# $M1$ decay rates of heavy quarkonia with a nonsingular potential

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We use a nonsingular-potential model for heavy quarkonia proposed by Gupta, Repko, and Suchyta to calculate the transition amplitudes for the magnetic-dipole ($M1$) one-photon radiative decays of the $c\bar{c}$ and $b\bar{b}$ bound systems. The wave functions of the bound systems are calculated by a nonperturbative treatment. The results are in better agreement with the experimental data than those predicted using other potential models.

Recently Gupta, Repko, and Suchyta [1] (GRS) proposed a nonsingular-potential model first suggested by Gupta [2] for heavy quarkonia. They pointed out that all other potential models have highly singular interaction terms which make it impossible to obtain the energy levels by a nonperturbative treatment. Moreover, large contributions to the energy levels arising from these singular terms make the perturbative treatment questionable. GRS have used this model to calculate the energy levels and the leptonic annihilation rates of the $c\bar{c}$ and $b\bar{b}$ bound systems. The energy levels and the wave functions were calculated by using the variational method and without resorting to any perturbative treatment. Their results were in excellent agreement with experimental data. It is of interest to compute the results of this model for the $M1$ decay rates of charmonium where the experimental data are available for comparison. In this note we consider the magnetic-dipole ($M1$) one-photon transition rates of the $c\bar{c}$ and $b\bar{b}$ systems using this model. It is a fact that transition matrix elements and, hence, radiative decay rates are very sensitive to the wave functions while in general the energy spectrum is not. Since in this model the wave functions are obtained as eigenfunctions of the full Hamiltonian (including all relativistic corrections) by a variational calculation, they may be better suited to the calculation of the decay rates than the ones obtained by perturbative methods.

The decay rate for the $M1$ transition between spin-triplet and spin-singlet $S$-wave states of quarkonium, including the leading relativistic corrections, can be written as [3,4]

$$W^{M1}=\frac{4k^3\alpha}{(2S_i+1)m^2}\left[\frac{e_q}{e}\right]^2|I_1+I_2+I_3+I_4|^2\ , \qquad (1)$$

where $S_i$ is the spin of the initial state of quarkonium, $\alpha$ the fine-structure constant, $k$ is the angular frequency of the emitted photon, and $m$ is the mass of the quark. The dimensionless integrals $I_1$, $I_2$, $I_3$, and $I_4$ are given by the expressions

$$I_1=\left\langle\phi_f\left|(1+a)j_0(kr/2)+\frac{k(1+2a)}{4m}\right|\phi_i\right\rangle\ , \qquad (2)$$

$$I_2=\left\langle\phi_f\left|-\frac{(1+a)\pi^2}{2m^2}-\frac{\pi^2}{3m^2}\right|\phi_i\right\rangle\ , \qquad (3)$$

$$I_3=\left\langle\phi_f\left|\frac{ar}{6m}\frac{\partial(V_p^{(0)}+V_c^{(0)})}{\partial r}\right|\phi_i\right\rangle\ , \qquad (4)$$

$$I_4=\left\langle\phi_f\left|-\frac{V_s^{(0)}}{m}j_0(kr/2)\right|\phi_i\right\rangle\ , \qquad (5)$$

where $a$ is the anomalous magnetic-moment parameter of the quark, which we take to be zero. The term $I_3$ will then be zero and we will not discuss it further. The wave functions $\phi_i$ and $\phi_f$ are the spatial parts of the quarkonium wave functions. In the expression for $I_4$, $V_s^{(0)}$ is the scalar part of the nonrelativistic potential.

We first calculated the wave functions of the initial and the final states in the decays nonperturbatively by using the GRS model. For the linear confining potential, it is not clear whether its spin dependence arises from scalar exchange or from vector exchange. GRS have calculated three sets of results which correspond to spin dependence in the linear confining potential as a scalar exchange, a scalar-vector exchange, and an arbitrary form. Since the scalar-vector exchange generated a much better result than the scalar exchange or the arbitrary forms, we used the "scalar-vector-exchange" GRS model to calculate the wave functions and the $M1$ decay rates. For the "scalar-vector-exchange" model we have $V_s^{(0)}=(1-B)(Ar+C)$, where $Ar+C$ is the confining potential and $B$ is a parameter [1]. In Table I, we give the results for the $M1$ decay rates of charmonium. We also give the range of the experimental results which were taken from Ref [5]. The corresponding results for $b\bar{b}$ are given in Table II. The parameters used in our calculation are those of Ref. [1].

From Table I we see that all decay rates agree with the experimental data except for $\psi\rightarrow\eta_c+\gamma$. The rate for this decay is a little larger than the experimental value.

