Phenomenology of quark-lepton-symmetric models

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Quark-lepton-symmetric models are a new class of gauge theories which unify the quarks and leptons. In these models the gauge group of the standard model is extended to include a "color" group for the leptons, and consequently the quarks and leptons can then be related by a Z_2 discrete quark-lepton symmetry. The phenomenological implications of these theories are explored. Two varieties are analyzed, one being the simplest quark-lepton-symmetric model, and the other also containing conventional left-right symmetry. Each theory has a Z' boson, whose masses are constrained at 90% C.L. to be greater than 700 and 650 GeV, respectively. Phenomenological constraints from rare decays are also examined. An examination of the phenomenological implications of the extended fermion spectrum is given.

I. INTRODUCTION

There are two important questions one may ask of a scientific theory. Is it consistent with experiment? Does it explain all that one would reasonably wish it to?

Upon asking these questions of the standard model (SM) of particle physics one essentially receives the answers "yes" and "no," respectively. The SM is beautifully in accord with all direct laboratory experiments on particles (with the possible exception of the β-decay experiments of Simpson and other experiments [1]). The only other doubts we have about this come via two rather indirect routes: namely, from experiments indicating a solar-neutrino deficit and from the dark-matter problem. It is possible that both, one, or neither of these problems result from the SM being phenomenologically inadequate.

A subjective element enters into answers to the second question. It is possible there are no satisfactory answers for some questions humans can pose, or the questions themselves may turn out to be meaningless. However, the working hypothesis that "all problems have solutions" seems for the moment a reasonable one to adopt, at least for particle physics.

What the SM does not adequately explain can be usefully divided into two (probably overlapping) categories. (i) What is the origin of the gauge group and the representations of this group utilized by quarks, leptons and Higgs bosons? (ii) What is the origin of the many parameters in the SM which are *a priori* arbitrary? We will refer to (i) as the "quantum-number problem" and (ii) as the "parameter problem."

Many ideas have been put forward in attempts to partially or completely answer these questions. It is not at present known if any of these ideas are correct. New thoughts are therefore to be welcomed.

In this paper we will be primarily concerned with the quantum-number problem, though we should keep in mind that the parameter problem may also need to be addressed in the process. We will examine an idea put forward in a previous paper [2], henceforth referred to as paper I, called "quark-lepton symmetry" or "QL symmetry" for short. Theories employing this concept simplify the quark and lepton spectrum by unifying their properties through a discrete symmetry. Note that a *simplification* of the fermion spectrum is achieved; a definitive solution to the whole quantum-number problem is not entertained. This simplification, however, may well prove to be an important step along the possibly rather long road to "complete" understanding. We will also discover that QL-symmetric models can solve the electric-charge quantization problem of the SM [3]. Electric-charge quantization is, of course, an important part of the quantum-number problem.

It is useful to make a further dichotomic subdivision. Ideas which have been proposed in attempts to provide partial or complete solutions to the quantum-number and parameter problems may be divided into two classes: "bottom-up" where new physics is introduced at a relatively nearby scale only, and "top-down" where physics somewhere near the Planck scale is utilized. Examples of bottom-up ideas are technicolor, spontaneous P and/or CP violation, radiative fermion mass generation, most composite models and many more, while examples of top-down notions are grand unified theories (GUT's), random dynamics and string theory.

Top-down ideas are generally much more ambitious than those of the bottom-up variety. String theory, as an extreme example, has as its aim a complete understanding of the whole of particle physics and gravitation. Bottom-up concepts usually attempt to shed light on a restricted selection of puzzles. For instance, left-right-symmetric models [4] tackle the issues of parity violation and the relationship between the quantum numbers of left- and right-handed fermions. The parameter problem is not addressed, although it *may* turn out that left-right symmetry is a necessary precondition for the understanding of other issues at some future stage of development. No one can say at the moment.

"Quark-lepton symmetry" is a bottom-up concept. It is a radically different approach to the unification of quarks and leptons, and may be viewed as a low-energy alternative to the orthodox top-down idea embodied in GUT's. In GUT's, quark and lepton properties are unified by placing them in, usually, one or two irreducible representations of the GUT group (typically on a generation-by-generation basis). Quark-lepton unity then becomes evident at energies greater than about 10^{16} GeV. Quark-lepton-symmetric models, by contrast, utilize a reducible representation of its gauge group, but interrelate the irreducible pieces through discrete symmetries. The scale at which this increased symmetry becomes manifest can be as low as a few 100 GeV's, as we will see.

These two scenarios have quite different phenomenological implications. We find the idea of a quark-lepton symmetry operating at low energies phenomenologically fascinating, because it is amenable to experimental investigation in the near future.

This paper is structured as follows: Having motivated quark-lepton symmetry in the Introduction we go on to define it rigorously in Section II. In Section III we then show how gauge models employing quark-lepton symmetry are constructed. Two theories are presented, one being the simplest such model, and the other also employing conventional left-right symmetry. Section IV then details a phenomenological analysis of the neutral-current sector of the models, and bounds on their Z' bosons are obtained. Other phenomenological issues, such as rare decays, and the basic properties of a new strongly interacting sector predicted by the theories are explored in Section V. We then conclude in Section VI.

II. DEFINITION OF QUARK-LEPTON SYMMETRY

Consider the classification of quarks and leptons under the SM gauge group $G_{\rm SM}$ where $G_{\rm SM} = {\rm SU}(3)_q \otimes {\rm SU}(2)_L \otimes {\rm U}(1)_Y$:

$$\begin{aligned} &Q_L \sim (3,2)(1/3), \quad \ell_L \sim (1,2)(-1), \\ &u_R \sim (3,1)(4/3), \quad \ell_R \sim (1,1)(-2), \\ &d_R \sim (3,1)(-2/3). \end{aligned} \tag{1}$$

There are two suggestive "similarities" evident in this spectrum. The first is a similarity between left- and right-handed particles and the second is between quarks and leptons. The former is characterized by the equality of electric charges and color representations between left- and right-handed partners. The latter is characterized by identical ${\rm SU}(2)_L$ representations for quarks and leptons. Both also have an imperfect correspondence between degrees of freedom. Left-right similarity would be more evident if a right-handed neutrino (ν_R) were to exist. Quark-lepton similarity would be assisted by both a ν_R and a tripling of the number of leptons.

Both of these *similarities* can be elevated to *symmetries* by extending the gauge group and adding the required fermions. The idea of left-right symmetry has been widely studied [4]. $G_{\rm SM}$ is enlarged to G_{LR} where $G_{LR} = {\rm SU}(3)_q \otimes {\rm SU}(2)_L \otimes {\rm SU}(2)_R \otimes {\rm U}(1)_{B-L}$ and ν_R is added to the spectrum:

$$\begin{aligned} &\ell_L \sim (1,2,1)(-1), \quad \ell_R \sim (1,1,2)(-1), \\ &Q_L \sim (3,2,1)(1/3), \quad Q_R \sim (3,1,2)(1/3). \end{aligned} \tag{2}$$

The Z_2 discrete symmetry

$$\ell_L \leftrightarrow \ell_R, \quad Q_L \leftrightarrow Q_R, \quad W_L^\mu \leftrightarrow W_R^\mu \tag{3}$$

can now be defined [where $W^\mu_{L,R}$ are the gauge bosons of ${\rm SU}(2)_{L,R}$]. Note that in enlarging $G_{\rm SM}$ to G_{LR}, ${\rm U}(1)_Y$ has been extended to ${\rm SU}(2)_R \otimes {\rm U}(1)_{B-L}$ with ${\rm SU}(3)_q \otimes SU(2)_L$ remaining untouched. Standard hypercharge is identified as the linear combination $2I_{3R} + (B-L)$ where $I_{3R} = {\rm diag}(1/2, -1/2)$ is the diagonal generator of ${\rm SU}(2)_R$.

The idea of quark-lepton symmetry has recently been proposed [2]. $G_{\rm SM}$ is enlarged to $G_{q\ell}$ where

$$G_{q\ell} = {\rm SU}(3)_\ell \otimes {\rm SU}(3)_q \otimes {\rm SU}(2)_L \otimes {\rm U}(1)_X. \tag{4}$$

Here ${\rm SU}(3)_q$ is the usual color group and ${\rm SU}(3)_\ell$ is its leptonic partner. This enlargement requires a tripling in the number of leptons, which when added to ν_R plus its ${\rm SU}(3)_\ell$ partners yields the spectrum

$$\begin{aligned} &Q_L \sim (1,3,2)(1/3), \quad F_L \sim (3,1,2)(-1/3), \\ &u_R \sim (1,3,1)(4/3), \quad E_R \sim (3,1,1)(-4/3), \\ &d_R \sim (1,3,1)(-2/3), \quad N_R \sim (3,1,1)(2/3). \end{aligned} \tag{5}$$

The standard lepton doublet ℓ_L is embedded in F_L, ℓ_R in E_R, and ν_R in N_R. The Z_2 discrete symmetry

$$\begin{aligned} &F_L \leftrightarrow Q_L, \quad E_R \leftrightarrow u_R, \quad N_R \leftrightarrow d_R, \\ &G_q^\mu \leftrightarrow G_\ell^\mu, \; C^\mu \leftrightarrow -C^\mu \end{aligned} \tag{6}$$

can now be defined [where $G^\mu_{q,\ell}$ are the gauge bosons of ${\rm SU}(3)_{q,\ell}$ and C^μ is the gauge boson of ${\rm U}(1)_X$]. Note that in enlarging $G_{\rm SM}$ to $G_{q\ell}$, ${\rm U}(1)_Y$ has been extended to ${\rm SU}(3)_\ell \otimes {\rm U}(1)_X$ with ${\rm SU}(3)_q \otimes {\rm SU}(2)_L$ remaining untouched. Standard hypercharge is identified as the linear combination $X + \frac{1}{3}T$ where $T = {\rm diag}(-2,1,1)$ is a generator of ${\rm SU}(3)_\ell$.

As should be obvious from the above discussion, the motivation for quark-lepton symmetry is similar to that for left-right symmetry. Both result in interesting and pleasing simplifications of the fermion spectrum. In addition to simplifying the fermion spectrum, left-right symmetry also introduces parity invariance to the Lagrangian. The parity violation observed in experiments then emerges as a result of spontaneous symmetry breaking. Remarkably, it is possible to define a version of quark-lepton symmetry to also yield a parity-invariant Lagrangian.

