Multiplicity distributions, transverse momenta, and nonstationary effects

E. M. Friedlander
Nuclear Science Division, Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720

X. C. He and C. C. Shih
Department of Physics, University of Tennessee, Knoxville, Tennessee 37916
and Department of Physics, University of Marburg, D-3550 Marburg, Germany

R. M. Weiner
Department of Physics, University of Marburg, D-3550 Marburg, Germany
(Received 26 July 1990; revised manuscript received 4 March 1991)

The application of the quantum statistical formalism to multiplicity distributions has been generalized to include corrections for "nonstationarity" in rapidity as well as the transverse-momentum dependence. The identical-particle data available so far can be explained in this formalism with a quite broad range of parameters of chaoticity and correlation lengths.

I. INTRODUCTION

Quantum statistical results in general and coherent states in particular have proven quite useful in the analysis of fluctuations in many fields of physics and in particular in optics and in high-energy physics (for reviews on this subject cf., e.g., [1–4]). One appealing feature of this formalism is that when applied, e.g., to multiplicity distributions, the latter can be interpreted in terms of a few physical quantities, the most important of which are the chaoticity p which plays the role of an order parameter and the rapidity coherence (correlation) length ξ_y [5]. Both of these parameters are important tools in the investigation of phase transitions. Thus, e.g., in solid-state physics, the temperature dependence of these parameters gives important information about the structure of the corresponding system, and, in high-energy physics, it is probably the energy dependence that contains the equivalent knowledge. In particular the broadening of the multiplicity distributions $P(n)$ with energy $\sqrt{s}$ [Kuba-Nielsen-Olesen (KNO) scaling violation] [6] has been interpreted as due to the increase of p and ξ_y with $\sqrt{s}$ suggesting the approach of a phase transition [3,7]. Another important phenomenon observed in the investigation of $P(n)$ is the dependence of $P(n)$ on (a) the width of the rapidity window δy [6] and (b) the position of the center y_c of the rapidity window, $|y-y_c|<\delta y/2$ [6,8]. Observation (a) in particular has given rise to many speculations related to the phenomena of intermittency [9] known from other branches of physics. On the other hand many features of (a) could be explained within the quantum statistical (QS) formalism as due to the interference of a coherent and a chaotic field leading to a scaling of $P(n)$ in terms of the ratio $\delta y/\xi_y$, without any need of invoking intermittent behavior [10]. (b) has been interpreted as due to two independent sources: one totally chaotic and contributing mainly to the very central region, and another one totally coherent contributing in the entire rapidity region [11]. The above-mentioned interpretations of the experimental observations on multiplicity distribution may imply far-reaching consequences. In addition to the possible approach to a phase transition already mentioned above and reiterated recently in the context of intermittency [12], it was conjectured in [11] and then in [12] that the shape of $P(n)$ as reflected in ξ_y may signal a quark-gluon plasma. Given these facts a new appraisal of the QS formalism as applied to multiplicity fluctuations and which contains intermittency as a limiting case appears worthwhile, especially because in the past several approximations were made [7,11], the justification of which may be in certain cases questionable. This last point got new emphasis in a recent study by Ochs [13] who proved how important a three-dimensional momentum-space analysis of multiplicity distributions may be when applied to intermittency studies. This appraisal will be done in the present paper.

The first question that arises when confronting observations (a) and (b) is whether there does not exist a common mechanism underlying both. Indeed, although both (a) and (b) could be interpreted within the QS formalism, the mechanism assumed in [7] is different from that of [11]. The difference is that [7] used a superposition of fields whereas [11] involved a convolution of probabilities. Moreover, in [7] "stationarity" in rapidity space y was assumed, meaning that the two-particle correlation function of the chaotic field was a function of the rapidity differences $|y_1-y_2|$ only. This last assumption, which is quite standard in quantum optics, where the role of rapidity is played by the time variable, may not always hold in particle physics. Finally, both in [7] and [11] the role of the transverse-momentum distribution in the correlation was ignored. The importance of including the q_T dependence has been stressed recently by Gyulassy (cf. below). These three aspects of the y dependence of multiplicity distributions will be discussed in the following.

