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ρ parameter and pseudo-Goldstone-boson couplings to the Z

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The possibility that neutral pseudo Goldstone bosons of "enhanced" technicolor theories can imitate neutral-Higgs-boson couplings to the Z is considered in light of ρ-parameter constraints. A rough estimate indicates that couplings of neutral pseudo Goldstone bosons to the Z induced by extended technicolor interactions are unlikely to be of conventional Higgs strength.

I. INTRODUCTION

One facet of the effort to unravel the mechanism of electroweak symmetry breaking (EWSB) is the search for physical spin-0 particles associated with EWSB. Possible pseudo Goldstone bosons[1,2] (PGB's) of technicolor[3] theories, fundamental Higgs scalars,[4,5] and composite Higgs scalars[6] are probably the most prominent examples of such particles. In the absence of other input (e.g., the discovery of superpartners) the discrimination among these sorts of particles based on the interpretation of data would have to rely on an understanding of their allowed couplings and coupling strengths. Couplings to the Z are of particular relevance in this context given ongoing experiments at CERN LEP and the SLAC Linear Collider (SLC) and planned experiments at the Superconducting Super Collider (SSC) and CERN Large Hadron Collider (LHC). For instance, the coupling $Z^\mu Z_\mu H_0$ has been used at LEP to search for the standard-model Higgs boson up to masses of about 40 GeV, and will be used at LEP 200 to search up to masses of about 80 GeV.

In this paper we use "technicolor" to refer exclusively to theories in which the technicolor interactions are vectorlike (as opposed to chiral, as in, e.g., the "technicolor limit" of a composite Higgs model), and the technicolor condensates necessarily spontaneously break $SU(2)_L \times U(1)_Y$. The results in this paper are to be taken to apply to such theories. It is possible that a general chiral theory could behave quite differently, and our results certainly do not apply in general to composite Higgs models where the (ultracolor) condensates do not necessarily break $SU(2)_L \times U(1)_Y$. In technicolor theories without extended technicolor (ETC) interactions, the couplings of neutral PGB's to the Z are considerably different than those of Higgs scalars.[7–11] By explicit computation, in general there is no $Z_\mu Z^\mu P_0$ coupling;[10] and in a "one-generation" ("n-generation") model of technifermions there is no $Z_\mu P_0 \partial^\mu P_0'$ coupling—see Ref. 10 (Ref. 12). In Ref. 13 the most prominent symmetries restricting these couplings, present in models with $SU(N_{TC})$ technicolor gauge groups, were identified to be *CP* and "doublet parity," where doublet parity is a product of Z_2-flavor symmetries.[14] Since it is possible to form *CP*-even neutral PGB's in technicolor theories in analogy to K_1 of the neutral kaon system (see Ref. 15), *CP* alone is not sufficient to forbid such couplings in general. The two symmetries together are sufficient to forbid any $Z_\mu Z^\mu P_0$ coupling. It was found, though, that the $Z_\mu P_0 \partial^\mu P_0'$ coupling can arise at full strength in atypical models in which neutral PGB's carry nonzero I_3 of vector SU(2), but is otherwise forbidden. (See Ref. 16 for a recent complementary study of PGB-γ-Z couplings.)

In any case ETC interactions (or their analogs) are required in order to give quarks and leptons mass.[17,18] ETC-induced four-technifermion interactions potentially can make the desired discrimination difficult because they are not expected to respect *CP* or doublet parity, and so can induce couplings "Higgs-like" at least in form. Until the advent of technicolor theories with enhancement,[19,20] ETC-induced perturbations were thought to influence PGB masses by at most $\sim$40 GeV,[17,21] and apparently to have little effect on PGB couplings.[12] Enhancement of technifermion condensates was proposed in order to raise ordinary fermion masses above common expectations for a given ETC scale, and thereby alleviate difficulties satisfying flavor-changing neutral-current bounds.[17,22] It was realized that by the same token PGB masses would be raised as well.[20] Reference 13 contains a detailed examination of PGB interactions to *first* order in ETC perturbations. It was found that neutral PGB-Z couplings, Higgs-like both in form and strength, even for relatively light neutral PGB's, might be induced in ETC theories with enhancement. Furthermore, the W and Z masses were not unduly shifted under the circumstances of Higgs-strength ETC-induced PGB-Z interactions. Because the results were obtained from a phenomenological Lagrangian approach, they are arguably of more general applicability, in distinguishing the description of EWSB by means of a nonlinear rather than a linear σ model.