 

TABLE I. $M1$ decay rates of charmonium. GRS scalar-vector-exchange model was used in this calculation. See Ref. [1] for details. The experimental data were taken from Ref. [5]. We used the experimental photon energies for the decays $\psi' \to \eta_c + \gamma$ and $\psi \to \eta_c + \gamma$. The values of these two photon energies were also taken from Ref. [5].

| | Decay | | | |
|---|---|---|---|---|
| | $\psi' \to \eta_c + \gamma$ | $\eta_c' \to \psi + \gamma$ | $\psi' \to \eta_c' + \gamma$ | $\psi \to \eta_c + \gamma$ |
| Photon energy (GeV) | 0.639 | 0.468 | 0.093 | 0.115 |
| $I_1$ | 0.166 | −0.123 | 0.982 | 0.996 |
| $I_2$ | −0.210 | −0.152 | −0.244 | −0.223 |
| $I_4$ | 0.081 | 0.016 | 0.168 | 0.279 |
| $\sum_i I_i$ | 0.037 | −0.258 | 0.906 | 1.053 |
| $\sum_i I_i - I_4$ | −0.044 | −0.274 | 0.738 | 0.774 |
| Decay rate (keV) | 0.386 | 22.1 | 0.681 | 1.795 |
| Decay rate (keV) (without $I_4$) | 0.531 | 24.9 | 0.451 | 0.969 |
| Expt. data (keV) | 0.4–1.0 | | 0.4–3.7 | 0.5–1.3 |

In Table III we compare our results with those of Zambetakis and Byers [4] and the previous work by two of us [3]. From Table III we see that the GRS model gives a very different result for the hindered decay $\psi' \to \eta_c + \gamma$. This indicates that the nonperturbative treatment is very important. We can also see that the decay rates of the GRS model are in better agreement with the experimental data.

As a reference and comparison, in Tables IV and V we give the $M1$ decay rates by using the GRS scalar-exchange model and in Tables VI and VII we give the results by using the "singular form" of the same potential

TABLE II. $M1$ decay rates of $b$ quarkonium. GRS scalar-vector-exchange model was used in this calculation. See Ref. [1] for details.

| | Decay | | | |
|---|---|---|---|---|
| | $\Upsilon' \to \eta_b + \gamma$ | $\eta_b' \to \Upsilon + \gamma$ | $\Upsilon' \to \eta_b' + \gamma$ | $\Upsilon \to \eta_b + \gamma$ |
| Photon energy (GeV) | 0.595 | 0.523 | 0.028 | 0.048 |
| $I_1$ | 0.069 | −0.047 | 0.997 | 0.999 |
| $I_2$ | −0.058 | −0.051 | −0.065 | −0.074 |
| $I_4$ | 0.014 | 0.001 | 0.107 | 0.130 |
| $\sum_i I_i$ | 0.025 | −0.097 | 1.039 | 1.055 |
| $\sum_i I_i - I_4$ | 0.011 | −0.098 | 0.932 | 0.924 |
| Decay rate (keV) | 0.005 | 0.141 | 0.001 | 0.004 |
| Decay rate (keV) (without $I_4$) | 0.001 | 0.144 | 0.001 | 0.003 |

TABLE III. Comparison of $M1$ decay rates of charmonium. The number in the parentheses is the decay rate including the coupled-channel mixing effect. The decay-rates of Grotch-Owen-Sebastian (GOS) and Zambetakis and Byers (ZB) were calculated using Eq. (1), and the values of $\sum_i I_i$ and the quark masses were taken from Refs. [3] and [4].

| | Decay | | |
|---|---|---|---|
| | $\psi' \to \eta_c + \gamma$ (keV) | $\psi' \to \eta_c' + \gamma$ (keV) | $\psi \to \eta_c + \gamma$ (keV) |
| ZB (Ref. [4]) | 4.5(0.34) | 0.78 | 2.3 |
| GOS (Ref. [3]) | 8.0 | 0.19 | 1.1 |
| This work | 0.39 | 0.68 | 1.8 |
| Expt. data | 0.4–1.0 | 0.4–3.7 | 0.5–1.3 |

which is actually the Gupta-Radford-Repko (GRR) [6] potential model. From Tables IV and V we see that the pure scalar-exchange model gives a result which is not very different from that of the scalar-vector-exchange model. The result in Table VI indicates that the GRR model, which is expressed in singular form, like other singular potential models, gives a too large decay rate for the transition $\psi' \to \eta_c + \gamma$. It seems that the nonsingular form of the potential, not the nature of the spin dependence in the linear confining potential, is crucial in order to yield better $M1$ decay rates.