Consider a field $\pi(y,\mathbf{q}_T)$ which has a coherent component $\pi_{\rm coh}$ and a chaotic one $\pi_{\rm chao}$:

$$\pi(y,\mathbf{q}_T)=\pi_{\rm coh}(y,\mathbf{q}_T)+\pi_{\rm chao}(y,\mathbf{q}_T)\ , \qquad (1)$$

where $(y,\mathbf{q}_T)$ are the rapidity and transverse momentum at which the field is measured. The chaotic component $\pi_{\rm chao}$ determines the correlation function

$$\Gamma(y_1,\mathbf{q}_{T1},y_2,\mathbf{q}_{T2})=\langle \pi_{\rm chao}(y_1,\mathbf{q}_{T1})^\dagger\pi_{\rm chao}(y_2,\mathbf{q}_{T2})\rangle_{\rm ensemble} =\Gamma(1,2)\ , \qquad (2)$$

with

$$\langle \pi_{\rm chao}(y,\mathbf{q}_T)\rangle_{\rm ensemble}=0\ . \qquad (3)$$

The average multiplicity is

$$\langle n\rangle=\int dy\, dq_T^2\langle \pi(y,\mathbf{q}_T)^\dagger\pi(y,\mathbf{q}_T)\rangle_{\rm ensemble} \qquad (4)$$

$$=\int dy\, dq_T^2[\pi_{\rm coh}(y,\mathbf{q}_T)^\dagger\pi_{\rm coh}(y,\mathbf{q}_T)r +\Gamma(y,\mathbf{q}_T,y,\mathbf{q}_T)]\ . \qquad (5)$$

In quantum statistics, the ensemble average is performed with the help of the density matrix ρ:

$$\Gamma(y_1,\mathbf{q}_{T1},y_2,\mathbf{q}_{T2})={\rm Tr}[\rho\pi_{\rm chao}(y_1,\mathbf{q}_{T1})^\dagger\pi_{\rm chao}(y_2,\mathbf{q}_{T2})]\ . \qquad (6)$$

Whenever there is no ambiguity, we shall abbreviate the three-dimensional variables by their index [writing, e.g., for the left-hand side (LHS) of Eq. (2), $\Gamma(1,2)$]. We shall parametrize the above correlation function as follows:

$$\Gamma(1,2)=\sqrt{\Gamma(1,1)\Gamma(2,2)}\gamma(1,2)\ , \qquad (7)$$

with

$$\gamma(1,2)=\gamma_y(y_1,y_2)\gamma_T(\mathbf{q}_{T1},\mathbf{q}_{T2})\ . \qquad (8)$$

It is useful to introduce also the chaoticity

$$p_{\rm chao}(y)=\Gamma(1,1)/[\Gamma(1,1)+|\pi_{\rm coh}(1)|^2] =p_{\rm chao}(1)\ , \qquad (9)$$

meaning that the $\mathbf{q}_T$ dependences of the coherent and the chaotic components are assumed to be the same. This kind of parametrization assumes factorization in y and $\mathbf{q}_T$. Partial support for these assumptions comes from the observation that in a first-order approximation the inclusive cross sections factorize [14] as

$$d^3\sigma/dy\, dq_T^2\sim f(y)g(\mathbf{q}_T)\ . \qquad (10)$$

For the particular case of a flat rapidity plateau without a $\mathbf{q}_T$ dependence

