Cornering color SU(5)

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Planned collider experiments will decisively test the color SU(5) model of Foot and Hernández, in which an extended QCD group is broken at the TeV scale. Constraints from cosmology and from neutral-kaon mixing imply that exotic charge-$\frac{1}{2}$ fermions of this model cannot all be given masses above about 1 TeV. These "quirks" carry a new strong confining force, from the surviving unbroken gauge symmetry. Searches for the leptonic decay products of quirkonium at CERN LEP II will probe quirk masses up to near the beam energy, while searches at planned hadron colliders will be sensitive to quirk masses all the way up to the TeV upper bound.

I. INTRODUCTION

A basic belief which underlies much contemporary particle physics is that a new level of physics will emerge in the TeV domain. The best-motivated schemes are those which address the origin of the weak scale. However, the elaborate and well-studied schemes of technicolor and supersymmetry may not be correct. It is then of interest to pose another question: are there extensions of the standard model which are sufficiently simple that they can be tested in the near future? Although such models do not give answers to presently perceived theoretical difficulties of the standard model, they do provide pictures of how physics in the TeV domain may appear. The vast majority of such models add new fermions or scalars within the framework of $SU(3)\otimes SU(2)\otimes U(1)$, or extend the $SU(2)\otimes U(1)$ electroweak gauge group. In this paper we follow a much less traveled route: that of extending the color SU(3) interaction. In the course of our investigation, we encounter new primordial relics: glueball-like metastable states whose cosmological implications are interesting in their own right as well as serving to constrain the model.

We study the color SU(5) model of Foot and Hernández [1]. Under plausible assumptions, those authors show that SU(5) is the uniquely favored choice for such an extended color group. Models of this sort can arise from Planck-scale models through the breaking of a fundamental gauge group containing $SU(5)_C \otimes SU(2)_L \otimes U(1)'$, as has been discussed elsewhere [2], but our emphasis will be confined to accessible or near-accessible energies throughout this paper.

An interesting feature of the model is that the color SU(5)-breaking scale can be roughly the same as the weak-breaking scale. Despite the presence of new gauge bosons and fermions, whose interactions are largely constrained by the theory, present experimental data do not provide stringent tests of the model. For example, the model contains a heavy neutral Z' gauge boson that can be much lighter than typical Z' bosons from such schemes as E_6 unification. Precise electroweak experiments provide only very mild constraints on the Z' because it naturally has a small mixing angle with the Z and because it couples predominantly to quarks and not leptons [2]. Searches for bumps in the dijet spectrum at hadron colliders provide the best constraints on the Z' mass [3]: $M_{Z'} > 100$ GeV from Collider Detector at Fermilab (CDF) and $M_{Z'} > 280$ GeV from UA2, which are quite mild. Furthermore, it is natural in this model for the Z' to be heavier than the Z: the Z' originates predominantly from the SU(5) color group, so its mass is proportional to the strong gauge coupling constant.

In this paper we argue that the masses of the new gauge bosons and fermions of the color SU(5) model cannot all be made very large. We show that there are exotic fermions of charge $\frac{1}{2}$ which lie in the TeV range or below. We consider production rates and signatures for such particles at CERN LEP II and at hadron colliders.

The first constraint on the spectrum of the new physics comes from primordial nucleosynthesis. When the extended color SU(5) group breaks at the weak scale it leaves two gauge groups which are never broken: the usual color SU(3) and a new SU(2) force which is also confining. This new force produces glueball-like states which are long lived because there are no light fermions which carry this SU(2) force. If the lifetime of these glueballs exceeds 1 sec they contribute to the energy density of the Universe during the nucleosynthesis era and exclude the model. A shorter lifetime can only be achieved if the new heavy gauge bosons and fermions are not too massive.

A complementary constraint can be imposed from kaon mixing, since the exotic quantum numbers can also run around an internal loop between asymptotic neutral kaon states. Here the forbidden combination is heavy fermions [which preclude a Glashow-Iliopoulos-Maiàna-(GIM) like cancellation between flavors] in conjunction with light broken gauge generators (whose propagators fail to suppress the loop diagrams). Since flavor-changing neutral currents are so highly suppressed, this test is stringent enough to rule out very-low-energy symmetry breaking for any tolerable fermion masses, in the absence of some unexplained family symmetry that would make those masses nearly degenerate. Better still, in conjunction with the cosmological limit, it completely ties down the fermionic sector. If the color gauge group is extended in the way envisaged by Foot and Hernández, then there must be exotic fermions in the TeV range or below. Tracking them down at LEP and the Superconducting Super Collider (SSC) is then an exciting possibility.

The organization of our paper is as follows: in Sec. II we review the particle content of the model, and propose some new nomenclature; in Sec. III, we find expressions for the mass of the lightest confined bound states under various assumptions. In Sec. IV we examine the evolution of these states in the early Universe, and delimit the conditions under which their energy density is cosmologically troublesome. In Sec. V we study their possible decay modes, and in each case find a region in parameter space for which the decays are fast enough to save the standard nucleosynthesis results. In Sec. VI we discuss the limits from the neutral kaon system. In Sec. VII we examine the fate of heavy fermion pairs produced at colliders, and argue that they form nonrelativistic bound states which are usually forced to decay to visible particles of sharply defined energy; the characteristic signatures of these decays are compared to their standard-model backgrounds in Sec. VIII. Finally, we sketch out the progress that has been made in understanding these models, and speculate briefly about directions for future research.

II. THE MODEL

Foot and Hernández [1] have speculated that the standard model (SM) arises from a larger gauge group, specifically

$$SU(5)_C \otimes SU(2)_L \otimes U(1)' \ . \tag{2.1}$$

The reader is referred to previous work [2] for details about this model and its low-energy implications; we shall only summarize the contents of the model in this section. The theory contains three families of fermions, each of which transforms under the gauge groups as

$$\begin{aligned} &\tilde{Q}_L:(5,2)_{1/10},\quad \tilde{U}^c_R:(\bar{5},1)_{-3/5},\quad \tilde{D}^c_R:(\bar{5},1)_{2/5}\ ,\\ &l_L:(1,2)_{-1/2},\quad e^c_R:(1,1)_1\ . \end{aligned} \tag{2.2}$$

It also contains two scalars, which transform as

$$\chi:(10,1)_{1/5},\quad \phi:(1,2)_{1/2}\ . \tag{2.3}$$

The Yukawa interactions take the form

$$\begin{aligned} \mathcal{L}_{\rm Yuk} = &\frac{\sqrt{2}M_L}{w}\chi^c\bar{\tilde{Q}}{}^c_L\tilde{Q}_L+\frac{\sqrt{2}M_R}{w}\chi\bar{\tilde{U}}_R\tilde{D}^c_R\\ &+\frac{\sqrt{2}M_U}{v}\phi^c\bar{\tilde{Q}}_L\tilde{U}_R+\frac{\sqrt{2}M_D}{v}\phi\bar{\tilde{Q}}_L\tilde{D}_R\\ &+\frac{\sqrt{2}M_e}{v}\phi\bar{l}_Le_R+\text{H.c.} \end{aligned} \tag{2.4}$$

When χ acquires a vacuum expectation value w at some large scale, it breaks the group structure of the theory down to

$$SU(3)_C\otimes SU(2)_H\otimes SU(2)_L\otimes U(1)_Y\ . \tag{2.5}$$

[Hypercharge arises from a linear combination of SU(5) and U(1)′ generators.] Under this group, the fermions transform as

$$\begin{aligned} &q_L;(3,1,2)_{1/6},\quad u^c_R:(\bar{3},1,1)_{-2/3}\ ,\\ &d^c_R:(\bar{3},1,1)_{1/3},\quad l_L:(1,1,2)_{-1/2}\ ,\\ &e^c_R:(1,1,1)_1,\quad Q_L:(1,2,2,)_0\ ,\\ &U_R:(1,2,1)_{1/2},\quad D_R:(1,2,1)_{-1/2}\ . \end{aligned} \tag{2.6}$$

The first five fields are those of the standard model (SM); the remaining three are new fermions which feel a new colorlike force. We will call these fermions *quirks*, and their new quantum number *hue*.

At the first stage of symmetry breaking, all twenty-five gauge bosons of $SU(5)_C\otimes U(1)'$ will acquire mass except for the eight gluons g of $SU(3)_C$, the three huons h of $SU(2)_H$, and the linear combination which generates hypercharge. The orthogonal linear combination will be (up to small mixing with the Z) a heavy Z-like particle which we call the Z'. The remaining twelve degrees of freedom are massive gauge bosons

$$X:(3,2,1)_{-1/6} \tag{2.7}$$

which mediate quirk-quark transitions. Of the twenty real degrees of freedom of χ, thirteen become the longitudinal pieces of the X's and the Z', one becomes a heavy Higgs-like particle which couples only to quirks, and the remaining six form a heavy scalar

$$\chi_3:(3,1,1)_{-1/3}\ . \tag{2.8}$$

The other scaler ϕ is just the standard Higgs doublet. It acquires a vacuum expectation value $v\simeq 246$ GeV, giving quarks and quirks $SU(2)_L$-breaking masses. The symmetry breaking at this stage is identical to the SM except that the Z and Z' mix to a small degree. If the end, the unbroken gauge group is $SU(3)_C\otimes SU(2)_H\otimes U(1)_Q$; the $SU(2)_H$ acts nontrivially only on the nonstandard particles.

