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Rare B decays, rare τ decays, and grand unification

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In multi-Higgs-boson extensions of the standard model, tree-level flavor-changing neutral currents exist naturally, unless suppressed by some symmetry. For a given rate, the exchanged scalar or pseudoscalar mass is very sensitive to the flavor-changing coupling between the first two generations. Since the Yukawa couplings of the first two generations are unknown and certainly very small, bounds which rely on some assumed value of this flavor-changing coupling are quite dubious. One might expect the size (and reliability) of the Yukawa couplings involving the third generation to be greater. In this paper, we consider processes involving τ's and B's, and determine the bounds on the flavor-changing couplings which involve third-generation fields. The strongest bound in the quark sector comes from B-$\bar{B}$ mixing and in the lepton sector, surprisingly, from $\mu\to e\gamma$. It is then noted that the flavor-changing couplings in the quark sector are related to those in the lepton sector in many grand unified theories, and one can ask whether an analysis of rare τ decays or rare B decays will provide the strongest constraints. We show that rare B decays provide the strongest bounds, and that *no* useful information can be obtained from rare τ decays. It is also noted that the most promising decay modes are $B\to K\mu\tau$ and $B_s\to\mu\tau$, and we urge experimenters to look for rare decay modes of the B in which a τ is in the final state.

I. INTRODUCTION

The standard model of the electroweak interactions has been extremely successful phenomenologically, and yet the large number of free parameters, as well as the large number of unanswered questions, has led to a strong belief that the standard model is incomplete. For this reason, there have been many studies of possible extensions of the standard model, ranging from simple extensions such as additional Higgs doublets to more complicated extensions such as supersymmetry and technicolor.

One feature that tends to occur in most extensions of the standard model is the presence of tree-level flavor-changing neutral currents (FCNC's). In fact, even in the simplest extension, with just the addition of a Higgs doublet, such currents will occur. When analyzing such models, virtually all theorists *require* that tree-level FCNC's, in both the quark and lepton sectors, are absent. This requirement is imposed in different ways; often a discrete symmetry is added to the model which eliminates these unwanted currents. In fact, many have examined the effects of virtual particles on one-loop FCNC's to constrain physics beyond the standard model, again *assuming* that this new physics does not give tree-level FCNC's.

The elimination of tree-level FCNC's often requires additional assumptions. Why do model builders insist on it so frequently? Many point to the small value of the K_L-K_S mass difference, arguing that any tree-level contribution must be suppressed by making the exchanged particle very heavy; the small value of muon-electron transitions (either in $\mu\to e\gamma$ or $\mu N\to eN$) extends this argument to the lepton sector. Another reason is more psychological—the requirement that tree-level FCNC's be absent led to the prediction of the charmed quark and to the general acceptance of the standard model, and it is natural to suppose that it applies to the entire model.

We feel that the assumption of no tree-level FCNC's may not be as necessary as generally believed. In a model with an extra Higgs doublet, for example, it is often stated that the mass of the extra scalar must be greater than 100 TeV, to avoid too large a contribution to the K_L-K_S mass difference [1,2]. This statement assumes, however, that the flavor-changing coupling is as large as the b-quark Yukawa coupling. A more natural value for the coupling would be [3] the geometric average of the d-quark and s-quark Yukawa couplings, which gives a bound on the exchanged particle mass of 1 TeV. Even that applies only to a pseudoscalar exchange; for a scalar, the bound is 300 GeV. Given the uncertainty in the Yukawa couplings of the first two generations, and the fact that Yukawa couplings in the standard model span six orders of magnitude, it is not implausible that the coupling would be somewhat smaller, thus making the bound even smaller. (In many grand unified theories, effective nonrenormalizable interactions at the Planck scale [4] give $\sim M_W M_X/M_{\rm Pl}\sim 10$ MeV uncertainties in all masses, making reliance on the value of the down-quark Yukawa coupling quite dubious anyway.) In addition, the information one obtains from this result would apply only to mixing between the first and second generations. Since Yukawa couplings in the standard model vary with mass, one might expect FCNC's couplings to also vary with mass, and thus FCNC's involving the third generation (flavor-changing B decays or τ decays) could be considerably larger. Yet virtually all analyses of the effects of tree-level FCNC's in extended Higgs models have only addressed the first two generations [5].

In this paper, we examine all the bounds that arise on flavor-changing couplings in extended Higgs models from an analysis of rare τ and B decays. These bounds will all consist of an upper limit on the couplings (which are proportional to the exchanged scalar or pseudoscalar mass). The objective here will be to determine which of the many possible processes are most sensitive to these decays, and thus offer the greatest chance of success.

We will then note that in grand unified theories, the quarks and leptons are often in the same representation. This implies that their FCNC's couplings could be related. In other words, a τ to μ transition would be related to a b-quark to s-quark transition. Thus, one will be able to eliminate the b-quark flavor-changing couplings in favor of the τ flavor-changing coupling. In the simplest grand unified theories, the couplings will be equal at the unification scale. The principle question we will address is: which set of decays (b or τ) will give stronger bounds? In other words, would one be more likely to detect them in τ decays or in B decays? The relevance of this question to the current discussion over whether to build a τ factory or a B factory is obvious. Furthermore, by examining the various bounds, we will be able to determine which processes are most important, and which (in the context of this model) are not.

In Sec. II, we examine the model itself, and discuss the most reasonable value for the couplings; we also examine the relationship between the flavor-changing τ and b-quark couplings. Section III contains an analysis of leptonic decays, including three-body decays, radiative decays, and μ-e conversion in nuclei. In Sec. IV, we consider B and B_s decays, including three-body decays (which are sensitive to scalar exchange), two-body decays (which are sensitive to pseudoscalar exchange), as well as B-$\bar{B}$ mixing. In Sec. V, our results are discussed and in Sec. VI, we give our conclusions.

II. FLAVOR-CHANGING NEUTRAL CURRENTS

We first consider the simplest possible extension of the standard model—the addition of a Higgs doublet. Since we are interested in neutral currents only, effects of the charged Higgs field will be ignored. The most general Yukawa couplings are given by

$$(\lambda_{ija}\bar{d}'_{iL}d'_{jR}\phi_a+\lambda_{ijb}\bar{d}'_{iL}d'_{jR}\phi_b)+\text{H.c.} ,$$

where $d'_i=(d',s',b')$, ϕ_a and ϕ_b are complex neutral fields and the λ_{ijk} are arbitrary. Similar terms can be written for the charge 2/3 quarks and for the charged leptons. In general, the real components of the Higgs fields will acquire vacuum expectation values v_a and v_b. We can then redefine two new scalar fields H and ϕ as

$$H\equiv\cos\beta\,\phi_a+\sin\beta\,\phi_b,\quad \phi\equiv-\sin\beta\,\phi_a+\cos\beta\,\phi_b ,$$

where $\tan\beta\equiv v_b/v_a$. The new fields H and ϕ have real components with vacuum expectation values $v=\sqrt{v_a^2+v_b^2}$ and zero, respectively. Note that in the standard model, $v=246$ GeV. The Yukawa couplings can be rewritten in terms of these new fields:

$$(f_{ij}\bar{d}'_{iL}d'_{jR}H+g_{ij}\bar{d}'_{iL}d'_{jR}\phi)+\text{H.c.} ,$$

where the f_{ij} and g_{ij} are still arbitrary. The mass matrix is then given by

$$M_{ij}=f_{ij}v .$$

When this matrix is diagonalized, we find, in terms of quark mass eigenstates,

$$[m_d\bar{d}_Ld_R(\sqrt{2}H/v)+m_s\bar{s}_Ls_R(\sqrt{2}H/v)+m_b\bar{b}_Lb_R(\sqrt{2}H/v)+h_{ij}\bar{d}_{iL}d_{jR}\phi]+\text{H.c.} ,$$

where again, the h_{ij} are arbitrary. We see that the H field is the Higgs field of the standard model; the ϕ field is simply an additional scalar which does not contribute to symmetry breaking or to quark and lepton masses; its couplings are, of course, completely arbitrary.

For simplicity, we will neglect mixing between the H fields and the ϕ field. This will not affect our bounds significantly if the mixing is small; the effects of such mixing on the results are discussed in the Appendix. The H field is then identical to the standard-model Higgs field (with the imaginary component being the Goldstone boson absorbed by the Z). The complex ϕ field is composed of a scalar ϕ_S and a pseudoscalar ϕ_P. The couplings of the scalar are given by

$$\frac{h_{ij}}{\sqrt{2}}\bar{d}_id_j\phi_S$$

and those of the pseudoscalar by

$$\frac{h_{ij}}{\sqrt{2}}\bar{d}_i\gamma_5d_j\phi_P$$

with similar terms for the leptons. For simplicity, we will assume here that the Yukawa coupling matrices are Hermitian (or at least that the deviations from hermiticity are small).

