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Penguin trapping with isospin analysis and CP asymmetries in B decays

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Isospin relations are used to eliminate hadronic uncertainties in various CP asymmetries in B^0 decays. In addition to the simple triangle relations for the $\pi\pi$ mode, we study quadrilateral relations for $K\pi$ and pentagon relations for $\rho\pi$. A combined angular and isospin analysis is required for $\rho\rho$. These methods are useful also for three-body decays such as $K\pi\pi$. The magnitude of the penguin amplitude can be extracted in various modes. The theoretical principles behind this analysis can be experimentally tested through sum rules for decay rates prior to the measurement of CP asymmetries.

INTRODUCTION

CP asymmetries in B^0 decays into a final CP eigenstate are free of hadronic uncertainties if amplitudes which depend on only a single Cabibbo-Kobayashi-Maskawa (CKM) phase dominate the decay process. A clean measurement of the three angles of the unitarity triangle (see Fig. 1) is thus made possible [1]. Within the standard model, most processes get contributions from both tree-level and penguin amplitudes [2]. In $b \to c\bar{c}s$ processes (e.g., $B \to \psi K_S$) both amplitudes carry the same CKM phase; extracting $\sin 2\beta$ from this asymmetry is free of hadronic uncertainties. In $b \to u\bar{u}d$ processes (e.g., $B \to \pi\pi$) the two amplitudes carry different CKM phases. It is expected that the contribution from the penguin amplitude is small (a few percent), but it could be larger than the naive expectation if the matrix element for the penguin operator is enhanced; extracting $\sin 2\alpha$ from this asymmetry may suffer from hadronic uncertainties if this is indeed the case. For $b \to u\bar{u}s$ processes (e.g., $B \to K\pi$) the situation is even worse: not only do the tree and penguin amplitudes carry different CKM phases, but also they are expected to be comparable in magnitude (the tree process is strongly CKM suppressed). It has often been stated that clean information on CKM parameters cannot be extracted from this asymmetry.

Gronau and London [3] have shown how to separate the CKM phase of the tree-level $B \to \pi\pi$ process from any penguin contamination. This is done by means of isospin analysis of various (charged and neutral) B decays into $\pi\pi$. This will allow a determination of the phase α completely free of hadronic uncertainties, independent of the size of the penguin amplitude [4]. Nir and Quinn [5] have later shown that a more complicated isospin analysis could be used to separate the tree contribution to the asymmetry in modes such as $B \to K\pi$, thus allowing a clean measurement of α from these modes as well.

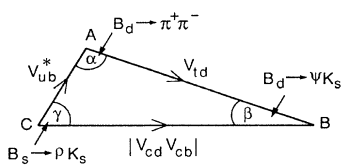

FIG. 1. The unitarity triangle. A relevant B^0 decay mode is indicated for the angle involved in the corresponding CP asymmetry.

In this work we attempt to complete the discussion for all relevant types of decay modes. For completeness, we briefly review previous results, namely, the isospin analysis of the $\pi\pi$ and $K\pi$ modes, and combine them with our own analysis of additional modes. We explicitly discuss $\rho\rho$, $\rho\pi$, and $K\pi\pi$. These analyses include various new ingredients: combinations of angular and isospin analyses; pentagon relations among decay amplitudes; and the study of three-body decay modes. In addition we show how to measure the magnitude of the penguin contribution (for example, in the $\pi\pi$ mode) and we suggest ways of testing the validity of our approach through sum rules which will be subject to experimental check long before CP asymmetries are actually measured.

$B \to \pi\pi$

We begin by reviewing the analysis of Gronau and London [3] for $\pi\pi$. We use a notation that is convenient for the general case.

The B^+ and B^0 decays into final $\pi\pi$ states proceed through the quark subprocess

$$\bar{b} \to \bar{u}u\bar{d} \ . \tag{1}$$

The Hamiltonian is of the form

$$\mathcal{H} = A_{3/2}|\tfrac{3}{2}, +\tfrac{1}{2}\rangle + A_{1/2}|\tfrac{1}{2}, +\tfrac{1}{2}\rangle \ , \tag{2}$$

giving

$$\mathcal{H}|B^+\rangle = \mathcal{H}|\tfrac{1}{2}, +\tfrac{1}{2}\rangle = \sqrt{\tfrac{3}{4}}A_{3/2}|2,1\rangle + (A_{1/2} - \tfrac{1}{2}A_{3/2})|1,1\rangle \ , \tag{3}$$

$$\mathcal{H}|B^0\rangle=\mathcal{H}|\tfrac{1}{2},-\tfrac{1}{2}\rangle$$
$$=\sqrt{\tfrac{1}{2}}A_{3/2}|2,0\rangle+\sqrt{\tfrac{1}{2}}(A_{1/2}+A_{3/2})|1,0\rangle$$
$$+\sqrt{\tfrac{1}{2}}A_{1/2}|0,0\rangle \ .$$

There are three relevant final $\pi\pi$ states:

$$|\pi^+\pi^0\rangle=|2,1\rangle \ ,$$
$$|\pi^+\pi^-\rangle=\sqrt{\tfrac{1}{3}}|2,0\rangle+\sqrt{\tfrac{2}{3}}|0,0\rangle \ , \qquad (4)$$
$$|\pi^0\pi^0\rangle=\sqrt{\tfrac{2}{3}}|2,0\rangle-\sqrt{\tfrac{1}{3}}|0,0\rangle \ .$$

We are interested in calculating the amplitudes A^{+0}, A^{+-}, and A^{00} where

$$A^{ij}\equiv\langle\pi^i\pi^j|\mathcal{H}|B^{i+j}\rangle \ . \qquad (5)$$

While the isospin states in Eq. (3) are four-quark states, the states in Eq. (4) are two-meson states. The transition between the two necessarily involves hadronization and other rescattering effects, which introduce both a phase shift and a form factor. In the general case, there is one independent amplitude A_{I_t,I_f} for each possible combination of $\{I_t,I_f\}$, transition isospin, and final-state isospin (including the spectator quark), respectively. The amplitudes A_{I_t,I_f} differ from the transition amplitudes A_{I_t} in that they include the effects of rescattering and hadronization processes.