The quirk mass terms are of the form

$$\mathcal{L}_{\rm mass}=(\bar{U}_L\bar{D}^c_R)\begin{bmatrix}M_L & M_U\\ M_D & M_R\end{bmatrix}\begin{bmatrix}D^c_L\\ U_R\end{bmatrix}+\text{H.c.} \tag{2.9}$$

For three generations, the entries in the above matrix are themselves 3×3 matrices. M_U and M_D are simply the SM up and down mass matrices, while M_L and M_R are arbitrary except that M_L must be symmetric in an $SU(2)_L$ eigenstate basis. We denote the 6×6 mass matrix of (2.9) by $\mathcal{M}$, and observe that $\mathcal{M}$ can be diagonalized by unitary transformations U and V, so that

$$\mathcal{M}_{ij}=U^{\dagger}_{iK}M_K V_{Kj} \ , \tag{2.10}$$

where M_K are the masses of the physical quirks.

III. CONFINEMENT AND THE HUEBALL MASS

Since there are fewer then eleven flavors of quirks, $SU(2)_H$ is asymptotically free but confining at low energies, so it will be useful to determine the confinement scale Λ_2. We start with the experimentally determined value of the strong-interaction gauge coupling α_3 at the 100-GeV scale, and use the renormalization group to evolve it up to the $SU(5)_C$ unification scale. We then match the $SU(2)_H$ and $SU(3)_C$ gauge couplings, and run the former down below the lightest quirk mass to determine Λ_2. We relegate the details of this straightforward but lengthy calculation to Appendix A, where we find an expression (A.4) for Λ_2 in the modified minimal subtraction ($\overline{\mathrm{MS}}$) scheme in terms of the various masses and gauge couplings.

To estimate an upper bound on Λ_2 we will assume that all of the quirks have perturbative Yukawa couplings, which guarantees $M_Q/w \lesssim \sqrt{4\pi}$. Since the X mass is given by $\frac{1}{2}g_5 w$, this translates into the bound

$$\frac{M_Q^2}{M_X^2}<\frac{4}{\alpha_5(M_X)} \ . \tag{3.1}$$

In writing $\alpha_5(M_X)$ we can neglect the fact that α_2 and α_3 run at different rates between M_X and M_Q, where the extended color group is no longer unified. This follows from (3.1) and the renormalization-group equation, since their ratio at M_X is one up to corrections of order $\alpha\ln\alpha$. For the same reason we will set $\alpha_2(M_X)=\alpha_3(M_x)=\alpha_5(M_X)=\alpha_5(M_Q)$. The top quark is at most a factor of 2 above 100 GeV, so we can set $\alpha_3(m_t)=\alpha_3(100)$. The χ_3 scalar is not likely to be lighter than 100 GeV, and in any case hardly influences the result. Finally, $\alpha_3(100)$ is measured [4] to be approximately $\alpha_3(100)=0.110\pm0.010$, so we can conservatively assume $\alpha_3(100)<0.125$. Combining these estimates we find that the confinement scale for $SU(2)_H$ is limited by

$$\Lambda_2<0.7\ \mathrm{GeV}\left[\frac{m_t}{100}\right]^{-1/11}\left[\frac{M_X}{\mathrm{TeV}}\right]^{-5/44}\left[\frac{mQ}{\mathrm{TeV}}\right]^{2/11}\left[\frac{\alpha_5(M_X)}{0.125}\right]^{-159/616}\left[\frac{\alpha_2(m_Q)}{0.125}\right]^{-89/968} . \tag{3.2}$$

Since no quirks have a mass below or near the confinement scale Λ_2, we expect the lightest particles in the (long-distance) $SU(2)_H$ spectrum to be bound hue-singlet states of the massless huons. We call these *hueballs H*, in analogy with the glueballs of QCD. Lattice gauge calculations [5] indicate that the lightest hueball has $J^{PC}=0^{++}$ and a mass $m_H=(3.60\pm0.35)\Lambda_{\mathrm{mom}}$, where Λ_{mom}, the value of Λ in the momentum regularization scheme, is related to the $\overline{\mathrm{MS}}$ value by [6] $\Lambda_{\mathrm{mom}}\simeq3\Lambda_{\overline{\mathrm{MS}}}$. We therefore find, using (3.2), the upper bound

$$m_H<8.3\ \mathrm{GeV}\left[\frac{m_t}{100}\right]^{-1/11}\left[\frac{M_X}{\mathrm{TeV}}\right]^{-5/44}\left[\frac{m_Q}{\mathrm{TeV}}\right]^{2/11}\left[\frac{\alpha_5(M_X)}{0.125}\right]^{-159/616}\left[\frac{\alpha_2(m_Q)}{0.125}\right]^{-89/968} . \tag{3.3}$$

Equation (3.3) is valid if there are four heavy quirks and two light ones. If there are five heavy quirks and a single light quirk the dependence on M_X becomes very weak, so we may set $M_X=1$ TeV and obtain

$$m_H<10\ \mathrm{GeV}\left[\frac{m_Q}{\mathrm{TeV}}\right]^{1/11}\left[\frac{\alpha_2(m_Q)}{0.125}\right]^{-801/19360} . \tag{3.4}$$

IV. HUEBALLS AND COSMOLOGY

Because the 0^{++} hueball is the lightest particle in the $SU(2)_H$ sector, it cannot decay strongly [here "strongly" means via $SU(2)_H$ interactions]. Its decay, through loops of quirks or heavy gauge bosons, will be considerably suppressed. If its mass is a few GeV, and if it is long lived, then it could dominate the Universe at the time of nucleosynthesis. The cosmological constraints on this model follow from the requirement that the highly successful predictions of primordial nucleosynthesis calculations not be upset by the contribution of the hueballs to the energy density of the Universe. We first calculate this contribution relative to the contribution of one neutrino species, using the various entropies to relate the temperatures in the SM and hueball sectors. We then impose agreement with the astrophysical determination of ^{4}He abundance to set an upper bound of 1 sec on the hueball lifetime.

In the big-bang scenario at very early times, all particles are in thermal equilibrium. In particular, at temperatures above the lightest quirk mass m_Q, quirks can mediate energy exchange between photons and huons, as shown in Fig. 1. While the lightest quirks annihilate, the temperature T' of the hue sector tracks the temperature T of ordinary matter, $T'=T$. As the temperature drops, ordinary particles eventually decoupled from the huons. Suppose that this occurs at a temperature of 30 GeV, which is roughly correct if the lightest quirk weighs a TeV. We can calculate the entropy present in the two sectors:

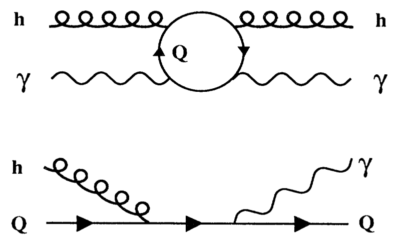

FIG. 1. Typical diagrams leading to quirk-mediated energy exchange between huons and photons in the early Universe.

$$S=\frac{2\pi^2}{45}g_{\text{eff}}T^3 \quad\text{and}\quad S'=\frac{2\pi^2}{45}q'_{\text{eff}}T'^3 , \tag{4.1}$$

where g_{eff} and g'_{eff} are the effective number of degrees of freedom in the two sectors at decoupling. If we count the top quarks and Higgs boson as contributing only about half their normal amount and the W and Z particles as contributing about two-thirds their normal amount (because $T\simeq m_t\simeq m_W\simeq m_Z\simeq m_{\text{Higgs}}$), then $g_{\text{eff}}=98$ and $g'_{\text{eff}}=6$. Since $T=T'$ at this point, the ratios of entropy will just be

$$\frac{S'}{S}=\frac{g'_{\text{eff}}}{g_{\text{eff}}}=\frac{3}{49} . \tag{4.2}$$

From now on the two sectors are never again in thermal contact. As the Universe expands, the entropy per comoving volume will remain constant in each sector, assuming that thermal equilibrium is maintained at all times within each sector. This is probably false, but not because the interactions involved are too slow. Rather, we recall that during a first-order phase transition the sector undergoing this transition is briefly out of thermal equilibrium while the false-vacuum state is maintained ("supercooling"), and then entropy is increased in the transition to the true vacuum ("reheating"). The QCD phase transition is probably first order while a pure-gauge SU(2) theory is believed to be second order [7]. Hence S (but not S') may be increased by some factor ξ during the transition era: below both critical temperatures the ratio of entropies will be given by

$$\frac{S'}{S}=\frac{3}{49\xi} . \tag{4.3}$$

By the time the temperature has dropped to about an MeV, the SM sector consists primarily of photons, electrons, and positrons, and three generations of neutrinos, yielding $g_{\text{eff}}=10.75$. At this temperature the neutrinos decouple. Each neutrino species contributes an entropy density S_ν and an energy density ρ_ν given by

$$S_\nu=\frac{2\pi^2}{45}\frac{7}{4}T^3,\quad \rho_\nu=\tfrac{3}{4}S_\nu T . \tag{4.4}$$

The ratio of entropy in the $SU(2)_H$ sector to the entropy per neutrino species becomes

$$\frac{S'}{S_\nu}=\frac{S'}{S}\frac{S}{S_\nu}=\frac{3}{49\xi}\frac{10.75}{1.75}\simeq\frac{3}{8\xi} . \tag{4.5}$$

Let us now focus on the hueball sector. As we drop below the $SU(2)_H$ confinement scale, the huon energy is clumped into hueballs, and as the temperature drops further eventually only 0^{++} hueballs remains. These cannot annihilate strongly into lighter particles (we will discuss hueball decay below), though three of them can annihilate into two. Such number-changing processes ensure that the hueball field will not acquire a chemical potential. As the Universe expands the temperature of the hueballs drops until they may be treated as a nonrelativistic, noninteracting gas of spinless particles, for which the entropy and energy densities are given by

$$S'=\frac{m_H^{5/2}T'^{1/2}}{(2\pi)^{3/2}}\exp\left[-\frac{m_H}{T'}\right],\quad \rho'=S'T' . \tag{4.6}$$