These extra scalars will lead to tree-level flavor-changing neutral currents through scalar exchange. The rate for such processes will generally be proportional to $h_{ij}^2h_{kl}^2/m_\phi^4$. It is important to note that some processes, such as two-body B decays, will only occur through pseudoscalar exchange, and others, such as three-body B decays, will only occur through scalar exchange. Some processes, such as τ decays, occur through both. This has led to some misunderstandings in the literature. In the classic work of Shankar [1], many processes (again, involving only the first two generations) were listed in a table with the accompanying bound on the scalar mass (assuming the couplings were all equal to the b or τ Yukawa coupling). In some cases, the bound refers to the scalar mass and in some, it refers to the pseudoscalar mass. In processes (such as μ decays) with both, it was assumed that the masses were equal. Although this was stated clearly in the text, the table gave the impression that the various modes were competing with each other. This is not the case—the process $K\to\mu e$ for example, only bounds the pseudoscalar mass, whereas the process $K\to\pi\mu e$ only bounds the scalar mass. Since the masses are expected to be different, these two processes do *not* compete with each other. In this paper, we will consider bounds on the scalar mass and bounds on the pseudoscalar mass to be completely separate, and give results for each. In experiments looking for rare decays, it is crucial to keep this distinction in mind when quoting bounds on scalar masses.

We now turn to the value of the coupling constants, h_{ij}^{quark} for the charge $-1/3$ sector and the corresponding couplings h_{ij}^{lepton} for the lepton sector. Although they are in principle arbitrary, we do have some theoretical guidance. When citing bounds, experimenters calculate the bound using couplings of the order of the gauge coupling; their bound is then cited in the form $m_S h_{ij}/g$. This makes the mass scale appearing in the bound quite large. However, not only is there no reason to expect these Yukawa couplings to be as large as gauge couplings, but there is every reason to expect them to be much smaller. After all, fourteen of the fifteen Yukawa couplings in the standard model are orders of magnitude smaller than the gauge couplings, and those involving the first generation are five orders of magnitude smaller.

What is the most reasonable value for these couplings? Some early authors [1,5] chose the following approach: since the most conservative approach is to take all couplings to be comparable, and since in some sense the heaviest fermion sets the scale for the whole matrix, we can assume that each element is given by the Yukawa coupling of the heaviest quark or lepton times some mixing angle. As we do not know these mixing angle factors, we set all of them to 1. Thus, all of the h_{ij} are given by the Yukawa coupling of the b or τ. Many of the bounds cited in the literature for the mass scale of the exchanged scalar assume this coupling. This approach was strongly criticized in Ref. [3]. They argued that the assumption that all of the couplings are comparable was not reliable, since one of the most conspicuous features of the fermion mass spectrum is its hierarchical structure. They showed that if one assumes that there is no fine-tuning (in which large terms add together to make a small term), then there is a small set of phenomenologically sound Yukawa matrices, and that *all* of these possibilities lead to Yukawa couplings of the form

$$h_{ij}^{\text{quark}} = \sqrt{(g_y)_i (g_y)_j}, \tag{1}$$

where $(g_y)_i$ is the Yukawa coupling of d_i. A similar term arises for the leptons. In other words, the flavor-changing coupling of the additional scalar to, for example, the b and s quarks, should be of the order of the geometric mean of the Yukawa couplings of the b and s quarks. This assumption gives the observed Kobayashi-Maskawa (KM) angles without fine-tuning.

Although we will keep our results general, we will consider the choice in Eq. (1) to be a "preferred" value, and will also express the results in terms of this value. To this end, we define

$$\eta_{ij}^{\text{quark}} \equiv \frac{h_{ij}^{\text{quark}}}{(g_y)_b} \ , \quad \eta_{ij}^{\text{lepton}} \equiv \frac{h_{ij}^{\text{lepton}}}{(g_y)_\tau} \ . \tag{2}$$

The early estimates will correspond to $\eta_{ij}=1$. Substituting Eq. (1) into Eq. (2) gives the "most natural value" for the couplings. This value for η_{ij}^{quark} is $\sqrt{m_i m_j}/m_b$ and that for $\eta_{ij}^{\text{lepton}}$ is $\sqrt{m_i m_j}/m_\tau$.

Is there any connection between the flavor-changing neutral-current couplings in the quark sector and those in the lepton sector? In general, there is not, but one might expect a connection to exist in grand unified theories. In SU(5), for example, the b and the τ are in the same representation and have the same Yukawa couplings (at the unification scale M_X). If one adds a Higgs 5-plet to the model, then the flavor-changing neutral-current couplings in the quark sector and in the lepton sector will be identical; i.e., the h_{bs} coupling will be equal to the $h_{\mu\tau}$ coupling, etc. How generic is this result? In models with a "grand desert," the b to τ mass ratio at M_X (obtained by extrapolating the observed low-energy value to high energies) is unity; i.e., the Yukawa couplings of the b and of the τ are equal at M_X. If this occurs for group theoretic reasons [as it does in minimal SU(5) and SO(10)], then FCNC's couplings in simple extensions of the Higgs sector will be equal at M_X. Even in many intermediate scale models, as well as in supersymmetric models, the successful prediction of the low-energy b to τ mass ratio is not significantly affected, thus the equality of the FCNC's couplings also should not be. However, in models with family group symmetries, or in models with much more complicated Higgs structures [such as SU(5) with 5-plets and 45-plets], one would expect a different relationship between the couplings, if any. Throughout this paper, we will assume that the flavor-changing neutral-current couplings of the quarks equal those of the leptons at M_X, as expected in the simplest grand unified theories (GUTs).

If the couplings are equal at the GUT scale, we must renormalize them down to the electroweak scale. The renormalization-group equation for each coupling will be of the general form

$$\mu \frac{dh_{ij}}{d\mu} = h_{ij}(C_y \alpha_y + C_s \alpha_s) + C_{w_{ijkl}} h_{kl} \alpha_w + C_{h_{ijkl}} h_{jk} h_{il} h_{kl} \ ,$$

where the α's are the gauge couplings and the C's are easily calculable coefficients. In the cases of interest, the h_{ij} will always be smaller than the gauge couplings (especially smaller than the strong coupling), so the last term can be dropped. The α_w term is identical for both quarks and leptons, so it will drop out of the ratio. The α_s term, of course, only applies to h_{ij}^{quark}. The remaining equation is *identical* to the renormalization-group equation for the conventional Yukawa couplings in the standard model (under the same approximations). As a result, the ratio of h_{ij}^{quark} to h_{ij}^{lepton} should be the same as the ratio of the b to τ Yukawa couplings, i.e., the ratio of the b mass to the τ mass (see Ref. [6] for an explicit derivation):

$$\frac{h_{ij}^{\text{quark}}}{h_{ij}^{\text{lepton}}} = \frac{m_b}{m_\tau} \ . \tag{3}$$

Virtually all of the contribution to this ratio comes from the effects of the SU(3) coupling.

One minor caveat must be mentioned. In deriving the b to τ mass ratio in grand unified theories, one runs the couplings down to $Q^2=4m_b^2$, since the b mass is "measured" by the threshold for b-pair production. Here, we only need to run the coupling down to $Q^2=m_\phi^2$. This introduces a correction to the right-hand side of Eq. (3) which is given by

$$\left[\frac{\alpha_s(m_\phi^2)}{\alpha_s(4m_b^2)}\right]^{12/23} .$$

The factor of 12/23 is related to the anomalous dimension and β function of the QCD coupling, see Ref. [6] for details. For the range of m_ϕ which is of interest (40 to 1000 GeV), this factor ranges from 82% to 92%. Since the uncertainty in matrix elements in b decays is typically a factor of 2, this correction will be smaller than the uncertainty in the results. We will, nonetheless, include a 10% correction in our final results (for each $h_{ij}^{\rm quark}$), although for simplicity, we will ignore it in the text.

