Mesons as bilocal fields in the harmonic approximation: A reassessment

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The spectrum and lepton decay constants of the pion, the kaon, and their radial excitations, as well as the pion decay width in two photons, are computed in a recently proposed framework that combines the bilocal approach to mesons and the potential method treated covariantly.

I. INTRODUCTION

The treatment of relativistic bound states is an important but notoriously difficult problem of quantum field theory, especially in quantum chromodynamics (QCD), where perturbation theory cannot be used for low energies. In addition, all kinds of nonperturbative approaches to QCD also have their limitations, so that the understanding of the dynamics of hadrons composed of light quarks (u,d,s) is still unsatisfactory.

Over the years there have been many attempts to get a more precise feeling for the dynamics of bound light quarks by using the potential approach (e.g., Refs. [1]). Recently, Pervushin and co-workers [2–5] have formulated an improved potential approach which (loosely speaking) amounts to redoing the work of Le Yaouanc *et al.* [6,7] covariantly, and which we use in this work to study the pion (π), the kaon (K), and their radial excitations π' and K'. In particular, we perform a critical reassessment of their recent application of this method to the case of harmonic interaction between quarks and finite quark masses [8] where they claimed excellent results for meson decay constants, while simultaneously fitting the meson masses to their experimental values. We disagree with them and conclude that while this method is indeed explicitly covariant, more realistic interaction kernels must be used in order to obtain results of such quality. We also compute the width for the decay $\pi^0 \to 2\gamma$ and reach the same conclusion.

II. SCHWINGER-DYSON AND BETHE-SALPETER EQUATIONS FOR THE HARMONIC POTENTIAL

Pervushin and co-workers started from a fermion effective action where quarks interact via an interaction kernel $K(x,y)$ which is supposed to mimic QCD. By eliminating quark bilinears $q(x)\bar{q}(x)$ in favor of bilocal fields [9–12], they obtained the effective action $W_{\text{eff}}[\chi]$ rewritten in terms of quark propagators and bilocal fields $\chi(x,y)$ [10]. From it they obtained the Schwinger-Dyson equation (SDE) determining the classical solution $\chi_0(x,y)$ or, equivalently, the dynamically generated quark self-mass operator $\Sigma(x,y)$ and therefore also the "dressed" quark propagator G_Σ. The fluctuations $\mathcal{M}(x,y)$ around the classical solution $\chi_0(x,y)$ represent mesons [2–5,8,9–13], so that the part of the action pertinent in this paper is the part containing $\mathcal{M}(x,y)$, denoted by $\widetilde{W}_{\text{eff}}$:

$$\widetilde{W}_{\text{eff}}[\mathcal{M}] = \frac{N_c}{2}(\mathcal{M}, K^{-1}\mathcal{M}) - iN_c \sum_{n=2}^{\infty} \frac{(-1)^n}{n} \text{Tr}\Phi^n , \quad (1)$$

$$\Phi(x,y) \equiv \int d^4z\, G_\Sigma(x,z)\mathcal{M}(z,y) , \quad (2)$$

$$\text{Tr}\Phi^n \equiv \text{tr} \int d^4x_1 d^4x_2 \cdots d^4x_n \Phi(x_1,x_2) \times \Phi(x_2,x_3) \cdots \Phi(x_n,x_1) , \quad (3)$$

$$(\mathcal{M}, K^{-1}\mathcal{M}) = \int d^4x\, d^4y\, \mathcal{M}_{\beta_1\alpha_2}(x,y) \times K^{-1}_{\alpha_1\beta_1;\alpha_2,\beta_2}(x,y)\mathcal{M}_{\beta_2\alpha_1}(y,x) . \quad (4)$$

From $\widetilde{W}_{\text{eff}}$ one derives the Bethe-Salpeter equation (BSE) for the bilocal field $\mathcal{M}(x,y)$. This BSE Fourier transformed to momentum space is

$$\Gamma_{(ab)}(q|P) = i \int \frac{d^4q'}{(2\pi)^4} K(q-q') G_{\Sigma_{(a)}}\left[q' + \frac{P}{2}\right] \times \Gamma_{(ab)}(q'|P) G_{\Sigma_{(b)}}\left[-q' + \frac{P}{2}\right] , \quad (5)$$

where $\Gamma_{(ab)}(q|P)$ is the vertex function in momentum space of the quark-antiquark pair a,b and $2q \equiv p_a - p_b$, $P \equiv p_a + p_b$. Note that this is the BSE equation in the somewhat improved ladder approximation, since the quark propagators it contains are not the bare ones, but G_Σ, containing the nontrivial self-energy function Σ.