$$p_{\rm chao}(y)=p_0={\rm const}\ ,$$

and

$$\gamma_T(\mathbf{q}_{T1},\mathbf{q}_{T2})=1\ ,$$

Eq. (7) reduces to

$$\Gamma(1,2)=n_0p_0\gamma_y(y_1,y_2)\ , \qquad (11)$$

where n_0 is the average number of particles within the rapidity window $|y-y_c|<\delta y/2$, i.e., $n_0=\langle n\rangle/\delta y$, as was assumed in [7]. In the quantum statistical approach, the fluctuations of multiplicity are most directly expressed theoretically by the factorial cumulants μ_j. They are defined through the generating function

$$\sum_{n=0}^{\infty}P_n(1-s)^n=\exp\left[\sum_{j=1}^{\infty}\frac{(-s)^j}{j!}\mu_j\right]\ ,$$

where P_n is the probability of finding n particles. The other statistical moments associated with the multiplicity distribution P_n can then be calculated in a simple way, in particular, the factorial moments which may be more readily calculable in certain dynamical models [15] and have become again fashionable recently in intermittency studies [9].

II. ONE-DIMENSIONAL FACTORIAL CUMULANTS IN RAPIDITY

In the limiting case of a one-dimensional distribution in y with $p_{\rm chao}(y)=p_0$, the expressions for the normalized factorial cumulants read [16]

$$\nu_j=\mu_j/(\mu_1)^j=(j-1)!B_{yj}^{(1)}+j!\bar{B}_{yj}^{(1)}\ , \qquad (12)$$

where

$$B_{yj}^{(1)}=p_{\rm chao}^jB_j\ , \qquad (13)$$

$$B_j=\frac{1}{(\delta y)^j}\prod_{i=1}^{j}\int_0^{\delta y}dy_i\gamma(y_1,y_2)\cdots\gamma(y_{j-1},y_j)\gamma(y_j,y_1)\ , \qquad (14)$$

$$\bar{B}_{yj}^{(1)}=(1-p_{\rm chao})p_{\rm chao}^{j-1}\bar{B}_j\ , \qquad (15)$$

$$\bar{B}_j=\frac{1}{(\delta y)^j}\prod_{i=1}^{j}\int_0^{\delta y}dy_i\gamma(y_1,y_2)\cdots\gamma(y_{j-1},y_j)\ . \qquad (16)$$

If we further assume a Lorentzian profile for the rapidity correlation,

$$\gamma(y_1,y_2)=\exp\left[-\frac{|y_1-y_2|}{\xi_y}\right]\ , \qquad (17)$$

we obtain analytical expressions for the functions B_j and $\bar{B}_j$ given in Ref. [16]. For example,

$$B_2=(e^{-2\beta_y}+2\beta_y-1)/(2\beta_y^2)\ , \qquad (18)$$

$$\bar{B}_2=2(e^{-\beta_y}+\beta_y-1)/\beta_y^2 \qquad (19)$$

where $\beta_y=\delta y/\xi_y$. For higher orders of j, B_j and $\bar{B}_j$ have been integrated in the Lorentzian case up to the eighth order explicitly [17].

III. THREE-DIMENSIONAL FACTORIAL CUMULANTS IN RAPIDITY AND TRANSVERSE MOMENTUM

We generalize now the expressions for the normalized factorial cumulants to the three-dimensional case and obtain

$$\nu_j=\mu_j/(\mu_1)^j=(j-1)!B_j^{(3)}+j!\bar{B}_j^{(3)} , \tag{20}$$

where

$$B_j^{(3)}\sim \prod_{i=1}^{j} \int dy_i \int dq_{Ti}^2\, \Gamma(1,2)\cdots\Gamma(j-1,j)\Gamma(j,1) , \tag{21}$$

$$\bar{B}_j^{(3)}\sim \prod_{i=1}^{j} \int dy_i \int dq_{Ti}^2\, \Gamma(1,2)\cdots\Gamma(j-1,j) \times \pi_{\rm coh}(y_1,\mathbf{q}_{T1})\pi_{\rm coh}^{\dagger}(y_j,\mathbf{q}_{Tj}) . \tag{22}$$