Note, however, that in the $\pi\pi$ case there is no $I_f=1$ state because it is forbidden by Bose symmetry for an angular momentum zero system of two pions. This fact simplifies the discussion considerably, because $I_t=\frac{3}{2}$ transitions lead to $I_f=2$ states only, while $I_t=\frac{1}{2}$ transitions lead to $I_f=0$ states only. Therefore, we have two independent amplitudes only, which we define as

$$A_2\equiv\tfrac{1}{2}\sqrt{\tfrac{1}{3}}A_{3/2,2}, \quad A_0\equiv\sqrt{\tfrac{1}{6}}A_{1/2,0} \ . \qquad (6)$$

The various decay amplitudes are thus given by

$$A^{+0}=3A_2 \ ,$$
$$\sqrt{\tfrac{1}{2}}A^{+-}=A_2-A_0 \ , \qquad (7)$$
$$A^{00}=2A_2+A_0 \ .$$

Similarly, $\bar{B}^0$ and B^- decay to final $\pi\pi$ states through the quark subprocess

$$b\to u\bar{u}d \ . \qquad (8)$$

The $\bar{B}$ and B^- decay amplitudes are given by

$$\bar{A}^{+0}=3\bar{A}_2 \ ,$$
$$\sqrt{\tfrac{1}{2}}\bar{A}^{+-}=\bar{A}_2-\bar{A}_0 \ , \qquad (9)$$
$$\bar{A}^{00}=2\bar{A}_2+\bar{A}_0 \ ,$$

where $\bar{A}^{ij}$ is the amplitude for the CP-conjugated process of A^{ij}, e.g., $\bar{A}^{+0}$ corresponds to $B^-\to\pi^-\pi^0$. The $\bar{A}_i$ amplitudes carry weak phases opposite to those of A_i, but unchanged strong phases. Notice that for each case, the set of amplitudes forms a triangle in the complex plane, as seen from the relationships:

$$\sqrt{\tfrac{1}{2}}A^{+-}=A^{+0}-A^{00} \ ,$$
$$\sqrt{\tfrac{1}{2}}\bar{A}^{+-}=\bar{A}^{+0}-\bar{A}^{00} \ . \qquad (10)$$

Measuring the total rates for $B^\pm\to\pi^\pm\pi^0$ gives $|A^{+0}|$ and $|\bar{A}^{+0}|$. As for the neutral modes, the decay rate into final $\pi^+\pi^-$ is [6]

$$\Gamma(B^0_{\rm phys}(t)\to\pi^+\pi^-)=e^{-\Gamma|t|}[(|A^{+-}|^2+|\bar{A}^{+-}|^2)-(|A^{+-}|^2-|\bar{A}^{+-}|^2)\cos(\Delta mt)+2|A^{+-}|^2a^{+-}\sin(\Delta mt)] \ ,$$
$$\Gamma(\bar{B}^0_{\rm phys}(t)\to\pi^+\pi^-)=e^{-\Gamma|t|}[(|A^{+-}|^2+|\bar{A}^{+-}|^2)+(|A^{+-}|^2-|\bar{A}^{+-}|^2)\cos(\Delta mt)-2|A^{+-}|^2a^{+-}\sin(\Delta mt)] \ , \qquad (11)$$

and similarly for $\pi^0\pi^0$. Here $B^0_{\rm phys}(\bar{B}^0_{\rm phys})$ is a time-evolved state such that $B^0_{\rm phys}(t=0)$ and $\bar{B}^0_{\rm phys}(t=0)$ are the interaction eigenstates B^0 and $\bar{B}^0$, respectively. We are interested in determining $|A^{+-}|$, $|\bar{A}^{+-}|$, and a^{+-}. In any given experiment, the *total* rates give two quantities, but additional experimental information, e.g., *time-dependent* measurements, is needed to fix all three. For example, in experiments conducted at the $\Upsilon(4S)$, t is the time difference between the tagging decay of one neutral B and the decay into the CP eigenstate of the other, and it runs in the range $-\infty\le t\le\infty$. The contribution of the $\sin(\Delta m\,t)$ term vanishes in the total rate, but not that of the $\cos(\Delta m\,t)$ term. Thus, measuring the total rates for the charged and neutral B decays gives all six magnitudes, $|A^{ij}|$ and $|\bar{A}^{ij}|$, and consequently the shapes of the two triangles can be determined. In addition, the time-dependent decay rates into $\pi^+\pi^-$ give the CP asymmetry

$$a^{+-}={\rm Im}\left[e^{-2i\phi_M}\frac{\bar{A}^{+-}}{A^{+-}}\right] \ . \qquad (12)$$

The phase ϕ_M is the CKM phase in the $B-\bar{B}$ mixing amplitude. In principle, one could also measure a time-dependent rate and extract an asymmetry for the $\pi^0\pi^0$ channel. However, to determine the CKM phase we need only the asymmetry for the charged-pion channel. This is fortunate, since in the $\pi^0\pi^0$ channel time-dependent measurements will be very difficult.

