)^2 c^2} \right] \right]^{1/2} \exp\left[-\frac{\alpha^2(q)c^2}{2\langle v_z^2 \rangle} \right] \ . \tag{21}$$

If q is not too close to the end-point energy, then the extra term I is very small. In order that I be less than 10^{-6}, it is sufficient that

$$\alpha \gtrsim 3 \times 10^{-3} \ , \tag{22}$$

or

$$q \lesssim 0.997 q_{0,\rm max} \ . \tag{23}$$

I have performed numerical integrations of the β decay spectra of ^{8}B, ^{13}N, ^{15}O, and ^{17}F which show that

$$\int_{0.997q_{0,\max}}^{\infty} dq\, P(q) < 10^{-4} . \quad (24)$$

Hence, in all practical situations in which good counting statistics are available, the inequality expressed by Eq. (23) will be satisfied.

There is also a finite probability, linear in v/c, that a neutrino will be blueshifted beyond the end-point energy of the laboratory spectrum. This is a systematic effect that could conceivably be searched for in a specially designed experiment. However, the probability of this happening is small because the Doppler velocities are small and because very few neutrinos are produced with energies close to the maximum energy. The neutrino energy in the rest frame of the decaying particle must be close to the laboratory maximum $q_{0,\max}$ since the Doppler velocities of the β decaying nuclei are much less than the velocity of light. For example, one can show, using Eqs. (10), (11), and (20), that only a fraction of order 5×10^{-4} of the decaying nuclei have velocities large enough to shift the rest energy q_0 to the slightly larger value q where

$$|q-q_0| \simeq 10^{-3}\left[\frac{8}{A}\right]^{1/3} q_0 . \quad (25)$$

In order to have any significant chance of being blueshifted beyond the end point, the neutrino energy in the rest frame of the decaying particle must lie within keV of the end-point energy, i.e., in an energy region in which practically no neutrinos are created. The fraction F of emitted neutrinos that have observed energies in excess of the laboratory maximum energy is

$$F=\pi^{-1/2}\int_0^{\infty} dx\, \exp(-x^2)\int_{q_{\min}}^{q_{0,\max}} dq\, P(q) , \quad (26)$$

where

$$q_{\min}=q_{0,\max}(1-v/c) \simeq q_{0,\max}[1-3\times10^{-4}(8/A)^{1/2}x] . \quad (27)$$

I have integrated numerically Eq. (26) for the β decay spectra of interest and have found that the fraction F is much less than 10^{-6}.

Therefore,

$$P_{\odot}(q)=P_{\text{lab}}(q)[1+O(v^2/c^2)] =P_{\text{lab}}(q)[1+O(10^{-6})] . \quad (28)$$

B. General-relativistic correction

The potential well of the solar interior causes a redshift of the neutrino energies and therefore changes slightly the emergent neutrino spectrum. The neutrino energy that is observed at Earth, q_{observed}, is related to the energy with which the neutrino is produced in the solar interior $q_{\odot}$ by [17]

$$q_{\text{observed}}=(1+2\phi)^{1/2}q_{\odot}\cong(1+\phi)q_{\odot} . \quad (29)$$

Here ϕ is the gravitational potential at the point at which the neutrino is produced and I have assumed the weak-field approximation, which is sufficiently accurate for this problem. Since ϕ is negative, neutrinos are shifted down in energy as they emerge from the Sun. The spectrum of neutrinos that emerges from the Sun $P_{\odot}(q)$ is related to the energy spectrum that is produced in the laboratory $P_{\text{lab}}(q)$ by the relation

$$P_{\odot}(q)=P_{\text{lab}}(q(1-\phi)) . \quad (30)$$

The potential at any point r within the sun can be expressed in terms of the potential at the surface $\phi(R_{\odot})$ as follows:

$$\frac{\phi(r)}{\phi(R_{\odot})}=\left[1+c^{-2}G\int_r^{R_{\odot}} dx\, M(x)x^{-2}\right] , \quad (31)$$

where $R_{\odot}$ is the solar radius. Figure 1 shows the potential as a function of radius r that is derived from the standard solar model [18] by using Eq. (31). The value of the gravitational potential on the surface of the Sun is

$$\phi(R_{\odot})=-\frac{GM_{\odot}}{R_{\odot}c^2}=-2.1\times10^{-6} , \quad (32)$$

where $M_{\odot}$ is the solar mass.