Substituting the factorized form of the correlation function Eq. (7) into the above expression, the B_j and $\bar{B}_j$ are also reduced to the products

$$B_j^{(3)}=B_{yj}^{(1)}B_{Tj} , \tag{23}$$

$$\bar{B}_j^{(3)}=\bar{B}_{yj}^{(1)}\bar{B}_{Tj} . \tag{24}$$

In the above expressions B_{Tj} and $\bar{B}_{Tj}$ are the analogon of B_j and $\bar{B}_j$, but in $\mathbf{q}_T$ space and $B_{yj}{}^{(1)},\bar{B}_{yj}{}^{(1)}$ are given by Eqs. (13) and (15). In the case of a constant $p_{\rm chao}$, one obtains the exact expressions such as those given by Eq. (18). Notice that the overlapping integrals possess structures that are quite similar to the linked-pair structures recently proposed by Carruthers and Sarcevic [18]. There are, however, two types of diagrams. Equations (15) and (22) are formally linked-pair diagrams, while Eqs. (13) and (21) are linked-ring diagrams.

We discuss now the $\mathbf{q}_T$ dependence of the cumulants. Two technical differences arise. (a) $\mathbf{q}_T$ is in a two-dimensional space, (b) the $\mathbf{q}_T$ space is usually not bounded above. A form of correlation suitable for analytical calculation is the Gaussian profile, i.e.,

$$d^2\sigma/dq_T^2\sim\exp(-|\mathbf{q}_T|^2/q_{T0}^2) , \tag{25}$$

$$\Gamma(\mathbf{q}_{T1},\mathbf{q}_{T2})\sim\left[\frac{d^2\sigma}{d^2q_{T1}}\frac{d^2\sigma}{d^2q_{T2}}\right]^{1/2} \times\exp(-|\mathbf{q}_{T1}-\mathbf{q}_{T2}|^2/4\xi_T^2) , \tag{26}$$

through which we get the corresponding B_{Tj} and $\bar{B}_{Tj}$. Analytical expressions for B_{Tj} were given by Gyulassy [19]. They can be generalized to some more complicated forms for $\bar{B}_{Tj}$. We have, for example,

$$B_{T2}(\beta_T)=\frac{1}{1+\beta_T^2} , \tag{27}$$

$$B_{T3}(\beta_T)=\frac{1}{(1+3\beta_T^2/4)^2} , \tag{28}$$

$$\bar{B}_{T2}(\beta_T)=\frac{1}{1+\beta_T^2/2} , \tag{29}$$

$$\bar{B}_{T3}(\beta_T)=\frac{1}{(1+\beta_T^2/4)(1+3\beta_T^2/4)} , \tag{30}$$

where

$$\beta_T=q_{T0}/\xi_T . \tag{31}$$

Expressions for higher j's are given in Appendix A.

At this point, one may argue that the true profile of the inclusive $\mathbf{q}_T$ spectrum is more likely to be a simple exponential rather than a Gaussian. In Appendix B, we discuss the expressions for this alternative choice. Given a reasonable set of values for β_T, however, the differences between these two choices are small.

It is interesting to remark that the assumption of a positive two-particle correlation in $\mathbf{q}_T$ is at variance with those cluster decay models, in which the clusters are assumed to be at rest before decaying into particles. The conservation of transverse momentum would then display an anticorrelation instead. The reason for this difference is the fact that our QS approach deals with identical Bose-Einstein particles where, because of the Bose-Einstein bunching effect, positive two-particle correlations must exist.