To be able to solve the SDE and BSE, one must restrict oneself to a tractable interaction kernel. Choosing an instantaneous interaction leads to a potential model. For the covariant generalization of the potential approach to the bound states, Pervushin and co-workers [4,5,13,14] found that the kernel should be of a special form [15], $K \to K^\eta$:

$$[K^\eta(x-y)]_{\alpha_1,\beta_1;\alpha_2,\beta_2} = [K^\eta(z,X)]_{\alpha_1,\beta_1;\alpha_2\beta_2} = \not{\eta}_{\alpha_1,\beta_1} V(z_\perp)\delta(z_P)\not{\eta}_{\alpha_2,\beta_2} , \quad (6)$$

where $z = x-y$, $X = (x+y)/2$, and $\eta^\mu = P^\mu/\sqrt{P^2}$. For any vector $x^\mu = x_\parallel^\mu + x_\perp^\mu$, the parallel and perpendicular components are

$$x_\parallel^\mu = \eta^\mu x_P, \quad x_P = x\cdot\eta = x\cdot P/\sqrt{P^2} ,$$
$$x_\perp^\mu = x^\mu - x_\parallel^\mu, \quad x_\perp \cdot P = 0 . \quad (7)$$

$V(r)$ is a scalar function of $r=z_\perp$. Below we shall use

$$V(r)=(\tfrac{4}{3})V_0 r^2,\quad V_0=\text{const}\ . \tag{8}$$

For the interaction kernel of the form (6), the dressed propagator in momentum space, $G_\Sigma(q)$, is conveniently parametrized through the functions $E(k_\perp)$ and $\varphi(k_\perp)$ (depending only on the transversal momentum $k_\perp$) as follows:

$$G_{\Sigma_{(a)}}(q)=\frac{1}{\not q-\Sigma_{(a)}(qt_\perp)-i\epsilon}=-S_{(a)}(q_\perp)\left[\frac{\Lambda_+^P}{E_{(a)}(q_\perp)-q_P-i\epsilon}+\frac{\Lambda_-^P}{E_{(a)}(q_\perp)+q_P-i\epsilon}\right]S_{(a)}(q_\perp)\ , \tag{9}$$

$$S_{(a)}^{-2}(k_\perp)=\exp\left[\hat{\not k}_\perp\left[\varphi(k_\perp)-\frac{\pi}{2}\right]\right]\ , \tag{10}$$

where $\Lambda_\pm^P\equiv\frac{1}{2}(1\pm\not P/\sqrt{P^2})$. The function $E_{(a)}(k_\perp)$ plays the role of the energy of the quark a and the vector components along P the role of the time components. Inserting (9) in the SDE yields coupled integral equations for $E(k_\perp)$ and $\varphi(k_\perp)$. In the rest frame, for the harmonic potential (8) these integral equations reduce to the *differential* equations

$$E(k)=m\sin\varphi(k)+k\cos\varphi(k)-\tfrac{1}{2}\varphi'(k)-\frac{1}{k^2}\cos^2\varphi(k)\ , \tag{11}$$

$$(k^2\varphi')'=2k^3\sin\varphi(k)-2k^2 m\cos\varphi(k)-\sin 2\varphi(k)\ . \tag{12}$$

Having solved the SDE for E and φ, one can turn to solving the BSE, conveniently written in terms of the quarkonium wave function $\Psi_{(ab)}^P(q_\perp)$:

$$\begin{aligned}\Psi_{(ab)}^P(q_\perp)&=i\int\frac{dq_P}{(2\pi)}\left[G_{\Sigma_{(a)}}\left[q+\frac{P}{2}\right]\Gamma_{(ab)}(q_\perp|P)\times G_{\Sigma_{(b)}}\left[-q+\frac{P}{2}\right]\right]\\&\equiv S_{(a)}(q_\perp)\psi_{(ab)}^P(q_\perp)S_{(b)}(q_\perp)\ .\end{aligned} \tag{13}$$

Pervushin and collaborators [2–5,13] have shown that with kernels of the form (6), this BSE is also manifestly Lorentz covariant.