In this article we examine constraints on PGB couplings from the ρ parameter,[23] working to *second* order in ETC perturbations. Technicolor without ETC was argued to give $\rho=1$ at the outset.[3] Analysis of deviations from $\rho=1$ begins with Ref. 24, and includes Refs. 25–27 in enhanced theories. Many analyses have appeared recently,[28] which bound also weak-isospin symmetric radiative corrections. We will argue that ρ-parameter constraints imply that it is unlikely that neutral PGB's can mimic Higgs particles through ETC-induced couplings in Z interactions. Thus in the event of experimental observation of neutral spin-0 particles with Higgs-strength couplings to the Z, we expect that the particles will not be PGB's of technicolor.

II. THE BOUND

Our goal is to relate the strengths of neutral PGB-Z couplings in a given theory to $\delta\rho$, computing to lowest order in electroweak couplings. For simplicity, we work with technicolor gauge group $SU(N_{TC})$, with N_D weak doublets of left-handed and $2N_D$ weak-singlet right-handed technifermions. Our reasoning should be valid in any (vectorlike) technicolor model because our conclusions follow basically from simple symmetry considerations and power counting, in factors of 4π as well as dimensional scales. The ETC-induced four-technifermion operators of most interest are of the form $J_L^\mu J_{R\mu}$ where $J^\mu_{L(R)}$ is formed from left- (right-) handed technifermion fields.[29] Here, "techniflavor" indices have been suppressed. At first order current-current operators of this form alone induce couplings of the Z to one or two (color and charge) neutral PGB's in theories without exotic hypercharge assignments.[13] Also they are the only current-current operators which, as argued later, give rise to low-energy effects which almost certainly can be enhanced strongly. Perturbations corresponding to operators of the form $J_L^\mu J_{R\mu}$ cannot affect the ρ parameter until second order when working with corrections lowest order in electroweak gauge couplings. In the limit $g'=0$, this is evident from the fact that $J_L^\mu J_{R\mu}$ is $\Delta I=1$ or 0 while, e.g., $m_{W^\pm}-m_{W^3}$ is $\Delta I=2$ (cf. $m_{\pi^\pm}-m_{\pi^0}$). The isospin referred to is the vector (custodial[30]) SU(2) related to $SU(2)_L$. The ρ parameter, here defined as the relative strength of charged and neutral weak currents at $q^2=0$, is approximately[31]

$$\rho_*(0)\simeq 1-\frac{g^2}{m_W^2}\delta\pi(0) , \tag{1}$$

where $\delta\pi(0)=\pi_{33}(0)-\pi_{11}(0)$ is the difference of vacuum polarizations for the W_3 and W_1. Clearly, $\delta\rho_*(0)=\rho_*(0)-1$ can be calculated here by working in the limit $g'=0$.

It is unlikely that custodial-SU(2)-violating interactions can be nonperturbative while $\delta\rho_*(0)$ is $\lesssim 1\%$, unless there is some form of fairly extreme fine-tuning. Therefore, it will be assumed that $\Delta I\neq 0$ interactions can be treated perturbatively.