In the above expressions, single inclusive cross sections are treated as constant (independent of the rapidity y). To overcome this restriction, we start with the more general expression of μ_j in terms of overlapping integrals of the correlation functions of y, i.e.,

$$\Gamma(y_1,y_2)=\left[p_{\rm chao}(1)p_{\rm chao}(2)\frac{dn_y(1)}{dy_2}\frac{dn_y(2)}{dy_2}\right]^{1/2} \times e^{-|y_1-y_2|/\xi_y} . \tag{32}$$

Nonstationary values of n_y taken from experiments can then be used. We also have taken the $\mathbf{q}_T$ dependence of the single inclusive distribution as Gaussian. As we discuss in Appendix B, with our values of β_T, an alternative choice of a simple exponential may introduce an uncertainty of about 15%. Since the $\mathbf{q}_T$ correlation length is, in general, not well known, there is no real necessity to use alternative inputs for $\mathbf{q}_T$.

IV. APPLICATION TO DATA

Given a set of cumulants μ_j, it is easy to convert them to an equivalent set of normalized factorial moments f_j,

$$f_j\equiv\frac{\langle n(n-1)\cdots(n-j+1)\rangle}{\langle n\rangle^j} ,$$

through the relationships

$$f_2=1+\nu_2 , \tag{33}$$

$$f_3=1+3\nu_2+\nu_3 , \tag{34}$$

$$f_4=1+6\nu_2+3\nu_2^2+4\nu_3+\nu_4 , \tag{35}$$

$$f_5=1+10\nu_2+15\nu_2^2+10\nu_3+10\nu_2\nu_3+5\nu_4+\nu_5 , \tag{36}$$

where $\nu_j=\mu_j/(\mu_1)^j$ as defined before. Experimental values of the f_j of negative-charge data at the NA22 energy [20] are plotted in Fig. 1 as a function of the width of the rapidity window $-\ln(\delta y)$. The resultant cumulants μ_j are computed with a typical set of parameters given by [21]

$$\beta_T=1.1 , \tag{37}$$

$$\xi_y=1.7 , \tag{38}$$

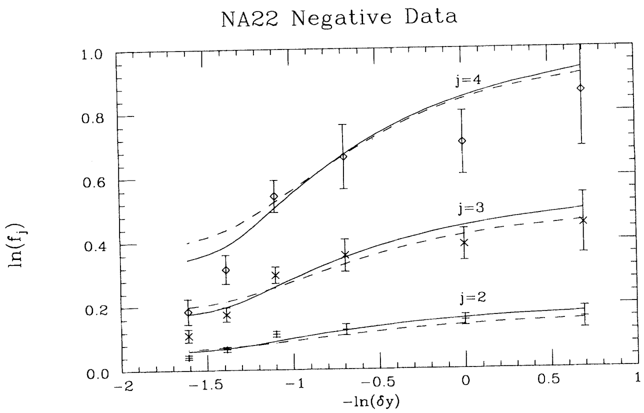

FIG. 1. Comparison of the experimental negative-charge data of the NA22 experiment [20] with values derived from Eqs. (33)–(36). The solid curve corresponds to the parametrization from Eqs. (37)–(42), and the dashed curve corresponds to the fully chaotic case (i.e., $p_{\rm chao}=1.0$) with $\beta_T=2.2$ and $\xi_y=1.6$.

and a nonstationary $p_{\rm chao}(y)$ taking value $p_a(p_b)$ at $y_a(y_b)$, i.e.,

$$p_{\rm chao}(y)=p_b\frac{|y|-y_a}{y_b-y_a}+p_a\frac{|y|-y_b}{y_a-y_b}\quad \text{for } |y|<\frac{p_a y_b-p_b y_a}{p_a-p_b}\,, \tag{39}$$

$$p_{\rm chao}(y)=0 \quad \text{for } |y|>\frac{p_a y_b-p_b y_a}{p_a-p_b}\,, \tag{40}$$

$$p_a=0.10,\quad y_a=1.0\,, \tag{41}$$

$$p_b=0.01,\quad y_b=2.0\,. \tag{42}$$

In Fig. 1, the experimental values of the factorial moments f_j are constructed from the multiplicity distributions $P(n)$ read from figures of [20]. The associated errors of f_j are obtained through the reported errors in the $1/k$ values of the NA22 negative-binomial fits to the negative-charge data [22]. For the smaller rapidity windows, the data can be satisfactorily fitted with a wide range of parameters, as indicated by the solid curves with the parameters quoted above in Eqs. (37)–(42). Even with a totally chaotic representation ($p_{\rm chao}=1$), the factorial moments f_j can still be reproduced. However, this is achieved through a choice of a rather large correlation length ξ_T of $\mathbf{q}_T$ (which gives a substantial reduction factor for the factorial cumulant). A completely chaotic solution would imply that the value of ξ_T is significantly higher than what is known from Bose-Einstein correlations [23].