Let us replace the barred amplitudes by rotated amplitudes:

$$\tilde{A}_i=e^{2i\phi_T}\bar{A}_i \ . \qquad (13)$$

The phase ϕ_T is the CKM phase in the tree diagram. The crucial point to notice next is that the penguin diagram is purely $I=\frac{1}{2}$, and consequently only tree diagrams contribute to A_2. Hence $\tilde{A}_2=A_2$ and the triangle formed by the $\tilde{A}$'s shares a common side with that formed by the A's (see Fig. 2):

$$A^{+0}=\tilde{A}^{+0} . \tag{14}$$

The figure thus formed allows us to measure the angle between A^{+-} and $\tilde{A}^{+-}$, up to an overall ambiguity which arises from the four possible orientations of the two triangles relative to their common side. (Figure 2 shows one possible orientation.)

We can rewrite Eq. (12) for the CP asymmetry as

$$a^{+-}=\text{Im}\left[e^{-2i(\phi_M+\phi_T)}\frac{\tilde{A}^{+-}}{A^{+-}}\right] . \tag{15}$$

Were A_0 dominated by the tree-level diagram, we would have $(\tilde{A}^{+-}/A^{+-})=1$, and Eq. (15) would reduce to the usual $\sin2(\phi_M+\phi_T)$ expression. However, if we can construct the triangles we know both the magnitude and the phase of $(\tilde{A}^{+-}/A^{+-})$; we need not make the assumption of a small penguin amplitude anymore. We are able to disentangle the value of $(\phi_M+\phi_T)$ without any uncertainty from the unknown penguin contribution to A_0. The fourfold ambiguity from the different possible orientations of the triangles could, in general, be resolved if the asymmetry in the $\pi^0\pi^0$ mode were also measured, but this is unlikely.

In addition, if we assume the standard model, we can actually extract a measure of the penguin contribution to A_0. Let us distinguish the tree and penguin contribution explicitly. We use three generation CKM unitarity and the approximation that the penguin contributions with u and c quarks in the loop are equal in magnitude to $\mathcal{O}(m_c2/m_b2)$, apart from their CKM factors. Rescattering effects that change quark flavor are included in the penguin contributions. Terms proportional to $e^{i\phi_P}$ thus correspond to pure penguin contributions, while those proportional to $e^{i\phi_T}$ correspond, to a good approximation, to tree contributions. We define

$$P_0=(A_0)_P , \tag{16}$$

where the subscript P means contributions from penguin processes only. The A^{00} and $\tilde{A}^{00}$ amplitudes can be rewritten as

$$A^{00}=T^{00}+P_0 ,$$
$$\tilde{A}^{00}=T^{00}+\tilde{P}_0 , \tag{17}$$

where T^{00} contains no penguin contribution (and therefore $T^{00}=\tilde{T}^{00}$). Note that $|P_0|=|\tilde{P}_0|$. We obtain the relation

$$|P_0|=\frac{|A^{00}-\tilde{A}^{00}|}{\sqrt{2[1-\cos2(\phi_T-\phi_P)]}} . \tag{18}$$

The phase ϕ_P is the CKM phase in the penguin diagram. This result is readily seen from Fig. 2: The distance between the vertices opposite the common side of the two triangles is $|A^{00}-\tilde{A}^{00}|$. It is the basis of an equilateral triangle (dotted in Fig. 2) with the angle opposite to this basis $=2(\phi_T-\phi_P)$. The length of its other two sides is $|P_0|$. The quantities on the right-hand side of Eq. (18) can be determined from the figure, except for $(\phi_T-\phi_P)$. However, within the standard model, both the penguin amplitude and the mixing amplitude depend on the same CKM combination, $V_{tb}^*V_{td}$. Consequently, $\phi_T-\phi_P=\phi_T+\phi_M=\alpha$ determined from the CP asymmetry. We conclude that the full isospin analysis allows a determination of $|P_0|$ and is, therefore, useful not only for our understanding of CP violation but also to test estimates of hadronic physics.

There are two kinds of discrete ambiguities in the determination of $|P_0|$ from Eq. (18).

(i) The fourfold ambiguity mentioned above reduces to a twofold ambiguity in the magnitude of P_0. The two solutions will probably differ significantly. Since theoretical estimates indicate that $|P_0|$ should be small, this will suggest which solution is preferred.

(ii) There is an additional discrete ambiguity in the determination of $\phi_M+\phi_T$ even in the case of $\tilde{A}^{+-}=A^{+-}$. However, this ambiguity can, in principle, be resolved using additional data [7].

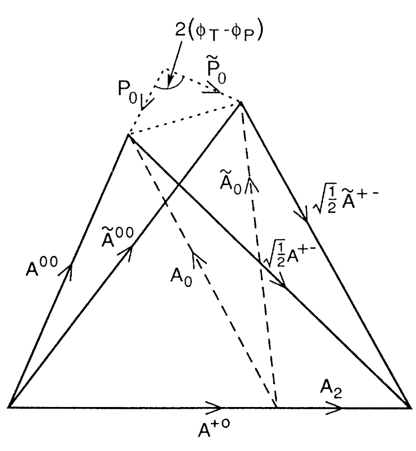

FIG. 2. The two triangles of $B\to\pi\pi$. Note that A^{+0} is a common basis. The dotted triangle serves to find the magnitude of the penguin contribution.