Hence the ratio of energy density in hueballs to that in one-neutrino species at or below an MeV may be expressed as

$$\frac{\rho'}{\rho_\nu}=\frac{4}{3}\frac{T'}{T}\frac{S'}{S_\nu}\simeq\frac{4}{3}\frac{T'}{T}\frac{3}{8\xi}=\frac{T'}{2\xi T} . \tag{4.7}$$

Using our previous expressions for S_ν, S' and S'/S_ν, we can relate the temperature in the two sectors:

$$\exp\left[\frac{m_H}{T'}\right]=\frac{60\sqrt{2}}{7\pi^3\sqrt{\pi}}\xi\left[\frac{m_H}{T}\right]^3\left[\frac{m_H}{T'}\right]^{1/2} . \tag{4.8}$$

[Note that this formula has no solution for $T>T_{\max}\equiv 0.456\xi^{1/3}m_H$, while for $T\simeq T_{\max}$ it gives $T'\simeq 2m_H$ for which our nonrelativistic approximation fails. If we are to use this expression we must demand, say, $T'(T)<\frac{1}{5}m_H$ which means we can only consider neutrino temperatures, $T<0.149\xi^{1/3}m_H$. In particular, at the onset of nucleosynthesis we will consider only hueballs of mass $m_H>4.7\xi^{-1/3}$ MeV.] Eliminating T' between (4.7) and (4.8), we find

$$\frac{\rho'}{\rho_\nu}\simeq\frac{m_H}{5T\xi}\left[\ln\left[\frac{m_H}{1.6T}\right]+0.6\ln\xi+0.2\ln\left[\frac{\rho'}{\rho_\nu}\right]\right]^{-1} , \tag{4.9}$$

which can be solved recursively for the desired ratio ρ'/ρ_ν at any desired temperature T. Recall that this equation assumes that $T\lesssim\text{MeV}\ll\Lambda,\Lambda'$ and that the hueballs have not yet decayed.

Examination of (4.9) shows that the mass density of hueballs compared to neutrinos increases as the temperature decreases. Indeed, setting $T=T_0$ (the present neutrino temperature) shows that hueballs of mass $m_H\simeq 2.7\xi(1+0.09\ln\xi)$ keV would have just the critical density at the present to close the Universe: they would comprise the dark matter. However, from the scaling of α_2 we learn that such a small Λ_2 requires the six quirks to have masses of order $10^{-12}M_X$, which drives the unification scale w to the grand-unified-theory (GUT) scale. Such a scenario would have few low-energy phenomenological consequences, and Yukawa couplings of order 10^{-12} seem unappealing. We prefer to insist that w be far below the GUT scale; as a result the hueballs must weigh more than a few keV and therefore must decay if only to avoid overclosing the Universe.

How quickly must they decay? That depends on the mass of the hueball and the factor ξ of entropy dumping that occurs in the SM sector. One critical time is during nucleosynthesis, when the Universe is approximately 1 sec old and has a temperature of ~ 0.7 MeV. This is when the processes which interconvert neutrons and protons freeze out. The freeze-out temperature is increased if the energy density is increased because the Universe expands faster and so freeze-out is reached sooner, that is, at a higher temperature. A higher freeze-out temperature implies more neutrons are left, which in turn raises the predicted ^{4}He abundance. The increase in ρ that would result from an additional neutrino species already strains the agreement between the calculated and observed abundance of ^{4}He in the Universe [8]. Hence, the hueball density must satisfy

$$\frac{\rho'}{\rho_\nu}(T=0.7\text{ MeV}) \lesssim 1 \; . \tag{4.10}$$

The value obtained for the ratio in (4.10) depends both on m_H and on ξ. Demanding $\rho' < \rho_\nu$ and $T=0.7$ MeV in (4.9) translates approximately into

$$\xi > 0.09 \left[\frac{m_H}{\text{MeV}}\right]^{(0.85)} \exp\left[\frac{6.9\text{MeV}}{m_H}\right] \tag{4.11}$$

for m_H of at least 10 MeV. The actual value of ξ is very difficult to determine. For our purposes, however, a crude estimate will suffice. Such an estimate can be made by requiring that the degree of supercooling during the transition be insufficient to inflate the Universe, since such inflation would leave most of the Universe today in the QCD plasma state [9]. From this requirement it follows that entropy is at most doubled during reheating. Of course, since our knowledge of the dynamics involved in the QCD phase transition is far from complete, various unexpected complications may have arisen during this period of the early Universe, such as condensation of quark matter matter droplets [10] or formation of primordial black holes [11]. Barring such "nonstandard" conjectures, however, we can take as a conservative upper bound $\xi < 10$. From (4.11) we see that we cannot meet this bound; i.e., we will have difficulties with primordial nucleosynthesis, if the hueball mass is heavier than about 250 MeV. These problems can be avoided if the hueball rots in less than 1 sec.

As will be demonstrated in the next section, the hueball decay rate scales as at least the seventh power of its mass, so light hueballs will last much longer than heavy ones. If the hueball weighs ~ 250 MeV–10 GeV, then it must decay in 1 sec or less, for the high value of ξ that would be needed to avoid nucleosynthesis problems seems highly implausible. If it weighs a few hundred MeV or less, then perhaps it is allowed to live longer than a second without upsetting the neutron-to-proton ratio. However, 10^4 sec later, when the temperature drops to 10 keV, 250-MeV hueballs would certainly dominate the Universe and spoil later nucleosynthesis calculations. Moreover, the decay of such a light hueball into $\gamma\gamma$ or pions will distort the microwave background unless it occurs before 10^6 sec. Because of the strong dependence of the lifetime on the mass, forcing a 250-MeV or lighter hueball to decay in less than 10^4 or even 10^6 sec is a much stronger constraint than demanding that a hueball in the upper part of the 250-MeV to 10-GeV mass range decay in less than 1 sec. Since the hueball *could* weigh as much as 10 GeV we can only impose this weakest constraint, namely $\tau_H < 1$ sec.

V. HUEBALL DECAY

The hueball decays only through loops of heavy gauge bosons and quirks. Some of the diagrams which contribute to hueball decay are shown in Fig. 2. At low energies hueball decay is mediated by effective interactions of the form

$$\begin{aligned} &\mathcal{O}_{q\bar{q}} \sim H_a^2 \bar{q} q, \quad \mathcal{O}'_{\bar{q}q} \sim H_a^2 \bar{q} i\gamma^\mu \partial_\mu q \; , \\ &\mathcal{O}_{GG} \sim H_a^2 G_b^2, \quad \text{and} \quad \mathcal{O}_{FF} \sim H_a^2 F^2 \; , \end{aligned} \tag{5.1}$$

where H_a, G_a, and F are the huon, gluon, and photon field strengths, respectively.

Since the first of these is of lower dimension, it presumably dominates the decay rate. However, it also violates $SU(2)_L$, and hence must have an amplitude proportional to the $SU(2)_L$-breaking masses which may suppress it relative to the other operators through small masses and/or small mixing angles. The lowest-order diagram contributing to $\mathcal{O}_{\bar{q}q}$ is shown in Fig. 2(a). We assume that at least one of the six quirks is lighter than the X, and perform the calculation in four steps: integrating out the X; scaling down to the quirk mass; integrating out the quirk; and scaling down to the hueball mass. We treat the external quark legs as massless to be conservative. For concreteness, we also assume that the external quarks are down type; a similar calculation can be used to determine

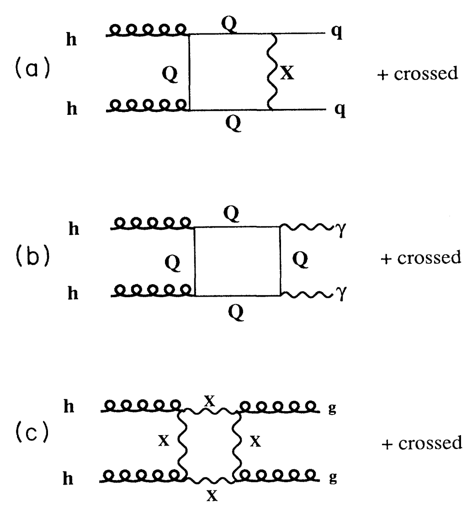

FIG. 2. Loop diagrams which lead to hueball decay into (a) quark-antiquark pairs, (b) photons, or (c) gluons.

the decay rate to up-type quarks. The various steps are carried out in detail in Appendix B, with the result

$$\Gamma(H\to\bar{d}_i d_j)=\frac{m_H^4|\psi(0)|^2}{12\pi M_X^4}|\mathcal{M}_{ij}^{-1}|^2\alpha_3(m_H)^{24/23}\times\alpha_2(m_H)^2\alpha_3(m_t)^{16/161}\alpha_2(m_Q)^{3/4}\times\alpha_5(M_X)^{3/28}\,, \tag{5.2}$$

where $\psi(0)$ is the overlap wave function for the two huons.

What do we substitute for $|\psi(0)|^2$? Presumably this will be something like the reciprocal of the "volume" of the hueball. Lattice calculations [12] indicate that the hueball has a charge-density radius of at least $4/m_H$, so assuming a hydrogenlike wave function, we can expect that the decay rate will be something like

$$\Gamma(H\to\bar{d}_i d_j)=\frac{\mathcal{F}m_H^7}{M_X^4}|\mathcal{M}_{ij}^{-1}|^2\alpha_3(m_H)^{24/23}\alpha_2(m_H)^2\times\alpha_3(m_t)^{16/161}\alpha_2(m_Q)^{3/4}\alpha_5(M_X)^{3/28}\,, \tag{5.3}$$

where $\mathcal{F}\simeq 10^{-4}$. Note that (5.3) displays the advertized seventh-power dependence on the hueball mass.