The production of ^{8}B neutrinos peaks near $0.05R_{\odot}$, at which point the gravitational potential is

$$\phi(0.05R_{\odot})=-4.9\frac{GM_{\odot}}{R_{\odot}c^2}\cong-1\times10^{-5} . \quad (33)$$

The value at the center of the Sun is $\phi(0)\cong5.02\phi(R_{\odot})$, in good agreement with the value of $5.08\phi(R_{\odot})$ calculated by Gould [19] using an earlier version of the standard solar model.

Combining Eq. (30) and Eq. (33), we see that

$$P_{\odot}(q)=P_{\text{lab}}^{(q)}(q)[1+O(\phi)]=P_{\text{lab}}(q)[1+O(10^{-5})] , \quad (34)$$

which is the same as Eq. (1). Initially, one might be surprised that the gravitational potential produces a correction that is an order of magnitude larger than the thermal, v^2/c^2 terms. Indeed, the viral theorem states

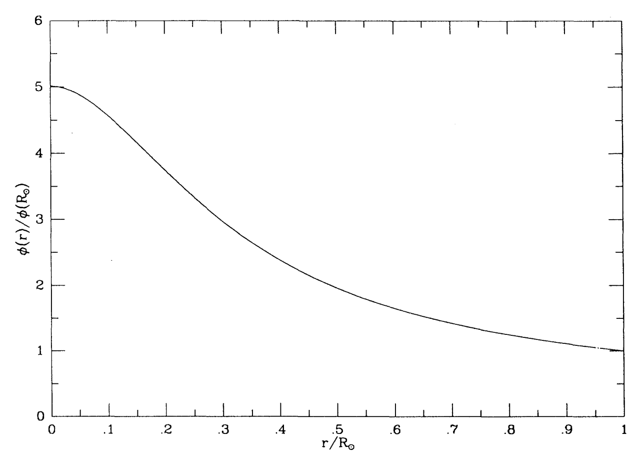

FIG. 1. The ratio of the gravitational potential at a point r to the potential at the solar surface is shown as a function of the ratio $r/R_{\odot}$.

that the average thermal energy is related to the average potential energy. The reason for the order-of-magnitude difference in this case between v^2/c^2 and ϕ is that the former contains a factor of A^{-1}, where A is the atomic mass number of the decaying particle. For the β decays of interest here, A is much larger than the average mass of the solar interior particle that appears in the virial theorem.

IV. FUSION REACTIONS: *pp*, hep

There are two important nuclear fusion reactions that directly produce solar neutrinos. The first is the basic proton-proton (pp) reaction,

$$p+p \to {}^2\text{H}+e^+ +\nu_e, \quad q_{\max}=0.420 \text{ MeV} , \tag{35}$$

which is a copious source of low-energy neutrinos. The second is the rare ${}^3\text{He}+p$ (i.e., hep) reaction,

$$p+{}^3\text{He}\to {}^4\text{He}+e^+ +\nu_e, \quad q_{\max}=18.77 \text{ MeV} , \tag{36}$$

which produces the most energetic neutrinos from the solar interior. The spectrum of neutrinos produced by a fusion reaction is $P_{\text{lab}}(q)\propto q^2 pWF(Z,W,Q)$, where Z is the charge of the nucleus in the initial state [e.g., $+1$ for deuterium, see Eq. (35), and $+2$ for helium, see Eq. (36)], and p, W are the momentum and energy of the positron created.

The probability of penetrating the Coulomb barrier, which exists between the charged ions in the initial state, increases rapidly with increasing relative kinetic energy of the ions. However, the number of interacting ions that have energies above some specified value is an exponentially decreasing function because the distribution of thermal energies is Maxwellian. For interactions that lead to fusion, the distribution of relative kinetic energies is strongly peaked about an average value of [16,20]

$$E_0=1.220(Z_1^2 Z_2^2 A T_*^2)^{1/3} \text{ keV} , \tag{37}$$

where T_* is the ambient temperature expressed in units of 10^6 deg. The numerical values of E_0 are

$$E_0(pp)=5.91 \text{ keV } T_{15}^{2/3} \tag{38}$$

and

$$E_0(\text{hep})=10.73 \text{ keV } T_{15}^{2/3} , \tag{39}$$

where T_{15} is the temperature in units of 15×10^6 K. In the center-of-mass frame, the maximum neutrino energy is increased by E_0, i.e.,

$$q_{\text{c.m.,max}}=q_{\text{lab,max}}+E_0 , \tag{40}$$

where $q_{\text{lab,max}}=Q-m_e c^2$ and Q is the exothermic energy release. The shape of the neutrino energy spectrum from the pp reaction, Eq. (35), and from the hep reaction, Eq. (36), can be determined by the argument given in Sec. III. For both cases, one imagines a decaying nucleus that has a maximum neutrino energy given by Eq. (40), with the total decay energy composed of the relative kinetic energy of the fusing particles and the exothermic energy release that applies to the laboratory experiments. The neutrino energy spectrum that is produced in the Sun corresponding to a given value of Q is the same as the spectrum that would be produced in the laboratory for an energy $Q+E_0$. Thus,

$$P_{\odot}(q,Q)=P_{\text{lab}}(q,Q+E_0) . \tag{41}$$

Here Q is the energy difference between initial and final nuclear states as measured in the laboratory. The result shown in Eq. (41) is the same as Eq. (2).