$B\to\rho\rho$

B decays into final $\rho\rho$ states proceed via the same quark subprocess, $\bar{b}\to\bar{u}u\bar{d}$ or $b\to u\bar{u}d$, as into final $\pi\pi$ states. Thus, the Hamiltonian is the same as in Eq. (2). With an appropriate angular analysis [8], one can separate the CP-even final states from the CP-odd ones. By Bose symmetry, the $J=0$ $\rho\rho$ states are isospin-even since $L=S$ and $(-1)^{(L+S+I)}$ is the symmetry of the system under particle interchange. Thus there is no $I=1$ final state for $B\to\rho\rho$, and an isospin analysis of the $\rho\rho$ system can be done in exactly the same way as for $\pi\pi$. There are four CP asymmetries to be measured: $\rho^0\rho^0(CP=+)$, $\rho^0\rho^0(CP=-)$, $\rho^+\rho^-(CP=+)$, and $\rho^+\rho^-(CP=-)$. In each of these channels, time-integrated B^0_{phys} and $\bar{B}^0_{\text{phys}}$ decay rates are needed to construct the triangles, but a time-dependent measurement is needed for the CP asymmetry. We would then obtain four measurements of the CKM phase α that are completely free of hadronic uncertainties.

In the $\rho\rho$ case there is not as much difficulty in measuring the time-dependent rate for any channel as there is in the $\pi^0\pi^0$ case. Possibly, all four measurements can be made, thus resolving the discrete ambiguities. However, angular analysis is needed for the neutral channels, which will require somewhat higher statistics than for the $\pi\pi$ case. To decide which channel is more useful, we need to know the branching ratios.

$B \to \pi K$

The underlying quark subprocess for this channel is $\bar{b} \to \bar{u}u\bar{s}$. Here the tree diagrams are strongly CKM suppressed, so contributions from tree and penguin processes are believed to be comparable, or possibly the penguins may even dominate. A similar analysis to that applied above for $\pi\pi$ can be made. The transition isospin can be either $I_t=0$ or 1. The final four quarks can have isospin $I_f=\frac{1}{2}$ or $\frac{3}{2}$. Thus there are three independent amplitudes: $A_{0,1/2}$, $A_{1,1/2}$, and $A_{1,3/2}$. These A_{I_t,I_f} amplitudes again incorporate the change in magnitude as well as the strong phase shift corrections to A_{I_t} due to hadronization and rescattering effects. We find it convenient to define the following amplitudes which absorb Clebsch-Gordan coefficients:

$$\begin{aligned} W &\equiv \sqrt{\tfrac{1}{3}}A_{0,1/2} \,, \\ U &\equiv \tfrac{1}{3}(2A_{1,3/2}+A_{1,1/2}) \,, \\ V &\equiv \tfrac{1}{3}(A_{1,3/2}-A_{1,1/2}) \,. \end{aligned} \tag{19}$$

There are two possible charge assignments for the π and the K for each decay, so there are four amplitudes for B^0 and B^+ decays:

$$A^{ij} \equiv \langle \pi^i K^j | \mathcal{H} | B^{i+j} \rangle \,. \tag{20}$$

They can be written as

$$\begin{aligned} A^{0+} &= U - W, \quad \sqrt{\tfrac{1}{2}}A^{+0} = V + W \,, \\ A^{00} &= U + W, \quad \sqrt{\tfrac{1}{2}}A^{-+} = V - W \,. \end{aligned} \tag{21}$$

Since these four amplitudes are given in terms of only three isospin amplitudes there is once again a single relationship between them, and a similar relation holds for the $\tilde{A}$ amplitudes defined from the barred amplitudes by Eq. (13):

$$\begin{aligned} A^{00}+\sqrt{\tfrac{1}{2}}A^{-+} &= A^{0+}+\sqrt{\tfrac{1}{2}}A^{+0} \,, \\ \tilde{A}^{00}+\sqrt{\tfrac{1}{2}}\tilde{A}^{-+} &= \tilde{A}^{0+}+\sqrt{\tfrac{1}{2}}\tilde{A}^{+0} \,. \end{aligned} \tag{22}$$

Thus, each set of four amplitudes forms a quadrilateral in the complex plane.

Measuring the various decay rates gives all eight magnitudes, $|A^{ij}|$ and $|\tilde{A}^{ij}|$. In addition, the time-dependent decay rates into $\pi^0 K_S$ give the CP asymmetry

$$a^{00} = \mathrm{Im}\left[e^{-2i(\phi_M+\phi_K+\phi_T)} \frac{\tilde{A}^{00}}{A^{00}} \right] \,. \tag{23}$$

The phase ϕ_K is the CKM phase in the K-$\bar{K}$ mixing amplitude [9]. To extract $\alpha=\phi_M+\phi_T+\phi_K$, one needs to determine $\tilde{A}^{00}/A^{00}$. While $|\tilde{A}^{00}/A^{00}|$ is known from the decay rates, $\arg(\tilde{A}^{00}/A^{00})$ can be determined only with further isospin analysis, as explained below.

The crucial point to notice now is that the penguin amplitude contributes only to $I_t=0$ transitions. Thus, only W has any penguin contribution while U and V have contributions from tree diagrams only. This gives two further relationships between the two quadrilaterals:

$$\begin{aligned} A^{00}+A^{0+} &= \tilde{A}^{00}+\tilde{A}^{0+} \,, \\ A^{00}+\sqrt{\tfrac{1}{2}}A^{-+} &= \tilde{A}^{00}+\sqrt{\tfrac{1}{2}}\tilde{A}^{-+} \,. \end{aligned} \tag{24}$$

As can be seen from Fig. 3, the relationships (22) and (24) are sufficient to determine $\tilde{A}^{00}/A^{00}$ and hence to extract the CKM phase of the tree diagram from the measured CP asymmetry (23). A more detailed analysis of the πK mode is given in Ref. [5]. It also explains how to extract the magnitudes of penguin and tree amplitudes for this mode.