We now can see the advantage of the notation used in Eq. (2). Plugging in Eq. (3), we find that

$$\eta_{ij}^{\rm quark}=\eta_{ij}^{\rm lepton} . \quad (4)$$

As in grand unified theories, this relation should be most reliable for second- and third-generation fields. We will use this relation (modulo the correction mentioned in the last paragraph) and express our results entirely in terms of $\eta_{ij}^{\rm lepton}$. Note that the only assumption we have made is that the quark FCNC's and lepton FCNC's are identical at some grand unified scale—an assumption which is true in the simplest grand unified models. Our statement that the most natural value for $\eta_{ij}^{\rm lepton}$ is $\sqrt{m_i m_j}/m_\tau$, although plausible, is less reliable [7], and is based on the "no fine-tuning" arguments of Ref. [3].

Let us summarize the results of this section. In the simplest extension of the standard model, the addition of another scalar multiplet, one generally has tree-level flavor-changing neutral currents. If the flavor-changing couplings are taken to be the same as the b-quark Yukawa coupling, then the resulting lower bound on the exchanged scalar mass is very large. However, it has been argued that a more natural value for this coupling is the geometric mean of the Yukawa couplings of the two quarks (or leptons), which leads to much lower couplings. We have noted that the Yukawa couplings of the first two generations are very small and uncertain, and have pointed out that bounds based on mixing with the third generation should be more reliable. We have also noted that in many grand unified theories the τ and b flavor-changing couplings are identical at the unification scale. When they are renormalized, we find that $\eta_{ij}^{\rm lepton}=\eta_{ij}^{\rm quark}$, where $\eta_{ij}^{\rm quark\ (lepton)}$ is the ratio of the flavor-changing coupling between the ith and jth quark (lepton) to the Yukawa coupling of the $b(\tau)$. (This relation has a 10% correction which we include.) This relation will be used throughout, as we determine the bounds on the η_{ij} from various rare decays. The "most natural value" for the η_{ij} will not be explicitly used, but should be kept in mind in determining how strong the various constraints are.

III. CONSTRAINTS FROM RARE τ DECAYS

A. Three-body decays

The flavor-changing interactions of the ϕ_S and ϕ_P will lead to lepton-number-violating τ decays, as shown in Fig. 1. There are six rare τ decays which will occur:

$$\tau\to e^-e^-e^+,\quad \tau\to\mu^-\mu^-\mu^+,\quad \tau\to e^-e^-\mu^+ ,$$
$$\tau\to\mu^-\mu^-e^+,\quad \tau\to e^-\mu^-e^+,\quad \tau\to e^-\mu^-\mu^+ . \quad (5)$$

The latter two can occur through two different processes; for example, $\tau\to e^-\mu^-e^+$ can occur either through a $h_{\mu\tau}h_{ee}$ term or through a $h_{e\tau}h_{e\mu}$ term. For example, the matrix element can be written as

$$M=\frac{i}{m_s^2}(\bar u_\mu h_{e\mu}u_e\bar u_e h_{e\tau}u_\tau+\bar u_e h_{ee}u_e\bar u_\mu h_{\mu\tau}u_\tau) , \quad (6)$$

where we have ignored the momentum dependence of the propagator (since the scalar mass is so much larger than the momentum transfer). A similar term will exist from pseudoscalar exchange (with a γ_5 in the vertices); as discussed in the preceding section, since only the lighter of the scalar and pseudoscalar will contribute much, we are considering the two cases separately. Note that if the scalar and pseudoscalar masses were similar, then interference between the matrix elements would be important. However, the masses come from different terms in the Higgs potential, and will generally be different; we are assuming that they are sufficiently different that the interference term will not drastically change the results. The momentum dependence of the spinors has not been explicitly shown. Neglecting the mass of the muon, this gives a decay rate of

$$\omega=\frac{m_\tau^5}{3072\pi^3 m_S^4}(h_{e\tau}^2h_{e\mu}^2+h_{ee}^2h_{\mu\tau}^2+\tfrac{1}{2}h_{ee}h_{e\mu}h_{e\tau}h_{\mu\tau}) . \quad (7)$$

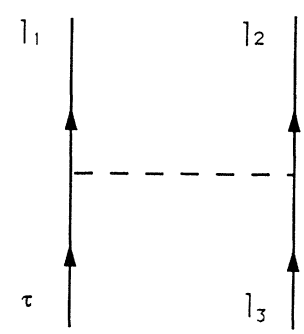

FIG. 1. Lepton-number-violating τ decays can occur through exchange of an intermediate scalar. l_1, l_2, and l_3 are either electrons or muons. If two identical fermions are in the final state, an exchange diagram must be subtracted. In some cases, such as $\tau\to e^+e^-\mu^-$, the process can occur with either $l_1=\mu^-$, $l_2=e^-$ and $l_3=e^+$ or $l_1=e^-$, $l_2=\mu^-$ and $l_3=e^+$; these two diagrams have different coupling constant dependences and are added.

TABLE I. Bounds on the flavor-changing couplings which arise from three-body leptonic decays. Here, η_{ij} is defined in the text in Eq. (2). The numerical values should be understood as multiplied by $(m_S/m_W)^4$ $[(m_P/m_W)^4]$. The contribution is the same for scalar exchange and for pseudoscalar exchange, and so leads to identical bounds on the masses. All experimental bounds in this paper are from Ref. [8], unless explicitly stated otherwise.

Decay process	Expt. limit	Bound
$\tau\to e^-e^-e^+$	3.8×10^{-5}	$\eta_{ee}^2\eta_{e\tau}^2<2000$
$\tau\to\mu^-\mu^-\mu^+$	2.9×10^{-5}	$\eta_{\mu\mu}^2\eta_{\mu\tau}^2<1600$
$\tau\to e^-e^-\mu^+$	3.8×10^{-5}	$\eta_{e\tau}^2\eta_{e\mu}^2<2000$
$\tau\to\mu^-\mu^-e^+$	3.8×10^{-5}	$\eta_{\mu\tau}^2\eta_{e\mu}^2<2000$
$\tau\to e^-\mu^-e^+$	3.3×10^{-5}	$\eta_{e\tau}^2\eta_{e\mu}^2+\eta_{ee}^2\eta_{\mu\tau}^2<1800$
$\tau\to e^-\mu^-\mu^+$	3.3×10^{-5}	$\eta_{\mu\tau}^2\eta_{e\mu}^2+\eta_{\mu\mu}^2\eta_{e\tau}^2<1800$
$\mu\to e^-e^-e^+$	1.0×10^{-12}	$\eta_{ee}^2\eta_{e\mu}^2<10^{-5}$

The rate for psuedoscalar exchange is identical, with $m_S\to m_P$. The observed limit on the branching ratio is 3.3×10^{-5}. With this limit, we then find that

$$\eta_{e\tau}^2\eta_{e\mu}^2+\eta_{ee}^2\eta_{\mu\tau}^2+\tfrac{1}{2}\eta_{e\tau}\eta_{ee}\eta_{e\mu}\eta_{\mu\tau}<1400\left[\frac{\min(m_S,m_P)}{m_W}\right]^4$$

using the definition of η_{ij} given in the preceding section. Completing the square, and assuming maximal interference, gives

$$\eta_{e\tau}^2\eta_{e\mu}^2+\eta_{ee}^2\eta_{\mu\tau}^2<1800\left[\frac{\min(m_S,m_P)}{m_W}\right]^4 . \tag{8}$$

A similar calculation can be done for each of the above six processes. The results are given in Table I, where we have also included the bound from the $\mu\to3e$ process.

Note how poor the bounds from τ decays are. As discussed in the preceding section, the most natural values for the η_{ij} are much less than one, and thus these processes do not give any significant limits, even for a very light scalar or pseudoscalar. Improvement in the experimental bounds of at least three orders of magnitude (and generally four or five orders of magnitude) would be needed to approach the interesting region [9]. We now turn to radiative decays.