Figure 2 illustrates the small difference in the shape of the pp neutrino energy spectrum that results from including the kinetic energy of fusing particles in the solar interior [see Eq. (40) and Eq. (41)]. The dashed curve is the neutrino energy spectrum computed in the usual way [16] by ignoring E_0; the continuous curve was computed by taking account of E_0 according to Eq. (40).

The change in the shape of the spectrum, $\Delta S(E_0)$, that results from adding the relative kinetic energy to the Q value can be represented formally by the relation

$$\Delta S(q,E_0)\equiv E_0 \frac{d}{dQ} \ln P_{\text{lab}}(q,Q) . \tag{42}$$

The definition given in Eq. (42) results from considering the shapes of two energy spectra with slightly different Q values. For neutrino energies that are not too close to $q_{\text{max,lab}}$

$$\Delta S(q,E_0)=O\left[\frac{E_0}{Q}\right], \quad Q-q \gg E_0 . \tag{43}$$

Inserting the values of E_0 given in Eq. (38) and Eq. (39) and using the appropriate Q values, the computed changes in shapes are

$$\Delta S(pp)\sim 10^{-2}, \quad Q-q \gg E_0 \tag{44}$$

and

$$\Delta S(\text{hep})\sim 10^{-3}, \quad Q-q \gg E_0 . \tag{45}$$

The estimate given in Eq. (44) is in excellent agreement

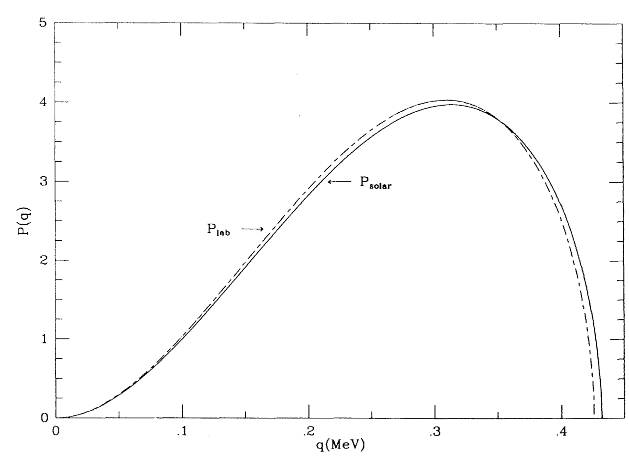

FIG. 2. The pp neutrino energy spectrum. This figure compares the energy spectrum that would be observed in the laboratory, P_{lab}, with the neutrino spectrum, $P_{\odot}$, that is produced in the Sun.

with the results of the detailed numerical evaluation illustrated in Fig. 2. The energy spectra shown in Fig. 2 were computed by using at each energy point accurate expressions for the various factors that determine the energy spectrum, including the Coulomb interaction with the nucleus, electron screening, the finite size of the nucleus, and special-relativistic effects [16].

V. THERMAL CORRECTIONS FOR NEUTRINO ENERGY LOSS

This section presents equations that can be used to calculate the average energy loss via neutrinos produced in nuclear reactions in a stellar interior. The numerical results are summarized in Table I. Until recently [18], the thermal energy of fusing nuclei was not included in this computation. The numerical results given here for the *pp* and hep reactions agree with those given previously, but the present results for the electron capture reactions

$$e^- + {}^7\mathrm{Be} \rightarrow \nu_e + {}^7\mathrm{Li} \quad (46)$$

and

$$p + e^- p \rightarrow \nu_e + {}^2\mathrm{H} \quad (47)$$

are more accurate.