To solve the BSE, we should decompose $\psi_{(ab)}^P$ over the Dirac γ matrices [4,5,9,13]. Since in this work we are interested only in the pseudoscalar mesons π,π',K,K', we simply have

$$\psi_{(ab)}^P(q_\perp)=\gamma_5\left[L_1(q_\perp)+\frac{\not P}{\sqrt{P^2}}L_2(q_\perp)\right]\ . \tag{14}$$

It is simplest to solve the BSE in the rest frame, $q_\perp=(0,\mathbf{q}),P=(M,0)$, where M is the mass of the bound system, the pion or the kaon in this case. With the harmonic interaction, the integral BSE reduces to the differential eigenvalue equations

$$-ML_2(k)=\left[E_T(k)-\frac{d^2}{dk^2}-\frac{2}{k}\frac{d}{dk}+\frac{1}{4}\left[\frac{d\tilde\varphi_-}{dk}\right]^2+\frac{2}{k^2}\sin^2\left[\frac{\tilde\varphi_-}{2}\right]\right]L_1(k)\ , \tag{15}$$

$$-ML_1(k)=\left[E_T(k)-\frac{d^2}{dk^2}-\frac{2}{k}\frac{d}{dk}+\frac{1}{4}\left[\frac{d\tilde\varphi_+}{dk}\right]^2+\frac{2}{k^2}\sin^2\left[\frac{\tilde\varphi_+}{2}\right]\right]L_2(k)\ , \tag{16}$$

where $E_T=E_{(a)}+E_{(b)}$, $\tilde\varphi_a\equiv\varphi_{(a)}-\pi/2$, and $\tilde\varphi_\pm\equiv\tilde\varphi_{(a)}\pm\tilde\varphi_{(b)}$.

Our solutions for $\varphi(k)$, $E_T(k)$, $L_1(k)$, and $L_2(k)$ (obtained by the Adams-Bashfort method) agree with the ones in Ref. [8] and in an earlier paper [4], where their connection with the Goldstone mode ($M=0$ solution) in the chiral limit and implications for spontaneous chiral-symmetry breaking in potential models [6,7] were discussed. In this work we shall use these solutions to calculate some weak and electromagnetic decays to test whether the harmonic interaction indeed yields results as good as are claimed [8].

III. DECAY CONSTANTS OF π,K AND THEIR RADIAL EXCITATIONS π' AND K'

In the preceding sections we have talked only about describing the hadronic structure, which is here determined by the effective quark-quark interaction kernel $K(x,y)$. In this context, weak and radiative decays can be described by an external local operator $L(x)$. For example, the leptonic weak decay is described by coupling the $V-A$ quark current to the leptonic current:

$$L(x)=\frac{G_F}{\sqrt2}l_\mu\gamma^\mu\frac{1-\gamma_5}{2}\ . \tag{17}$$

Retracing the steps in the derivation of the bilocal action (1) it is easy to see that the presence of such an external operator can be consistently introduced by the substitution $\mathcal{M}(x,y)\rightarrow\mathcal{M}(x,y)+L(x)\delta^{(4)}(x-y)$ in W_{eff}.

The leptonic decay of $\pi^{\pm}$ or $K^{\pm}$ is then caused by the term $(iN_c/2)\mathrm{Tr}[G_{\Sigma}(\mathcal{M}+L)]^2$ in the effective action (1) with $L(x)$ given by (17). More precisely (see Fig. 1),

$$\langle l^{\pm}\nu_l|\tilde{W}_{\text{eff}}|\pi^{\pm}\rangle=\langle l^{\pm}\nu_l|iN_c\mathrm{Tr}(G_{\Sigma_{(b)}}\mathcal{M}_{(ba)}G_{\Sigma_{(a)}}L)|\pi^{\pm}\rangle\ . \tag{18}$$