B. Radiative decays

The flavor-violating couplings of the ϕ_S and ϕ_P will also lead to lepton-number-violating radiative decays of the μ and τ, through the one-loop diagrams shown in Fig. 2. One expects that these will give better bounds than flavor-changing radiative decays of the b, since the latter already occur at one loop in the standard model. The sum of the amplitudes of the diagrams in Fig. 2 is given by

$$M=-e\epsilon^{\mu}h_{l_1l_2}h_{l_2l_3}\bar{u}_{l_3}(p')\times\left[\Lambda_\mu+i\Sigma_L\frac{(\not{p}'+m_1)\gamma_\mu}{p'^2-m_1^2}+i\frac{\gamma_\mu(\not{p}+m_3)}{p^2-m_3^2}\Sigma_R\right]\times u_{l_1}(p) , \tag{9}$$

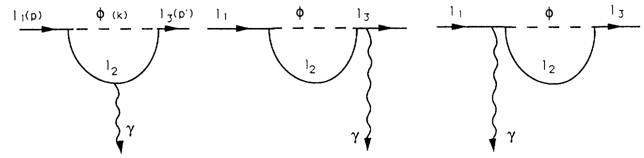

FIG. 2. Diagrams which lead to lepton-number-violating radiative decays. If l_1 and l_3 are identical, these diagrams give contributions to the anomalous magnetic moment.

where

$$\Lambda_\mu=-\int\frac{d^nk(\not{p}-\not{k}+m_2)}{(2\pi)^n(p-k)^2-m_2^2}\frac{\gamma_\mu}{k^2-m_s^2}\frac{(\not{p}'-\not{k}+m_2)}{(p'-k)^2-m_2^2} ,$$

$$\Sigma_L(p')=i\int\frac{d^nk}{(2\pi)^n}\frac{\not{p}'-\not{k}+m_2}{(p'-k)^2-m_2^2}\frac{1}{k^2-m_s^2} , \tag{10}$$

$$\Sigma_R(p)=\Sigma_L(p\to p') .$$

Here, we have only shown the result when the scalar is in the loop; if the pseudoscalar is in the loop, appropriate γ_5's must be inserted. The divergences all cancel, as they should. The finite part can then be computed.

First, consider $\tau\to e\gamma$ decay. Performing all of the integrations and expanding in powers of m_τ^2/m_S^2, we find that the leading term is $O(m_\tau^2/m_S^2)$ and get

$$|M|^2=\frac{e^2}{256\pi^4}\left|\bar{u}_e(p')\left[h_{\tau\tau}h_{e\tau}\frac{m_\tau^2}{6m_S^2}\left[\not{\epsilon}-2\frac{p\cdot\epsilon}{m_\tau}\right]+(h_{e\tau}h_{ee}+h_{\mu\tau}h_{e\mu})\frac{m_\tau^2}{3m_S^2}\times\left[\not{\epsilon}-2\frac{p\cdot\epsilon}{m_\tau}\right]\right]u_\tau(p)\right|^2 . \tag{11}$$

Unless there is fine-tuning, the interference terms will be negligible. Ignoring them, we get the overall decay rate

$$\omega=\frac{e^2m_\tau^5}{2^{13}9\pi^5m_S^4}(h_{\tau\tau}^2h_{e\tau}^2+4h_{ee}^2h_{e\tau}^2+4h_{\mu\tau}^2h_{e\mu}^2) . \tag{12}$$

Comparing with the standard τ decay, we find that

$$\frac{h_{\tau\tau}^2h_{e\tau}^2}{g^4}+4\left[\frac{h_{ee}^2h_{e\tau}^2+h_{\mu\tau}^2h_{e\mu}^2}{g^4}\right]<1.4\left[\frac{m_S}{m_W}\right]^4 . \tag{13}$$

The terms in the square brackets are negligible, compared with the right-hand side, because of the bounds from three-body decays from Table I. Dropping these and expressing the results in terms of the η_{ij} finally gives the result in Table II. A similar calculation can be done for the process $\tau\to\mu\gamma$.

We can also calculate the process $\mu\to e\gamma$. Here, the dominant contribution (by many orders of magnitude) comes when the fermion line in the loop is a τ. The calculation is similar; the matrix element is

TABLE II. Bounds on flavor-changing couplings which arise from radiative decays of the τ and μ. The numerical values should be understood as multiplied by $(m_S/m_W)^4$ $[(m_P/m_W)^4]$. The bounds for the case of scalar exchange are slightly different from those for the case of pseudoscalar exchange; the number in parentheses gives the bound when the pseudoscalar mass is used.

Decay process	Expt. limit	Bound
$\tau\to e\gamma$	2.0×10^{-4}	$\eta_{\tau\tau}^2\eta_{e\tau}^2<2.2\times10^7(8.8\times10^5)$
$\tau\to\mu\gamma$	5.5×10^{-4}	$\eta_{\tau\tau}^2\eta_{\mu\tau}^2<6.7\times10^7(2.7\times10^6)$
$\mu\to e\gamma$	5.0×10^{-11}	$\eta_{\mu\tau}^2\eta_{e\tau}^2<3.6\times10^{-4}(3.6\times10^{-4})$

$$|M|^2=\frac{e^2}{256\pi^4}\frac{m_\tau^2m_\mu^2}{4m_S^4}h_{\mu\tau}^2h_{e\tau}^2 \times\left|\bar{u}_e(p')\left[\not\epsilon-2\frac{p\cdot\epsilon}{m_\mu}\right]u_\mu(p)\right|^2 . \quad (14)$$

which then gives the bound listed in Table II:

$$\eta_{\mu\tau}^2\eta_{e\tau}^2<3.6\times10^{-4}\left[\frac{m_S(m_P)}{m_W}\right]^4 . \quad (15)$$

Suppose we choose the "most natural" values for the η_{ij}, i.e., $\eta_{\mu\tau}=\sqrt{m_\mu/m_\tau}$, $\eta_{e\tau}=\sqrt{m_e/m_\tau}$. Then the bound on m_S and m_P is 40 GeV. Yet one expects the values of these masses to be of the order of the weak scale. We see that the experimental limit on $\mu\to e\gamma$ is just beginning to probe the most interesting range of masses. An improvement of three orders of magnitude in the experimental bound, which is expected in the next few years, would cover the region of m_S and m_P from 40 to 210 GeV, just the region where one might expect them to be.

Let us restate this point. In the simplest extension of the standard model, with what we believe to be the most natural values for the additional flavor-changing couplings, one expects $\mu\to e\gamma$ to occur at a rate not much below the current limit. If the extra scalars have masses below about 210 GeV, as one might expect, then the decay will be observed within the next few years. Note that here, observation of $\mu\to e\gamma$ *does not* indicate mixing between the muon and the electron, but rather between the muon and the tau, and between the electron and the tau. As we will see later, the bound from this process is the one of the most severe, and thus this decay may be the first signature of this simple extension.

Suppose the decay is seen. At that time, all theorists will come up with their particular models. Is there any way to distinguish between these models? As can be seen from Eq. (14), the interaction is a tensor interaction. This will distinguish the model from some of the other possibilities. The clearest way to determine which model is correct, of course, is to observe additional signatures. Although $\mu\to e\gamma$ is the first signature likely to be observed, we will see in the next section that there are other signatures in rare B decays that may not be far behind. First, we consider other lepton-number-violating processes.

C. Other processes

It has been pointed out [10] that bounds from muon to electron conversion in nuclei are very often stronger than bounds from $\mu\to e\gamma$. The reason is that the "exchanged particle" often couples coherently to the nucleus. Here, however, the bound from muon conversion will be weaker. The reason is that we are interested in bounds in couplings involving the third generation, i.e., it is still necessary to have a τ in a loop; the relevant diagrams simply involve attaching the nucleus to the photon in the $\mu\to e\gamma$ diagrams. The photon will couple coherently to the nucleus (the cross section will vary as Z^2), but the loop is still necessary. We have calculated the rate for muon to electron conversion in titanium (which gives the strongest bound) and found the bound to be two orders of magnitude weaker than that from $\mu\to e\gamma$. We have not included QCD enhancements, finite-size effects, etc.; should these enhance the rate by a factor of 100, then muon to electron conversion would give bounds competitive with $\mu\to e\gamma$ (at least until the latter is improved).

Bounds can also be calculated from the contribution of scalar exchange to the anomalous magnetic moments of the electron and muon. Nor surprisingly (since the standard electroweak contribution is too small to have been seen), these bounds are also much, much weaker than the other processes we have considered. Finally, one could also consider two-body τ decays, such as $\tau\to\mu K^0$. These processes will all involve couplings involving the first generation fields, and are expected to be small; it turns out that the bounds are much weaker than those from B decays.