To see this we first reexpress the fermion spectrum in terms of antileptons rather than leptons:

$$\begin{array}{ll} Q_L \sim (1,3,2)(1/3), & (F_L)^c \sim (\bar{3},1,2)(1/3), \\ u_R \sim (1,3,1)(4/3), & (E_R)^c \sim (\bar{3},1,1)(4/3), \\ d_R \sim (1,3,1)(-2/3), & (N_R)^c \sim (\bar{3},1,1)(-2/3). \end{array} \quad (7)$$

The charge conjugate of the left-handed lepton field $(F_L)^c$ is of course a right-handed field and $(E_R)^c, (N_R)^c$ are left handed. Instead of the discrete symmetry of Eq. (6) we can use

$$\begin{array}{l} (F_L)^c \leftrightarrow Q_L, \;\; (E_R)^c \leftrightarrow u_R, \;\; (N_R)^c \leftrightarrow d_R, \\ -T^{i*} G^{\mu}_{i\ell} \leftrightarrow T^i G^{\mu}_{iq}, \;\; C^{\mu} \leftrightarrow C^{\mu}, \end{array} \quad (8)$$

where the T^i generate the SU(3)-triplet representation. This is manifestly a left-right symmetry, and any Lagrangian invariant under it will be a parity singlet. This provides a realization of the idea of spontaneous parity breaking which is an alternative to the conventional left-right-symmetric model approach. The quarks and leptons are then related physically as "mirror" images of each other.

The two different forms of the quark-lepton discrete symmetry given by Eqs. (6) and (8) are related by a CP transformation. If one were to impose CP invariance on a Lagrangian which was also invariant under one of these discrete symmetries, then it would automatically be invariant under the other discrete symmetry. The slightly different properties of a theory that follow from symmetry (6) will not affect the physics we will explore in this paper. For definiteness, we will henceforth take (6) as our worked example of quark-lepton symmetry.

In paper I a variant of symmetry (8) was employed, wherein the $\bar{\mathbf{3}}$'s of $SU(3)_\ell$ were replaced by $\mathbf{3}$'s. The physical consequences of either version will be identical provided one may redefine the $SU(3)_\ell$ gauge-boson fields by $-T^{i*}G^{\mu}_{i\ell} \to T^i G^{\mu}_{i\ell}$. This will always be possible for us. An example of when $\mathbf{3}$'s would be distinguishable from $\bar{\mathbf{3}}$'s occurs if one wanted to break $SU(3)_q \otimes SU(3)_\ell$ down to its diagonal subgroup. We will of course never want to do this.

Both quark-lepton and left-right symmetry simplify the fermion spectrum, each in a different though analogous fashion. It is interesting to impose these two symmetries simultaneously, thus obtaining a fermion spectrum in which any fermion multiplet (within a generation) can be mapped into any other one by a combination of these two discrete transformations. The gauge group of the quark-lepton left-right-symmetric model [5, 6] is $G_{q\ell LR}$ where

$$G_{q\ell LR} = SU(3)_\ell \otimes SU(3)_q \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)_V, \quad (9)$$

and the fermion spectrum is

$$\begin{array}{ll} F_L \sim (3,1,2,1)(-1/3), & F_R \sim (3,1,1,2)(-1/3), \\ Q_L \sim (1,3,2,1)(1/3), & Q_R \sim (1,3,1,2)(1/3). \end{array} \quad (10)$$

The discrete symmetry of the model may now be represented as

$$\begin{array}{ccccc} F_L \leftrightarrow F_R & V^{\mu} & G^{\mu}_{\ell} & W^{\mu}_L \leftrightarrow W^{\mu}_R \\ \updownarrow \quad\;\; \updownarrow & \updownarrow & \updownarrow & \\ Q_L \leftrightarrow Q_R & -V^{\mu} & G^{\mu}_{q} & \end{array} \quad (11)$$

where left-right-symmetry acts horizontally and quark-lepton symmetry acts vertically.

It is possible to use the discrete symmetry (8) instead of (6) in the left-right-symmetric case as well. There would then be two discrete symmetries interchanging left- and right-handed fermions in the theory. One would thus have a choice as to what was the parity partner of a given Weyl fermion. We will focus on the theory invariant under Eq. (11) in this paper for definiteness, but the particular issues covered here will not depend on this choice, and the reader should understand that the alternative theory is at least as interesting as the one specifically analyzed.

Each of the three possibilities of (i) left-right symmetry, (ii) quark-lepton symmetry, and (iii) simultaneous left-right and quark-lepton symmetries yield a pleasing simplification of the fermion spectrum. They afford greater understanding of the tantalizing regularities observed in quark and lepton quantum-numbers by upgrading vague associations to rigorous symmetries. In the rest of this paper, we will construct gauge models which use both quark-lepton symmetry and simultaneous left-right and quark-lepton symmetries, and then examine their phenomenology.

III. CONSTRUCTION OF MODELS WITH QUARK-LEPTON SYMMETRY

A. The basic quark-lepton-symmetric model

Having defined and motivated quark-lepton symmetry, we now return to first principles and examine the logic of the construction of models invariant under, firstly, $G_{q\ell}$ and (6) and, secondly, $G_{q\ell LR}$ and (11). In this subsection we focus on the first theory. During the process of explaining how the models are constructed we intend to answer many of the questions which the skeptical reader would have formulated.

The most general quark-lepton spectrum under $G_{q\ell}$ consistent with (6) is

$$\begin{array}{ll} Q_L \sim (1,3,2)(-\alpha), & F_L \sim (3,1,2)(\alpha), \\ u_R \sim (1,3,1)(-\beta), & E_R \sim (3,1,1)(\beta), \\ d_R \sim (1,3,1)(-\gamma), & N_R \sim (3,1,1)(\gamma), \end{array} \quad (12)$$

where α, β, and γ are for the moment arbitrary X charges. An important question is whether the construction of the model will force these charges to take on unique values.

We now introduce Higgs bosons in order to break $SU(3)_\ell$, the quark-lepton discrete symmetry (6), and electroweak symmetry, and also to give masses to the fermions. All the Higgs-boson multiplets will be introduced through Yukawa coupling Lagrangians. This also occurs in the standard model and is an important feature since it means the Higgs bosons may not be elementary fields, but may in fact be dynamical bound states of the existing fermions in the model.

Electroweak symmetry breaking is achieved through a SM Higgs doublet, which is defined through the analogue of the standard Yukawa Lagrangian:

$$\mathcal{L}^{(1)}_{\text{Yuk}} = \lambda_1(\bar{F}_L E_R \phi + \bar{Q}_L u_R \phi^c) + \lambda_2(\bar{F}_L N_R \phi^c + \bar{Q}_L d_R \phi) + \text{H.c.} \tag{13}$$

This Lagrangian has the same purpose as in the SM. The Higgs field ϕ has quantum numbers given by

$$\phi \sim (1,1,2)(1). \tag{14}$$

We have used the normalization freedom to fix the X charge of ϕ. Under quark-lepton symmetry ϕ has to transform into its charge conjugate field, due to the change in sign of the gauge field C^μ of $\text{U}(1)_X$:

$$\phi \leftrightarrow \phi^c. \tag{15}$$

Equation (13) also provides the classical constraints

$$\beta = \alpha - 1, \quad \gamma = \alpha + 1 \tag{16}$$

on the fermion X charges. These constraints make the fermion spectrum (12) free from $G_{q\ell}$ gauge anomalies. This property of the quark-lepton-symmetric model is not shared by the SM. In the SM some of the anomaly-cancellation equations are independent of the classical constraints arising from the invariance of the Yukawa Lagrangian.

The Yukawa Lagrangian (13) yields the tree-level mass relations

$$m_u = m_e, \quad m_d = m_\nu. \tag{17}$$

Here $m_{u,e,d,\nu}$ refer to the 3×3 mass matrices (u refers to charge 2/3 uplike quarks, e refers to the charged leptons, etc.). These mass relations arise as a consequence of, (i) the assumption that quark-lepton symmetry is a symmetry of the Yukawa Lagrangian and, (ii) using the minimal Higgs sector of only one doublet. Mass relations between quarks and leptons are generally obtained in theories which unify their properties, provided their Higgs sectors are simple enough. This is true of the Pati-Salam SU(4) model [7] as well as the minimal SU(5) GUT [8], for instance. While it is true that the known fermion masses are not random numbers, the similarities in the fermion mass spectrum do not seem to obey any simple mass relations. It is also true that radiative effects will modify the simple mass relations of Eq. (17), but not enough to be phenomenologically viable. It is an open problem as to whether it is possible to obtain successful mass *predictions* for the known fermions in some scheme employing quark-lepton symmetry. Note, however, that the mass relations can be *evaded* at the expense of predictivity by using a nonminimal Higgs sector. Perhaps the simplest possibility is to introduce two Higgs doublets $\phi_1 \sim (1,1,2)(1)$, $\phi_2 \sim (1,1,2)(-1)$ which transform into each other under quark-lepton symmetry: $\phi_1 \leftrightarrow \phi_2$. It is easy to see that the analogue of $\mathcal{L}^{(1)}_{\text{Yuk}}$ now yields no fermion mass relations at all. Details of the origin of the actual values for the fermion masses will not be necessary for the issues discussed in this paper. It should also be noted that the equality between neutrino and down-quark mass eigenvalues may be altered by the seesaw mechanism (see later).

In order to spontaneously break $\text{SU}(3)_\ell$ and the quark-lepton discrete symmetry, and also to give masses to the $\text{SU}(3)_\ell$ partners of the leptons, we introduce the Higgs bosons χ_1 and χ_2 through the Yukawa Lagrangian $\mathcal{L}^{(2)}_{\text{Yuk}}$ where

$$\mathcal{L}^{(2)}_{\text{Yuk}} = h_1[\overline{(F_L)^c} F_L \chi_1 + \overline{(Q_L)^c} Q_L \chi_2] + h_2[\overline{(E_R)^c} N_R \chi_1 + \overline{(u_R)^c} d_R \chi_2] + \text{H.c.} \tag{18}$$

The quantum numbers of the Higgs fields, and their behavior under the discrete symmetry, are

$$\chi_1 \sim (3,1,1)(-2\alpha), \quad \chi_2 \sim (1,3,1)(2\alpha); \qquad \chi_1 \leftrightarrow \chi_2. \tag{19}$$

In order to understand the role of, in particular, χ_1 we need to look at the transformation properties of F_L, E_R, N_R, and χ_1 under

$$\text{SU}(3)_\ell \to \text{SU}(2)' \otimes \text{U}(1)_T, \tag{20}$$

where $T = \text{diag}(-2,1,1)$ is one of the diagonal generators of $\text{SU}(3)_\ell$. The branching rules are

$$F_L \sim (3,1,2)(\alpha) \to (1,1,2)(-2,\alpha) \oplus (2,1,2)(1,\alpha), \tag{21}$$

$$E_R \sim (3,1,1)(\alpha-1) \to (1,1,1)(-2,\alpha-1) \oplus (2,1,1)(1,\alpha-1), \tag{22}$$

$$N_R \sim (3,1,1)(\alpha+1) \to (1,1,1)(-2,\alpha+1) \oplus (2,1,1)(1,\alpha+1), \tag{23}$$

$$\chi_1 \sim (3,1,1)(-2\alpha) \to (1,1,1)(-2,-2\alpha) \oplus (2,1,1)(1,-2\alpha). \tag{24}$$

We will identify the standard leptons with the $T = -2$ components of F_L, E_R, and N_R. The $T = 1$ components, which form doublets under $\text{SU}(2)'$, are exotic fermions we will call "liptons." We require the $T = -2$ component of χ_1 to develop a nonzero VEV. This does three things. (i) It spontaneously breaks quark-lepton symmetry because, of course, the VEV of its partner χ_2 is required to be zero to ensure color conservation. (One can check that there is a range of parameters for which the Higgs potential yields this pattern of breaking.) (ii) It breaks $\text{SU}(3)_\ell \otimes \text{U}(1)_X$ down to $\text{SU}(2)' \otimes \text{U}(1)_{X-\alpha T}$. We will identify $X - \alpha T$ with standard hypercharge Y. And, (iii) it gives $\text{SU}(2)_L \otimes \text{U}(1)_Y$-invariant masses to the liptons.