The GRS model neglects the effect of the coupling of the quark-antiquark system to the virtual decay channels. Although the coupled-channel mixing effect on the energy levels may be small below the bottom and charm thresholds, it may have a large effect on the hindered $M1$ decays [4]. Since the purpose of this work is only to extend the application of the GRS model to the $M1$ decays,

TABLE IV. $M1$ decay rates of charmonium. GRS scalar-exchange model was used in this calculation. See Ref. [1] for details. The experimental data were taken from Ref. [5]. We used the experimental photon energies for the decays $\psi' \to \eta_c + \gamma$ and $\psi \to \eta_c + \gamma$. The values of these two-photon energies were also taken from Ref. [5].

| | Decay | | | |
|---|---|---|---|---|
| | $\psi' \to \eta_c + \gamma$ | $\eta_c' \to \psi + \gamma$ | $\psi' \to \eta_c' + \gamma$ | $\psi \to \eta_c + \gamma$ |
| Photon energy (GeV) | 0.639 | 0.470 | 0.092 | 0.115 |
| $I_1$ | 0.178 | −0.135 | 0.979 | 0.993 |
| $I_2$ | −0.231 | −0.159 | −0.265 | −0.238 |
| $I_4$ | 0.102 | 0.032 | 0.141 | 0.299 |
| $\sum_i I_i$ | 0.050 | −0.262 | 0.855 | 1.053 |
| $\sum_i I_i - I_4$ | −0.053 | −0.294 | 0.714 | 0.755 |
| Decay rate (keV) | 0.752 | 25.2 | 0.669 | 1.982 |
| Decay rate (keV) (without $I_4$) | 0.853 | 31.7 | 0.467 | 1.018 |
| Expt. data (keV) | 0.4–1.0 | | 0.4–3.7 | 0.5–1.3 |

TABLE V. $M1$ decay rates of $b$ quarkonium. GRS scalar-exchange model was used in this calculation. See Ref. [1] for details.

| | Decay | | | |
|---|---|---|---|---|
| | $\Upsilon'\rightarrow\eta_b+\gamma$ | $\eta_b'\rightarrow\Upsilon+\gamma$ | $\Upsilon'\rightarrow\eta_b'+\gamma$ | $\Upsilon\rightarrow\eta_b+\gamma$ |
| Photon energy (GeV) | 0.594 | 0.522 | 0.028 | 0.048 |
| $I_1$ | 0.022 | −0.018 | 0.986 | 0.994 |
| $I_2$ | −0.034 | −0.029 | −0.076 | −0.070 |
| $I_4$ | 0.011 | 0.004 | 0.103 | 0.185 |
| $\sum_i I_i$ | −0.001 | −0.042 | 1.013 | 1.109 |
| $\sum_i I_i-I_4$ | −0.012 | −0.046 | 0.910 | 0.924 |
| Decay rate (keV) | 0.000 | 0.030 | 0.001 | 0.005 |
| Decay rate (keV) (without $I_4$) | 0.001 | 0.036 | 0.001 | 0.004 |

we are not going to discuss the effect of coupled-channel mixing further.

There is some ambiguity as to whether or not we should include $I_4$ in Eq. (1). The dimensionless integrals $I_1$ and $I_2$ have very simple origins independent of any interaction models between the quark and the antiquark. The operator responsible for $I_1$ originates from the nonrelativistic interaction of the quark and the antiquark magnetic moments with the magnetic field of the quantized radiation field. This term would have been there even if we had neglected the internal interaction between the quark and the antiquark. As for $I_2$, the operator responsible for it, $-\frac{1}{2}(1+a)\pi^2/m^2-\pi^2/6m^2$, originates [7] from a relativistic correction to the nonrelativistic interaction of the magnetic moments with the magnetic field. The piece $-\pi^2/6m^2$ in the $I_2$ integral is due to the $\mathbf{p}\cdot\mathbf{A}$ interaction when the recoil momentum of the quarkonium due to the photon emission is included. These terms are also independent of any internal interaction. On the other hand, the integral $I_4$ has an entirely different origin [8,9]. If the scalar potential is in fact due to the exchange of a scalar meson between the quark and the antiquark and an external photon line is inserted on the quark or the antiquark external line in such an exchange diagram [8] we do get $I_4$ in the $M1$ decay amplitude. Since no scalar-meson exchange can give rise to a linear confining potential, it is quite clear the scalar exchange line can only be thought of as a simulation of a sum of a large number of diagrams in QCD where there are many internal gluon and quark lines. In the QCD diagrams we should also attach an external photon line to every charged-particle line, including the internal quark lines, and not just the external quark lines. While there are only four external quark lines, there are an arbitrarily large number of internal quark lines. Under these circumstances, it is not at all clear whether the $V_s$ term of $I_4$ would then have this form in the $M1$ transition amplitude. From Tables I and IV we can see that if we drop the term $I_4$ all decay rates of charmonium are in agreement with the experimental data for both the scalar-vector exchange and the scalar-exchange confining potentials in the nonsingular GRS model with the nonperturbative wave functions. On the other hand, including the $I_4$ term gives a value which is outside the experimental range for the decay rate of the transition $\psi\rightarrow\eta_c+\gamma$.