The neutrino energy spectrum for both nuclear fusion reactions [cf. Eq. (35) and Eq. (36)] and for electron capture reactions [cf. Eq. (46) and Eq. (47)] can be written in the form

$$P_\odot = N_0 p W q^2 F(Z,W) \ . \quad (48)$$

Here p, W are the momentum and total energy, respectively, of the associated electron or positron and q is the energy of the neutrino emitted. The quantity Z equals the charge on the initial nucleus for electron capture and equals minus the charge on the final nucleus for fusion reactions. The computation of the relativistic Fermi function $F(Z,W)$ including electron screening and finite nuclear size effects is described elsewhere [16]. The conservation of energy,

$$q = \pm W + \Delta E_{\mathrm{nuclear}} + E \equiv q_0 + E \ , \quad (49)$$

determines the value of the neutrino energy. For electron capture, the plus sign applies in Eq. (49), $W = m_e c^2$, and E is the electron kinetic energy. For fusion reactions, the minus sign applies and $E \approx E_0$ [see Eq. (37)]. The average energy loss by neutrinos is

$$\langle q \rangle = \int_0^{q_{\max}} dq\, q P_\odot(q) \ . \quad (50)$$

The extra energy loss that results from the thermal correction is

$$\Delta q(T) = \langle q \rangle - \langle q \rangle_0 \ . \quad (51)$$

Here $\langle q \rangle_0$ is the average energy loss that is computed using Eq. (50), but omitting the thermal energy that appears in Eq. (49).

I have performed accurate numerical integrations which show that the extra energy loss in the *pp* reaction is

$$\Delta q(p-p) \simeq 3.6 T_{15}^{2/3} \ \mathrm{keV} \ . \quad (52)$$

The result given in Eq. (52) corresponds to a 1.3% increase in the energy loss by *pp* neutrinos. The correction for the energy loss in the hep reaction is

$$\Delta q(\mathrm{hep}) = 5.4 T_{15}^{2/3} \ \mathrm{keV} \ , \quad (53)$$

which amounts to only a small fractional correction, approximately 0.05%.

For electron capture, the average energy loss $\langle q \rangle_T$ is given by the expression

$$\langle q \rangle_T = \frac{\int_0^\infty dE\, E^{1/2}(q_0+E)^3 F(Z,W)\exp(-E/T)}{\int_0^\infty dE\, E^{1/2}(q_0+E)^2 F(Z,W)\exp(-E/T)} \ , \quad (54)$$

where q_0 is defined by Eq. (49).

With the appropriate nonrelativistic approximations,

$$\frac{\langle q \rangle_T}{q_0} = \frac{\int_0^\infty dx\, \exp(-x)(1+xT/q_0)^3[1-\exp(-0.643 Z T_{15}^{-1/2} x^{-1/2})]^{-1}}{\int_0^\infty dx\, \exp(-x)(1+xT/q_0)^2[1-\exp(-0.643 Z T_{15}^{-1/2} x^{-1/2})]^{-1}} \ . \quad (55)$$

Detailed numerical integrations of Eq. (55), yield values for the additional energy loss of

$$\Delta q(pep) = 1.57 T_{15} \ \mathrm{keV} \quad (56)$$

and

$$\Delta q({}^7\mathrm{Be}) = 1.41 T_{15} \ \mathrm{keV} \ . \quad (57)$$

There are two thermal corrections to the *pep* neutrino energy loss: the extra contribution from the center-of-mass energy of the two fusing protons, which is described by Eq. (52), and the extra contribution from the kinetic energy of the electron, which is described by Eq. (56). In Table I, both contributions are included in the total energy loss for the *pep* reaction.

The thermal correction is negligible for nuclear β decays. In the center-of-mass frame of the decaying particle, the average energy of the neutrino emitted has the same value in the solar interior as it does in a terrestrial laboratory. The first-order Doppler correction to the neutrino energy cancels out since the Boltzmann distribution of the velocities of the decaying particles is symmetric between positive and negative velocities. Only the negligibly small second-order Doppler correction, which is $\sim 10^{-6}$, survives for nuclear β decay.

VI. OTHER EFFECTS

In this section, I discuss briefly some additional effects that must be evaluated in order to establish the independence of neutrino energy spectra from solar interior conditions. Some of these affects are significant for β decays in the interiors of evolved stars [21].

A. Atomic binding energies

Light element nuclei in the solar interior are essentially fully ionized. On the other hand, atoms that β decay in the laboratory are generally not ionized. The difference in these two situations causes a slight shift in the solar-neutrino energy spectra relative to the laboratory spectra, analogous to—but of opposite sign to—the shift in neutrino spectra caused by the kinetic energy of fusing particles [see Sec. IV, especially Eq. (41)]. For the β decays of interest, the change in atomic binding energies, ΔB, between initial and final atomic states in the laboratory is

$$\Delta B \sim 10^2 \text{ eV} , \quad (58)$$

which results in a change ΔS in the shape of neutrino energy spectra of order [cf. Eq. (43)]

$$\Delta S(q\Delta B) \sim \frac{\Delta B}{Q} < 10^{-4} \text{ to } 10^{-5} , \quad (59)$$

for ^{8}B, ^{13}N, ^{15}O, and ^{17}F decays.