IV. B AND B_s DECAYS

A. Three-body decays

The flavor-changing neutral-current interaction will also lead to anomalous B decays. We will only consider semileptonic decays; nonleptonic decays are much more difficult to calculate and the experimental bounds are much, much worse. Of course, some processes, such as $B\to K\mu^+\mu^-$ occur at the one-loop level in the standard model, but some, such as $B\to K\mu^+\tau^-$, do not. In all cases that we consider, the standard-model processes will occur at a rate far below the current experimental limit. For example, the process $B^-\to K^-\mu^+e^-$ occurs through the diagram in Fig. 3. Unlike τ decays, this process cannot occur through either scalar or pseudoscalar exchange. The reason is simply that the parity of the B and

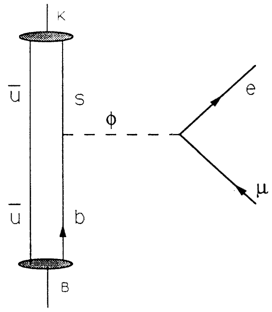

FIG. 3. Contributions to three-body decays of the B meson. The exchanged particle, in this case, must be a scalar.

the K are the same, and the interaction does not change the spin, thus only a scalar can be exchanged. (In two-body decays, only the pseudoscalar can be exchanged, as we will see.)

The matrix element is

$$M = i\frac{h_{e\mu}h_{sb}}{m_S^2}\bar{u}_e v_\mu f_+(q^2) , \tag{16}$$

where $f_+(q^2)$ is a Lorentz-invariant form factor which is only a function of $q^2=(p-p')^2$. (Note that another possible form factor, often referred to as f_-, does not contribute because of conservation of the vector current; see Okun [11] for a discussion.) This form factor can be calculated with the nonrelativistic approximation of Isgur and Scora [12], which should cause an error somewhat less than a factor of 2. Their procedure can be outlined as follows.

We can write

$$f^+(q^2) = \langle K^-(\mathbf{p}')|\bar{s}b|B^-(\mathbf{p})\rangle . \tag{17}$$

The nonrelativistic state vectors for the B^- and K^- bosons are given by

$$\begin{aligned}|B^-(\mathbf{p})\rangle &= \sqrt{2m_B}\int d^3k\,\phi_B(\mathbf{k})\sum\chi_{s\bar{s}}\left|b\left(\frac{m_b}{m_B}\mathbf{p}_B+\mathbf{k},s\right)\right\rangle\left|\bar{u}\left(\frac{m_u}{m_B}\mathbf{p}_B-\mathbf{k},\bar{s}\right)\right\rangle ,\\ |K^-(\mathbf{p}')\rangle &= \sqrt{2m_K}\int d^3k'\,\phi_K(\mathbf{k}')\sum\chi_{s'\bar{s}'}\left|s\left(\frac{m_s}{m_K}\mathbf{p}_K'+\mathbf{k}',s'\right)\right\rangle\left|\bar{u}\left(\frac{m_u}{m_K}\mathbf{p}_K'-\mathbf{k}',\bar{s}'\right)\right\rangle ,\end{aligned} \tag{18}$$

where $\chi_{s\bar{s}}$ couples the spins s and $\bar{s}$ to the total spin zero and $\phi(\mathbf{k})$ is the relative momentum-space wave function. Isgur and Scora chose Schrödinger wave functions that are appropriate to a Coulomb plus linear potential and used variational solutions based on harmonic-oscillator wave functions:

$$\phi^{1s}(r) = \frac{\beta_S^{3/2}}{\pi^{3/4}}\exp(-\beta_S^2 r^2/2) ,$$

in which β_S is the variational parameter, whose value turns out to be $\beta_S \sim 0.3$ GeV. We now compute the Fourier transform of these wave functions and substitute the result into Eqs. (18) and (17). Note that the form factor vanishes if a γ_5 is present, so that pseudoscalar exchange does not contribute. The result for the form factor is

$$f_+(q^2) = 2\sqrt{m_B m_K}\frac{\beta_B^{3/2}\beta_K^{3/2}}{\beta_{BK}^3}\exp\left[-\frac{m_u^2(t_m-t)}{4\beta_{BK}^2 m_B m_K}\right] , \tag{19}$$

where $\beta_{BK}^2=(\beta_B^2+\beta_K^2)/2$ and $t_m=(m_B-m_K)^2$. The decay rate is

$$\frac{h_{sb}^2 h_{e\mu}^2 \beta_B^3 \beta_K^3 m_B^2 m_K^3}{16\pi^3 \beta_{BK}^4 m_S^4 m_u^2}\left[1-\frac{m_K\beta_{BK}\sqrt{2\pi}}{m_u m_B}\times\text{erf}\left[\frac{m_u m_B}{2\sqrt{2}m_K\beta_{BK}}\right]\right] , \tag{20}$$

where erf(x) is the error function [normalized so that erf(∞)=1].

Since we know that the lifetime of the B^- is 10^{-12} sec, we can compute the branching ratio for the process. The results are identical for all processes of the form $B^-\to K^- l_1 l_2$ for any two leptons, with the obvious change in the couplings. If there is one τ in the final state, there is a phase-space factor of 0.7; if there are two τ's, the phase-space factor is 0.4. The resulting bound on the η's is given by (with inclusion of the factor of 20% in the conversion from η^{quark} to η^{lepton})

$$\eta_{\mu\tau}^2\eta_{l_1 l_2}^2 < 7\times 10^4(\text{branching ratio}) \times\left[\frac{m_S}{m_W}\right]^4\Big/(\text{phase-space factor}) . \tag{21}$$

All we need to do now is to put in the various branching ratios. Note that a similar calculation can be done for $B^-\to\pi^- l_1 l_2$, with an identical result (with $m_K\to m_\pi$), although the nonrelativistic approximation is a bit more suspect in this case. Experimental bounds

TABLE III. Bounds on the flavor-changing couplings from three-body B decays. Since only the scalar contributes, the bound only applies to the scalar mass, and not the pseudoscalar mass. The numerical values should be understood as multiplied by $(m_S/m_W)^4$. We have used Eq. (4) to relate the quark flavor-changing couplings to the lepton-number-changing couplings, and included the 10% correction factor discussed in Sec. II. Processes marked with parentheses are not firm experimental bounds, but simply our estimate of the bound that could be obtained from the results in Eq. (22).

Decay process	Expt. limit	Bound
$B\to K\mu\mu$	5×10^{-5}	$\eta_{\mu\tau}^2\eta_{\mu\mu}^2<3$
$B\to\pi\mu\mu$	5×10^{-5}	$\eta_{e\tau}^2\eta_{\mu\mu}^2<330$
$B\to Kee$	5×10^{-5}	$\eta_{\mu\tau}^2\eta_{ee}^2<3.6$
$B\to\pi ee$	5×10^{-5}	$\eta_{e\tau}^2\eta_{ee}^2<360$
$B\to K\mu e$	10^{-3}	$\eta_{\mu\tau}^2\eta_{e\mu}^2<70$
$B\to\pi\mu e$	10^{-3}	$\eta_{e\tau}^2\eta_{e\mu}^2<7000$
$B\to K\mu\tau$	(3×10^{-4})	$\eta_{\mu\tau}^4<30$
$B\to\pi\mu\tau$	(3×10^{-4})	$\eta_{e\tau}^2\eta_{\mu\tau}^2<3000$
$B\to Ke\tau$	(3×10^{-3})	$\eta_{\mu\tau}^2\eta_{e\tau}^2<300$
$B\to\pi e\tau$	(3×10^{-3})	$\eta_{e\tau}^4<30000$
$B\to K\tau\tau$	(2×10^{-3})	$\eta_{\mu\tau}^2\eta_{\tau\tau}^2<350$
$B\to\pi\tau\tau$	(2×10^{-3})	$\eta_{e\tau}^2\eta_{\tau\tau}^2<35000$
$K\to\pi\mu\mu$	2.3×10^{-7}	$\eta_{e\mu}^2\eta_{\mu\mu}^2<200$
$K\to\pi\mu e$	2.1×10^{-10}	$\eta_{e\mu}^4<0.18$
$K\to\pi ee$	1.0×10^{-8}	$\eta_{ee}^2\eta_{e\mu}^2<9$

have been given for decays in which the two leptons are muons and/or electrons, but no bounds have been cited when one or both is a τ. Nonetheless, one can make a rough estimate of the bounds from two processes which have been cited [13,14]:

$$\frac{\Gamma(B\to e^+e^-X)+\Gamma(B\to\mu^+\mu^-X)}{\Gamma(B\to\text{all})}<2.4\times10^{-3}\,,$$
$$\frac{\Gamma(B\to\mu^+\mu^-X)}{\Gamma(B\to\text{all})}<5.3\times10^{-5}\,, \tag{22}$$