We note that the eight decay rates are functions of four independent complex amplitudes, U, V, W, and $\tilde{W}$. Since one overall phase is irrelevant, there are seven parameters determining eight decay rates. Therefore, this model predicts a relation between these eight decay rates. To derive this sum rule, we first write the expressions for the differences between CP-conjugate decay rates:

$$\begin{aligned} |A^{0+}|^2-|\tilde{A}^{0+}|^2 &= |W|^2-|\tilde{W}|^2+2U\cdot(\tilde{W}-W) \,, \\ \tfrac{1}{2}(|A^{+0}|^2-|\tilde{A}^{+0}|^2) &= |W|^2-|\tilde{W}|^2-2V\cdot(\tilde{W}-W) \,, \\ |A^{00}|^2-|\tilde{A}^{00}|^2 &= |W|^2-|\tilde{W}|^2-2U\cdot(\tilde{W}-W) \,, \\ \tfrac{1}{2}(|A^{-+}|^2|-|\tilde{A}^{-+}|^2) &= |W|^2-|\tilde{W}|^2+2V\cdot(\tilde{W}-W) \,, \end{aligned} \tag{25}$$

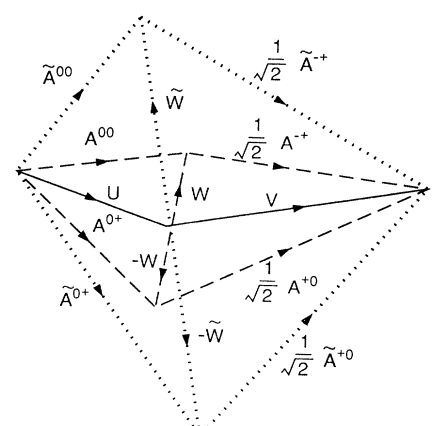

FIG. 3. The two quadrilaterals of $B \to K\pi$. Note that $U+V$ is a common diagonal, while the noncommon diagonals bisect each other.

where the dot product denotes the symmetrized scalar product of two complex vectors:

$$2A\cdot B\equiv A^*B+B^*A \ . \tag{26}$$

The four *CP*-conjugate differences vanish if *CP* is conserved in the decays, but they are different from zero for the general case of *CP* violation. Note that the form of Eq. (25) depends only on the following very general assumptions: (1) isospin invariance for all strong final-state interactions; (2) the weak Hamiltonian can produce only $I_t=0$ and $I_t=1$ transitions; (3) the weak phase is the same for all $I_t=1$ transitions.

For the standard model, where there are contributions from penguin and tree diagrams, the above assumptions hold since penguin diagrams contribute only to $I_t=0$ transitions and all tree contributions have the same weak phase. Combining these equations gives the sum rule

$$|A^{0+}|^2-|\tilde{A}^{0+}|^2+|A^{00}|^2-|\tilde{A}^{00}|^2 = \tfrac{1}{2}(|A^{+0}|^2-|\tilde{A}^{+0}|^2+|A^{-+}|^2-|\tilde{A}^{-+}|^2) \ . \tag{27}$$

All the *CP*-conjugate differences vanish if there is no penguin contribution, since then all the amplitudes have the same weak phase. In that case the sum rule is trivially satisfied. If there are appreciable penguin contributions, the sum rule becomes nontrivial. Since branching ratios will be measured before time-dependent asymmetries, these data will show immediately whether there is an appreciable penguin contribution. If indeed there are appreciable penguins, the sum rule will test whether the very general assumptions above are valid [10].

All the above discussion can obviously be applied equally well to the channels ρK and πK^*. For the channel ρK^*, one needs to use angular analysis in the neutral channels (to separate a definite *CP* contribution) in combination with this isospin analysis. Note that the K^{*0} is observed both as $K_S\pi^0$ and $K^+\pi^-$ and only $K_S\pi^0$ is a *CP* eigenstate which exhibits *CP* asymmetry. The $K^\pm\pi^\mp$ decay modes provide another determination of $|A^{00}|$ and $|\tilde{A}^{00}|$ and a check on systematics in time-dependent measurements. Unfortunately, the branching ratios for any of these channels (including πK) are likely to be very small: a combined angular and isospin analysis will be so dominated by the errors in the several measurements that it would probably be rendered useless. Additional channels that can be similarly analyzed are those with an additional isosinglet meson in the final state. Such a particle does not affect the isospin structure of the amplitudes. As long as the *CP* of the neutral system can be fixed, using angular analysis where necessary, such channels provide further possible measurements of the CKM parameter α. Unfortunately, none of these channels is expected to have a large branching ratio, so it is unlikely that sufficient data will be available to accurately construct the quadrilaterals and extract α in this way.

The isospin structure of the channels πD, ρD, and πD^* is exactly the same as in the πK case. (*CP* asymmetries will be measured with *CP* eigenmodes of D^0 and $\bar{D}^0$). However, here no penguin contributions are expected. Differences between A^{ij} and $\tilde{A}^{ij}$ processes cannot be explained within the standard model. The $|A^{ij}|=|\tilde{A}^{ij}|$ relation can be checked channel by channel and does not require any isospin analysis [11].