TABLE VI. $M1$ decay rates of charmonium. The Gupta, Radford, and Repko (GRR) model was used in this calculation. See Ref. [6] for details. The experimental data were taken from Ref. [5]. We used the experimental photon energies for the decays $\psi'\rightarrow\eta_c+\gamma$ and $\psi\rightarrow\eta_c+\gamma$. The values of these two photon energies were also taken from Ref. [5].

| | Decay | | | |
|---|---|---|---|---|
| | $\psi'\rightarrow\eta_c+\gamma$ | $\eta_c'\rightarrow\psi+\gamma$ | $\psi'\rightarrow\eta_c'+\gamma$ | $\psi\rightarrow\eta_c+\gamma$ |
| Photon energy (GeV) | 0.639 | 0.455 | 0.092 | 0.115 |
| $I_1$ | 0.211 | 0.232 | 1.029 | 0.805 |
| $I_2$ | −0.213 | −0.213 | −0.347 | −0.250 |
| $I_4$ | 0.103 | 0.103 | −0.724 | −0.505 |
| $\sum_i I_i$ | 0.101 | 0.122 | −0.041 | 0.050 |
| $\sum_i I_i-I_4$ | −0.002 | 0.019 | 0.683 | 0.555 |
| Decay rate (keV) | 6.610 | 10.44 | 0.003 | 0.009 |
| Decay rate (keV) (without $I_4$) | 0.003 | 0.253 | 0.902 | 1.163 |
| Expt. data (keV) | 0.4–1.0 | | 0.4–3.7 | 0.5–1.3 |

TABLE VII. $M1$ decay rates of $b$ quarkonium. GRR model was used in this calculation. See Ref. [6] for details.

| | Decay | | | |
|---|---|---|---|---|
| | $\Upsilon'\rightarrow\eta_b+\gamma$ | $\eta_b'\rightarrow\Upsilon+\gamma$ | $\Upsilon'\rightarrow\eta_b'+\gamma$ | $\Upsilon\rightarrow\eta_b+\gamma$ |
| Photon energy (GeV) | 0.590 | 0.510 | 0.039 | 0.045 |
| $I_1$ | 0.070 | 0.072 | 0.592 | 0.844 |
| $I_2$ | −0.050 | −0.050 | −0.069 | −0.065 |
| $I_4$ | 0.018 | 0.018 | −0.104 | −0.056 |
| $\sum_i I_i$ | 0.038 | 0.040 | 0.418 | 0.723 |
| $\sum_i I_i-I_4$ | 0.020 | 0.022 | 0.522 | 0.779 |
| Decay rate (keV) | 0.014 | 0.030 | 0.000 | 0.002 |
| Decay rate (keV) (without $I_4$) | 0.004 | 0.009 | 0.001 | 0.003 |

In conclusion, if the coupled-channel mixing effect is neglected, the $M1$ decay rates of the GRS model are in better agreement with the experimental data than those of other potential models. We stress, however, that there are still uncertainties in such calculations since the derivation of the term $I_4$ is ambiguous and also coupled-channel mixing may have a large effect, especially for hindered $M1$ decay rates.

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[1] S. N. Gupta, W. W. Repko, and C. J. Suchyta III, Phys. Rev. D **39**, 974 (1989).
[2] S. N. Gupta, Phys. Rev. D **35**, 1736 (1987).
[3] H. Grotch, D. A. Owen, and K. J. Sebastian, Phys. Rev. D **30**, 1924 (1984).
[4] R. McClary and N. Byers, Phys. Rev. D **28**, 1692 (1983); V. Zambetakis and N. Byers, *ibid.* **28**, 2908 (1983).
[5] Particle Data Group, J. J. Hernández *et al.*, Phys. Lett. B **239**, 1 (1990).
[6] S. N. Gupta, S. F. Radford, and W. W. Repko, Phys. Rev. D **34**, 201 (1986).
[7] H. Grotch and K. J. Sebastian, Phys. Rev. D **25**, 2944 (1982).
[8] J. Sucher, Rep. Prog. Phys. **41**, 1781 (1978); J. S. Kang and J. Sucher, Phys. Rev. D **18**, 2698 (1978).
[9] K. J. Sebastian, Phys. Rev. A **23**, 2810 (1981).