where the charge of the B is undetermined. For example, if $B\to K\mu\tau$ occurs, it will give a signal in the above process 17% of the time (the percentage of τ's which decay into muons). Consider the first of these bounds. We have made two modifications to it. First, in extracting their bound, the authors chose many different possible matrix elements to model the decay, and cited the one that gave the most conservative bound. Unfortunately, none of these matrix elements was a scalar. We have chosen to model the decay with a constant matrix element, resulting in a bound which is a factor of 2 smaller than the one they cite (virtually all of their choices gave a factor within 10% of this one). Second, they also searched for $B\to e^+\mu^-X$, *assumed* this was zero, and used that to check their background calculation. Since τ's decay into electrons and muons with equal enthusiasm, we will also get a signal here, so we have included these data in extracting the bound (they give the number of events seen). Regarding the second bound, we have not yet seen the detailed analysis, and will simply take the number at face value. Note that it gives no information on decays with an electron in the final state. From these values, we estimate that the limit on $B\to\pi e\mu$ and $B\to Ke\mu$ is 10^{-3}; the limit on $B\to e\tau X$ is 3×10^{-3}; the limit on $B\to\mu\tau X$ is 3.2×10^{-4}, and the limit on $B\to\tau\tau X$ is 2×10^{-3}. The other bounds are given in Ref. [8]. It is important to emphasize that these bounds involving final state τ's are only rough estimates, and should not be considered firm experimental limits. Experimental limits could be obtained from the above experiments if the appropriate Monte Carlo calculations were done, and we have not done so. The estimates have been done to give an idea of the bounds that can be obtained from such decays; we urge experimenters to determine limits on these branching ratios so that more precise bounds can be found.

The results are given in Table III. Note that the bounds are much stronger than the corresponding bounds on τ decays. Some of the processes, such as $B\to Ke\tau$, are proportional to the same couplings as in $\mu\to e\gamma$. The latter bound is so strong that these processes would be unobservable. Other processes depend on first-generation couplings and are expected to be small. Perhaps the most interesting process is $B\to K\mu\tau$. This decay depends only on $\eta_{\mu\tau}$, which is expected to be the largest flavor-changing coupling. The right-hand side will reach unity with an improvement of a factor of 30 in the rate. This may seem extremely difficult, but the process has never been looked for. Such an improvement seems quite possible.

These processes all depend on scalar exchange. If the scalar were much heavier than the pseudoscalar, these decays would be negligible, while τ decays would still occur. We now turn to two-body decays, which are not only sensitive to pseudoscalar exchange, but offer much more realistic prospects for experimental improvement in the bounds.

B. Two-body decays

Two-body decays of the B and B_s mesons occur through the diagram of Fig. 4. Since these mesons have negative parity, the decay can only occur through a pseudoscalar interaction. As an example, the matrix element for the decay $B_s\to e\mu$ can be written as

$$M=i\frac{h_{e\mu}h_{sb}}{m_P^2}\bar{u}_e v_\mu f_+(q^2),$$

where $f_+(q^2)=\langle 0|\bar{s}\gamma_5 b|B_s\rangle$. The form factor can be evaluated by the method of the preceding subsection [15], and is given by

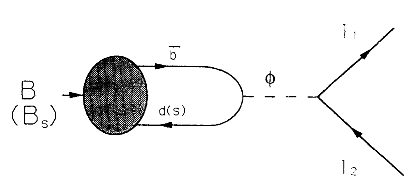

FIG. 4. Contributions to two-body decays of the B (or the B_s) meson. The exchanged particle must be a pseudoscalar.

$$f_{+}(q^2)=\sqrt{4m_B}\,\frac{\beta_B^{3/2}}{\pi^{3/4}}$$

so that the decay rate is

$$\omega=\frac{h_{e\mu}^2 h_{sb}^2 \beta_B^3 m_B^2}{4\pi^{5/2} m_P^4} . \tag{23}$$

The same decay rate (with the obvious change in the subscripts on the coupling constants) applies to all other processes. With one τ (two τ's) in the final state, a phase-space factor of 0.76 (0.36) must be included. This will give a bound for $B\to l_1 l_2$

$$\eta_{l_1 l_2}^2 \eta_{e\tau}^2 < 3.0\times 10^4 \text{ (branching ratio)} \times \left[\frac{m_P}{m_W}\right]^4 \Big/ \text{(phase-space factor)} . \tag{24}$$

For B_s decays, one obtains an identical result (the small mass difference between the B and B_s gives corrections much smaller than the factor-of-2 uncertainty in the form factor), with $\eta_{e\tau}^2 \to \eta_{\mu\tau}^2$.

To determine the branching ratio, we compare this rate with the observed B lifetime. For processes involving τ's, we use the results which followed Eq. (22). This does not work well for B_s decays, since the lifetime of the B_s has not been measured. However, one expects the lifetime of the B_s to be the same as that of the B, since the standard-model decay proceeds through the weak decay of the b quark. Thus, we will take the lifetime of the two to be equal. In determining the branching ratio for the B_s, we note that the UA1 result [14] does not distinguish between B's and B_s's. We will assume that the relative production rate for B_s's is a factor of 4 smaller than that for B's (since the probability of popping an s pair out of the vacuum is about one-fourth that of d pairs [16]), and thus the bounds on the B_s branching ratio into $\mu\mu$, $\mu\tau$, and $\tau\tau$ is four times as large as that for B's. Note that no bounds currently exist for $B_s\to e\mu$ or $B_s\to ee$.

The results are given in Table IV. As in the three-body case, the bounds from two-body B decays are much strong than the corresponding bounds from τ decays.

C. K-$\bar{K}$ and B-$\bar{B}$ mixing

The strongest bounds on scalar- and/or pseudoscalar-mediated tree-level FCNC's quoted in the literature come from K-$\bar{K}$ mixing. We now discuss the constraints from this and similar processes.

As discussed earlier, it has generally been recognized that the most stringent bounds on flavor-changing couplings (involving the first two generations) come from K-$\bar{K}$ mixing. Here, this result is extended to include B-$\bar{B}$ mixing.

A discussion of the calculation of K-$\bar{K}$ mixing due to Higgs-scalar exchange can be found in Ref. [3], and references therein. The relevant matrix element discussed in these papers is $\langle \bar{K}|(\bar{s}\gamma_5 d)(\bar{s}\gamma_5 d)|K\rangle$, which has a value [2] of 0.085 GeV^3. With a value for the coupling of $\sqrt{(g_y)_d(g_y)_s}$, a bound of 1.0 TeV on the mass of the exchanged particle is obtained.

The γ_5 in the above matrix element shows that *pseudoscalar* exchange only was treated in these papers. If one considers scalar exchange, the matrix element will be different. It is easy to see why the matrix element with scalars will be smaller: If one uses the vacuum-insertion method, and inserts the vacuum state in the matrix element, then the fact that the kaon is a pseudoscalar implies that the matrix element for scalar exchange will vanish. The scalar matrix element has been calculated [17] and is smaller by a factor of 12. This will lower the bound on the mass by a factor of $\sqrt{12}$. One thus finds

TABLE IV. Bounds on flavor-changing couplings which arise from two-body decays of the B and B_s. Since only the pseudoscalar contributes, the only bounds apply to m_P. The numerical values should be understood as multiplied by $(m_P/m_W)^4$. The bounds on $B(B_s)\to\mu\mu$ comes from a recent result of Ref. [15]. Processes marked with parentheses are not firm experimental bounds, but simply our estimate of the bound that could be obtained from the results in Eq. (22). For the processes $B_s\to ee$, $B_s\to e\mu$, and $B_s\to e\tau$, there is currently no experimental limit; once a limit X is determined, the bound given in the third column follows.