Phenomenology requires the quark-lepton symmetry-breaking scale to be larger than the electroweak scale, thus requiring the liptons to be quite massive (unless one were to introduce tiny Yukawa couplings $h_{1,2}$). There are thus two contributions to lipton masses: the $SU(2)_L \otimes U(1)_Y$-invariant masses obtained from the large vacuum expectation value (VEV) of χ_1, and the relatively small $\Delta I_{3L} = 1/2$ masses from the usual electroweak symmetry breaking. We expect the former to dominate over the latter. One of the consequences of this is that liptons may be very massive without any significant constraint from the ρ parameter of weak interactions, since the dominant contribution to the masses is electroweak invariant, and the electroweak variant part is equal to lepton masses rather than, say, a large top-quark mass. It is, however, possible for the neutrino Dirac masses to be large, which would invalidate these qualitative observations. The reader should also note that, contrary to naive expectations, Eq. (18) does *not* induce Majorana masses.

The gauge group $SU(2)'$ is unbroken. It is also asymptotically free and would thus be expected to confine liptons in the infrared region. It is easy to work out from renormalization-group arguments that the confinement scale for $SU(2)'$ (neglecting scalars) should be approximately equal to the QCD scale provided the $SU(3)_\ell$-breaking scale is not too much larger than the lipton mass scale. [If the $SU(3)_\ell$ breaking scale is larger than the lipton masses, then the $SU(2)'$ confinement scale would be smaller than the QCD scale.] Lipton bound states are thus expected to be heavy but nonrelativistic.

This new strongly interacting sector is an interesting prediction of quark-lepton symmetry, yielding some quite striking phenomenology. The lightest lipton bound states may well be lighter than the exotic heavy gauge bosons, and may be the first experimental infestation of an underlying quark-lepton symmetry in nature. We return to liptonic physics in Sec. V.

There is one more piece of physics we may introduce. Our model contains right-handed neutrinos which form Dirac mass terms with their left-handed counterparts as a result of electroweak symmetry breaking. In order to avoid a fine-tuning problem it is attractive to induce a seesaw mechanism by giving large Majorana masses to the ν_R's [9]. This is achieved through a Higgs multiplet Δ_1 defined through $\mathcal{L}^{(3)}_{\rm Yuk}$ where

$$\mathcal{L}^{(3)}_{\rm Yuk} = n_1[\overline{(N_R)^c}N_R\Delta_1 + \overline{(d_R)^c}d_R\Delta_2] + \text{H.c.}, \quad (25)$$

and which transforms like

$$\Delta_1 \sim (\bar{6},1,1)(-2\alpha-2), \quad \Delta_2 \sim (1,\bar{6},1)(2\alpha+2). \quad (26)$$

Here Δ_2 is Δ_1's quark-lepton-symmetric partner. By choosing the quantum numbers of the Higgs field Δ_1 so that it couples to $\overline{(N_R)^c}N_R$ we have recognised that ν_R is the $T=-2$ component of N_R rather than E_R. Under the breakdown $SU(3)_\ell \to SU(2)' \otimes U(1)_T$, Δ_1 decomposes as

$$\begin{aligned}\Delta_1 \sim (\bar{6},1,1)(-2\alpha-2) \to\ & (3,1,1)(-2,-2\alpha-2)\\ &\oplus(2,1,1)(1,-2\alpha-2)\\ &\oplus(1,1,1)(4,-2\alpha-2). \end{aligned} \quad (27)$$

We require the $(1,1,1)(4,-2\alpha-2)$ component of Δ_1 to develop a large nonzero VEV. Like the VEV for χ_1, this breaks quark-lepton symmetry and $SU(3)_\ell$. It also induces a Majorana mass for the ν_R as is evident from Eqs. (23), (25), and (27). Indeed $\langle\Delta_1\rangle \neq 0$ breaks $SU(3)_\ell \otimes U(1)_X$ down to $SU(2)' \otimes U(1)_{X+\frac{1}{2}(\alpha+1)T}$.

It is necessary to identify standard hypercharge Y as $X + \frac{1}{2}(\alpha+1)T$. From the breakdown induced by $\langle\chi_1\rangle \neq 0$ we had previously noted that $Y = X - \alpha T$. For these two equations to be consistent we need

$$\alpha = -\frac{1}{3}, \quad (28)$$

which yields

$$Y = X + \frac{1}{3}T. \quad (29)$$

Adopting $\alpha = -1/3$ is tantamount to choosing a particular way of arranging X-charge quantization, and it reproduces the spectrum of Eq. (5). X-charge quantization in turn implies standard hypercharge quantization through the symmetry breaking induced by $\langle\chi_1\rangle, \langle\Delta_1\rangle \neq 0$ and Eq. (29). This in turn implies electric-charge quantization through $\langle\phi\rangle \neq 0$ (which of course yields $Q = I_{3L} + \frac{Y}{2}$). It is easy to check that the ordinary leptons have the correct charges provided Eq. (28) holds. The choice $\alpha = -1/3$ is thus pivotal in ensuring the success of the quark-lepton-symmetric model. Equation (28) also implies that the liptons have electric charges $\pm 1/2$. These exotic fermions are then confined by $SU(2)'$ into integer-charged bound states. (The spectrum and dominant decay modes of these bound states will be studied in Section V.) Quarks obviously have the correct electric charges because X is equal to Y for quarks. It would be pleasing if something in the theory itself required Eq. (28) to be true. If so, then the logic of the construction of the model would explain electric-charge quantization. This would be a very important result. There is a simple requirement one can make on the Lagrangian which forces $\alpha = -1/3$ on us. It is simply to demand that the Higgs potential term $\chi_1^2\Delta_1$ exists. This is only true if $\alpha = -1/3$. Like Eq. (16), Eq. (28) follows from a classical constraint derived from the Lagrangian.

The quark-lepton-symmetric model therefore explains electric-charge quantization, provided the above Higgs potential term exists. Now, this term is associated with some very interesting phenomenology because its quark-lepton-symmetric partner $\chi_2^2\Delta_2$ violates baryon-number conservation by 2 units [note that $(-1)^B$ is still conserved, so $\Delta B = \pm 1$ processes such as proton decay are forbidden]. This can be easily seen from the Yukawa Lagrangians defined in Eqs. (18) and (25). Thus the observation of neutron-antineutron oscillations would be experimental evidence for the existence of this term.

This completes the construction of the simplest quark-lepton-symmetric model. By way of summary, we list its interesting features. (i) Nonzero VEV's for χ_1 and Δ_1 perform the symmetry breaking

$$\mathrm{SU}(3)_\ell \otimes \mathrm{SU}(3)_q \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_X \to \mathrm{SU}(2)' \otimes \mathrm{SU}(3)_q \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y.$$

(ii) Electroweak symmetry breaking then proceeds via a nonzero VEV for ϕ. (iii) $\mathrm{SU}(2)'$ remains unbroken; it is an asymptotically free gauge force which confines the exotic charge-1/2 fermions (liptons) into integer-charged bound states. (iv) Right-handed neutrinos may gain Majorana masses from $\langle\Delta_1\rangle$ through Eq. (25) and thus induce a seesaw mechanism. (v) Liptons gain $\mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$-invariant masses from $\langle\chi_1\rangle$ through Eq.(18). (vi) The standard fermions and the liptons gain electroweak symmetry-violating masses from $\langle\phi\rangle$ through Eq. (13).

Note that the seesaw mass scale and the lipton mass scale are set by VEV's for different fields. This is important for phenomenological reasons because of the rather stringent lower bound of 50 PeV which naturalness and standard cosmology imply for the seesaw mass scale [10]. (These bounds are only valid if the massive neutrinos do not decay fast enough. For reasons which we will discuss later, it is unclear whether the massive neutrinos in quark-lepton-symmetric models decay in a way inconsistent with standard cosmology. Thus the 50-PeV bound may be evaded in quark-lepton symmetric models.) This is a depressing result because it suggests that new physics associated with the seesaw mechanism should not be observable in the next generation of accelerators. If one takes this lower bound seriously, then in the present model both quark-lepton symmetry and $\mathrm{SU}(3)_\ell$ should be broken at a high scale of 50 PeV or more. However, new physics may still be observable in the 100-GeV to 10-TeV region provided $\langle\chi_1\rangle \ll \langle\Delta_1\rangle$, because then lipton masses would be much lighter than the quark-lepton-symmetry-breaking scale. The exotic $\mathrm{SU}(2)'$ strongly interacting sector, which is a remnant of quark-lepton symmetry, may be relevant for the CERN e^+e^- collider LEP 200, the CERN Large Hadron Collider (LHC), and the Superconducting Super Collider (SSC), even if full quark-lepton symmetry is not.