B. Excited nuclear states

In principle, β decay can occur from excited nuclear states that are in thermal equilibrium with the ground nuclear states. However, for the solar interior, the equilibrium populations of excited nuclear states are negligibly small. The ratio of excited to ground populations is of order

$$\frac{n_{\text{ex}}}{n_{\text{ground}}} \sim 10^{-(336/T_{15})} , \quad (60)$$

for a state at an excitation energy of order 1 MeV.

C. Electron capture

For all solar interior β decays, there is a finite but small probability that the decaying nucleus captures an electron from an initial continuum or bound state instead of producing a positron in the final state. For example, instead of the ^{8}B β decay described by Eq. (12), the decay may occur by electron capture:

$$e^- + {}^8\text{B} \rightarrow {}^8\text{Be}^* + \nu_e . \quad (61)$$

The ratio of electron capture to β decay under solar interior conditions has been calculated [22]; the result is capture/decay$\sim 10^{-7}$ for ^{8}B, $\sim 10^{-4}$ for the CNO β-decays, and $\sim 10^{-8}$ for the hep reaction. The ratio of the rates of the *pep* and the *pp* reactions is $\sim 10^{-3}$. Electron capture reactions produce energy spectra that are lines of fixed energy, not the familiar continuum energy spectra (cf. Figure 2). Also, the lines have an energy that exceeds the end point of the continuum spectra $q_{\max}$ by $2m_e c^2 = 1.02$ MeV.

D. Collisional broadening

The collisions of decaying ions with other particles in the ambient plasma will broaden any potential narrow features in the energy spectra and will smooth out the spectrum over an energy scale that is inversely proportional to the coherence time. The characteristic coherence time in the solar interior is [23] of order 5×10^{-17} s. This corresponds to smoothing over energy scales of about $\Delta E/E \sim 10^{-5}$. For the slowly varying energy spectra considered in this paper, the estimated small amount of smoothing will not lead to discernible effects.

E. Exclusion principle

β decays that produce electrons and antineutrinos are affected in stellar interiors by the Pauli exclusion principle since many of the continuum electron states may already be filled. For example, neutrons in neutron stars do not decay into protons, electrons, and neutrinos because the energetically accessible electron states are occupied. However, the Pauli principle does not inhibit the β decays of interest for solar neutrino studies, which involve the creation of positrons (whose equilibrium population is negligible) and neutrinos.

F. Scattering and absorption

Inelastic scattering by electrons, as well as absorption and charged-current interactions with nuclei, also change the neutrino spectra by a small amount. The neutrino spectra are gradually modified, as the neutrinos make their way out of the Sun, by energy-changing scattering interactions with electrons, by preferential removal of higher-energy neutrinos due to charged-current reactions with nuclei, and by degradation in neutrino energies that results from neutral-current interactions with nuclei. The largest effect is caused by inelastic electron scattering, but the total probability of an interaction is only $\tau=7\times10^{-8}(q/10 \text{ MeV})$.

VII. SUMMARY AND DISCUSSION

The corrections to the shape of the neutrino energy spectra from the thermal motions of the ions in the solar interior are small because the thermal velocities are small compared to the velocity of light. The computed corrections in the shape are of order E_0/Q ($\sim 10^{-2}$) for fusion reactions and of order v^2/c^2 ($\sim 10^{-6}$) for β decays of single nuclei. The gravitational redshift is the dominant solar effect for nuclear β decays and is $\simeq 10^{-5}$. There are no currently practical experiments that can detect effects as small as the ones calculated in this paper. In a high counting rate solar neutrino experiment, for example, with the kiloton deuterium detector, the total capture rate due to electron neutrinos from ^{8}B is expected to be at best of order 10^4 events per year. In order to check a correction to the shape of order 10^{-5}, one would need more than 10^{10} events. This would require about 10^6 yr. For the lower-energy *pp* neutrinos, there is no demonstrated feasible experiment for measuring the spectrum shape, although several have been proposed with conceivable counting rates of 1 per day. The biggest difference between the two curves shown in Figure 2 amounts to about 0.7% in the energy range between 0.2 and 0.3 MeV. To make a 3σ detection of this difference would require more than 100 years if the average rate were one event per day. If one could develop an experiment with low noise and with high-energy resolution, the effect of the finite-thermal energy E_0 on the shape of the spectrum near the end point could be detected with fewer events [24].

Any measured difference between the observed shape of a solar neutrino energy spectrum and the shape of the corresponding laboratory neutrino spectrum that is found with currently available experimental techniques must be due to a departure from the standard electroweak model.

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