This matrix element is expressed through the axial-vector-current matrix element $\langle 0|A_{\mu}(0)|\Pi\rangle$, which is in turn parametrized by the pseudoscalar leptonic decay constants $F_{\Pi}(\Pi=\pi,K,\pi',K')$. The computation of (18) therefore yields [8]

$$F_{\Pi}=\frac{4N_c}{M_{\Pi}}\int\frac{d^4q_{\perp}}{(2\pi)^3}L_2(q_{\perp})\sin\left[\frac{\varphi_{(a)}(q_{\perp})+\varphi_{(b)}(q_{\perp})}{2}\right]\ . \tag{19}$$

The isosymmetric version of this formula $[m_{(a)}=m_{(b)}$, i.e., $(a)=(b)$, appropriate for the pion] had already been obtained in Ref. [4], but Amirkhanov *et al.* [8] were the first to present concrete results not only for L and K', but also for the simpler case of π and π'. Their results are surprisingly close to experiment, considering that in (19) they used the solutions L_2, φ, and the eigenvalue M_{π} resulting from the usage of the harmonic color-singlet interaction kernel (8), which one would not expect to be realistic, but just an illustrative and oversimplified, "toy" quark-antiquark interaction.

The parameters of the model are the "strength of the interaction" $\frac{4}{3}V_0$, which sets the overall energy scale $(\frac{4}{3}V_0)^{1/3}$, and the current quark masses $m_{(a)}=\tilde{m}_{(a)}(\frac{4}{3}V_0)^{1/3}$, where the tilde denotes the dimensionless quantities out of which we have factored the overall dimensionful scale $(\frac{4}{3}V_0)^{1/3}$. The dimensionless current masses govern the dimensionless ratios of dimensionful quantities.

Reference [8] fixes the parameters in such a way that the pion and kaon masses take their experimental values $M_{\pi}=140$ MeV and $M_K=497$ MeV:

$$\begin{aligned} m_{ud}&=\tilde{m}_{ud}(\tfrac{4}{3}V_0)^{1/3}=0.007(\tfrac{4}{3}V_0)^{1/3}\ ,\\ m_s&=\tilde{m}_s(\tfrac{4}{3}V_0)^{1/3}=0.21(\tfrac{4}{3}V_0)^{1/3}=30m_{ud}\ ,\\ (\tfrac{4}{3}V_0)^{1/3}&=289\text{ MeV}\ , \end{aligned} \tag{20}$$

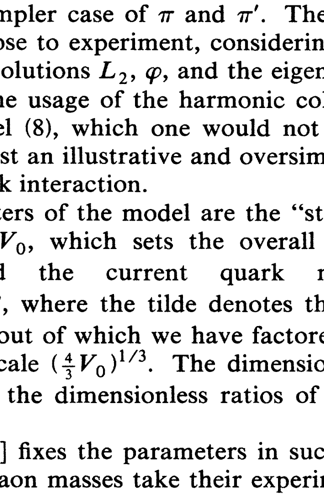

FIG. 1. Decay of the meson bilocal $\mathcal{M}$ into a lepton $l^{\pm}$ end its neutrino ν_l. Dressed quark propagators on the internal lines are given by (9).

where $m_{ud}=(m_u+m_d)/2$ stands for both u- and d-quark masses, which are taken to be the same, i.e., isospin symmetric. With this potential strength, also used in earlier potential-model calculations [6,7], the current masses are $m_{ud}\approx 2$ MeV, $m_s\approx 60$, MeV, i.e., roughly three times smaller than most widely accepted values. However, $m_s/m_{ud}\approx 30$ is close to the widely accepted ratio of 25 [16]. We agree with their solutions $\varphi(q_{\perp})$, $E(q_{\perp})$ for the quarks and solutions $L_1(q_{\perp})$, $L_2(q_{\perp})$, and eigenvalues $M_{\pi'},M_{K'}$. We also reproduce their solutions $L_{1,2}^{\pi'},L_{1,2}^{K'}$ and masses $M_{\pi'},M_{K'}$ for the radial excitations of the pion and the kaon. We get $M_{\pi'}=1604.6$ MeV and $M_{K'}=1653$ MeV, which, in our opinion, can be identified as the experimentally found state $\pi(1300)J^{PC}=0^{-+}$ [with a possible admixture of $\pi(1770)J^{PC}=0^{-+}$] and $K(1460)J^P=0^-$ [with a possible admixture of $K(1830)J^P=0^-$], respectively. $\pi(1770)$, $K(1460)$, and $K(1830)$ still await confirmation [17].