$B\to\rho\pi$

We can continue to play yet more arcane versions of this game. For $B\to\rho\pi$ we have again $I_t=\frac{1}{2}$ or $\frac{3}{2}$ and Eqs. (1)–(3) hold. However, here $I_f=0,1,2$ are all allowed. Thus we have four independent isospin amplitudes ($A_{1/2,0}$, $A_{1/2,1}$, $A_{3/2,1}$, and $A_{3/2,2}$) for B^0 and B^+ decays, and a corresponding set for $\bar{B}^0$ and B^- decays. There are five different possible charge assignments for the $\rho\pi$ system, so once again there is a single relation between the A^{ij} amplitudes and a corresponding one between the $\tilde{A}^{ij}$ amplitudes. Consequently, each set forms a pentagon in the complex plane. Here the penguins have $I_t=\frac{1}{2}$ only and thus do not contribute to the two $A_{3/2,I_f}$ amplitudes. Following the steps of previous sections we now obtain

$$\begin{aligned}
|\rho^+\pi^0\rangle &= \sqrt{\tfrac{1}{2}}|2,1\rangle+\sqrt{\tfrac{1}{2}}|1,1\rangle \ , \\
|\rho^0\pi^+\rangle &= \sqrt{\tfrac{1}{2}}|2,1\rangle-\sqrt{\tfrac{1}{2}}|1,1\rangle \ , \\
|\rho^+\pi^-\rangle &= \sqrt{\tfrac{1}{6}}|2,0\rangle+\sqrt{\tfrac{1}{2}}|1,0\rangle+\sqrt{\tfrac{1}{3}}|0,0\rangle \ , \\
|\rho^-\pi^+\rangle &= \sqrt{\tfrac{1}{6}}|2,0\rangle-\sqrt{\tfrac{1}{2}}|1,0\rangle+\sqrt{\tfrac{1}{3}}|0,0\rangle \ , \\
|\rho^0\pi^0\rangle &= \sqrt{\tfrac{2}{3}}|2,0\rangle-\sqrt{\tfrac{1}{3}}|0,0\rangle \ .
\end{aligned} \tag{28}$$

Now let

$$A^{ij}\equiv\langle\rho^i\pi^j|\mathcal{H}|B^{i+j}\rangle \ . \tag{29}$$

Then

$$\begin{aligned}
A^{+0} &= \tfrac{1}{2}\sqrt{\tfrac{3}{2}}A_{3/2,2}-\tfrac{1}{2}\sqrt{\tfrac{1}{2}}A_{3/2,1}+\sqrt{\tfrac{1}{2}}A_{1/2,1} \ , \\
A^{0+} &= \tfrac{1}{2}\sqrt{\tfrac{3}{2}}A_{3/2,2}+\tfrac{1}{2}\sqrt{\tfrac{1}{2}}A_{3/2,1}-\sqrt{\tfrac{1}{2}}A_{1/2,1} \ , \\
A^{+-} &= \tfrac{1}{2}\sqrt{\tfrac{1}{3}}A_{3/2,2}-\tfrac{1}{2}A_{3/2,1}+\tfrac{1}{2}A_{1/2,1}-\sqrt{\tfrac{1}{6}}A_{1/2,0} \ , \\
A^{-+} &= \tfrac{1}{2}\sqrt{\tfrac{1}{3}}A_{3/2,2}+\tfrac{1}{2}A_{3/2,1}-\tfrac{1}{2}A_{1/2,1}-\sqrt{\tfrac{1}{6}}A_{1/2,0} \ , \\
A^{00} &= \sqrt{\tfrac{1}{3}}A_{3/2,2}+\sqrt{\tfrac{1}{6}}A_{1/2,0} \ .
\end{aligned} \tag{30}$$

As before, the amplitudes $\bar{A}^{ij}$ correspond to the *CP*-conjugated processes and $\tilde{A}^{ij}=e^{2i\phi_T}\bar{A}^{ij}$. For convenience we define the quantities

$$\begin{aligned}
S_1 &\equiv \sqrt{2}A^{+0}, \quad S_2\equiv\sqrt{2}A^{0+} \ , \\
S_3 &\equiv A^{+-}, \quad S_4\equiv A^{-+}, \quad S_5\equiv 2A^{00} \ ,
\end{aligned} \tag{31}$$

and similarly for $\tilde{S}_i$. Then

$$\begin{aligned}
S_1+S_2 &= S_3+S_4+S_5 \ , \\
\tilde{S}_1+\tilde{S}_2 &= \tilde{S}_3+\tilde{S}_4+\tilde{S}_5 \ .
\end{aligned} \tag{32}$$

These are the two pentagon relations.

Now let us distinguish tree and penguin contributions explicitly. The penguin operator is purely $I=\frac{1}{2}$, so we define

$$P_1=\tfrac{1}{2}(A_{1/2,1})_P\ ,$$
$$P_0=-\sqrt{\tfrac{1}{6}}(A_{1/2,0})_P\ , \tag{33}$$

where the subscript P means contributions from penguin processes only. The five vectors can be rewritten as

$$\begin{aligned} S_1&=T^{+0}+2P_1\ ,\\ S_2&=T^{0+}-2P_1\ ,\\ S_3&=T^{+-}+P_1+P_0\ ,\\ S_4&=T^{-+}-P_1+P_0\ ,\\ S_5&=T^{0+}+T^{+0}-T^{-+}-T^{+-}-2P_0\ , \end{aligned} \tag{34}$$

where the quantities T^{ij} contain no penguin contributions. Similar relations hold for $\tilde{S}_i$. Note that $\tilde{T}^{ij}=T^{ij}$. Therefore, linear combinations of S_i which are P_j independent equal the corresponding $\tilde{S}_i$ combinations:

$$\begin{aligned} S_1+S_2&=\tilde{S}_1+\tilde{S}_2\ ,\\ S_1-2S_3-S_5&=\tilde{S}_1-2\tilde{S}_3-\tilde{S}_5\ . \end{aligned} \tag{35}$$