B. Simultaneous left-right- and quark-lepton-symmetric models

We now turn to the construction of a theory invariant under $G_{q\ell LR}$ and discrete symmetry (11). Our experience with the procedure explained above will allow us to write down the Lagrangian immediately. The fermion spectrum under $G_{q\ell LR}$ is

$$F_L \sim (3,1,2,1)(-1/3), \quad F_R \sim (3,1,1,2)(-1/3), \tag{30}$$
$$Q_L \sim (1,3,2,1)(1/3), \quad Q_R \sim (1,3,1,2)(1/3).$$

Note that all the fermion V charges are related to each other by either quark-lepton or left-right symmetry, where we have set the arbitrary normalization of V so that the quark V charges equal their $B-L$ eigenvalues. All $G_{q\ell LR}$ anomalies cancel as a result of these two discrete symmetries. Instead of Eq.(13) we now have

$$\mathcal{L}^{(1)}_{LR1} = \lambda_1(\bar{F}_L F_R \Phi + \bar{Q}_L Q_R \Phi) + \lambda_2(\bar{F}_L F_R \Phi^c + \bar{Q}_L Q_R \Phi^c) + \text{H.c.}, \tag{31}$$

or, alternatively,

$$\mathcal{L}^{(1)}_{LR2} = \lambda_1(\bar{F}_L F_R \Phi + \bar{Q}_L Q_R \Phi^c) + \lambda_2(\bar{F}_L F_R \Phi^c + \bar{Q}_L Q_R \Phi) + \text{H.c.}, \tag{32}$$

where Φ is the usual Higgs bidoublet of left-right-symmetric models:

$$\Phi \sim (1,1,2,2)(0). \tag{33}$$

Whether Eq. (31) or Eq. (32) is adopted depends on how Φ transforms under quark-lepton symmetry. These equations correspond, respectively, to the choices

$$\Phi \leftrightarrow \Phi \tag{34}$$

or

$$\Phi \leftrightarrow \Phi^c. \tag{35}$$

Equation (34) leads to the mass relations

$$m_u = m_\nu, \quad m_d = m_e, \tag{36}$$

while Eq. (35) leads to the relations in Eq. (17). The transformation law of Eq. (35) is the analogue in this left-right-symmetric theory of Eq. (15). These phenomenologically unsuccessful relations can be evaded, as in the previous theory, by complicating the Higgs sector and by using the seesaw mechansim. Or, as before, one may regard these tree-level results as an excellent basis on which to build predictive models for fermions masses and mixing angles, though such work is beyond the scope of this paper. These considerations will not affect discussion in the present work, though we will need to discuss the seesaw mecahnism later on. Liptons gain mass from the analogue of Eq. (18):

$$\mathcal{L}^{(2)}_{LR} = h[\,\overline{(F_L)^c} F_L \chi_1 + \overline{(F_R)^c} F_R \chi_1 + \overline{(Q_L)^c} Q_L \chi_2 + \overline{(Q_R)^c} Q_R \chi_2] + \text{H.c.}, \tag{37}$$

where

$$\chi_1 \sim (3,1,1,1)(2/3), \quad \chi_2 \sim (1,3,1,1)(-2/3) \tag{38}$$

and $\chi_{1,2}$ interchange under quark-lepton symmetry:

$$\chi_1 \leftrightarrow \chi_2. \tag{39}$$

A nonzero VEV for χ_1 also breaks quark-lepton symmetry and $\mathrm{SU}(3)_\ell$.

Right-handed neutrinos gain Majorana masses from the Lagrangian

$$\mathcal{L}^{(3)}_{LR} = n[\,\overline{(F_L)^c} F_L \Delta_{1L} + \overline{(F_R)^c} F_R \Delta_{1R} + \overline{(Q_L)^c} Q_L \Delta_{2L} + \overline{(Q_R)^c} Q_R \Delta_{2R}] + \text{H.c.}, \tag{40}$$

where

$$\Delta_{1L} \sim (\bar{6},1,3,1)(2/3), \quad \Delta_{1R} \sim (\bar{6},1,1,3)(2/3), \tag{41}$$

$$\Delta_{2L} \sim (1,\bar{6},3,1)(-2/3), \quad \Delta_{2R} \sim (1,\bar{6},1,3)(-2/3).$$

The transformation laws under left-right symmetry and quark-lepton symmetry should be immediately apparent for these Higgs fields.

Several interesting observations can now be made

(1) Suppose we again take the cosmological/naturalness lower bound of 50 PeV for the seesaw mass scale seriously. This requires the VEV for Δ_{1R} to be large. This nonzero VEV induces the symmetry breaking

$$G_{q\ell LR} \to \mathrm{SU}(2)' \otimes \mathrm{SU}(3)_q \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y \otimes \mathrm{U}(1)_{Y'}, \tag{42}$$

where standard hypercharge Y is given by

$$Y = 2I_{3R} + V + \tfrac{1}{3}T \tag{43}$$

and the orthogonal generator Y' is given by

$$Y' = 2\eta I_{3R} + \zeta V + \frac{\lambda}{\sqrt{3}}T, \tag{44}$$

where the explicit form of Y' will be given in the next section. The effective theory after this stage of symmetry breaking is thus just the SM, together with the massless lipton/SU(2)$'$ sector, and an additional U(1) gauge symmetry generated by Y'.

If we assume that $\langle\chi_1\rangle \ll \langle\Delta_{1R}\rangle$, then lipton masses and the mass of the Z' boson coupling to Y' would be typically much smaller than the quark-lepton- and SU(3)$_\ell$-symmetry-breaking scale. Therefore in addition to the new strongly interacting lipton sector, the left-right symmetric version of the quark-lepton symmetric model could also have a relatively light Z' boson, even though the seesaw scale is high. This is in contrast with the simplest quark-lepton symmetric model, which necessarily has its Z' boson mass set by the seesaw scale. Actually, the left-right symmetric version has a Z'' boson as well as a Z' boson. This is due to the additional diagonal generator in the theory. However the Z'' boson's mass is set by the seesaw scale.

(2) It should be also be mentioned that the QL-LR model contains unbroken global baryon- and lepton-number symmetries. These symmetries are not imposed but are consequences of the gauge structure of the theory (as in the SM). This was not the case for the basic QL-symmetric model discussed in the last section. The baryon number B has the values 1/3 for the left- and right-handed quarks and $-2/3$ for the Δ_{2L}, Δ_{2R}, and χ_2 Higgs fields. Lepton number is given by

$$L = L' - T - 6(I_{3L} + I_{3R}), \tag{45}$$

where L' has the value 1 for left- and right-handed leptons and -2 for the χ_1 and $\Delta_{1L,R}$ Higgs fields. It is easy to check that there are no Lagrangian terms consistent with gauge invariance and renormalizability which break these global symmetries.

(3) Finally, one should also note that electric-charge quantization is explained in the left-right symmetric version of the QL symmetric model as well as in the basic model.

In the next section we will make a detailed study of the Z' bosons of both models, and obtain bounds from known neutral current data. Of course, for the first model this would be an uninteresting analysis if one took the 50-PeV lower bound as completely unquestionable. However, we feel it is prudent to adopt the attitude that, despite the successes of standard cosmology, one should not take cosmological constraints as seriously as direct laboratory measurements. We therefore ignore the cosmological/naturalness bound for the purpose of studying the Z' physics of the simplest quark-lepton symmetric model. For the left-right symmetric version, we will actually assume that the VEV for Δ_{1R} is much larger than a few TeV. This will allow us to ignore the presence of the Z'' boson in the theory; we will obtain bounds on the Z' boson coupling to Y' assuming that Z'' effects are negligibly small.

IV. NEUTRAL-CURRENT PHENOMENOLOGY

In this section we will proceed to work out the details of the neutral-current sector of the two quark-lepton symmetric models discussed in this paper. Our aim will be to fit the parameters of the model to the existing low-energy data so that a lower bound for the QL-symmetry-breaking scale can be determined. Note that no upper bound on this mass scale can be derived from the neutral-current data alone for either of these QL-symmetric models. These models will reduce to the SM, in the neutral-current sector, when the QL-symmetry-breaking scale is pushed to infinity.

In the following two subsections we derive the interaction Lagrangians between the fermions and neutral gauge bosons in terms of the fundamental parameters. In subsection C we derive bounds from neutral current data for these parameters.

A. The basic quark-lepton-symmetric model

Recall that the relevant part of the covariant derivative required for the neutral current analysis is given by

$$\mathcal{D}^\mu = \partial^\mu + ig_2 I_{3L} W^\mu_{3L} + i\frac{g'}{2}Y'B^\mu + ig_s T_\ell C^\mu, \tag{46}$$

where $T_\ell = \frac{1}{2\sqrt{3}}\mathrm{diag}(-2,1,1)$ in SU(3)$_\ell$ space and g_2, g' and g_s are the SU(2)$_L$, U(1)$_{Y'}$, and SU(3)$_{q,l}$ coupling constants respectively. Then from the kinetic terms of the Higgs scalars the neutral-gauge-boson mass matrix can be written down as

$$M^2 = \begin{pmatrix} g_2^2 u^2/2 & -g_2 g' u^2/2 & 0 \\ -g_2 g' u^2/2 & g'^2 u^2/2 + 4g'^2 w^2/9 & 4\sqrt{3} g' g_s w^2/9 \\ 0 & 4\sqrt{3} g' g_s w^2/9 & 4g_s^2 w^2/3 \end{pmatrix}, \tag{47}$$

where $\langle\phi\rangle = u$ and $\langle\chi_1\rangle = w$.

By solving the eigenvalue problem for M^2 one obtains the mass of the Z' boson and the VEV w in terms of the SM parameters:

$$M_{Z'}^2 = \frac{2\kappa_1 M_W^2}{g_2^2}\left(1 - \frac{g_2^2}{2\kappa_1}\frac{M_Z^2}{M_W^2}\right)\left(1 - \frac{\kappa_2 g_2^2}{2\kappa_3}\frac{M_Z^2}{M_W^2}\right)^{-1}, \tag{48}$$

$$w^2 = \frac{g_2^2}{2\kappa_3}\frac{M_Z^2}{M_W^2}M_{Z'}^2, \tag{49}$$

where

$$\begin{aligned} \kappa_1 &= \frac{1}{2}(g_2^2 + g'^2), \\ \kappa_2 &= \frac{4}{9}(g'^2 + 3g_s^2), \\ \kappa_3 &= \frac{2}{9}(g_2^2 g'^2 + 3g_2^2 g_s^2 + 3g'^2 g_s^2). \end{aligned} \tag{50}$$

Furthermore, the electromagnetic coupling constant is related to the other gauge coupling constants by

$$\frac{1}{e^2} = \frac{1}{g_2^2} + \frac{1}{g'^2} + \frac{1}{3g_s^2}. \tag{51}$$

Having done this, the neutral-current interactions can be written in the form

$$\mathcal{L}_{\rm NC} = \sum_i g_2 \bar{f}\gamma_\mu (V_i^f - A_i^f\gamma_5) f Z_i^\mu, \tag{52}$$

where the indices f and i denote the types of fermions and neutral gauge bosons, respectively, with

$$\begin{aligned} V_Z^f &= \frac{1}{2N_Z\cos^2\theta_W}\left(I_{3L} - 2\sin^2\theta_W Q + \frac{1}{3}\sin^2\theta_W \xi_Z T_\ell\right), \\ A_Z^f &= \frac{1}{2N_Z\cos^2\theta_W}I_{3L}, \\ V_{Z'}^f &= \frac{M_{Z'}^2}{2N_{Z'}M_W^2}\left[I_{3L} - 2\left(1 - \frac{M_W^2}{M_{Z'}^2}\right)Q + \frac{1}{3}\left(1 - \frac{M_W^2}{M_{Z'}^2}\right)\xi_{Z'}T_\ell\right], \\ A_{Z'}^f &= \frac{M_{Z'}^2}{2N_{Z'}M_W^2}I_{3L}, \end{aligned} \tag{53}$$

where

$$\xi_Z = \left(1 - \frac{3M_Z^2}{4g_s^2 w^2}\right)^{-1} - 1, \tag{54}$$

$$\cos^2\theta_W \equiv \frac{M_W^2}{M_Z^2}, \tag{55}$$

and

$$\begin{aligned} N_Z^2 &= 1 + \tan^4\theta_W\left(\frac{g_2^2}{g'^2} + \frac{1}{3}\frac{g_2^2}{g_s^2}(1+\xi_Z)^2\right), \\ N_{Z'}^2 &= 1 + \left(\frac{M_{Z'}^2}{M_W^2} - 1\right)^2\left(\frac{g_2^2}{g'^2} + \frac{1}{3}\frac{g_2^2}{g_s^2}(1+\xi_{Z'})^2\right). \end{aligned} \tag{56}$$

Note in the above expressions that $\xi_{Z'}$ is the same as ξ_Z but with Z replaced by Z'.