If the quirk masses are large compared to the quark masses, then $\mathcal{M}_{ij}^{-1}$ will tend to be small. Since $m_t<250$ GeV and $m_Q>43$ GeV we can use $m_Q^2>m_t m_b$ to expand $\mathcal{M}^{-1}$ in powers of m_{quark}/m_Q. To leading order,

$$\mathcal{M}_{ij}^{-1}=(M_L^{-1})_{ik}(M_U)_{kl}(M_R^{-1})_{lj}\,. \tag{5.4}$$

By far the largest mass in the up-quark mass matrix is the top-quark mass. The magnitude of $\mathcal{M}_{ij}^{-1}$ will depend on the masses and mixing angles of the quirks, but we can get a bound on this matrix element by noting that no mixing angle can be greater than unity, so that

$$\mathcal{M}_{ij}^{-1}<m_L^{-1}m_t m_R^{-1}=\frac{m_t(m_Q)}{m_Q^2}\,, \tag{5.5}$$

where m_L and m_R are the lightest eigenstates of the matrices M_L and M_R and m_Q^2 is their product. We see that for this mode of hueball decay to be efficient, there must be at least two light quirks. There must also be large mixing between the generations—although this seems unlikely, we must allow for this possibility.

In (5.5) we need to evaluate $m_t(\mu)$ at the quirk mass; because of renormalization, this will not necessarily be equal to the physical top-quark mass m_t. The relation between the two is given by

$$m_t(m_Q)=m_t\left[\frac{\alpha_5(M_X)}{\alpha_3(m_t)}\right]^{4/7}\left[\frac{\alpha_2(m_Q)}{\alpha_5(M_x)}\right]^{3/8}\,. \tag{5.6}$$

Substituting Eqs. (5.6), (5.5), and (3.3) into Eq. (5.3), we find

$$\Gamma<5.6\times10^5\ \text{sec}^{-1}\left[\frac{m_t}{100}\right]^{15/11}\left[\frac{\text{TeV}}{M_X}\right]^{211/44}\times\left[\frac{\text{TeV}}{m_Q}\right]^{30/11}\alpha_3(m_H)^{24/23}\alpha_2(m_H)^2\times\left[\frac{\alpha_2(m_Q)}{0.125}\right]^{-218/484}\left[\frac{\alpha_2(m_Q)}{\alpha_5(M_X)}\right]^{115/88}\,. \tag{5.7}$$

Since m_H is several times the confinement scale, substitution indicates that $\alpha_2(m_H)$ is much less than one. However, the lightest hueball is, by definition, in the nonperturbative regime for the $SU(2)_H$ theory, so we conservatively estimate $\alpha_2(m_H)<1$. Since we are approximating $\alpha_3(100)\simeq 0.125$, we also use $\alpha_3(m_H)\simeq 0.2$. Finally, we assume $m_t<200$ GeV and $\Gamma>1\ \text{sec}^{-1}$ and obtain the limit

$$\left[\frac{m_Q}{2\ \text{TeV}}\right]\left[\frac{M_X}{10\ \text{TeV}}\right]^{7/4}<1\quad[\text{for }\Gamma(\to\bar{q}q)>1\ \text{sec}^{-1}]\,. \tag{5.8}$$

This limit can be strengthened considerably if we assume that small angles occur in M_L and M_R connecting the top quirk with the down (or strange) quirk. Our experience with quarks implies that such small mixings are likely. If all of the angles are less than twice those appearing in the standard Cabibbo-Kobayashi-Maskawa matrix (0.12 or smaller), this limit is improved to

$$\left[\frac{m_Q}{90\ \text{GeV}}\right]\left[\frac{M_X}{10\ \text{TeV}}\right]^{7/4}<1\quad[\text{for }\Gamma(\to\bar{q}q)>1\ \text{sec}^{-1}]\,. \tag{5.9}$$

This is the limit if hueballs decay predominantly by diagrams of the type in Fig. 2(a). To calculate the rate through diagrams such as those in Fig. 2(b) requires two steps, which again we leave for Appendix B. The resulting decay rate, calculated using the same assumptions as before, is

$$\Gamma(H\to\gamma\gamma)=\frac{\mathcal{F}'\alpha(m_H)^2\alpha_2(m_H)^2m_H^9}{m_Q^8}\,, \tag{5.10}$$

where we expect $\mathcal{F}'\simeq 2.7\times10^{-8}$. Note that for this decay we need only one light quirk, and that no mixing angles are involved. Substituting (3.4) for the hueball mass, we find

$$\Gamma(H\to\gamma\gamma)<2.2\times10^{-3}\ \text{sec}^{-1}\left[\frac{m_Q}{\text{TeV}}\right]^{-79/11}\times\left[\frac{\alpha_2(m_Q)}{0.125}\right]^{-7209/19\,360}\,. \tag{5.11}$$

Assuming the hueball decays in less than 1 sec, this gives a limit

$$m_Q<430\ \text{GeV}\quad[\text{for }\Gamma(\to\gamma\gamma)>1\ \text{sec}^{-1}]\,. \tag{5.12}$$

We have also made an estimate of the decay rate using

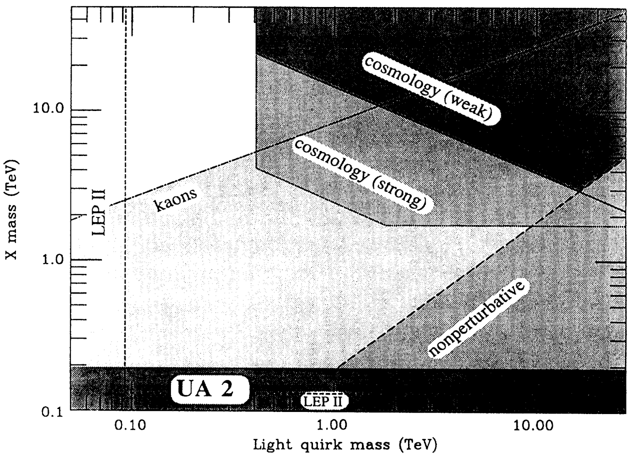

FIG. 3. The two-dimensional parameter space spanned by the quirk and X masses, showing the various astrophysical and experimental constraints. The shaded regions are excluded by nucleosynthesis bounds, by requiring perturbativity, by kaon mixing or by UA2 jet data. The region below and to the left of the dashed line will be accessible to CERN LEP II, whereas the regions below and to the left of the axes in this figure are already excluded by collider results.

diagrams such as those in Fig. 2(c), where the hueball decays into two gluons which must then hadronize. If this is the decay mode chosen by the hueball, then its lifetime is less than 1 sec if

$$M_X < 1.7 \text{ TeV} \quad [\text{for } \Gamma(\to gg) > 1 \text{ sec}^{-1}] . \tag{5.13}$$

It should be emphasized that *only one* of the inequalities (5.8), (5.12), or (5.13) need be satisfied for there to be no conflict with big-bang cosmology. These three limits [together with the strengthened limit (5.9)] are graphed together with Fig. 3.

VI. KAON MIXING

Many extensions of the standard model have potentially detectable effects in the kaon sector, and the $SU(5)_C$ model is no exception. Several diagrams contribute to the K_L-K_S mass difference. In general these lead to quite complicated expressions for this difference. To help simplify the expressions, we will assume that all quirks are heavy compared to the quarks, but much lighter than the X. If the quirks are much heavier than the quarks, then the mass matrix (2.9) takes on an essentially block-diagonal form, and the quirk sector decouples into separate Q_L and Q_R mass eigenstates. The matrices U and V which diagonalize the full quirk mass matrix will also have a nearly block-diagonal form. These assumptions are not essential to estimating kaon mixing, but they make the equations manageable.

Several diagrams contribute to kaon mixing, but the contribution which is naively the largest comes from box diagrams of the type shown in Fig. 4, where a Q_L goes around one side of the box and a Q_R appears on the other side. These diagrams lead to an effective four-quark interaction:

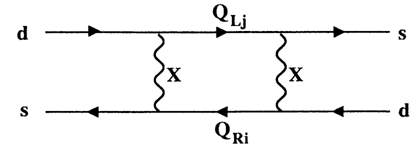

FIG. 4. The quirk and X contribution to the K_L-K_S mass difference.

$$\mathcal{L}_{\text{eff}} = \frac{4\alpha_2^2}{M_X^2} (\bar{s}P_+ d)(\bar{s}P_- d) \sum_{I=1}^{3} \sum_{J=1}^{3} \xi_I \zeta_J \frac{x_I y_J}{x_I - y_J} \ln\left[\frac{x_I}{y_J}\right] , \tag{6.1}$$

where $\xi_I = U_{sI}^\dagger U_{1d}$, $\zeta_J = V_{dJ}^\dagger V_{Js}$, $x_I = M_{RI}^2/M_X^2$, and $y_J = M_{LJ}^2/M_X^2$. The sum on I (J) runs over only the three mass eigenstates of M_R (M_L). Because of unitarity, $\sum \xi_I = \sum \zeta_J = 0$, so that if all of the x_I's or all of the y_J's are equal, this expression will vanish. Such a degeneracy, however, is difficult to arrange naturally. If we use the vacuum saturation approximation, and treat the strange quark as heavy compared to the down quark but light compared to the kaon, then the operator (6.1) will contribute to the kaon mass difference an amount

$$\Delta m_K = \frac{2\alpha_2^2 f_K^2 m_K^2}{m_s^2 M_X^2} \text{Re}\left[\sum_I \sum_J \xi_I \zeta_J \frac{x_I y_J}{x_I - x_J} \ln\left[\frac{x_I}{y_J}\right]\right] . \tag{6.2}$$