Although we do not take this route here, we point out that in order to calculate $\delta\pi$, in principle we could proceed by calculating vacuum polarization to all orders in weak-isospin-conserving interactions, and then include two insertions of $\Delta I=1$ interactions of the form $(1/\Lambda^2_{ETC})[J_L^\mu J_{R\mu}]_{(\Lambda_{ETC})}$ $(\Delta I=1)$ into it. Notationally, $[J^\mu J_\mu]_{(\Lambda_{ETC})}$ is a current-current operator renormalized at scale Λ_{ETC}. In the vacuum-insertion approximation[32] the dominant dependence of $[J_L^\mu J_{R\mu}]_{(\Lambda_{ETC})}$ $(\Delta I=1)$ on Λ_{ETC} arising through anomalous scaling is the same as that of $\{[\bar{T}T]_{(\Lambda_{ETC})}\}^2$ under the circumstances of enhancement, T representing a technifermion field. (Note that in the same approximation $[J_L^\mu J_{L\mu}]$ and $[J_R^\mu J_{R\mu}]$ have vanishing anomalous dimensions.) Though one cannot trust the precise form of the result yielded by the vacuum-insertion approximation, beyond the vacuum-insertion approximation there is no known reason to believe that the dominant dependence on Λ_{ETC} will differ appreciably, i.e., that strong cancellations occur in the calculation of the anomalous dimension of $[J_L^\mu J_{R\mu}]$ to leave a net dependence on Λ_{ETC} much weaker than the above. Therefore, assuming that vacuum polarization at zero momentum is controlled by momenta of order Λ_{TC}, the dominant scaling of $\delta\rho_*(0)$ with Λ_{ETC} is roughly

$$\delta\rho_*(0)\sim\{\langle 0|[\bar{T}T]|0\rangle_{(\Lambda_{ETC})}\}^4/(\Lambda^4_{ETC}\Lambda^8_{TC}) . \tag{2}$$

Remarks of this type first appear in Ref. 26. This form can be inaccurate in theories with PGB's if the $\Delta I=0$ component of PGB masses is due predominantly to ETC interactions, because perturbative expansion in PGB masses of the relevant diagrams is infrared singular. See Eq. (4) below for illustration. At present, it is not possible to calculate $\delta\rho_*(0)$ directly in terms of technifermions and technigluons in a systematic approximation; there are, however, calculations in the Pagels-Stokar approximation[26,27] which provide useful information.

Instead, we adopt the following strategy. Because $\delta\rho_*(0)$ is a low-energy quantity, its calculation can instead be framed in terms of a phenomenological Lagrangian $\mathcal{L}_{ph}$. $\mathcal{L}_{ph}$ contains terms characterized by $\Delta I=0,1,2,\ldots$, which are to be used both for tree and loop calculations.[33] From the point of view of calculations using the elementary fields, i.e., technifermions and technigluons, loops of PGB's are associated with virtual modes which can propagate over relatively long distances. Now, we do not propose to calculate all contributions to $\delta\rho_*(0)$ in a general or specific model. Rather we will repeatedly take recourse to the position that different contributions to $\delta\rho_*(0)$ which are theoretically unrelated cannot cancel appreciably. Of course there is a danger in such an attitude in that two things theoretically unrelated today may be related tomorrow. So, being mindful of this type of possibility we shall proceed. What we do wish to estimate are contributions to $\delta\rho$ which follow purely from chiral symmetry and the presence of ETC-induced neutral PGB-Z couplings in the phenomenological Lagrangian. Then this will provide a rough lower bound on $\delta\rho$ because we are assuming no substantial cancellation from other contributions.

The calculations will be done to second order in electroweak couplings. Writing explicitly just two terms in the ETC-induced phenomenological Lagrangian, which are first-order reflections of $(1/\Lambda^2_{ETC})[J_L^\mu J_{R\mu}]$-type operators,

$$\mathcal{L}_{ph}^{(ETC)}= A_{ABCD}U_{BC}U^\dagger_{DA} + \tilde{B}_{ABCD}(D^\mu U)_{BC}(D_\mu U^\dagger)_{DA}+\cdots , \tag{3}$$

where $A,B,C,D=1,\ldots,2N_D$ are "techniflavor" indices.[13] The first-order ETC-induced PGB masses are contained in A_{ABCD}: $m_{PGB}^{(ETC)2}\sim(1/f^2)A$ with $m_{PGB}^{(ETC)2}$