Measuring the ten decay rates gives all ten $|S_i|$ and $|\tilde{S}_i|$. Thus we can (in principle at least) measure the lengths of all the sides of both pentagons. This plus the conditions above determine the figure up to one parameter, which we can take to be the length of the penguin-independent quantity S_1+S_2. There are also three time-asymmetric quantities which can be measured in the three channels for neutral B decays. One of these is the usual CP asymmetry:

$$a^{00}=\mathrm{Im}\left[e^{-2i(\phi_M+\phi_T)}\frac{\tilde{S}_5}{S_5}\right]\equiv\frac{|\tilde{S}_5|}{|S_5|}\sin(\delta_{00})\ . \tag{36}$$

The other two give [6]

$$\begin{aligned} a^{+-}&=\mathrm{Im}\left[e^{-2i(\phi_M+\phi_T)}\frac{\tilde{S}_4}{S_3}\right]\equiv\frac{|\tilde{S}_4|}{|S_3|}\sin(\delta_{+-})\ ,\\ a^{-+}&=\mathrm{Im}\left[e^{-2i(\phi_M+\phi_T)}\frac{\tilde{S}_3}{S_4}\right]\equiv\frac{|\tilde{S}_3|}{|S_4|}\sin(\delta_{-+})\ . \end{aligned} \tag{37}$$

Defining $\alpha\equiv-(\phi_M+\phi_T)$, we can determine

$$\begin{aligned} \delta_{00}&=2\alpha+\tilde{\phi}_5-\phi_5\ ,\\ \delta_{+-}&=2\alpha+\tilde{\phi}_4-\phi_3\ ,\\ \delta_{-+}&=2\alpha+\tilde{\phi}_3-\phi_4\ , \end{aligned} \tag{38}$$

where $\phi_i=\arg(S_i)$, $\tilde{\phi}_i=\arg(\tilde{S}_i)$. From the difference $\delta_{+-}-\delta_{-+}$ we can extract $\arg[(\tilde{A}^{-+}A^{-+})/(\tilde{A}^{+-}A^{+-})]$, which then allows us to fix the one remaining free parameter and hence determine $(\tilde{S}_5/S_5)$. This, in turn, will allow us to convert the measurement of a^{00} without approximation into a measurement of $\phi_M+\phi_T$. All this requires solving a number of higher-order algebraic equations with constants that are the various measured quantities. Errors on these quantities make this analysis inaccurate. In addition, discrete ambiguities will further complicate the analysis but, as in the $\pi\pi$ case, theoretical calculations suggest that the solution consistent with small penguin contributions is preferable. Perhaps the most likely outcome will be that the construction of the two pentagons will give a measure of the possible magnitude of penguin contributions; if they are in fact small compared to the tree amplitudes then the two pentagons will match within errors. In this case the naive analysis which assumes $a^{00}=\sin 2\alpha$ will be the best we can do. The figure can be used to provide an estimate of the possible error in that assumption.

$B\rightarrow K\pi\pi$

For three-body decay modes such as $B\rightarrow K\pi\pi$ there are many more amplitudes and more decay modes and the analysis is more complicated but possible. Such analyses may be useful because they include quasi-two-body final states such as $K^*\pi$ or $K\rho$ without the necessity of separating out nonresonant background. Here a combination of isospin and angular analyses can be useful. These final states can be classified in terms of their total isospin and the isospin of any two-particle subsystem, which we choose to be the two pions.

It is instructive to first consider the relationship between angular dependence and isospin in this system. In the $\pi\pi$ rest frame the z axis can be chosen along the K direction. States with isospin $I_{\pi\pi}=1$ have their amplitudes odd in $\cos\theta$ with respect to this axis, while states with $I_{\pi\pi}$=even have even angular momenta and hence their amplitudes are even in $\cos\theta$. Hence angular analysis can select contributions that are purely $I_{\pi\pi}=1$, purely $I_{\pi\pi}$=even or cross terms between them. This selection between odd and even $I_{\pi\pi}$ also selects quantities of definite CP for the $K_S\pi^+\pi^-$ channel. Angular analysis is unnecessary for the $K_S\pi^0\pi^0$ state which is already a CP eigenstate with $I_{\pi\pi}$=even.

There are six isospin amplitudes $A(I_t,I_{\pi\pi},I_f)$. The three amplitudes with $I_{\pi\pi}=1$: $A(0,1,\frac{1}{2})$, $A(1,1,\frac{1}{2})$, and $A(1,1,\frac{3}{2})$, form a system which is equivalent to the $K\pi$ system. The same method of analysis as for $K\pi$ can therefore be applied to all $K\pi\pi$ of $I_{\pi\pi}=1$ without separating out the ρ resonance. For the even $I_{\pi\pi}$ channels, the situation is even better. The three amplitudes $A(0,0,\frac{1}{2})$, $A(1,0,\frac{1}{2})$, and $A(1,2,\frac{3}{2})$ parametrize six channels: $B^0\rightarrow K^0\pi^+\pi^-$, $K^0\pi^0\pi^0$, $K^+\pi^-\pi^0$, and $B^+\rightarrow K^+\pi^+\pi^-$, $K^+\pi^0\pi^0$, $K^0\pi^+\pi^0$. The resulting relationships are

$$\begin{aligned} A(K^0\pi^+\pi^0)&=-A(K^+\pi^-\pi^0)=X\ ,\\ A(K^+\pi^+\pi^-)&=-\tfrac{1}{3}X-Y+Z\ ,\\ A(K^0\pi^+\pi^-)&=+\tfrac{1}{3}X+Y+Z\ ,\\ A(K^+\pi^0\pi^0)&=-\tfrac{2}{3}X+Y-Z\ ,\\ A(K^0\pi^0\pi^0)&=+\tfrac{2}{3}X-Y-Z\ , \end{aligned} \tag{39}$$

where

$$X=\sqrt{\tfrac{2}{5}}A(1,2,\tfrac{3}{2}),\quad Y=\tfrac{1}{3}A(1,0,\tfrac{1}{2})\ ,$$
$$Z=\sqrt{\tfrac{1}{3}}A(0,0,\tfrac{1}{2})\ . \tag{40}$$