We cannot reliably evaluate this expression because we know nothing about the masses or mixings of the quirks. If the quarks are any guide, it would be surprising if ξ_I or ζ_J were always very small. In the quark sector, we have values of order $\xi_I = 0.2$ coming from the Cabibbo angle. Suppose we assume that at least one of the Q_L and Q_R has a mixing of at least half the magnitude of that in the standard Cabibbo-Kobayashi-Maskawa matrix connecting the first two generations, so that $\xi_I \zeta_J > 0.01$ for some I and J. It is easily shown that

$$\frac{x_I y_J}{x_I - y_J} \ln\left[\frac{x_I}{y_J}\right] > \min(x_I, y_J) . \tag{6.3}$$

Then assuming there is no accidental cancellation occurring, there should be a contribution to the kaon mass difference of at least

$$\Delta m_K \gtrsim \frac{0.02\alpha_2^2 f_K^2 m_K^3 m_Q^2}{m_s^2 M_X^4} . \tag{6.4}$$

Assuming this does not exceed the measured kaon mass difference, $\Delta m_K = 3.5 \times 10^{-6}$ eV, implies that the quirk mass is limited by about

$$\frac{m_Q}{\text{TeV}} \lesssim 1.4 \left[\frac{M_X}{10 \text{ TeV}}\right]^2 . \tag{6.5}$$

This limit is also graphed in Fig. 3. Note that the limits graphed in Fig. 3 imply that the quirks are lighter (probably much lighter) than the gauge bosons. Experimentally, this implies that it is much easier to pair produce quirks than it is to look for the Z' resonance in hadron colliders. If we use both the kaon limit (6.5) and the strengthened decay limit (5.9), then the lightest quirk *must* be lighter than 420 GeV.

VII. QUIRKONIUM

The most promising way to probe the $SU(5)_C$ model at accelerators is to pair produce the lightest quirks. Since these are long lived, they will bind to form quirkonium resonances. In this section we first discuss the signatures such resonances might produce, and then estimate the relevant event rates at e^+e^- and hadron colliders.

It can be seen from Fig. 3 that it is possible for all the quirkonium resonances to lie above 10 TeV, and be inaccessible. However, the kaon naturality constraint strongly suggests that some will lie below 2 TeV, and probably below 0.5 TeV. These are very conservative bounds. If the heaviest quirks are not much heavier than the Z' then the renormalization-group scaling gives a lighter hueball and cosmology then forces the lightest quirkonium beneath about 1 TeV. Furthermore, if the central value for the strong gauge coupling is used for input, $\alpha_s(m_W)=0.11$, the lightest quirkonium will be lighter than about 200 GeV, and will be accessible to LEP II. Thus while it is not possible to exclude the possibility that all the quirkonia of this model are above 10 TeV, it is highly plausible that the lightest quirks will not be more than a factor of 2 heavier than the current limit from Z decay.

For simplicity we assume that one quirk is significantly lighter than the others. It is a Dirac fermion of charge $\frac{1}{2}$. Its mass is the smallest eigenvalue of the matrix in equation (2.9). We assume that this mass eigenstate is predominantly a "left quirk" $Q_L=(U_L,D_L^c)$ or a "right quirk" $Q_R=(D_R^c,U_R)$. [Of course, the handedness subscripts here serve only to denote the field degrees of freedom enumerated in Eq. (2.6). Each mass eigenstate has both chiralities as required for a Dirac fermion.] This occurs naturally if the mass splitting between the lightest eigenvalues of M_L and M_R is large compared to the top-quirk mass, or if the top quirk does not mix much with the lightest quirks. These assumptions are to simplify our calculations of the signatures; we do not expect our conclusions to be much changed if there were several light quirks with each mass eigenstate a mixture of left and right quirks.

What happens when a quirk pair is produced in a collision? Suppose that the pair-invariant mass is well above the threshold of $\sim 2m_Q$, but below $4m_Q$. The case of a heavy-quark pair is not analogous at all: the gluon string between the pair fragments by light-quark pair production. This cannot happen with quirks: there are not lighter quirks and hence the huon string cannot break. The system can be viewed as a highly excited quirkonium state. The deexcitation of this quirkonium will occur in two distinct stages. In the first stage the string is longer than Λ_2^{-1} and the quirkonium is in the linear regime of the potential. Deexcitation will begin by hueball emission. There is little we can calculate here; the string is long and the multipole expansion breaks down. However, since the string is made out of hue, we clearly expect huon emission to dominate, and to occur rapidly. Since the string is extended, the hueballs can carry away significant angular momentum leaving the quirkonium with very high J. Our expectation is that many hueballs will be emitted from the oscillating string and as many in any given direction as in the opposite one, so that there is no significant change in the laboratory velocity of the quirkonium. The hueball emission will lead to substantial missing energy but only small missing transverse momentum. Once the excitation energy drops beneath the hueball mass, $E_{ex}<m_H$, deexcitation occurs by photon emission from the oscillating charged quirks. As long as $E_{ex}>\Lambda_2$ the picture is still that of a string with large J. The linear potential has energy spacings characterized by the scale $\Lambda_2(\Lambda_2/m_Q)^{1/3}$. We are not sure whether many soft photons or a few photons with energy a sizable fraction of m_H will be emitted. Fortunately, this is not important since the signature we expect will come from the visible annihilation of the quirk pair. Such annihilation is very unlikely to occur during the era of a long oscillating string.

When E_{ex} drops beneath $9\alpha_2^2m_Q/64$, where $\alpha_2=\alpha_2(m_Q)$, and the spatial extent of the system becomes less than Λ_2^{-1}, the second stage begins: the quirks are now moving in the one-huon-exchange Coulomb potential $-3\alpha_2/4r$. The size restriction translates into a limit on the quantum numbers: $nl \lesssim \alpha_2 m_Q/\Lambda_2$. Throughout this section it will be useful to consider typical values of parameters as $\alpha_2(m_Q)=0.1$, $m_Q=100$ GeV, and $\Lambda_2=100$ MeV. For these values $nl\lesssim 100$ characterizes the Coulomb region.

Prior to entering the Coulomb region the discussion was qualitative; once in the Coulomb region calculations can be made. The picture we have sketched for the string era serves to justify the assumptions which we need for the initial conditions as the quirkonium enters the Coulomb domain. In particular we assume that the quirks have survived and not annihilated, that they have a high orbital quantum number l and that the spin states are statistically populated with the ratio of $S=1$ to $S=0$ states being 3:1. We now turn to the deexcitation in the Coulomb domain.

Annihilation of quirks to huons $Q\bar{Q}\to hh$ is small for high-l states because $\psi_l(r)\propto(\alpha_2 m_Q r)^l$ near the origin and the annihilation is proportional to $|\psi_l(r\lesssim 1/m_Q)|^2$. We find that annihilation from a quirkonium state l gives

$$\Gamma_l(hh)\simeq\alpha_2^{5+2l}m_Q \tag{7.1}$$

which is to be compared with a typical $E1$ electromagnetic transition rate

$$\Gamma_l(E1)\simeq\alpha\alpha_2^4 m_Q\ . \tag{7.2}$$

Annihilation is unimportant for $l\geq 2$ and need be considered only for the S and P states. The dominant $E1$ transitions have $|\Delta l|=1$, so that multiple emission occurs before the P states are reached. The crucial question then is what happens to quirkonia in the states $n\,^3P$ and $n\,^1P$.

One might worry that the eventual fate of the vast majority of all quirkonia is annihilation to invisible hueballs, so that the dominant signature is missing energy together with some soft transition radiation photons. This is completely incorrect: there is a parity which forbids many of the quirkonium states from annihilating to any number of huons. Charge conjugation, C, on the huons can be chosen to be $(h_1,h_2,h_3)\rightarrow(-h_1,h_2,-h_3)$. Define R to be a rotation by π about the T_2 axis of $SU(2)_H$. The h_i have the same R quantum numbers as C quantum numbers so that all are $G=RC$ even. The quirkonia are all $SU(2)_H$ singlets and therefore R even; hence, they have $G=C=(-1)^{l+s}$. Quirkonia with odd $l+s$ cannot decay to only huons. The G-odd singlet P states, $n\,^1P$, cascade predominantly to the singlet S states which are G even and are lost to hueballs. The important remaining question is the fate of the triplet P states, $n\,^3P$. These are G even and could annihilate to hh or cascade to the S states via an $E1$ transition. In this latter case they will end up annihilating visibly via virtual γ and Z.

The fate of the $n\,^3P_J$ states requires a more careful calculation than the estimates of Eqs. (7.1) and (7.2). Assuming equal populations of the (J,m_J) states we find average decay rates

$$\bar{\Gamma}(n\,^3P\rightarrow hh)=\frac{7}{8}\frac{\alpha_2^2}{m_Q^4}|R'_{n1}(0)|^2\,,$$
$$\bar{\Gamma}(n\,^3P\rightarrow 1^3S)=\frac{\alpha}{9}k^3|\langle 1|r|n\rangle|^2\,, \tag{7.3}$$

where k is the energy of the emitted photon, and $R'_{n1}(0)$ is the derivative of the radial wave function at the origin. Evaluating the matrix elements and taking the ratio, we find $\Gamma_{1S}/\Gamma_{hh}=\mathcal{F}(n)\alpha/\alpha_2^3$, where $\mathcal{F}(n)$ varies from 1.9 for $n=2$ to 1.2 for $n=\infty$. So the electromagnetic transition always dominates the huon decay. We conclude that the majority of ^3P states will eventually become ^3S states. These are G odd and cannot annihilate to hueballs, and will instead eventually give annihilations primarily via virtual γ and Z.