representing typical ETC-induced PGB mass squares. Using Dashen's theorem and the vacuum-insertion approximation on the resulting vacuum matrix elements of four-technifermion operators gives the ETC-induced PGB mass-squared matrix proportional to squares of $\langle 0|[\bar{T}_A T_B]|0\rangle_{(\Lambda_{ETC})}$.[20] Tree-level PGB-$Z$ couplings are contained in interactions such as the second term $\tilde{B}$. The chiral-symmetry-breaking scale, denoted Λ_{CSB}, is characteristic of the derivative expansion in $\mathcal{L}_{ph}$; roughly, $\tilde{B}\sim A/\Lambda_{CSB}^2$. ETC-induced PGB–gauge-boson couplings then are typically enhanced in essentially the same manner as PGB masses, and it is in enhanced theories that one might expect that these couplings could be large, perhaps of Higgs-like strengths.

As mentioned earlier, in the limit $g'=0$ in which we can work to calculate $\delta\rho$, the operators such as the $\tilde{B}$ operator which contain the neutral PGB-Z couplings have only $\Delta I=0$ or 1 pieces. Thus at the tree level the chiral symmetries implicit in the use of the phenomenological Lagrangian cannot link the presence of neutral PGB-Z couplings with any nonzero $\delta\rho$, a purely $\Delta I=2$ effect. But at one or more loops $\Delta I=2$ effects can emerge as a result of iterating $\Delta I=1$ parts of the $\tilde{B}$ and A operators. [Note that effects of operators of the form $[J_R^\mu J_{R\mu}]$, while unenhanced in the vacuum-insertion approximation, can influence $\delta\rho_*(0)$ at the tree level.[34]]

The graphs relevant for the calculation of one-loop $\Delta I=2$ effects can be grouped into three types. In the first set only $\Delta I=1$ PGB mass effects are included. In the second set both $\Delta I=1$ mass effects and couplings contribute, while in the third only $\Delta I=1$ couplings appear. The first set has been discussed in Ref. 24 in unenhanced theories and in Ref. 25 in enhanced theories. It typically dominates over the other two sets, as seen by the following power-counting argument. Generically, the first set gives

$$\delta\pi(0)\sim\frac{1}{(4\pi)^2}\frac{(m_{PGB}^{(1)(ETC)2})^2}{m_{PGB}^{(0)2}}\,, \tag{4}$$

where $m_{PGB}^{(1)(ETC)2}$ is a $\Delta I=1$ ETC-induced splitting of PGB mass squares, and $m_{PGB}^{(0)2}$ is $\Delta I=0$. Ignoring logarithms and cutting off the quadratic divergence of the third set at Λ_{CSB}, the second and third set are of the same order, giving

$$\delta\pi(0)\sim\frac{1}{(4\pi)^2}\frac{(m_{PGB}^{(1)(ETC)2})^2}{\Lambda_{CSB}^2}\,, \tag{5}$$

where the relation $\tilde{B}\sim A/\Lambda_{CSB}^2$ has been used in the form

$$\tilde{B}(\Delta I=1)\sim\frac{f^2}{\Lambda_{CSB}^2}m_{PGB}^{(1)(ETC)2}\,. \tag{6}$$

The effects of Eq. (5) scale as illustrated in Eq. (2). By comparison of Eqs. (4) and (5) the corrections from the second and third set are suppressed relative to those from the first set by the ratio $m_{PGB}^{(0)2}/\Lambda_{CSB}^2$. Note however that, in the limit $m_{PGB}^{(0)2}/\Lambda_{CSB}^2\sim 1$, appropriate for Higgs-strength couplings,[13] all three sets are roughly of the same order. Also, when examined in more detail it is difficult to make a specific connection between the PGB mass matrix and the couplings which govern, say, $Z_\mu Z^\mu P_0$ as an ETC-induced vertex.[13] Therefore, it is both convenient and adequate to work with the third set of graphs alone to obtain a rough, but useful bound on $\delta\rho_*(0)$.