On the other hand, we strongly disagree with the decay constants of Amirkhanov *et al.* They practically reproduce the experimental pion decay constant of 93 MeV, claiming $F_{\pi}=90.4$ MeV. Experimentally, $(F_K)_{\text{expt}}=1.22(F_{\pi})_{\text{expt}}=113$ MeV, and they get $F_K=133$ MeV, which is still very satisfactory. For π' and K', they get $F_{\pi'}=3.4$ MeV and $F_{K'}=52$ MeV, in good agreement with the duality estimates [18,19] $F_{\pi'}=3$–4 MeV, $F_{K'}=51$ MeV.

In contrast with that, for the parameters (20), we find $F_{\pi}=33.8$ MeV and $F_K=47.5$ MeV, which is, respectively, three and two and one half times smaller than the experimental values. Also, $F_{\pi'}=0.83$ MeV and $F_{K'}=12.4$ MeV, so that in fact there is no agreement with the duality estimates [18,19].

We are sure that Amirkhanov *et al.* (and not us) computed the decay constants erroneously since we have discovered internal inconsistencies in their results. For instance, they plotted the quantity $R_1^{\pi}(p)=L_1^{\pi}(p)F_{\pi}/\sin\varphi_{(u)}(p)$ and at $p=0$ it took the value $R_1^{\pi}(0)=1$. Since we agree with Amirkhanov *et al.* that $L_1^{\pi}(0)\simeq 0.87/(\frac{4}{3}V_0)^{1/3}$ and $\varphi_u(0)=\pi/2$, their result $R_1^{\pi}(0)=1$ yields $F_{\pi}\simeq 33.5$ MeV, consistent with our result and not their result.

Is it nevertheless possible to reproduce the experimental decay constants for some other set of parameters? F_{π} and F_K are too small for the quark masses (20) or, if we increase the scale $(\frac{4}{3}V_0)^{1/3}$ to fix this, the meson masses become too large. However, in this model, $M_{\pi}\rightarrow 0$ as $m\rightarrow 0$ [4]. Still more precisely, the correct behavior of a (pseudo) Goldstone boson such as our π is that its mass should behave like $\sqrt{m_{ud}}$ if F_{π} (and $\langle 0|\bar{q}q|0\rangle$) stay finite as $m_{ud}\rightarrow 0$. (See, e.g., Ref. [16].) It is a pleasing feature of the present model that it behaves exactly like that as shown in Table I: $M_{\pi}\sim\sqrt{m_{ud}}$, and F_{π} varies extremely slowly with m_{ud}. Therefore, there is some $\tilde{m}_{ud}<0.007$ for which the ratio $M_{\pi}/F_{\pi}=(M_{\pi}/F_{\pi})_{\text{expt}}=1.50$ and then the same scale V_0 will reproduce M_{π}^{expt} and F_{π}^{expt}. This indeed happens slightly below $\tilde{m}=0.001$ and $(\frac{4}{3}V_0)^{1/3}\approx 750$ MeV. However, although here the current quark mass has the role of a fitting parameter,

TABLE I. Pion mass M_π, pion decay constants F_π, and the $\pi^0 \to \gamma\gamma$ decay width $\Gamma(\pi^0 \to \gamma\gamma)$ for various values of current quark masses m_{ud}; all quantities are in units of MeV and for the scale $(\frac{4}{3}V_0)^{1/3} = 289$ MeV. Results for other values of $(\frac{4}{3}V_0)^{1/3}$ can be obtained by trivial rescaling.

m_{ud}	M_π	F_π	$\Gamma(\pi^0 \to \gamma\gamma)$
0.289	53.6	32.9	0.088×10^{-6}
2.023	140.4	33.8	1.6×10^{-6}
2.89	167.1	34.2	2.7×10^{-6}
6.647	249.3	36.1	9.0×10^{-6}
18.9	479.3	47.0	79.0×10^{-6}

this is such a small value ($m_{ud} = \tilde{m}_{ud}(\frac{4}{3}V_0)^{1/3} = 0.75$ MeV is ten times smaller than the standard value from [16]) that one would have to invoke additional, rather exotic possibilities that the standard current mass assignments might be far too large. Actually, this is not impossible: for instance, Kaplan and Manohar [20] pointed out that even a massless u quark could not be excluded. In the next section, however, we shall show that such small current quark masses lead to the so catastrophic $\pi^0 \to \gamma\gamma$ decay width that even this exotic assumption cannot salvage the claim of Ref. [8] that the covariant generalization of the potential approach in the harmonic approximation can yield a roughly good description of pseudoscalars.