In the above discussion we have ignored $M1$ spin-flip transitions because for all nl states we find the $M1$ transition is much smaller than the $E1$ transition. This is especially true for high n and l so that we can take the S quantum number to be fixed during the Coulomb deexcitation. We conclude that 75% of the total quirk pair-production rate will result in the annihilation of a quirkonium resonance involving standard-model particles. When the decay is to pairs of visible particles, the invariant mass of the resulting pair will be $2m_Q$ to an accuracy much better than the detector resolution.

The decay rates of the ^3S states are given by

$$\Gamma(^3S\rightarrow W^+W^-)=\frac{\pi\alpha^2|\psi(0)|^2P^3(4m_Q^4+20m_W^2m_Q^2+3m_W^4)}{6m_Q^5[(4m_Q^2-m_Z^2)^2+m_Z^2\Gamma_Z^2](C-\sin^2\theta_W)^2}\left[\frac{2m_q^2-m_Z^2\sin^2\theta_W}{2m_Q^2-m_W^2}\right]^{2-2C},$$
$$\Gamma(^3S\rightarrow Z^0\ \mathrm{Higgs})=\frac{2\pi\alpha^2|\psi(0)|^2(P^2+3m_Z^2)P}{3m_Q[(4m_Q^2-m_Z^2)^2+m_Z^2\Gamma_Z^2](C-\sin^2\theta_W)^2}\,,$$
$$\Gamma(^3S\rightarrow hh\gamma)=\frac{\pi^2-9}{2m_Q^2}\alpha_2^2\alpha|\psi(0)|^2\,, \tag{7.4}$$
$$\Gamma(^3S\rightarrow hhZ^0)=\frac{\pi^2-9}{2m_Q^2}\alpha_2^2\alpha|\psi(0)|^2\left[\frac{C-\cos^2\theta_W}{C-\sin^2\theta_W}\right],$$
$$\Gamma(^3S\rightarrow\bar{f}f)=\frac{16\pi\alpha^2m_Q^2|\psi(0)|^2}{3[(4m_Q^2-m_Z^2)^2+m_Z^2\Gamma_Z^2]}\sum\left[\frac{Q_fm_Z^2}{4m_Q^2}+\frac{T_{3f}-CQ_f}{C-\sin^2\theta_W}\right]^2,$$

where the last two expressions do not include threshold effects, h stands for a huon, P is the momentum of either of the decay products, the sum in the last term is over colors and chiralities of the outgoing fermions, and $C=0$ ($C=1$) for quirkonium made from Q_L (Q_R). Figure 5 shows the various branching ratios for quirks of mass up to 1 TeV. The easy identifiability of lepton pairs whose invariant mass will be consistently twice the quirk mass, together with the relatively large branching ratio, implies that lepton pairs will be the most easily identified signal of quirk production.

VIII. EXPERIMENTAL SIGNATURES

The unpolarized cross section for quirk pair production is given by

$$\sigma(\bar{f}f\rightarrow\bar{Q}Q)=\frac{\pi\alpha^2s(3-\beta^2)\beta}{g[(s-m_Z^2)^2+m_Z^2\Gamma_Z^2]}\times\sum\left[\frac{Q_fm_Z^2}{s}+\frac{T_{3f}-CQ_f}{C-\sin^2\theta_W}\right]^2, \tag{8.1}$$

where s is the square of the center-of-mass energy, β is

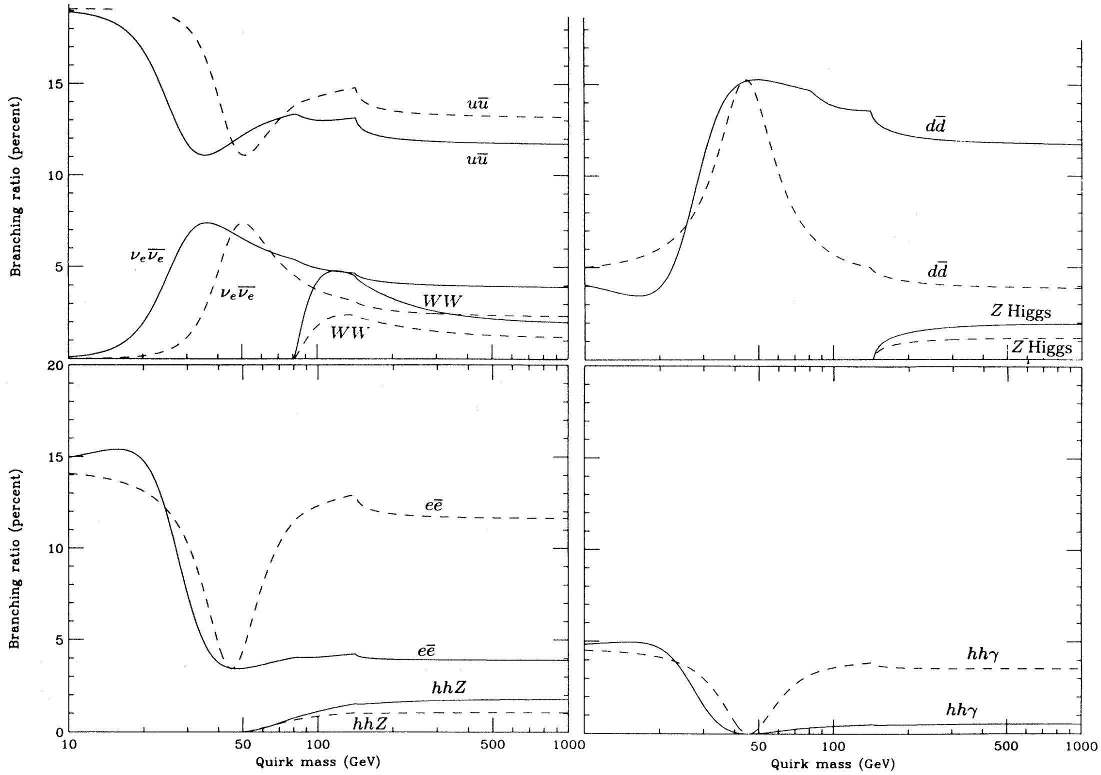

FIG. 5. The various branching ratios of the ^3S quirkonium state. The solid (dashed) lines correspond to left-handed (right-handed) quirks.

the velocity of the outgoing quirks, and the sum is carried out over the two chiralities of the fermions. If the quirks are light enough, they will appear in Z decay. Its partial width into quirks is $4(C-\cos^2\theta_W)^2$ times that for one neutrino species if the quirks are not too close to half the Z mass. All quirk events would result in quirkonium, which would decay primarily to pairs of particles with a mass twice the quirk mass. If the quirk mass is not much bigger than about 43 GeV, these events should be numerous and easily separated from standard Z decays. There is no significant physics background to worry about. We therefore conclude that LEP puts a limit on the lightest quirk of

$$m_Q > 43 \text{ GeV} . \tag{8.2}$$

When LEP II begins operation at $\sqrt{s}$ up to 200 GeV, higher quirk masses will be easily detectable. For quirk masses in the range 40–95 GeV, the cross section to quirks is between 1 and 3 pb. When we include the 75% probability that the quirkonium ends up in a ^3S state, together with the branching ratio to muons, the effective cross section to muons lies in the range 0.03–0.10 pb, which should be easily detectable. The background from W pairs decaying to like-type leptons is only about 0.16 pb, and the cross section from Z pairs, one of which decays invisibly while the other decays to leptons, is only about 0.01 pb. The cross section for τ pairs which then both decay to muons is roughly 0.07 pb. All these background events should be readily distinguishable from quirk events. If candidates are detected, the collider could be run in the neighborhood of twice the quirk mass, looking for the large resonance from the $1S$ state.

For larger quirk masses, only hadron colliders have the energy to produce quirks. Unlike a lepton collider, a hadron collider produces a large background from standard Drell-Yan production at all energies. The signal of quirk pair production would be a peak in the invariant mass of muon (or electron) pairs. Figure 6 shows the cross section for quirk pair production times the branching ratio to muon pairs at the Fermilab Tevatron and the SSC. The integrated luminosity for these machines is effectively doubled if electron and muon data can be combined. For left-type quirks, the Tevatron may be able to detect quirks with a mass in the range 55–70 GeV if the expected $14\%/\sqrt{E(\text{GeV})}$ resolution can be obtained on the mass of the lepton pairs and an integrated luminosity exceeding 50 pb^{-1} is reached. No effective improvement on the mass limit for right-type quirks is likely at the Tevatron.

In contrast to the Tevatron, the SSC can easily discover or exclude quirks of both right and left type. If the SSC can attain design luminosity, and if 1% resolution can be attained on the invariant mass of muon and electron pairs, there is no reason the SSC could not detect or discover quirk pairs throughout most of the probable range of parameter space. Within a decade, we could convincingly confirm or exclude this model.

IX. CONCLUSIONS

The phenomenological successes of the standard model at currently attainable energies carry with them no guarantee of uniqueness. Models such as color SU(5) demonstrate that even surviving unbroken symmetries can be concealed if their quantum numbers are not carried by the light fermions, and that natural hierarchies of symmetry breaking can occur at TeV scales without fine-tuning in the scalar sector. There is ample room for new and surprising physics far below the hypothetical GUT scale.

In earlier work on these models attention was focused on the gauge bosonic sector, which has proved so helpful to our quantitative understanding of the standard model.