These amplitudes represent any projection chosen to select only even $\pi\pi$ angular momentum and hence even $I_{\pi\pi}$. Note that penguins contribute only to the quantity Z, neither to X nor to Y. The relationships in Eq. (39) can be represented by triangles in the complex plane. These triangles are completely determined (as in the $\pi\pi$ case) once all decay rates and hence the magnitudes of the amplitudes are known. Similar relationships apply for the related amplitudes for $\bar{B}^0$ and B^- decays. Measuring the CP asymmetries in both $K_S\pi^0\pi^0$ and $K_S\pi^+\pi^-$ channels will leave no discrete ambiguities. Thus the asymmetry measurements in the $K_S\pi\pi$ system can again (at least in principle) be converted to clean measurements of CKM matrix elements, even though *a priori* the tree and penguin contributions are comparable in magnitude.

The cross terms between even and odd $I_{\pi\pi}$ can readily be selected by integrating against any odd function of $\cos\theta$, or by taking the forward-backward asymmetry. Let us denote such a quantity by $F(K\pi\pi)$ ($\bar{F}$ denotes the CP conjugate process). The isospin analysis implies further conditions on these quantities; e.g.,

$$F(K^0\pi^+\pi^0)-\bar{F}(K^0\pi^+\pi^0)=F(K^+\pi^-\pi^0)-\bar{F}(K^+\pi^-\pi^0)\ . \tag{41}$$

Both sides of Eq. (41) vanish separately if there are not both tree and penguin contributions.

The above relationship can also be expressed in an unintegrated form. We consider particular momenta for the pions in the final states containing two pions with total charge ±1. The state $\pi^+(p_1)\pi^0(p_2)$ is distinguished from the state $\pi^0(p_1)\pi^+(p_2)$. The relation (41) can be rewritten in the form

$$|A_{0+0}|^2-|\bar{A}_{0+0}|^2+|A_{+-0}|^2-|\bar{A}_{+-0}|^2 =|A_{00+}|^2-|\bar{A}_{00+}|^2+|A_{+0-}|^2-|\bar{A}_{+0-}|^2\ , \tag{42}$$

where A_{ijk} denotes $A[K^i\pi^j(p_1)\pi^k(p_2)]$. As with the sum rules given above for the πK system, the relationships (41) and (42) can be derived under a very general set of assumptions and become trivial if the penguin contributions vanish. Thus they provide tests that can probe the penguin contribution before sufficient data are available to make the CP violation studies.

As in the case of the $K\pi$ system, all the above discussion applies equally if the K is replaced by a D. However, in that case there are no penguin contributions. Consequently, no asymmetries are expected in the charged B decays, while those in the neutral B decays are directly related to the CKM parameters (once a state of definite CP is selected by angular analysis). Isospin analysis does not add new information in this case.

CONCLUSIONS

We have shown that the method of isospin analysis introduced by Gronau and London [3] to eliminate hadronic uncertainties in CP asymmetries in $B\to\pi\pi$ is, in fact, very general and can, in principle, be applied to many channels. Its usefulness depends on obtaining good measurements for a whole set of isospin-related quantities. This, in turn, will depend on branching ratios and on the available experimental techniques. We present here the generalizations of the method to other channels in the hope that there will be some channel in which a large enough branching ratio and a full set of observable decays will make it useful. It is clear from the cases presented here that the applicability is, in principle, very broad, but the practical usefulness remains to be seen. It is a tool worth considering once sufficient data are accumulated.

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[1] For definitions of the angles of the unitarity triangle, α, β, and γ, and predictions of their values, see, e.g., C. O. Dib, I. Dunietz, F. J. Gilman, and Y. Nir, Phys. Rev. D **41**, 1522 (1990).

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[4] The contribution of electromagnetic penguins is expected to be small and is neglected throughout this paper. The isospin analyis given here does not separate the effects of electromagnetic penguins.

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[7] Y. Nir and H. R. Quinn, Phys. Rev. D **42**, 1473 (1990).

[8] I. Dunietz *et al.*, Phys. Rev. D **43**, 2193 (1991).

[9] Note that the amplitude to produce K_S differs by a factor of $\sqrt{2}$ from those for K^0 or $\bar{K}^0$, used in our analysis.

[10] Note that in the channel π^+K_S, the hypothesis of factorization (J. D. Bjorken, in *Developments in High-Energy Physics*, Proceedings, Crete, Greece, 1988, edited by E. G. Floratos and A.Verganelakis [Nucl. Phys. B (Proc. Suppl.) **11**, 325 (1989)]) indicates that, since there are no d quarks in the naive tree diagrams, only penguin diagrams contribute. If this is indeed the case, then $|A^{+0}|=|\bar{A}^{+0}|$ while $\arg(A^{+0}/\bar{A}^{+0})=2(\phi_P-\phi_T)$, providing a measurement of the angle γ of the unitarity triangle directly from *charged* B decays. This measurement will be independent of the mechanism responsible for neutral meson mixing.

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