Decay process	Expt. limit	Bound
$B\to ee$	3×10^{-5}	$\eta_{ee}^2\eta_{e\tau}^2<1.0$
$B\to e\mu$	4×10^{-5}	$\eta_{e\mu}^2\eta_{e\tau}^2<1.4$
$B\to \mu\mu$	9×10^{-6}	$\eta_{\mu\mu}^2\eta_{e\tau}^2<0.3$
$B_s\to ee$	X	$\eta_{ee}^2\eta_{\mu\tau}^2<3\times 10^4 X$
$B_s\to e\mu$	X	$\eta_{e\mu}^2\eta_{\mu\tau}^2<3\times 10^4 X$
$B_s\to \mu\mu$	4×10^{-5}	$\eta_{\mu\mu}^2\eta_{\mu\tau}^2<1.2$
$B\to e\tau$	(3×10^{-3})	$\eta_{e\tau}^4<140$
$B\to \mu\tau$	(3×10^{-4})	$\eta_{e\tau}^2\eta_{\mu\tau}^2<14$
$B\to \tau\tau$	(2×10^{-3})	$\eta_{e\tau}^2\eta_{\tau\tau}^2<190$
$B_s\to e\tau$	X	$\eta_{e\tau}^2\eta_{\mu\tau}^2<4\times 10^4 X$
$B_s\to \mu\tau$	(1.2×10^{-3})	$\eta_{\mu\tau}^4<50$
$B_s\to \tau\tau$	(8×10^{-3})	$\eta_{\mu\tau}^2\eta_{\tau\tau}^2<640$
$K\to \mu\mu$	6×10^{-9}	$\eta_{e\mu}^2\eta_{\mu\mu}^2<0.02$
$K\to e\mu$	2.2×10^{-10}	$\eta_{e\mu}^4<0.0008$
$K\to ee$	3.2×10^{-10}	$\eta_{ee}^2\eta_{e\mu}^2<0.0012$

that the bound on the pseudoscalar mass is 1000 GeV, and the bound on the scalar mass is only 300 GeV.

The weakness of these bounds may surprise those who have always felt that the bounds from K-$\bar{K}$ mixing put very stringent constraints on the mass of flavor-changing scalars. Let us emphasize why this bound is so much smaller. The main difference is in the choice of coupling. The early authors chose a coupling equal to the b-quark Yukawa coupling; Cheng and Sher [3] then argued that choosing the geometric average of the d- and s-quark Yukawa couplings was much more natural and realistic. Finally, the scalar matrix element is much smaller than the pseudoscalar matrix element, leading to weaker bounds on the scalar mass. We wish to emphasize that this bound is highly uncertain, since it depends so heavily on mixing between the first two generations and on the light-quark Yukawa couplings.

Putting all of this together, we can extract the bound on the coupling:

$$\eta_{e\mu}^4 < 9.0\times 10^{-14}\left[\frac{m_P}{m_W}\right]^4 , \qquad \eta_{e\mu}^4 < 1.3\times 10^{-11}\left[\frac{m_S}{m_W}\right]^4 . \tag{25}$$

What about the bound on B-$\bar{B}$ mixing? In the case of K-$\bar{K}$ mixing, it was assumed that the contribution due to scalar exchange was no greater than the standard-model contribution, reflecting the factor-of-2 uncertainty in the standard-model contribution. The same uncertainty applies to B-$\bar{B}$ mixing. The ratio of B-$\bar{B}$ mixing to K-$\bar{K}$ mixing is given by

$$\frac{\Delta m_B}{\Delta m_K} = \frac{h_{bd}^2\langle \bar{B}|(\bar{b}\gamma_5 d)(\bar{b}\gamma_5 d)|B\rangle}{h_{sd}^2\langle \bar{K}|(\bar{s}\gamma_5 d)(\bar{s}\gamma_5 d)|K\rangle} . \tag{26}$$

Estimating the matrix elements by the vacuum-insertion method, we find [3]

$$\frac{\langle \bar{B}|(\bar{b}\gamma_5 d)(\bar{b}\gamma_5 d)|B\rangle}{\langle \bar{K}|(\bar{s}\gamma_5 d)(\bar{s}\gamma_5 d)|K\rangle} = \frac{f_B^2}{f_K^2}\frac{m_B^3}{m_K^3}\left[\frac{m_s+m_d}{m_b+m_d}\right]^2 . \tag{27}$$

Numerically, this ratio is 0.9. We will take the ratio of scalar matrix elements to be the same. We see that the ratio of Δm_B to Δm_K is almost entirely due to the difference in couplings. Taking the observed value of the mass splitting gives our bounds:

$$\eta_{e\tau}^4 < 2.0\times 10^{-9}\left[\frac{m_P}{m_W}\right]^4 , \qquad \eta_{e\tau}^4 < 3.0\times 10^{-7}\left[\frac{m_S}{m_W}\right]^4 . \tag{28}$$

It is interesting to note that our "most natural value" for $\eta_{e\tau}$ is $\sqrt{m_e/m_\tau}$, giving $\eta_{e\tau}^4 = 7.8\times 10^{-8}$, so that the bounds (in this case) on m_S and m_P are 60 and 200 GeV, respectively. These bounds should be more reliable than bounds from K-$\bar{K}$ mixing (since they do not involve mixing between the first two generations), but less reliable than bounds involving mixing between the second and third generations.

Finally, what about B_s-$\bar{B}_s$ bounds? In the standard model, this mixing is maximal, and adding extra contributions will make no difference. The only way in which scalar exchange could matter would be if it contributed with roughly the same magnitude and opposite sign to the standard-model contribution. The uncertainties in both calculations would make any bounds found from this meaningless.

V. RESULTS

Of all of the processes that have been considered, three stand out as giving very stringent bounds on flavor-changing neutral currents. Those three are $\mu \to e\gamma$, K-$\bar{K}$ mixing, and B-$\bar{B}$ mixing. The bounds are given in Eqs. (15), (25), and (28). As discussed earlier, the bound on $\mu \to e\gamma$ arises from a diagram in which a τ is on an internal line, and is thus sensitive to the (more reliable) couplings which mix the third generation, and it is also at the edge of the most interesting region of parameter space. From the tables, one can see immediately that these three bounds eliminate the possibility of seeing many other processes. For example, the bound in Eq. (28) is much, much more stringent than that from $B \to e\tau$ or $B \to \pi e\tau$; the bound in Eq. (25) is much more stringent than that from $K \to e\mu$ or $K \to \pi e\mu$; and the bound in Eq. (15) is much more stringent than that from $B \to \mu\tau$, $B_s \to e\tau$, $B \to \pi\mu\tau$, or $B \to K e\tau$.

The bounds from these three processes are so strong, in fact, that one can use perturbation theory to derive many additional constraints. In a grand-unified theory, the validity of perturbation theory forces all of the η_{ij} to be small at all scales between the electroweak and unification scales. This gives an upper bound on the η_{ij} at the electroweak scale. A similar calculation for the top-quark Yukawa coupling gives an upper bound on the top-quark mass of 230 GeV, i.e., a bound on the coupling of 1.3. The same bound will apply here, and thus we have $h_{ij} \lesssim 1.3$, corresponding to $\eta_{ij} \lesssim 45$. Combining this with Eqs. (25) and (28), and noting that we are interested in cases in which the exchanged scalar is heavier than its current limit of 40 GeV, we find

$$\begin{aligned} \eta_{ij}^2\eta_{e\tau}^2 &< 0.36\left[\frac{m_P}{m_W}\right]^4 , \\ \eta_{ij}^2\eta_{e\tau}^2 &< 4.4\left[\frac{m_S}{m_W}\right]^4 , \\ \eta_{ij}^2\eta_{e\mu}^2 &< 2.4\times 10^{-3}\left[\frac{m_P}{m_W}\right]^4 , \\ \eta_{ij}^2\eta_{e\mu}^2 &< 2.9\times 10^{-2}\left[\frac{m_S}{m_W}\right]^4 . \end{aligned} \tag{29}$$

This bound must hold for *any i and j*, and is easily seen to be a more stringent bound than many of the processes in Tables I, II, and III.

Let us now examine the various processes more explicitly to determine which offer the best possibilities in the future (as well as whether τ or B decays are more likely to be productive). We first consider the case of scalar exchange.

Consider the various three-body τ decays. It is easy to see that the bounds on the six τ decays in Table I are much weaker than other processes. In the order given in Table I, the processes which give better bounds are (i) $B \to \pi ee$, (ii) $B \to K\mu\mu$, (iii) Eq. (29), (iv) $B \to K\mu e$, (v) Eq. (29) and $B \to Kee$, and (vi) $B \to \pi\mu\mu$ and $B \to K\mu e$. Now consider the two radiative τ decays. The bound from $\tau \to e\gamma$ is weaker than that from Eq. (29), and the bound from $\tau \to \mu\gamma$ is weaker than that from $B \to K\tau\tau$. In all of these cases, the bound from τ decays is so much weaker that even a slight improvement in the bound will not help. We conclude that there is no useful information which can be obtained from τ decays in these models in which a scalar mediates flavor-changing neutral currents.