The one-loop diagrams contributing to $\delta\rho_*(0)$ are quadratically divergent, requiring $\Delta I=2$ counterterms for renormalization. A rough lower bound on the renormalized $\Delta I=2$ couplings can be obtained by computing the one-loop counterterms induced by the iteration of $\Delta I=1$ couplings, and using Λ_{CSB} as the ultraviolet cutoff. (The situation is analogous to the problem of scalar field masses in theories with fundamental scalars, where rough lower bounds can be put on scalar masses by computing the quadratically divergent mass counterterms and requiring naturalness.) Just as discussed in Refs. 35 and 36, higher-order loop corrections are not suppressed here, so one-loop results are only reliable in order of magnitude. The study of quadratic divergences arising in nonlinear σ models in calculations of $\delta\rho_*(0)$ at one loop was initiated in Ref. 37. That $\delta\rho_*(0)$ at one loop can be quadratically sensitive to scales above m_W in extensions of the standard model was first illustrated in Ref. 38.

Only the simplest neutral PGB-Z couplings will be discussed; such cases should be sufficient for the order-of-magnitude bounds we seek. These couplings are for Category-A neutral PGB's, in the nomenclature of Ref. 13, defined by the condition that $[T^J,Y]=0$. Here, $T^J\tau^j$ represents the broken generator corresponding to the given PGB, T^J being a Hermitian generator of $SU(N_D)$, and Y is the diagonal matrix of the hypercharges of the left-handed doublets.

The dimension-2 ETC-induced operators of the chiral Lagrangian which contribute to the Category-A neutral PGB-Z couplings[13] are the $\tilde{B}$ operator of Eq. (3) and

$$\begin{aligned}&C_{ABCD}U_{BC}U_{DA}^\dagger\frac{1}{2N_D}\mathrm{Tr}[(D^\mu U)(D_\mu U^\dagger)]\,,\\&E_{ABCD}U_{BC}[U^\dagger(D_\mu U)(D^\mu U^\dagger)]_{DA}+\mathrm{c.c.}\end{aligned} \tag{7}$$

We will obtain our lower bound by considering only the contributions of the $\tilde{B}$ operator to $\delta\rho$, with the familiar assumption that the contributions from the C and E operators do not significantly reduce the order of the estimate by cancellation. The one-loop graphs of interest are illustrated in Fig. 1. Because we are interested in a real order-of-magnitude bound with which to compare to the experimental bounds on $\delta\rho$, it is important not to omit possibly significant numerical factors by too rough an evaluation of these graphs. A precise calculation is facilitated by some expanded notation suited to electroweak interactions.

Let $\alpha,\beta,\gamma,\delta=1,\ldots,N_D$ label the "doublet," and $m_\alpha,\ldots,m_\delta=1,2$ label the member of a "doublet" (right-handed fields are also paired for notational ease). In this notation $\tilde{B}_{ABCD}$ corresponds to $\tilde{B}_{\alpha m_\alpha\beta m_\beta\gamma m_\gamma\delta m_\delta}$; expand

$$\tilde{B}_{\alpha m_\alpha\beta m_\beta\gamma m_\gamma\delta m_\delta}=\sum_{J,K=0}^{N_D^2-1}\sum_{j=0}^{3}\tilde{b}_{JK}^{0j}T_{\gamma\beta}^J T_{\alpha\delta}^K\delta_{m_\alpha m_\beta}\tau_{m_\gamma m_\delta}^j\,, \tag{8}$$

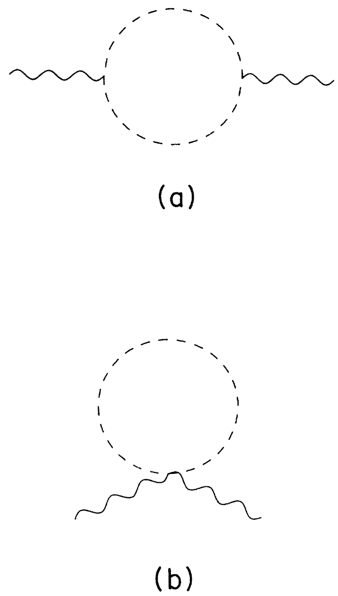

FIG. 1. One-loop vacuum-polarization diagrams.