B. The left-right and quark-lepton-symmetric model

The neutral part of the covariant derivative for the gauge symmetry $G_{q\ell LR}$ is

$$\begin{aligned} \mathcal{D}^\mu = \partial^\mu &+ ig_2 I_{3L} W_{3L}^\mu \\ &+ ig_2 I_{3R} W_{3R}^\mu + ig_s T_\ell C^\mu + i\frac{g_V}{2} V D^\mu, \end{aligned} \tag{57}$$

where $I_{3R} = \frac{1}{2}{\rm diag}(1,-1)$ in $SU(2)_R$ space and $T_\ell = \frac{1}{2\sqrt{3}}{\rm diag}(-2,1,1)$ in $SU(3)_\ell$ space. For reasons explained earlier we will assume in this model that the LR-symmetry-breaking scale is much larger than the other two scales in the model, i.e., $\langle\phi\rangle, \langle\chi\rangle \ll \langle\Delta_{1R}\rangle$. The VEV $\langle\Delta_{1R}\rangle$ will give mass to one combination of the W_{3R}^μ, C^μ, and D^μ neutral gauge fields. The two orthogonal combinations are massless in the limit $\langle\phi\rangle, \langle\chi\rangle = 0$. One of these bosons is the gauge field coupling to standard hypercharge, and the other one is the unmixed Z'. Explicitly, we find that the effective field theory which emerges from such a symmetry-breaking pattern is governed by the gauge symmetry $G_{\rm eff}$ where

$$G_{\rm eff} = SU(2)' \otimes SU(3)_q \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_{Y'}. \tag{58}$$

The corresponding covariant derivative relevant to the neutral current sector is given by

$$\mathcal{D}^\mu = \partial^\mu + ig_2 I_{3L} W_{3L}^\mu + i\frac{g_1}{2} Y B^\mu + i\frac{g'}{2} Y' B'^\mu, \tag{59}$$

where

$$Y = 2I_{3R} + \frac{2}{\sqrt{3}}T_\ell + V, \tag{60}$$

$$Y' = 2\eta I_{3R} + 2\lambda T_\ell + \zeta V, \tag{61}$$

with

$$\eta = g_2\left(\frac{2\sqrt{3}g_s}{g_V} - \frac{g_V}{\sqrt{3}g_s}\right),$$
$$\lambda = g_s\left(\frac{3g_2}{g_V} + \frac{g_V}{g_2}\right), \qquad (62)$$
$$\zeta = -g_V\left(\frac{2\sqrt{3}g_s}{g_2} + \frac{\sqrt{3}g_2}{g_s}\right).$$

The SM fermions transform under Eq. (58) as

$$l_L \sim (1,1,2)\left(-1, -\frac{2}{\sqrt{3}}\lambda - \frac{1}{3}\zeta\right),$$
$$\nu_R \sim (1,1,1)\left(0, \eta - \frac{2}{\sqrt{3}}\lambda - \frac{1}{3}\zeta\right),$$
$$e_R \sim (1,1,1)\left(-2, -\eta - \frac{2}{\sqrt{3}}\lambda - \frac{1}{3}\zeta\right),$$
$$Q_L \sim (1,3,2)\left(\frac{1}{3}, \frac{1}{3}\zeta\right), \qquad (63)$$
$$u_R \sim (1,3,1)\left(\frac{4}{3}, \eta + \frac{1}{3}\zeta\right),$$
$$d_R \sim (1,3,1)\left(-\frac{2}{3}, -\eta + \frac{1}{3}\zeta\right),$$

while the Higgs scalars responsible for subsequent symmetry breakings at lower energies transform as

$$\phi \sim (1,1,2)(1,\eta),$$
$$\chi_1 \sim (1,1,1)\left(0, -\frac{2}{\sqrt{3}}\lambda + \frac{2}{3}\zeta\right). \qquad (64)$$

When ϕ and χ_1 acquire nonzero VEV's the neutral gauge bosons gain masses of the form

$$\mathcal{L}_{\rm mass} = \frac{1}{4}u^2(g_2W_{3L} - g_1B - g'\eta B')^2 + \frac{1}{3}w^2g'^2\left(\lambda - \frac{1}{\sqrt{3}}\zeta\right)^2 B'^2, \qquad (65)$$

where Lorentz indices have been suppressed. By letting

$$\hat{Z} = \frac{1}{g_Z}(g_2W_{3L} - g_1B) \qquad (66)$$

and

$$v^2 = \frac{4}{3}\left(\lambda - \frac{1}{\sqrt{3}}\zeta\right)^2 w^2, \qquad (67)$$

where $g_Z^2 = g_1^2 + g_2^2$, the neutral-gauge-boson mass matrix can be rewritten as

$$\mathcal{L}_{\rm mass} = \frac{1}{2}\left(\hat{Z}\ B'\right)\begin{pmatrix} m_1^2 & \delta m^2 \\ \delta m^2 & m_2^2\end{pmatrix}\begin{pmatrix}\hat{Z} \\ B'\end{pmatrix}, \qquad (68)$$

where

$$m_1^2 = \frac{1}{2}g_Z^2u^2,$$
$$m_2^2 = \frac{1}{2}g'^2(\eta^2u^2 + v^2), \qquad (69)$$
$$\delta m^2 = -\frac{1}{2}g_Zg'\eta u^2.$$

The mass matrix can be diagonalized by an orthogonal transformation so that the interaction and mass eigenstates are related by

$$\begin{pmatrix}\hat{Z} \\ B'\end{pmatrix} = \begin{pmatrix}\cos\phi & -\sin\phi \\ \sin\phi & \cos\phi\end{pmatrix}\begin{pmatrix}Z \\ Z'\end{pmatrix}. \qquad (70)$$

Solving the eigenvalue problem results in

$$m_1^2 = \frac{g_Z^2}{g_2^2}M_W^2, \qquad (71)$$
$$M_{Z'}^2 = \frac{m_1^2}{M_Z^2 - m_1^2}\left[M_Z^2 - \left(1 + \frac{g'^2\eta^2}{g_Z^2}\right)m_1^2\right], \qquad (72)$$
$$v = \frac{\sqrt{2}}{g'}\frac{M_ZM_{Z'}}{m_1}, \qquad (73)$$
$$\tan\phi = \frac{g_Z^2M_W^2 - g_2^2M_Z^2}{g_Zg'\eta M_W^2}. \qquad (74)$$

In addition there are relations between the various coupling constants:

$$\frac{1}{g'^2} = \frac{1}{g_1^2}(9g_2^2 + 12g_s^2 + g_V^2),$$
$$\frac{1}{g_1^2} = \frac{1}{g_2^2} + \frac{1}{3g_s^2} + \frac{1}{g_V^2}, \qquad (75)$$
$$\frac{1}{e^2} = \frac{1}{g_1^2} + \frac{1}{g_2^2}.$$

The neutral-current coupling constants in the same notation as Eq. (52) are given by

$$V_Z^f = \frac{1}{2\sqrt{1-x}}(I_{3L} - 2xQ)\cos\phi + \frac{g'}{4g_2}(Y_L' + Y_R')\sin\phi,$$
$$A_Z^f = \frac{1}{2\sqrt{1-x}}I_{3L}\cos\phi + \frac{g'}{4g_2}(Y_L' - Y_R')\sin\phi,$$
$$V_{Z'}^f = -\frac{1}{2\sqrt{1-x}}(I_{3L} - 2xQ)\sin\phi + \frac{g'}{4g_2}(Y_L' + Y_R')\cos\phi, \qquad (76)$$
$$A_{Z'}^f = -\frac{1}{2\sqrt{1-x}}I_{3L}\sin\phi + \frac{g'}{4g_2}(Y_L' - Y_R')\cos\phi,$$

where $x \equiv (g_1/g_Z)^2$ and $Y_{L,R}' \equiv Y'(f_{L,R})$.

C. Phenomenological analysis

To extract the lower bounds for the Z' mass for the two models we will perform a χ^2 fit of the model parameters to current collider data. First we will present the expressions of the various observables necessary to do this.

1. Low-energy hadronic neutral-current parameters

The effective four-fermion interactions relevant to ν-hadron and parity-violating e-hadron neutral-current processes are presented as follows:

$$\mathcal{L}_{\rm eff}^{\nu q} = \frac{G_F}{\sqrt{2}}\overline{\nu}\gamma_\mu(1-\gamma_5)\nu \times[\epsilon_L^q\overline{q}\gamma_\mu(1-\gamma_5)q+\epsilon_R^q\overline{q}\gamma_\mu(1+\gamma_5)q], \quad (77)$$

where

$$\epsilon_L^q = \frac{2M_W^2}{M_Z^2}\left((V_Z^\nu + A_Z^\nu)(V_Z^q + A_Z^q) + \frac{M_Z^2}{M_{Z'}^2}(V_{Z'}^\nu + A_{Z'}^\nu)(V_{Z'}^q + A_{Z'}^q)\right),$$

$$\epsilon_R^q = \frac{2M_W^2}{M_Z^2}\left((V_Z^\nu + A_Z^\nu)(V_Z^q - A_Z^q) + \frac{M_Z^2}{M_{Z'}^2}(V_{Z'}^\nu + A_{Z'}^\nu)(V_{Z'}^q - A_{Z'}^q)\right). \quad (78)$$

and

$$\mathcal{L}_{\rm eff}^{eq} = \frac{G_F}{\sqrt{2}}(C_{1q}\overline{e}\gamma_\mu\gamma_5 e\ \overline{q}\gamma^\mu q + C_{2q}\overline{e}\gamma_\mu e\ \overline{q}\gamma^\mu\gamma_5 q + H_{AA}^q\overline{e}\gamma_\mu\gamma_5 e\ \overline{q}\gamma^\mu\gamma_5 q), \quad (79)$$

where

$$C_{1q} = \frac{8M_W^2}{M_Z^2}\left(A_Z^e V_Z^q - \frac{M_Z^2}{M_{Z'}^2}A_{Z'}^e V_{Z'}^q\right),$$

$$C_{2q} = \frac{8M_W^2}{M_Z^2}\left(V_Z^e A_Z^q - \frac{M_Z^2}{M_{Z'}^2}V_{Z'}^e A_{Z'}^q\right), \quad (80)$$

$$H_{AA}^q = -\frac{8M_W^2}{M_Z^2}\left(A_Z^e A_Z^q - \frac{M_Z^2}{M_{Z'}^2}A_{Z'}^e A_{Z'}^q\right). \quad (81)$$

2. Neutrino-electron scattering

The total cross section for ν_e-e scattering is given by

$$\sigma(\nu_e e) = \frac{E_\nu m_e}{\pi}\left((A+B)^2 + \frac{1}{3}(A-B)^2\right) \quad (82)$$

where

$$A = \frac{G_F}{\sqrt{2}}\left(8\cos^2\theta_W V_Z^\nu V_Z^e + 8\frac{M_W^2}{M_{Z'}^2}V_{Z'}^\nu V_{Z'}^e + 1\right),$$

$$B = \frac{G_F}{\sqrt{2}}\left(8\cos^2\theta_W V_Z^\nu A_Z^e + 8\frac{M_W^2}{M_{Z'}^2}V_{Z'}^\nu A_{Z'}^e + 1\right). \quad (83)$$

In this equation m_e is the mass of the electron and E_ν is the energy of the incident neutrino.