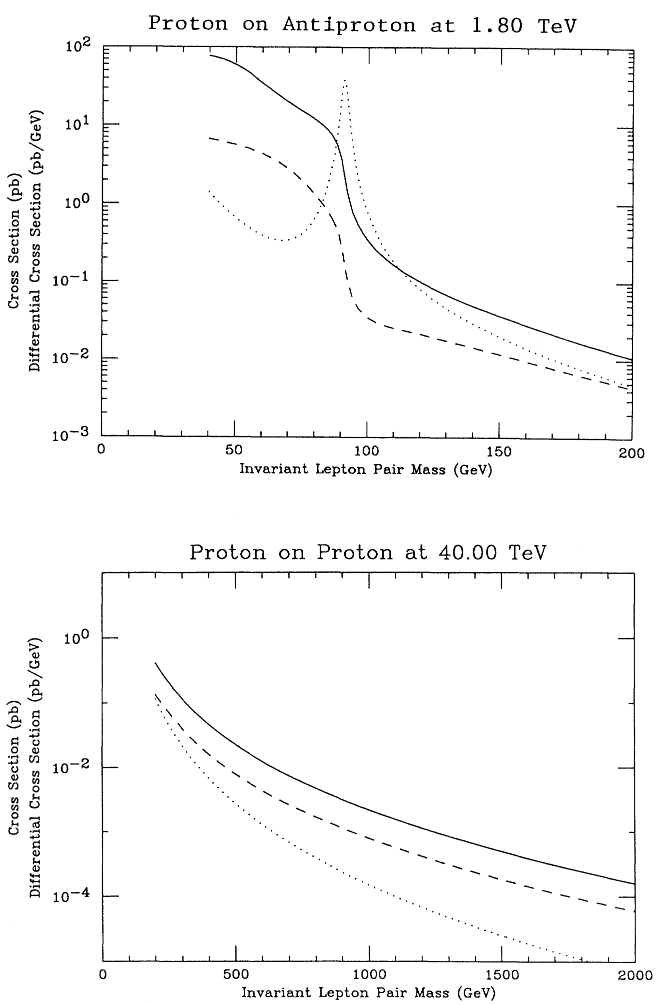

FIG. 6. Cross section times branching ratio into muons for quirk production at the Tevatron and at the SSC, as a function of the dimuon invariant mass. The solid curves are $\sigma \times B$ for left-handed quirks in pb, the dashed curves are the corresponding curves for right-handed quirks, while the dotted curves show the expected Drell-Yan background ($d\sigma/dm$) in pb/GeV, where m is the invariant dimuon mass.

However, the results of the current work suggest strongly that the quirks will be the lightest carriers of net hue, and that their production (or absence), will be the first and decisive test for models of this type.

If, on the other hand, and in defiance of our expectations, the exotic gauge bosons turn out to be as light, or nearly as light, as the lightest quirks, this will signal a cancellation between quirk masses (to evade the kaon bound). Such a cancellation might be a clue to some new underlying symmetry in the quirk mass matrix, and since the off-diagonal blocks of this matrix are simply the $SU(2)_L$-breaking quark masses, this symmetry might also explain why the first two generations of quarks are so light compared to the electroweak scale.

The color SU(5) model provides a simple extension of the standard model at the weak scale. The interactions of the new fermions and gauge bosons are predicted by the theory, but their masses are not. Masses in the range of the weak scale are to be expected, but the form of the interactions is such that it is not surprising that the new particles have not yet been found.

In this paper we have argued that cosmological and flavor-changing constraints on this model imply that it will be tested at planned colliders. Big-bang nucleosynthesis implies that the heavy new fermions and gauge bosons cannot all be made heavy. Mixing of the neutral kaons further indicates that the new fermions will be lighter than the new Z' gauge boson, and will be the most visible consequences of the theory. These quirks have charge $\frac{1}{2}$ and are bound together by a new strong confining force. Searches for the leptonic decay products of quirkonium at LEP II will probe quirk masses up to near the beam energy. Planned hadronic colliders will extend this search into the TeV region for quirk masses. Either quirkonium will be found or the model will become untenable.

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APPENDIX A. CALCULATION OF THE $SU(2)_H$ CONFINEMENT SCALE

In this appendix we use various experimental and theoretical inputs to determine the low-energy running of the $SU(2)_H$ gauge coupling constant, and hence the hue confinement scale Λ_2. This is, of course, renormalization-scheme-dependent quantity. We choose to work in the $\overline{\text{MS}}$ scheme. The gauge coupling $\alpha = g^2/4\pi$ depends on the renormalization point μ according to

$$\frac{d}{d\ln\mu^2}\left[\frac{\alpha(\mu)}{4\pi}\right] = -a\left[\frac{\alpha(\mu)}{4\pi}\right]^2 + b\left[\frac{\alpha(\mu)}{4\pi}\right]^3 + O\left[\left[\frac{\alpha(\mu)}{4\pi}\right]^4\right], \tag{A1}$$

where for an SU(N) theory with n_f fermions and no scalars [13], $a = \frac{11}{3}N - \frac{2}{3}n_f$ and $b = \frac{34}{3}N^2 - \frac{13}{3}Nn_f + n_f/N$. We define Λ by the approximate solution to the two-loop equation (A1):

$$\frac{4\pi}{\alpha(\mu)} \simeq a\ln(\mu^2/\Lambda^2) + \frac{b}{a}\ln[\ln(\mu^2/\Lambda^2)]$$

$$\simeq a\ln(\mu^2/\Lambda^2) - \frac{b}{a}\ln\left[a\left[\frac{\alpha(\mu)}{4\pi}\right]\right] \tag{A2}$$

or

$$\exp\left[\frac{-4\pi}{\alpha(\mu)}\right] \simeq \left[\frac{\Lambda^2}{\mu^2}\right]^a \left[\frac{\alpha(\mu)}{4\pi}\right]^{b/a} a^{b/a}.$$

At any scale μ we will adopt an effective theory by keeping only those particles with masses less than μ. Matching the theories below and above $\mu = M$ involves matching the values of α in the two theories at the scale M. If the theory above (below) M has coefficients a_+ (a_-) and b_+ (b_-) in Eq. (A1), making α continuous at the threshold M requires the two Λ's to be related by

$$\left[\frac{\Lambda_+^2}{M^2}\right]^{a_+}(a_+)^{b_+/a_+} = \left[\frac{\Lambda_-^2}{M^2}\right]^{a_-}(a_-)^{b_-/a_-} \times \left[\frac{\alpha(M)}{4\pi}\right]^{b_-/a_- - b_+/a_+}. \tag{A3}$$

By iteratively applying this formula at successive thresholds we can calculate the value of Λ at any scale. Suppose for simplicity that the quirks lighter than the X bosons have a common mass m_Q while the rest have a common mass M_Q, so $m_Q \le M_X \le M_Q$. We being by calculating Λ for QCD with five flavors, using the measured value of the QCD coupling. We then match at the top mass to obtain Λ for six-flavor QCD. The next threshold for QCD is at M_X, above which the running of the coupling is due to all the gauge bosons of SU(5) with six light flavors. Now above M_Q both the $SU(3)_C$ and $SU(2)_H$ couplings run as an SU(5) with six flavors, so if they are to be equal they must in particular match at M_Q. That is how we arrive at the value of Λ in the $SU(2)_H$ sector just below M_Q. From M_Q down to M_X the hue sector coupling also runs like an SU(5), but with only the light-quirk flavors. At M_X we match to $SU(2)_H$ with the same small number of quirks, and finally at m_Q we obtain the Λ_2 appropriate to $SU(2)_H$ without any light quirks; that is, the desired confinement scale.

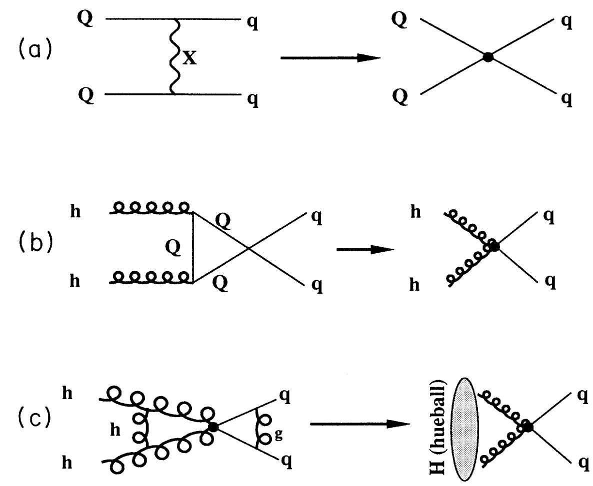

FIG. 7. The varous steps in deriving the effective operator leading to hueball decay into a quark-antiquark pair.

For concreteness, assume that four of the quirks are heavy and two are light. Then the value of Λ_2 at low energies can be expressed as

$$\frac{\Lambda_2}{100} = \exp\left[\frac{-3\pi}{11\alpha_3(100)}\right]\left[\frac{3}{22}\right]^{51/121}\left[\frac{100}{m_t}\right]^{1/11}\left[\frac{M_Q}{M_X}\right]^{4/11}\left[\frac{m_Q}{100}\right]^{2/11}\left[\frac{100}{M_X}\right]^{5/44}\left[\frac{M_{\chi 3}}{100}\right]^{1/44}\left[\frac{\alpha_3(100)}{4\pi}\right]^{-87/253}$$
$$\times\left[\frac{\alpha_3(m_t)}{4\pi}\right]^{107/1181}\left[\frac{\alpha_3(M_X)}{4\pi}\right]^{-7977/16555}\left[\frac{\alpha_5(M_Q)}{4\pi}\right]^{-1842/8041}\left[\frac{\alpha_2(M_X)}{4\pi}\right]^{4747/7480}\left[\frac{\alpha_2(m_Q)}{4\pi}\right]^{-89/968}, \tag{A4}$$

where we have included the one-loop dependence on the mass of the colored scalar χ_3 (and 100 means 100 GeV).