with the normalizations $\mathrm{Tr}(T^J T^K)=2\delta^{JK}$. The $\tilde{B}$-induced PGB-Z couplings of interest are[13]

$$2\sqrt{N_D}\,\frac{g^2+g'^2}{f}(P_0^K \mathrm{Im}\tilde{b}_{0K}^{00}+P_3^K \mathrm{Im}\tilde{b}_{0K}^{03})Z^\mu Z_\mu \quad (9)$$

and

$$-8\frac{\sqrt{g^2+g'^2}}{f^2}[(P_0^K\partial_\mu P_0^L+P_3^K\partial_\mu P_3^L)\mathrm{Im}\tilde{b}_{KL}^{03} + \tfrac{1}{2}(P_3^K\partial_\mu P_0^L-P_0^L\partial_\mu P_3^K)\mathrm{Im}\tilde{b}_{KL}^{00}]Z^\mu , \quad (10)$$

where the PGB field P_j^J corresponds to the generator $T^J\tau^j$. We work in the Landau gauge. With $g'=0$ only the divergence of Fig. 1(a) has a $\Delta I=2$ piece. Our final simplification will be to consider only contributions from the operators whose coefficients are $\tilde{b}_{JK}^{03}$. These should be representative, and by familiar logic should suffice in obtaining a lower bound. The counterterm quadratic in the $\tilde{b}_{JK}^{03}$ is quadratically divergent. (Quadratic divergences can be identified conveniently using dimensional regularization as simple poles at $D=2$.) Cutting off the divergence at Λ_{CSB}, the required $\Delta I=2$ counterterm is

$$\delta\mathcal{L}_{\mathrm{ph}}^{(\mathrm{ETC})}=-\left[\frac{4g}{f^2}\frac{\Lambda_{\mathrm{CSB}}}{4\pi}\right]^2\left[\sum_{J,K=1}^{N_D^2-1}(\mathrm{Im}\tilde{b}_{JK}^{03})^2+\sum_{K=1}^{N_D^2-1}(\mathrm{Im}\tilde{b}_{0K}^{03})^2\right]\times\frac{1}{2}(W_1^\mu W_{1\mu}+W_2^\mu W_{2\mu}) . \quad (11)$$

The complete counterterm involves not only the operator

$$\tilde{B}_{A'B'AD}\tilde{B}_{B'A'CB}(U^\dagger D^\mu U)_{BA}(U^\dagger D_\mu U)_{DC} , \quad (12)$$

of a form appearing in,[13] but also operators such as

$$\tilde{B}_{ABCD}\tilde{B}_{A'B'C'D'}U_{D'A}^\dagger(D^\mu U)_{BC'}(D_\mu U)_{B'C}U_{DA'}^\dagger . \quad (13)$$

These forms are consistent with spurion analysis (see, e.g., Refs. 36 and 39). The factors appearing in Eq. (11) are not surprising and are related in part to conventions in the definition of Eq. (8). The coefficient of the counterterm operator (12),

$$\frac{1}{2f^4}\frac{\Lambda_{\mathrm{CSB}}^2}{(4\pi)^2} ,$$

is typical; it exhibits no unusual factors. Recall again that a calculation for $g'=0$ is sufficient to calculate $\delta\rho_*(0)$ here. To obtain the final estimate take $m_W\simeq\frac{1}{2}gf\sqrt{N_D}$ and

$$\Lambda_{\mathrm{CSB}}\simeq 4\pi f/\sqrt{N_D} , \quad (14)$$

the presence of the additional factor $1/\sqrt{N_D}$ beyond the results of Refs. 35–37 being argued for in generality in Refs. 13 and 40. The bound is then

$$|\delta\rho_*(0)|\gtrsim\frac{1}{N_D^2}\left[\frac{8}{f^2}\right]^2\left[\sum_{J,K=1}^{N_D^2-1}(\mathrm{Im}\tilde{b}_{JK}^{03})^2+\sum_{K=1}^{N_D^2-1}(\mathrm{Im}\tilde{b}_{0K}^{03})^2\right] . \quad (15)$$