The cross section for ν_μ-e scattering is the same as for ν_e-e scattering but with

$$A \longrightarrow A - \frac{G_F}{\sqrt{2}},\quad B \longrightarrow B - \frac{G_F}{\sqrt{2}}. \quad (84)$$

Similarly, the cross section for $\overline{\nu_e}$-e scattering is given by

$$\sigma(\overline{\nu_e}e) = \frac{E_\nu m_e}{\pi}\left((A-B)^2 + \frac{1}{3}(A+B)^2\right), \quad (85)$$

where A and B are the same as in $\sigma(\nu_e e)$.

The cross section for $\overline{\nu_\mu}$-e scattering has the same form as $\overline{\nu_e}$-e scattering and A and B are the same as for $\sigma(\nu_\mu e)$.

3. Cross section for $e^+e^- \to \bar{f}f$ ($f = \mu, \tau$)

$$R(e^+e^- \to \bar{f}f) = \frac{\sigma(e^+e^- \to \bar{f}f)}{\sigma_{\rm QED}} = R_1 + R_2 + R_3 + R_{12} + R_{13} + R_{23}, \quad (86)$$

where

$$R_1 = 1,$$

$$R_2 = \left(\frac{G_F\sqrt{2}M_W^2}{\pi\alpha_{\rm em}}\right)^2 \frac{s^2}{(s-M_Z^2)^2 + \Gamma_Z^2 M_Z^2} \times[(V_Z^e)^2 + (A_Z^e)^2]^2,$$

$R_3 = R_2$ but with $Z \to Z'$,

$$R_{12} = \frac{G_F\sqrt{2}M_W^2}{\pi\alpha_{em}}\frac{s(s-M_Z^2)}{(s-M_Z^2)^2 + \Gamma_Z^2 M_Z^2}(V_Z^e)^2, \quad (87)$$

$R_{13} = R_{12}$ but with $Z \to Z'$,

$$R_{23} = 2\left(\frac{G_F\sqrt{2}M_W^2}{\pi\alpha_{\rm em}}\right)^2 \times\frac{s^2[(s-M_Z^2)(s-M_{Z'}^2) + \Gamma_Z M_Z \Gamma_{Z'} M_{Z'}]}{[(s-M_Z^2)^2 + \Gamma_Z^2 M_Z^2][(s-M_{Z'}^2)^2 + \Gamma_{Z'}^2 M_{Z'}^2]} \times(V_Z^e V_{Z'}^e + A_Z^e A_{Z'}^e)^2.$$

4. Forward-backward asymmetry

The forward-backward asymmetry for $e^+e^- \to \bar{f}f$ is given by

$$A_{\rm FB} = 3\frac{R^{\rm FB}}{R}, \quad (88)$$

where

$$R^{\rm FB} = R_1^{\rm FB} + R_2^{\rm FB} + R_3^{\rm FB} + R_{12}^{\rm FB} + R_{13}^{\rm FB} + R_{23}^{\rm FB} \quad (89)$$

with

$$R_1^{\rm FB} = 0,$$

$$R_2^{\rm FB} = \left(\frac{G_F\sqrt{2}M_W^2}{\pi\alpha_{\rm em}}\right)^2 \frac{s^2}{(s-M_Z^2)^2+\Gamma_Z^2M_Z^2}(V_Z^eA_Z^e)^2,$$

$$R_3^{\rm FB} = R_2^{\rm FB} \text{ but with } Z\to Z', \tag{90}$$

$$R_{12}^{\rm FB} = \frac{G_F\sqrt{2}M_W^2}{2\pi\alpha_{\rm em}}\frac{s(s-M_Z^2)}{(s-M_Z^2)^2+\Gamma_Z^2M_Z^2}(A_Z^e)^2,$$

$$R_{13}^{\rm FB} = R_{12}^{\rm FB} \text{ but with } Z\to Z',$$

$$R_{23}^{\rm FB} = \frac{1}{2}\left(\frac{G_F\sqrt{2}M_W^2}{\pi\alpha_{\rm em}}\right)^2 \times\frac{s^2[(s-M_Z^2)(s-M_{Z'}^2)+\Gamma_ZM_Z\Gamma_{Z'}M_{Z'}]}{[(s-M_Z^2)^2+\Gamma_Z^2M_Z^2][(s-M_{Z'}^2)^2+\Gamma_{Z'}^2M_{Z'}^2]} \times(V_Z^eA_{Z'}^e+A_Z^eV_{Z'}^e)^2.$$

5. *The Z width*

By assuming three light generations and the mass of the t quark to be heavier than the mass of the Z boson, then the leptonic, hadronic and total widths of the Z boson are given as

$$\Gamma_\ell = \frac{\sqrt{2}}{3\pi}G_FM_W^2M_Z[(V_Z^\ell)^2+(A_Z^\ell)^2], \tag{91}$$

$$\Gamma_{\rm had} = \frac{\sqrt{2}}{\pi}G_FM_W^2M_ZC_F\left([(V_Z^d)^2+(A_Z^d)^2] +\frac{2}{3}[(V_Z^u)^2+(A_Z^u)^2]\right), \tag{92}$$

$$\Gamma_Z = 3\Gamma_\ell+3\Gamma_\nu+\Gamma_{\rm had}, \tag{93}$$

where

$$C_F = 3\left(1+\frac{\alpha_s(M_Z^2)}{\pi}+\cdots\right). \tag{94}$$

A two-parameter χ^2 fit in (M_W, M_Z) was performed for the two QL-symmetric models to determine a lower bound for the mass of the Z' boson in each of them. The input parameters used were G_F, $\alpha_{\rm em}$, α_s, M_W, and M_Z [11] where

$$79.52 \le M_W \le 80.30 \text{ GeV},$$
$$91.156 \le M_Z \le 91.198 \text{ GeV}. \tag{95}$$

To incorporate the largest radiative corrections we have used the "improved Born approximation [12]." In our notation this amounts to

$$M_Z^2 \to \overline{\rho}M_Z^2,\quad g_i\to\overline{g_i}, \tag{96}$$

where

$$\overline{\rho}\simeq 1+\frac{3\sqrt{2}}{16\pi^2}G_Fm_t^2 \tag{97}$$

(m_t is the mass of the t quark) and $\overline{g_i}$ are the running coupling constants.

The results of the χ^2 fit for lower bounds on $M_{Z'}$ for the two models at 90% C.L. are given in Tables I and II. Note that the lower bound for both models is about the same ($\sim$ 700 GeV). The results presented are for $\alpha_s = 0.1$; they do not depend strongly on this choice. We thus see that QL-symmetry breaking is constrained to occur in the TeV region or higher.

TABLE I. 90%-C.L. lower bounds on the Z' boson in the basic QL-symmetric model for top-quark masses of 100 and 150 GeV. Also shown are the W and Z masses which yield this value, together with the χ^2 value. All masses are in GeV's.

m_t	100	150
M_W	79.97	80.19
M_Z	91.19	91.16
$M_{Z'}$	723	720
$\chi^2/N_{\rm DF}$	116/159	116/159

TABLE II. The Z' boson bounds for the QL-LR-symmetric model. (All masses are in GeV's.)

m_t	100	150
M_W	79.99	80.19
M_Z	91.16	91.16
$M_{Z'}$	665	943
$\chi^2/N_{\rm DF}$	109/159	110/159

V. LIPTONIC PHYSICS

A. Introduction

Quark-lepton-symmetric models can account for the known physics at low energies, just like the SM and many of its extensions. However quark-lepton symmetric models differ from the SM at low energies due to the existence of an additional unbroken gauge symmetry, namely the unbroken SU(2)$'$ subgroup of SU(3)$_\ell$. A necessary consequence of models with quark-lepton symmetry is the existence of new fermions. These new fermions are the extra leptonic degrees of freedom which gain SU(3)$_c\otimes$SU(2)$'\otimes$SU(2)$_L\otimes$U(1)$_Y$ invariant masses when the extended gauge symmetry is broken. Specifically, the new leptonic degrees of freedom are the right-handed neutrinos which are relatively familiar, and then there are the charged $\pm1/2$ liptons. The liptons could provide the most dramatic experimental implication of quark-lepton symmetry.

We will introduce the following notation for the liptons. The three "color" degrees of freedom of the fermion representations of SU(3)$_\ell$ will be expressed as

$$F_L = \begin{pmatrix} f_L \\ F_{1L} \\ F_{2L} \end{pmatrix}, \quad E_R = \begin{pmatrix} e_R \\ E_{1R} \\ E_{2R} \end{pmatrix}, \quad N_R = \begin{pmatrix} \nu_R \\ V_{1R} \\ V_{2R} \end{pmatrix}, \tag{98}$$

where f_L, e_R, ν_R are the familiar leptons and right-handed neutrino, and F_{iL}, E_{iR}, V_{iR} $(i = 1, 2)$ are the exotic fermions called liptons. The $\mathrm{SU}(2)_L$ degrees of freedom are denoted by

$$f_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}, \quad F_{iL} = \begin{pmatrix} V_{iL} \\ E_{iL} \end{pmatrix}. \tag{99}$$

For one generation, the mass matrix for the liptons has the form

$$L_{\rm mass} = \bar{X}_L \mathcal{M} X_R + \text{H.c.}, \tag{100}$$

where

$$X_L = \begin{pmatrix} V_L \\ (E_R)^c \end{pmatrix}, \quad X_R = \begin{pmatrix} (E_L)^c \\ V_R \end{pmatrix}, \tag{101}$$

and

$$\mathcal{M} = \begin{pmatrix} m_1 & m_\nu \\ m_e & m_2 \end{pmatrix}. \tag{102}$$

In the above equations, for the QL model, $m_e = \lambda_1 u$, $m_\nu = \lambda_2 u$ are the usual electron and neutrino Dirac mass terms, and $m_1 = 2h_1 w$, $m_2 = h_2^* w$. For the QL-LR model $m_1 = m_2$. Also note that the $\mathrm{SU}(2)'$ index is suppressed. For the realistic case of three generations, $V_{L,R}$ and $E_{L,R}$ acquire a generation index and the mass matrix becomes

$$\mathcal{M} = \begin{pmatrix} \mathcal{M}_1 & \mathcal{M}_\nu \\ \mathcal{M}_e^T & \mathcal{M}_2 \end{pmatrix}, \tag{103}$$

where $\mathcal{M}_1, \mathcal{M}_\nu, \mathcal{M}_e, \mathcal{M}_2$ are the 3×3 matrix generalizations of the one-generation case.