We have already noted that the most useful experiment in improving these bounds (or finding an effect) is $\mu \to e\gamma$. Which of the B decays is most likely to be productive? The decays which stand out here are $B \to Kee$, $B \to K\mu\mu$, $B \to K\mu\tau$, and $B \to K\tau\tau$. Using our "preferred" range of couplings, one can easily see that one needs to reach branching ratios of 3×10^{-10}, 3×10^{-9}, 3×10^{-8}, and 3×10^{-7}, respectively. In the case of $B \to K\mu\mu$ and $B \to K\tau\tau$, these branching ratios are below (barely below for the latter) the standard-model (one-loop) branching ratios. Keep in mind, however, that our "preferred range" is just a rough estimate, and the couplings could easily be somewhat higher (recall that a factor of 10 increase in a coupling corresponds to 10^4 in the rate). The process $B \to K\mu\tau$, however, vanishes in the standard model, and thus may offer the best (and least ambiguous) hope. Measuring its branching ratio to a level of a few times 10^{-8} obviously is difficult, although at a B factory, it may not be impossible.

Next, we consider the case of pseudoscalar exchange. The bound from the decay $\tau \to e^-e^-e^+$, $\tau \to \mu^-\mu^-\mu^+$, ($\tau \to e^-e^-\mu^+$, $\tau \to \mu^-\mu^-e^+$, $\tau \to e^-\mu^-\mu^+$) is much weaker than that from the decay $B \to ee$, $B_s \to \mu\mu$ [all the others are weaker than the bound from Eq. (29)]. The bound from $\tau \to e^-\mu^-e^+$ is still better than other bounds, however, if one can measure $B_s \to ee$ to have branching ratios less than 5%, then this process will set a better bound. It is hard to imagine that such a large branching ratio would have gone undetected (there would be many dramatic four electron events at UA1), and it is quite likely that this bound will be determined in the very near future. What about radiative decays? Again, the bound from $B \to \tau\tau$ is much better than that from $\tau \to e\gamma$. Similarly, the bound from $B_s \to \tau\tau$ is more stringent than that for $\tau \to \mu\gamma$. We thus conclude that improvement in rare τ decays will not be useful in setting bounds, even in the case of pseudoscalar exchange.

Finally, which of these B decays will be most productive? The decays which stand out are those of the B_s meson into $\tau\tau$, $\mu\tau$, $\mu\mu$, and μe. The branching ratios needed to reach the preferred range of parameter space are 7×10^{-7}, 8×10^{-8}, 6×10^{-9}, and 3×10^{-11}, respectively. Here the rate for $B_s \to \tau\tau$ is well below the standard-model prediction ($\sim10^{-6}$), and $B_s \to \mu\mu$ is slightly below the standard-model prediction. Again, our preferred range is just an estimate, and the couplings could be somewhat larger. The most intriguing decay is $B_s \to \mu\tau$, which only depends on the single $\eta_{\mu\tau}$ coupling. Measuring the branching ratio to get into the preferred range seems difficult, although the fact that it is a two-body decays with charged leptons may make it detectable at a B factory.

VI. CONCLUSIONS

The simplest extension of the standard model has an extra scalar field. This model will automatically have tree-level flavor-changing neutral currents, unless they are suppressed by some additional symmetry. It is often believed that the presence of tree-level flavor-changing neutral currents in this model is fatal, since it requires the exchanged scalar to be extremely heavy. This belief, however, is based on the assumption that the flavor-changing coupling is quite large. It has been pointed out that using a more natural value for the coupling (the geometric mean of the Yukawa couplings of the two fields) leads to much smaller bounds, closer to the range of several hundred GeV. Even this bound, however, is very sensitive to the precise value of the coupling. Given the uncertainty in assumptions involving the first generation Yukawa couplings (the couplings are five to six orders of magnitude smaller than gauge couplings, they are subject to uncalculable Planck mass corrections, etc.), even this bound of several hundred GeV certainly should not be considered particularly reliable.

With this in mind, we have calculated the bounds on the couplings of an additional scalar or pseudoscalar for processes involving the third-generation fields, which should be considerably more reliable. Since the masses of the scalar and the pseudoscalar are likely to be quite different, we have considered the bounds on each separately. The most stringent bound in the quark sector comes from B-$\bar{B}$ mixing; using our "most natural" value of the couplings, one gets a bound of about a hundred GeV on the exchanged scalar mass. In the lepton sector, the strongest bound comes from $\mu \to e\gamma$, in which a τ is on an internal line. This process is sensitive to mixing between the first and third generations as well as between the second and third generations (and is not as sensitive to mixing between the first and second generations, which is expected to be small). Using our most "natural" value, we get a bound of about 50 GeV on the exchanged scalar mass. Unlike the case of B-$\bar{B}$ mixing, however, this process does not exist in the standard model, and thus the bound will be improved considerably as the experimental bound is lowered. We thus feel that $\mu \to e\gamma$ is the best place to look for mixing involving the third generation.

In most grand unified theories, the τ and b are in the same representation, and thus we expect flavor-changing couplings in the quark sector to be related to those in the lepton sector. We have then asked the question: which processes, τ or B decays, give the strongest bounds? The answer, from Tables I–IV, is clear: B decays. We find no case in which τ decays give better bounds, nor in which they are likely to in the near future. The most promising B decays are $B \to K\mu\tau$ and $B_s \to \mu\tau$. In general, the interesting decays are those with τ's in the final state. A search for $B \to \mu e X$ would have relatively little background and could be quite productive; a search for exclusive processes with a final state τ, while more difficult, could also be quite useful.

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APPENDIX

In the text, we assume that the H field and the ϕ field do not mix. Here, we discuss the effects of our bounds if this assumption is relaxed. Our results for the case of pseudoscalar exchange will be completely unchanged by any mixing. The reason is that the basis has been chosen so that H gets a vacuum expectation value, and ϕ does not. In this basis, the imaginary part of H is the Goldstone boson which gives mass to the Z, and it does not mix with the imaginary part of the ϕ field. All results we have given then still hold, since the Z couplings are flavor diagonal.

The scalars will mix, in general. If the mass eigenstates are H_1 and H_2, then the couplings to H_1 are given by

$$(g_{y_d}\bar{d}d + g_{y_s}\bar{s}s + g_{y_b}\bar{b}b)H_1\cos\theta + (h_{ij}^{\text{quark}}\bar{d}_i d_j)H_1\sin\theta$$

and the couplings to H_2 are the same with the obvious replacement of $\cos\theta \to -\sin\theta$ and $\sin\theta \to \cos\theta$. Suppose we have a process in which both interactions are flavor changing (such as $B \to K\mu\tau$). Then the bound on m_S will change to

$$\min\left[\frac{m_{H_1}}{\sin\theta}, \frac{m_{H_2}}{\cos\theta}\right]^4 .$$

If one of the couplings is flavor diagonal, then the change is a bit more complicated, but straightforwardly calculated. Note that if the mixing is small, this gives the same result as before. Since all mixing angles known in the standard model have $\cos\theta > 0.85$, we do not expect mixing to give a significant effect, but one should certainly be aware of the possibility.

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[7] Note that we have expressed our results in terms of $\eta_{ij}^{\text{lepton}}$ instead of η_{ij}^{quark} since the "most natural value" for this will not be based on quark masses. The well-known problem of the SU(5) prediction $m_d/m_s = m_e/m_\mu$, which is presumably solved by the fact that nonrenormalizable Planck scale interactions (see Ref. [4]) give an uncertainty of ~ 10 MeV in the masses, makes this value much more reliable for second- and third-generation fields than for first-generation fields, although even for these fields it should not be more than an order of magnitude (in the square of the coupling) off.

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[9] Even the original assumption of Shankar gave $\eta_{ij} = 1$, which will not be probed by τ decays. Note, however, that the difference between our assumption regarding the most natural value of the η_{ij} and his does make a major difference in interpreting the $\mu \to 3e$ bound. His assumption that the couplings (even for the first and second generations) are all equal to the τ Yukawa coupling gives a lower bound on m_S of about 1 TeV. Our assumption, discussed in Sec. II, that $\eta_{ij} = \sqrt{m_i m_j}/m_\tau$, gives a lower bound on m_S of only about 1 GeV, i.e., no significant bound at all.

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[15] Of course, one could also evaluate the matrix element in terms of the B decay constant. Since the method of the preceding subsection gives the matrix element almost trivially (no integrals need to be done), we prefer that method. Comparing the two methods, one should be able to calculate the B decay constant. This has been done in Ref. [12], with the result that $f_B = 130$ MeV.

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