IV. DECAY WIDTH FOR $\pi^0 \to \gamma\gamma$

$\pi^0 \to \gamma\gamma$ is the simplest radiative decay that we can calculate in this model [when $L(x) = QA(x)$, $Q = \mathrm{diag}(Q_u, Q_d, Q_s) = e\,\mathrm{diag}(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3})$], so we do it as a further test of the harmonic approximation. It is already so well described by the Adler-Bell-Jackiw anomaly and PCAC (partial conservation of axial-vector current) that even computations with better kernels probably cannot compete with it. However, anomaly and other computation always contain the step when one actually parametrizes the unknown hadronic structure with the pion decay constant F_π. In this respect, the present calculation is more microscopic, not parametrizing but trying to describe the pion structure. We actually hope that we shall ultimately be able to tackle the two-photon decay of kaons (K_L) in this way, in order to elucidate the role of long-distance nonperturbative QCD effects in this case where PCAC does not work so well, and the calculation of $\pi^0 \to \gamma\gamma$ is a necessary prerequisite for this.

We carry out the explicit computation of the transition matrix element (where P, k, k' are the pion and photon momenta, and σ, σ' photon polarizations):

$$\mathcal{A}_{\pi^0\gamma\gamma} = \langle \gamma(k,\sigma)\gamma(k',\sigma') | \tilde{W}_{\mathrm{eff}}[\mathcal{M} + L] | \pi^0(P) \rangle \ . \tag{21}$$

More precisely, this decay is caused by the term $i(N_c/3)\mathrm{Tr}[G_\Sigma(\mathcal{M} + QA)]^3$ because it contains subterms with one meson bilocal $\mathcal{M}$ and two photon fields A^μ, so that

$$\mathcal{A}_{\pi^0\gamma\gamma} = \langle \gamma(k,\sigma)\gamma(k',\sigma') | iN_c \,\mathrm{tr} \int \prod_{i=1}^{4} d^4x_i \mathcal{M}(x_1,x_2) G_\Sigma(x_2 - x_3) QA(x_3) G_\Sigma(x_3 - x_4) QA(x_4) G_\Sigma(x_4 - x_1) | \pi^0(p) \rangle \ . \tag{22}$$

By transforming to momentum space, one sees that $\mathcal{A}_{\pi^0\gamma\gamma}$ corresponds to the triangle graph in Fig. 2 and its crossed mate ($k \leftrightarrow k', \sigma \leftrightarrow \sigma'$). Note, however, that it is not the usual perturbative γ_5 triangle graph. Indeed, in many respects it is very different. Not only do the propagator lines emanate out of a bilocal bound-state vertex, but these propagators are not free fermion propagators but dressed ones, given by (9) after solving the Schwinger-Dyson equations (11) and (12) for the energy E and the function φ. In other words, it is not bare quarks that appear on the internal lines, but quasiparticles which resulted from quarks being bound in our (albeit modeled) hadron.

Since we work in the isosymmetric limit, we take u and d propagators and vertex solutions Γ to be equal. Then

$$\mathcal{A}_{\pi^0\gamma\gamma} = \frac{(2\pi)^4 \delta^{(4)}(P + k + k')}{\sqrt{(2\pi)^3 2^3 P_0 k_0 k_0'}} \mathcal{T}_{\pi^0\gamma\gamma} \ , \tag{23}$$

$$\mathcal{T}_{\pi^0\gamma\gamma} \equiv 2iN_c e^2 \frac{Q_u^2 - Q_d^2}{\sqrt{2}} \epsilon_\mu(k,\sigma)\epsilon_\nu(k',\sigma') I^{\mu\nu} \ , \tag{24}$$

where

$$I^{\mu\nu} \equiv \int \frac{d^4q}{(2\pi)^4} \mathrm{tr}[\Gamma(q_\perp | P) G_\Sigma(q - P)\gamma^\mu G_\Sigma(q + k')\gamma^\nu G_\Sigma(q)] \ . \tag{25}$$