The results of the neutral current analysis of the previous section indicate that the QL-symmetry-breaking scale is somewhat larger than the electroweak breaking scale. This suggests that the heavy exotic gauge bosons will not be directly observable in the current collider experiments. However, the effects of these gauge bosons may be observable in rare decays of the known fermions. We therefore examine this possibility in the next subsection.

B. Rare decays

One of the immediate consequences of a QL symmetry is broken global family lepton-number symmetries due to the existence of right-handed neutrinos. Present experimental evidence suggests that family lepton number violation is highly suppressed, thus putting stringent limits on the branching ratios of these rare processes. The best limits come from rare decays such as $\mu \to e\gamma$ or $\mu \to 3e$ Their branching ratios are less than 5×10^{-11} and 1×10^{-12} respectively [13]. In this section we will investigate the new contributions to such processes arising in QL-symmetric models.

In addition to the usual contributions from the SM there will be one-loop diagrams which involve virtual liptons and virtual $\mathrm{SU}(3)_\ell/\mathrm{SU}(2)'$ massive charge-half gauge bosons. First, we need to consider the charged-current interaction between the ordinary leptons, the liptons, and the coset space gauge bosons. This can be extracted from the kinetic term of the $SU(3)_\ell$ fermion Lagrangian and is given by

$$\mathcal{L}_{\rm int} = \mathcal{J}_{j\mu} W'^{\mu}_j + \text{H.c.} \tag{104}$$

where

$$\mathcal{J}_{j\mu} = \frac{g_s}{\sqrt{2}} \sum_a (\overline{E^a_{jL}} \gamma_\mu e^a_L + \overline{V^a_{jL}} \gamma_\mu \nu^a_L) + (L \to R) \tag{105}$$

and W'^{μ}_j are the charged-half coset space gauge fields and a $(= e, \mu, \tau)$ is the flavor index. To obtain experimental consequences of the above interactions, we need to rewrite the above in the physical-mass-eigenstate basis. This is done by the diagonalization of the lipton mass matrix in Eq. (103). We denote the mass eigenstates as L_{j1}, L_{j2}, where L_{j1} (L_{j2}) are composed of left-handed fields (right-handed fields) in the limit where $u \to 0$. To simplify the notation the flavor index is suppressed. The interactions of these mass-eigenstate liptons then have the form

$$\mathcal{J}_{j\mu} = \frac{g_s}{2\sqrt{2}} [\, \overline{L_{j1}} \gamma_\mu (1 - \gamma_5) f + \theta \overline{L_{j1}} \gamma_\mu (1 + \gamma_5) f + \theta \overline{L_{j2}} \gamma_\mu (1 - \gamma_5) f + \overline{L_{j2}} \gamma_\mu (1 + \gamma_5) f], \tag{106}$$

where f is a generic ordinary lepton field and θ is a mixing angle which goes to zero when $u/w \to 0$. The above interaction term induces rare decays, such as $\mu \to e\gamma, \mu \to 3e$, etc. Such rare decays induced by exotic fermions have been examined previously [14], and it was concluded that if the exotic lepton is charged then the decay $\mu \to 3e$ is expected to be enhanced relative to $\mu \to e\gamma$. Note that it is difficult to give rigorous limits on the mass of the coset space gauge bosons from the experimental limits on the rare decays. The reason for this is that the rare decays depend on unknown mixing angles and masses of the liptons.

If we make the simplifying assumption of two generations, then there will be a 2×2 Kobayashi-Maskawa-(KM-) type matrix describing the interaction of the mass eigenstates L_1, L_2 with the leptons e and μ. Denoting this matrix by

$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}, \tag{107}$$

and by using the results of Ref. [14],

$$\frac{\Gamma(\mu \to 3e)}{\Gamma(\mu \to e\nu\nu)} \simeq \left(\frac{g_s M_W}{g_2 M_{W'}}\right)^4 \frac{\alpha^2 (1/2)^2}{12\pi^2} \times \sin^2\theta \cos^2\theta \ln^2 \left(\frac{M_{L_{11}}}{M_{L_{12}}}\right)^2, \tag{108}$$

The values of these mixing matrix elements are not predicted, although unitarity requires that their magnitude be less then 1. Using the experimental bound of 10^{-12}

on the above branching fraction, we find that

$$M_{W'} \geq (5\ \text{TeV})\sqrt{\sin\theta\cos\theta\ln\left(M_{L_{11}}/M_{L_{12}}\right)^2}. \qquad (109)$$

Thus, we see that rare decay limits suggest a lower bound on the mass scale of the W' coset gauge bosons of a few TeV. Note that the scale of the W' gauge bosons is set by the QL-symmetry-breaking scale. Thus the observation of family lepton-number-violating rare processes will suggest a relatively low QL-symmetry-breaking scale. However, such a low scale would be a problem for cosmology (unless the neutrinos decay fast enough in the model) since the W' gauge bosons are expected to have masses greater than about 50 PeV if the cosmological/naturalness constraints apply.

One should also note that there will be Higgs-induced flavor-changing neutral processes. Constraints can thus be derived on Higgs-boson masses and coupling constants, but we do not explicitly consider them here.

C. Exotic hadron spectroscopy

The liptons are expected to be confined by the unbroken SU(2)$'$ force into new exotic hadrons. These hadrons are expected to be nonrelativistic bound states of the liptons, since $\Lambda_{\text{SU}(2)'} \ll M_{\text{lipton}}$. Clearly, the hadrons built from the lightest lipton will be the most interesting phenomenologically (we denote the lightest lipton by L_1, with mass M_{L_1}). Furthermore these hadrons will have an approximate mass of twice the lipton mass ($2M_{L_1}$), since they are nonrelativistic bound states of two liptons.

If the quark-lepton symmetry idea is correct, then the physics associated with these new hadrons could be the first experimental hint of an underlying quark-lepton symmetry of nature. It is therefore important to examine the types of hadrons predicted in quark-lepton-symmetric models. Below we give a qualitative discussion of the interesting features of the exotic hadron spectrum.

We start by considering the hadrons made up of the lightest lipton (L_1). (Note that we are supposing for definiteness that the lightest lipton comes from the left-handed sector. We will comment on the right-handed case later.) The SU(2)$'$ strong-interaction Hamiltonian has an SU(2)-flavor (global) symmetry group, which we denote by SU(2)$_F$. The fundamental doublet of SU(2)$_F$ is given by

$$L = \begin{pmatrix} L_1 \\ L_1^c \end{pmatrix}_L . \qquad (110)$$

The lipton L_1 and the antilipton L_1^c have the same mass, so the SU(2)$_F$ symmetry is not broken by any mass difference. Notice that the situation here is quite different to QCD, since SU(2)$'$ does not have complex representations, so the lipton and antilipton both transform the same way under SU(2)$'$. The flavor group SU(2)$_F$ is broken only by the electromagnetic and the other electroweak interactions. Since the hadrons are formed from two liptons, the flavor structure of the hadrons follows from the SU(2)$_F$ decomposition:

$$\mathbf{2}\otimes\mathbf{2} = \mathbf{1}_A \oplus \mathbf{3}_S \qquad (111)$$

where the subscripts S, A denote the symmetry property under interchange of the liptons. Since the liptons are fermions we expect them to satisfy the Pauli exclusion principle, which means that the wave function of the two-lipton bound state will be antisymmetric under the interchange of the two liptons. The wave function can be expressed as

$$\psi = \psi_{\text{color}} \otimes \psi_{\text{space}} \otimes \psi_{\text{flavor}} \otimes \psi_{\text{spin}}. \qquad (112)$$

Here "color" refers to SU(2)$'$ quantum numbers, and is antisymmetric. For the ground state, the orbital angular momentum L is zero, which implies that the space wave function is symmetric. Thus, the Pauli principle implies that the flavor triplet will have spin 1 and the flavor singlet will have spin 0. We denote the states as follows. For the vector flavor triplet,

$$\rho^+ = L_1L_1, \quad \rho^0 = \frac{L_1L_1^c + L_1^cL_1}{\sqrt{2}}, \quad \rho^- = L_1^cL_1^c. \qquad (113)$$

The flavor singlet has the form

$$\zeta^0 = \frac{L_1L_1^c - L_1^cL_1}{\sqrt{2}}. \qquad (114)$$

If we neglect the electroweak interactions, then the flavor triplet will be degenerate. The flavor triplet will have a different mass from the singlet due to SU(2)$'$ spin-spin interaction proportional to $E_{ss} = -S_1\cdot S_2$ (where S_1 and S_2 are the spins of the two liptons which comprise the exotic hadron). The sign of this splitting implies that the flavor singlet ζ particle is heavier than the flavor-triplet ρ particles. The degeneracy of the flavor triplet will be broken by electroweak interactions. If the mass of the lightest hadron is not much greater then the Z mass, then the electromagetic effects will be the dominant perturbation on the masses of the flavor triplet. The electromagnetic contribution to the hadron masses has two types: the Coulomb interaction ($\sim e_ie_j$) and the magnetic interaction ($\sim e_ie_j/m_im_j$). These two interactions have the same sign, and so they can be treated together. These interactions imply a mass splitting between $\rho^\pm$ and ρ^0 of

$$M_{\rho^+} - M_{\rho^0} \simeq \frac{1}{2} < 1/R > \sim \alpha_s M_h, \qquad (115)$$

where $< R >$ is a measure of the separation between the constituent liptons, α_s is the SU(2)$'$ coupling (evaluated at M_h) and M_h is the hadron mass ($\approx 2M_{L_1}$). Thus we find that the lightest liptonic hadron should be the neutral state ρ^0.