This form is potentially misleading. In higher orders in PGB loops the various terms entering here do not necessarily appear with the same signs, as they do in Eq. (15). Also, the effect of the C and E interactions of Eq. (7), among others, should be included. In general, the counterterms do not depend precisely on the squares of PGB-Z couplings, as seen for example by the easily verified result that the one-loop counterterm has no dependence on C^2.

From Eqs. (9) and (10) the ETC-induced Category-A PGB-Z couplings of interest approach Higgs strength if $(8/f^2)(\mathrm{Im}\tilde{b}_{KL}^{03})$ or $(8/f^2)(\mathrm{Im}\tilde{b}_{KL}^{00})\to 1$ ($K,L=0$ allowed). Assuming that $SU(2)_C$ is badly broken in four-technifermion interactions, as is probable given the large splittings within the masses of ordinary fermion doublets, these two conditions are equivalent. However, it is possible that the four-technifermion interactions are by and large $SU(2)_C$ preserving, in which case the bound we have obtained would be weakened. We can identify two types of scenarios where this could be the case. First one might imagine that there are many technifermions which do not interact with ordinary fermions and their couplings among themselves are largely $SU(2)_C$ preserving. This possibility seems to us theoretically rather *ad hoc.* The second possibility entails having all ETC operators, including those which give ordinary fermions masses, being close to $SU(2)_C$ preserving, and all large violations of $SU(2)_C$ being due to hypercharge interactions. This scenario would, at the least, require enormous enhancement of hypercharge induced $SU(2)_C$ violation in order to explain the quark spectrum. Such enhancement may be possible in ETC-enhanced theories with a certain amount of fine-tuning. Having mentioned what appears to us as the possible singular exceptions to the applicability of our bound, we proceed.

From our bound, if one single coupling approaches Higgs strength, $|\delta\rho_*(0)| \gtrsim 1/N_D^2 \simeq 1\%$ for $N_D = 10$,[41] the reference value of Ref. 13. This is marginally acceptable by itself; it would correspond to nearly a two-σ effect.[42] However, there are $O(N_D^4)$ additional terms in Eq. (15). Although there are obvious uncertainties in the absence of a particular model of ETC interactions, it appears unlikely then that acceptable deviations from $\rho_*(0)=1$ can be found in models with ETC-induced neutral PGB-Z couplings of roughly Higgs strength, unless new theoretical considerations can suggest the possibility of large cancellations of the type we neglected.

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[40]It was shown in Ref. 13 that $\Lambda_{\rm CSB} \lesssim 4\pi f/\sqrt{N_D}$. The analysis there suggested that with very large $N_{\rm TC}$ this bound need not be saturated. However, with moderate $N_{\rm TC}$ one would expect Eq. (14) to hold based on analogy with soft-pion phenomenology. If $N_{\rm TC}$ is very large then the following estimate for $|\delta\rho_*(0)|$ would be weakened.
[41]$N_D > 4$ may not be consistent with other precision measurements at this point (Ref. 28). Note, however, that this con-

straint does not in itself rule out Higgs-like neutral PGB-Z couplings. Firstly if our rough estimate, Eq. (14), is high by even a factor 2, for example, one might very reasonably obtain Higgs-like couplings for the PGB's with just $N_D=4$. Also if a techni-Glashow-Iliopoulos-Maiani mechanism is in place, then one could have a very large (~ 1 TeV) scale for the PGB masses, which in a theory with $N_D=4$ doublets could result in Higgs-like couplings for the PGB's. The lightest of these PGB's might still lie considerably below 1 TeV.

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