Inserting Γ and G_Σ, rearranging, integrating over the parallel component q_P, and performing the spinor trace,

$$I^{\mu\nu} = 4\epsilon^{\alpha\beta\mu\nu} \frac{P_\alpha}{M_\pi} I_\beta \ , \tag{26}$$

$$\mathrm{Re} I_\beta = \int \frac{d^3 q_\perp}{(2\pi)^3} \mathcal{J}_\beta(q_\perp, k_\perp', \tilde{\varphi}) \frac{L_2(q_\perp)[E(q_\perp) + E((q + k')_\perp)] - L_1(q_\perp)(M_\pi/2)}{[E(q_\perp) + E((q + k')_\perp)]^2 - M_\pi^2/4} \ , \tag{27}$$

$$\mathrm{Im}I_\beta=-\frac{\pi}{2}\int\frac{d^3q_\perp}{(2\pi)^3}\mathcal{J}_\beta(q_\perp,k'_\perp,\tilde{\varphi})\left[\delta\left[E(q_\perp)+E((q+k')_\perp)+\frac{M_\pi}{2}\right][L_1(q_\perp)+L_2(q_\perp)]\right.$$
$$\left.-\delta\left[E(q_\perp)+E((q+k')_\perp)-\frac{M_\pi}{2}\right][L_1(q_\perp)-L_2(q_\perp)]\right], \tag{28}$$

$$\mathcal{J}_\beta(q_\perp,k'_\perp,\tilde{\varphi})=\frac{[(q+k')_\perp]_\beta}{|(q+k')_\perp|}\cos\tilde{\varphi}(q_\perp)\sin\tilde{\varphi}((q+k')_\perp)-\frac{(q_\perp)_\beta}{|q_\perp|}\cos\tilde{\varphi}((q+k')_\perp)\sin\tilde{\varphi}(q_\perp)\ . \tag{29}$$

The vector integral I_β is a function of the four-vector $(k'_\perp)_\beta$ and the pion mass M_π, and is a functional of the Schwinger-Dyson solution (11), (12) and the Bethe-Salpeter solutions (15), (16). (L_1, L_2, φ, and M_π depend in turn on the quark masses.) Except for the integration variable $q_\perp$,$(k'_\perp)_\beta$ is the only four-vector in the integral I_β. Thus I_β must be proportional to $(k'_\perp)_\beta$:

$$I_\beta=(k'_\perp)_\beta\mathcal{C}[E,L_1,L_2,\tilde{\varphi},M_\pi]\ , \tag{30}$$

where $\mathcal{C}$ is a dimensionless Lorentz-scalar functional of L_1,L_2 and a function of M_π [and of $(k'_\perp)_\beta$, i.e., of M_π again]. $\mathcal{C}$ must be extracted numerically from (30). After noting that

$$\epsilon^{\alpha\beta\mu\nu}P_\alpha(k'_\perp)_\beta=-\epsilon^{\alpha\beta\mu\nu}k_\alpha k'_\beta\ , \tag{31}$$

what remains is totally standard for any calculation of the 2γ decay via the γ_5 triangle. One sums $|\mathcal{A}_{\pi^0\gamma\gamma}|^2$ over the polarizations σ,σ' and integrates over the phase space of the two outgoing photons to finally get the decay width