This particle is a spin-1 state, and can be produced in an electron-positron collider via a virtual photon or Z. It will decay predominantly via virtual photon (and Z), to ordinary fermions, which should provide a clear experimental signature of this hadron. The charged state $\rho^\pm$ will mainly decay via the usual weak interactions:

$$\rho^\pm \to W^{*\pm} \to e\bar{\nu}, d\bar{u}, \ldots\ . \qquad (116)$$

The virtual W decays into the usual fermions with the usual branching fractions. The spin-0 hadron ζ^0 should decay (dominantly) via SU(2)$'$ interactions into two exotic gluons (which should hadronize into two exotic glueballs). This decay mode should be unobservable; however, there will be a significant branching fraction into two photons:

$$\frac{\Gamma(\zeta \to \gamma\gamma)}{\Gamma(\zeta \to g'g')} \simeq \frac{\alpha^2}{3\alpha_s^2}. \quad (117)$$

There may also be a significant branching fraction of ζ decaying via virtual Z boson.

Note that the discovery of a set of hadrons with the above properties will not necessarily imply that nature utilizes a fundamental quark-lepton symmetry. This is because there are other simple models which have a similar spectrum of hadrons, such as the SU(5)-color model [15, 16]. However, the careful study of the decays of these exotic hadrons should provide a way of testing quark-lepton symmetry against these similar models. This is because there will be decays mediated by the charged 1/2 gauge bosons [the W' bosons in the coset $SU(3)_\ell/SU(2)'$]. These gauge bosons will lead to enhanced leptonic decays of the ρ, ζ, and the other exotic hadrons. These decays depend on the fact that there are exotic gauge bosons which connect the liptons with the usual leptons and therefore provide a more significant test of the quark-lepton symmetry idea [17].

Note that the L_2-type liptons which are composed only of right-handed fields in the limit $u/w \to 0$ will have W-mediated decays suppressed. Thus the charged ρ particles consisting of L_2-type liptons will provide the ideal place to look for the enhanced decay modes. The strength of these decays will help determine the mass of the exotic gauge bosons, and hence the scale of quark-lepton symmetry breaking.

The lightest lipton is clearly the most important phenomenologically. However, if the two lightest liptons are nearly degenerate, then this will lead to some interesting physics. Below, we will work out the flavor symmetry group for this case.

The two lightest liptons may be nearly degenerate when usual left-right symmetry is combined with the quark-lepton symmetry as discussed in Section III B. In fact this is the prediction in the minimal model of this type. The left- and right-handed liptons will not be exactly degenerate due to the electroweak contributions to the lipton mass matrix, and also due to radiative effects. We therefore consider the case when the two lightest liptons L_1 and L_2 are approximately degenerate, so that the flavor symmetry will be enlarged. In order to work out the flavor group of the lipton Lagrangian mass terms and the $SU(2)'$ strong interactions, we ignore electroweak effects. The electroweak effects will be perturbations breaking the flavor group. Then the mass matrix of the two degenerate liptons has the form

$$L_{\text{mass}} = X_L^T C^{-1} \mathcal{M} X_L, \quad (118)$$

where

$$X_L = \begin{pmatrix} L_1 \\ L_2 \\ L_1^c \\ L_2^c \end{pmatrix}_L, \quad (119)$$

and

$$\mathcal{M} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix}. \quad (120)$$

In the above equation C is the charge-conjugation matrix defined by $(L_i)^c = C(\bar{L}_i)^T$. Notice that the symmetry group of the mass matrix is not SU(4), but rather the symplectic group Sp(4). To see this note that the L_{mass} is not preserved under SU(4) transformations (with the X_L transforming as the fundamental representation). However, L_{mass} is preserved under the subgroup of SU(4) which preserves the antisymmetric tensor proportional to the mass matrix. This is the definition of the symplectic group Sp(4). Thus the flavor group for the case of two degenerate liptons is Sp(4) and the fundamental four-dimensional representation is given by X_L above. It is easy to generalize this argument to show that for n degenerate liptons the flavor group is Sp(2n). [Note that Sp(2) = SU(2), which is the one-lipton case discussed earlier.]

Under the flavor group $Sp(4)_F$,

$$\mathbf{4} \otimes \mathbf{4} = \mathbf{10}_S \oplus \mathbf{5}_A \oplus \mathbf{1}_A. \quad (121)$$

Hence, in this case there will be a 10-plet of approximately degenerate spin-1 hadrons and six spin-0 hadrons. As before, the neutral spin-1 exotic hadrons will be the lightest states and they will decay via electroweak, $SU(2)'$, and W' interaction effects.

To conclude this section, we have discussed the physics of the exotic charged $\pm 1/2$ fermions, called liptons, predicted by the quark-lepton symmetry principle. We pointed out that the effects of the liptons can show up indirectly in rare lepton-number-violating processes. Alternatively, they may be light enough to be discovered experimentally. We therefore gave a survey of their expected properties. In particular we have shown that the flavor symmetry respected by the $SU(2)'$ interactions and the liptonic mass matrix is $Sp(2n)$ for n degenerate liptons. Thus, the discovery of liptonic bound states would necessitate the introduction of the symplectic groups into particle physics.

VI. CONCLUSION

We have performed a detailed analysis of models with quark-lepton discrete symmetry in this paper. This interesting idea unifies the quantum numbers of quarks and leptons in a radically different way from models of partial and grand unification. The gist of the idea is that a "color" group for leptons is introduced, which allows a Z_2 discrete symmetry to be defined that interchanges quarks and leptons. The leptonic color group and the discrete symmetry are then spontaneously broken by Higgs bosons which couple to fermion bilinears. We pointed out that one definition of this discrete symmetry also interchanges left- and right-handed fermions, thus offering an alternative way of achieving spontaneous parity breakdown. Quark-lepton symmetry is also compatible with orthodox left-right symmetry, which leads to a unification of all fermion quantum numbers within a generation, in the sense that every multiplet is related to every other multiplet through a discrete symmetry.

An important consequence of achieving quantum-number unification through discrete rather than continuous symmetries is the prediction of a significant amount of rather remarkable new physics, which may show up at the relatively low scale of 100 GeV or so. As well as nonstandard neutral-current effects induced by a Z' boson, the theories predict a new strongly interacting sector of charge $\pm 1/2$ fermions (liptons) which feel confining $SU(2)'$ gauge forces. The existence of these integrally charged bound states allows the elegant evasion of cosmological bounds on the presence of stable fractionally charged particles in the universe. We showed that these exotic hadrons may be classified under an approximate flavor symmetry of $Sp(2n)$ where n is a degeneracy number, and we described how they decay. We also pointed out that loop effects from virtual liptons and exotic charge $\pm 1/2$ heavy gauge bosons induce rare family-lepton-number-violating processes such as $\mu \to 3e$. Observation of such processes would imply a quite low scale of quark-lepton symmetry breaking, as well as allowing us to deduce whether the $SU(2)'$ sector comes from the present models, or from the closely related alternative of SU(5) color.

Finally, we wish to remark that a significant amount of progress can be made toward a deeper understanding of quark and lepton quantum numbers by the use of new physics at possibly a rather low energy scale. For instance, charge quantization can be understood either as a consequence of classical constraints alone, or classical constraints and gauge anomaly cancellation, in quark-lepton- or left-right-symmetric models, and in the SM with Majorana right-handed neutrinos. The similar properties of left- and right-handed fermions can be understood through orthodox left-right symmetry, while quarks and leptons can be unified by quark-lepton symmetry. It will be fascinating to see whether or not this bottom-up approach to some fundamental mysteries in particle physics is vindicated in the very important collider experiments to be performed in the next decade or two.

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used data from G. Costa *et al.*, Nucl. Phys. **B297**, 244 (1988). The ν-e scattering data came from U. Amaldi *et al.*, Phys. Rev. D **36**, 1385 (1987). The electron-positron cross sections and forward-backward asymmetries came from Amaldi *et al.* (above); AMY Collaboration, A. Bacala *et al.*, Phys. Lett. B **218**, 112 (1989); and TOPAZ Collaboration, I. Adachi *et al.*, *ibid.* **208**, 319 (1988). The Z-width data came from ALEPH Collaboration, D. Decamp *et al.*, Phys. Lett. B **231**, 509 (1989); DELPHI Collaboration, P. Aarnio *et al.*, *ibid.* **231**, 539 (1989); L3 Collaboration, B. Adeva *et al.*, *ibid.* **231**, 509 (1989); OPAL Collaboration, M. Z. Akrawy *et al.*, *ibid.* **231**, 519 (1989). Partial decay width data were taken from F. Dydak (above).

[12] D. C. Kennedy and B. W. Lynn, Nucl. Phys. **B321**, 83 (1989); **B322**, 1 (1989).

[13] Particle Data Group, J. J. Hernández *et al.*, Phys. Lett. B **239**, 1 (1990).

[14] W. J. Marciano and A. I. Sanda, Phys. Lett. **67B**, 303 (1977). Note that the term proportional to θ could lead to a large $\mu \rightarrow e\gamma$ decay rate due to lack of Glashow-Iliopoulos-Maiani suppression; however, the leading term vanishes because it is proportional to $Q - 2Q' = -1 - 2(-1/2) = 0$ (see Marciano and Sanda above). This is not the case for neutrino decay assuming that perturbation theory is valid here. In fact the perturbative calculation shows that the radiative decay ($\nu_{\text{heavy}} \rightarrow \nu_{\text{light}} + \gamma$) could be within the naive cosmological bound on the neutrino mass, provided that the exotic W' gauge bosons are not too heavy. However, for these radiative decays, there are much more stringent cosmological/astrophysical and laboratory constraints [see, for example, D. A. Dicus *et al.*, Phys. Rev. D **17**, 1529 (1978)]. It should be noted that, in addition to radiative decays, it may be possible for heavy neutrinos to decay into light neutrinos mediated by SU(2)$'$ glueballs in a manner consistent with standard cosmology provided the mass of the glueball is less than the mass of the heavy neutrino. This may be the dominant decay of a heavy neutrino because of the large SU(2)$'$ gauge coupling constant at low energy. This is a possible scenario because the many Higgs bosons in the model make the calculation of $\Lambda_{\text{SU}(2)'}$ uncertain.

[15] R. Foot and O. F. Hernández, Phys. Rev. D **41**, 2283 (1990); **42**, 948(E) (1990).

[16] The most obvious extension to the standard model which contains the exotic fermions can be obtained by appending an extra SU(2)$'$ gauge group to the standard model, together with the charged $\pm 1/2$ exotic fermions which transform under the gauge group $\text{SU}(2)' \otimes G_{\text{SM}}$ as $F_L \sim (2,1,2)(0)$, $V_L \sim (2,1,1)(1)$, $E_R \sim (2,1,1)(-1)$. In this case, the exotic fermions can have bare mass terms which are not fixed by any scale. This model is the limiting case of the QL [or SU(5)$_c$] models in which the QL-breaking scale [SU(5)$_c$-breaking scale] is taken to infinity. The exotic fermions can be kept light by imposing a global exotic lepton-number U(1) symmetry. In this case, the decay of ρ^+ via W^* does not occur, and ρ^+ would be absolutely stable, unless there is a degeneracy among the two lightest exotic fermions.

[17] The SU(5)-color models also have enhanced decays; however, in these models, the hadronic decays are enhanced rather than the leptonic decays.