APPENDIX B. EFFECTIVE HUEBALL DECAY OPERATORS

We first evolve the quark-antiquark decay operator $\mathcal{O}_{q\bar{q}}$ shown to lowest order in Fig. 2(a). At energies below the X mass, the X interaction appears an an effective four-point interaction of the quarks and quirks, as in Fig. 7(a). We keep only those terms which connect to $\bar{d}_{Li}d_{Rj}$, since only they can contribute to $\mathcal{O}_{q\bar{q}}$, and obtain the four-fermion interaction

$$\mathcal{O}_{4-\text{fermion}}(M_X)=\frac{-g_5^2}{2M_X^2}V_{iI}^\dagger U_{Jj}(\bar{Q}_J P_-\gamma^\mu d_j)(\bar{d}_i P_+\gamma_\mu Q_I)+\text{H.c.}$$
$$=\frac{4\pi\alpha_5}{M_X^2}V_{iI}^\dagger U_{Jj}(\bar{Q}_J P_- Q_I)(\bar{d}_i P_+ d_j)+\text{H.c.}\ , \tag{B1}$$

where $P_\pm(1\pm\gamma_5)/2$ is the chirality projection operator, d_i is the down-type quark field for family i, Q_I is the Ith quirk field, and g_5 is the gauge coupling of the X boson on the scale M_X.

At lower energies, we need to renormalize this effective interaction by considering the effects of attaching huons and gluons to the quirks and quarks. To one loop this operator runs according to

$$\frac{d\ \ln\mathcal{O}_{4-\text{fermion}}}{d\ \ln\mu^2}=-\frac{9}{16\pi}\alpha_2-\frac{1}{\pi}\alpha_3\ . \tag{B2}$$

With six quarks and two quirks lighter than the X, the couplings α_3 and α_2 run according to

$$\frac{d\ \ln\alpha_2}{d\ \ln\mu^2}=-\frac{3}{2\pi}\alpha_2 \quad\text{and}\quad \frac{d\ \ln\alpha_3}{d\ \ln\mu^2}=-\frac{7}{4\pi}\alpha_3\ , \tag{B3}$$

so

$$d\ \ln\mathcal{O}_{4-\text{fermion}}=\tfrac{3}{8}d\ \ln\alpha_2+\tfrac{4}{7}d\ \ln\alpha_3\ . \tag{B4}$$

Hence, $\mathcal{O}_{4-\text{fermion}}$ at any scale $\mu<M_X$ is given by

$$\mathcal{O}_{4-\text{fermion}}(\mu)=\frac{4\pi}{M_X^2}\alpha_3(\mu)^{4/7}\alpha_2(\mu)^{3/8}\alpha_5(M_X)^{3/56}V_{iI}^\dagger U_{Jj}(\bar{Q}_J P_- Q_I)(\bar{d}_i P_+ d_j)+\text{H.c.} \tag{B5}$$

Next, we integrate out the quirk which appears in the one-loop diagram of Fig. 7(b) to obtain the operator connecting two external huons to two quarks. Because we are interested in the decay of the 0^{++} glueball, we keep only the scalar part:

$$\mathcal{O}_{\bar{q}q}(m_Q)=\sum_I\frac{V_{iI}^\dagger U_{Ij}}{12M_I M_X^2}(\bar{d}_i P_- d_j)\alpha_2(m_Q)H_a^{\mu\nu}H_{a\mu\nu}\alpha_3(m_Q)^{4/7}\alpha_2(m_Q)^{3/8}\alpha_5(M_X)^{3/56}+\text{H.c.}$$
$$=\frac{(\mathcal{M}^{-1})_{ij}}{12M_X^2}(\bar{d}_i P_- d_j)\alpha_2(m_Q)H_a^2\alpha_3(m_Q)^{4/7}\alpha_2(m_Q)^{3/8}\alpha(M_X)^{3/56}+\text{H.c.} \tag{B6}$$

Finally, we run this operator down to the hueball mass according to loops such as those shown in Fig. 7(c). As demonstrated by Grinstein and Randall [14], the product of the β function with H_a^2 does not rescale. To leading order, this means that $H_a^2(m_H)=H_a^2(m_Q)\cdot\alpha_2(m_Q)/\alpha_2(m_H)$. When we also take into account the rescaling of the external quark lines, we find

$$\mathcal{O}_{q\bar{q}}(m_H)=\frac{(\mathcal{M}^{-1})_{ij}}{12M_X^2}(\bar{d}_i P_- d_j)H_a^2\alpha_2(m_H)\alpha_3(m_H)^{12/23}\alpha_3(m_t)^{8/161}\alpha_2(m_Q)^{3/8}\alpha(M_X)^{3/56}+\text{H.c.} \tag{B7}$$

The effective interaction (B7) will contribute to hueball decay, but it is difficult to determine the matrix element of H_a^2 between the 0^{++} hueball state and the vacuum. If we treat the hueball as a weakly bound state of two on-shell huons, then the decay rate to quarks will be given by

$$\Gamma(H\to\bar{d}_i d_j)=\frac{m_H^4|\psi(0)|^2}{12\pi M_X^4}|\mathcal{M}_{ij}^{-1}|^2\alpha_3(m_H)^{24/23}\alpha_2(m_H)^2\alpha_3(m_t)^{16/161}\alpha_2(m_Q)^{3/4}\alpha_5(M_X)^{3/28}\ , \tag{B8}$$

where $\psi(0)$ is the overlap wave function for the two huons. This approximation is not necessarily very accurate, but it does give us some idea of the decay rate.

We proceed similarly for the two-photon decay operator $\mathcal{O}_{FF}$ of Fig. 2(b). We integrate out the quirks, and then rescale the operator down to the hueball scale. After the first step we find the effective interaction

$$\mathcal{O}_{FF}(m_Q)=\frac{\alpha(m_Q)\alpha_2(m_Q)}{180m_Q^4}\left[\tfrac{1}{2}(F_{\mu\nu}F^{\mu\nu})(H_{a\alpha\beta}H_a^{\alpha\beta})+(F_{\mu\nu}H_a^{\mu\nu})^2+\tfrac{7}{16}(\epsilon_{\mu\nu\alpha\beta}F^{\mu\nu}H_a^{\alpha\beta})^2+\tfrac{7}{32}(\epsilon_{\mu\nu\alpha\beta}F^{\mu\nu}F^{\alpha\beta})(\epsilon_{\sigma\rho\gamma\tau}H_a^{\sigma\rho}H_a^{\gamma\tau})\right]\ . \tag{B9}$$

where α is the electromagnetic coupling.

Rescaling this operator down to the hueball mass is nontrivial because the effective Lagrangian contains several operators which rescale in different ways.

We first decompose the various interactions into irreducible parts, as follows:

$$F_{\mu\nu}F_{\alpha\beta}=\tfrac{1}{12}(g_{\mu\alpha}g_{\nu\beta}-g_{\mu\beta}g_{\nu\alpha})(F_{\sigma\rho}F^{\sigma\rho})-\tfrac{1}{24}\epsilon_{\mu\nu\alpha\beta}(\epsilon^{\rho\sigma\gamma\tau}F_{\rho\sigma}F_{\gamma\tau})+(\text{two-index tensor piece})+(\text{four-index tensor piece}) \ . \tag{B10}$$

The product of two H's can be similarly decomposed. Each of these operators will renormalize multiplicatively, and they will not mix with each other.

The symmetry properties of the pseudoscalar and four-index pieces assure that they cannot annihilate the 0^{++} hueball, but the two other operators can. The product of the two-index tensor parts gives zero, however, when we calculate the matrix element between a hueball and two on-shell photons. Hence we need only keep the scalar parts of F^2 and H^2:

$$\mathcal{O}_{FF}(m_Q)=\frac{\alpha_2(m_Q)\alpha_2(m_Q)}{480m_Q^4}H_a^2F^2 \ . \tag{B11}$$

It is relatively simple to rescale these, since $\alpha(\mu)F^2(\mu)$ and $\alpha_2(\mu)H_a^2(\mu)$ once again do not rescale:

$$\mathcal{O}_{FF}(m_H)=\frac{\alpha(m_H)\alpha_2(m_H)}{480m_Q^4}H_a^2F^2 \ . \tag{B12}$$

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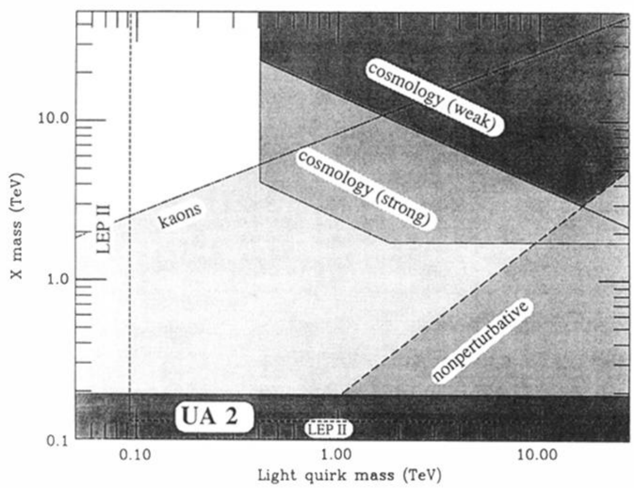

FIG. 3. The two-dimensional parameter space spanned by the quirk and X masses, showing the various astrophysical and experimental constraints. The shaded regions are excluded by nucleosynthesis bounds, by requiring perturbativity, by kaon mixing or by UA2 jet data. The region below and to the left of the dashed line will be accessible to CERN LEP II, whereas the regions below and to the left of the axes in this figure are already excluded by collider results.