$$\Gamma(\pi^0\to\gamma\gamma)=\alpha^2M_\pi 8\pi|\mathcal{C}|^2\ . \tag{32}$$

The value of Γ as a function of the average current m_{ud} quark mass is given in Table I [for $(\frac{4}{3}V_0)^{1/3}=289$ MeV]. Generally, the consistency with experiment, $\Gamma_{\rm expt}=(7.7\pm0.5)$ eV, is of the same quality as for meson masses and decay constants. For instance, for $(\frac{4}{3}V_0)^{1/3}=750$ MeV, the third line in Table I ($\tilde{m}_{ud}=0.010$) would yield the almost experimental $F_\pi=89$ MeV and reasonable $\Gamma(\pi^0\to\gamma\gamma)=7$ eV for a very acceptable [16] average u,d quark mass $m_{ud}=7.5$ MeV, but then M_π is three times too large. On the other hand, $\Gamma(\pi^0\to\gamma\gamma)$ eliminates the exotic possibility that nonstandardly small quark masses ($\tilde{m}_{ud}\simeq0.001$), which fit $M_\pi^{\rm expt},F_\pi^{\rm expt}$, save the harmonic approximation since then $\Gamma(\pi^0\to\gamma\gamma)=0.23\ {\rm eV}\simeq\Gamma^{\rm expt}/30$.

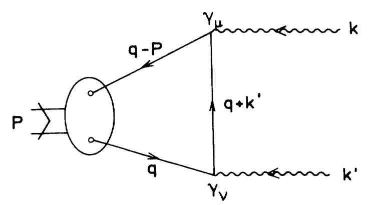

FIG. 2. Decay of the pseudoscalar bilocal (representing π^0) into two photons. Internal lines are dressed quark propagators.

At first sight, the dependence of $\Gamma(\pi^0\to\gamma\gamma)$ on quark masses seems peculiar: although the only dimensionalful quantity $\Gamma(\pi^0\to\gamma\gamma)$ is proportional to M_π, $\Gamma(\pi^0\to\gamma\gamma)$ varies much more rapidly ($\sim m_{ud}^{3/2}$) than M_π itself ($\sim m_{ud}^{1/2}$). However, this is only a concrete manifestation of consistency with PCAC; namely, the standard calculations use current algebra to express the unknown hadronic structure via the experimentally measured F_π, yielding

$$\Gamma(\pi^0\to\gamma\gamma)=\frac{\alpha^2}{64\pi^2}\frac{(M_\pi^{\rm expt})^3}{(F_\pi^{\rm expt})^2}=7.6\ {\rm eV}\ . \tag{33}$$

However, our $\Gamma(\pi^0\to\gamma\gamma)=\alpha^2 8\pi M_\pi|\mathcal{C}|^2$, when divided by our model (M_π^3/F_π^2), is roughly constant over a wide range of quark masses (see Table I) showing that the behavior of $\mathcal{C}$ is consistent with current algebra.

V. CONCLUSION

We have shown that the covariant generalization of the instantaneous potential model proposed in the framework of the effective bilocal Lagrangian [4] in the harmonic approximation does not yield such good results for the pseudoscalar-meson spectra and decay constants $F_\pi,F_K,F_{\pi'},F_{K'}$, as claimed in this model in the first concrete computation of measurable quantities [8]. We have found an improvement of only 50% over F_π obtained in a noncovariant case with quark masses equal to zero [6,7]. We have also computed the decay width for $\pi^0\to\gamma\gamma$ and found results of a similar quality. This is what we would expect. It would actually be very surprising if such a naive imitation of the QCD interaction as the simple harmonic potential should yield good results. One must use more sophisticated interactions. Of course, then one must solve integral equations, but this will not prevent a systematic improvement of interaction kernels. Along with covariance and some technical merits, this possibility of systematic improvement makes the perspective of this approach quite good. Before examining nonsinglet spin and color structures, one should first add the simple funnel (Coulomb-plus-linear) potential

$$V(r)=-\frac{4}{3}\frac{\alpha_s}{r}+ar \tag{34}$$

since (i) the short-distance interactions are dominated by the Coulomb interaction [where the momentum dependence $\alpha_s=\alpha_s(Q)$ can also be taken into account] and (ii) in the long-distance (or $k\to0$) regime, $\alpha_s(k^2)$ times the gluon propagator seems to behave as $1/k^4$ [21,22], from where a term linear in coordinates can arise [23] by Fourier transformation. Then we can hope to obtain simultaneously the approximately correct masses, pseudoscalar decay constants, and $\Gamma(\pi^0 \rightarrow \gamma\gamma)$, which would in turn indicate that our solutions of bound-state equations are of sufficient quality to be used for calculating the long-distance effects in other electroweak decays of light hadrons.

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