One possible way to distinguish the two extreme cases of a highly coherent system (very small $p_{\rm chao}$) and a fully chaotic system ($p_{\rm chao}=1$) is to examine the multiplicity distribution for shifted rapidity windows. For the nonstationary representation ($p_{\rm chao}$ not a constant), the amount of noncoherent contribution in the charged data becomes very small if we move away from the central region. This behavior of f_2 is given in Table I. Since negative charged data for shifted rapidity windows, i.e., $|y-y_c|<\delta y/2$, are not available, a direct comparison is not possible. The negative-charge data would be, however, quite consistent with the charged-particle data for shifted rapidity windows, if we would assume that the approximate relation $k_{\rm eff}^{-}\sim 2k_{\rm eff}^{\rm chao}$ found for the symmetrical windows is also valid for asymmetrical ones [20]. On the other hand if one assumes a totally chaotic case $p_{\rm chao}=1$, one finds that $\nu_2=1/k$ is independent on y_c, which is difficult to accept in view of the observed behavior of the charged data (cf. Table I).

It is interesting to observe that our results tend to overestimate the fluctuations of the multiplicity distribution of n (as reflected by its associated factorial moments) for the largest rapidity window, as long as a reasonable fit is established for the smaller windows. This we believe is the reflection of the effect of the overall conservation laws, such as energy and charge conservation, which were not taken into account in the preceding considerations. For example, given a large rapidity window, it is no longer possible to consider the negative and positive particles as independently fluctuating quantities. It is thus necessary to use more general quantum statistical formulations such as charged-coherent states [24].

The factorial moments f_j observed for "bin averaging" are in general somewhat smaller than that of the symmetrical windows with the same bin size. This can be partially understood by the fact that the particle fluctuations for a window away from the central rapidity region are smaller. The statistical errors of the "horizontally" averaged f_j are also smaller than the errors associated with a single symmetrical window, where no "horizontal" averaging is allowed.

The formalism developed above has been used to determine the parameters $p_{\rm chao}$, ξ_y, and ξ_T from the multiplicity distributions. If this formalism gives an adequate description of data, it must describe two-body correlation functions as well. This appears to be the case at least at a qualitative level [25]. Implications of "bin-average" and the subtle differences between "horizontal" and "vertical" averages [10] shall be discussed in the future.

The generalization of the QS formulation presented in this paper, assumes among other things that the y and $\mathbf{q}_T$ dependence of the chaotic and the coherent fields are the same. This assumption may be too strong. At the present stage, however, there is no further information available about this issue.

The inclusion of the $\mathbf{q}_T$ dependence and the "nonstationary" rapidity effects in the QS formulation as performed in the preceding considerations influences also the concrete behavior of the chaoticity $p_{\rm chao}$ and of the coherence lengths ξ_y and ξ_T as a function of energy, if the broadening of the multiplicity distribution with $\sqrt{s}$ is considered. Unfortunately no identical particle data exist so far at the collider energies, so that a more reliable analysis in this direction is not possible at this stage. This aspect of the application of the QS formulation is related to the well known problem of charged- versus identical-particle multiplicity distribution. The solution

TABLE I. "Nonstationary" dependence of the average multiplicity $\langle n \rangle$, factorial moment f_2, and cumulant ν_2, $\nu_2 = f_2 - 1 = 1/k_{\text{eff}}$ on the rapidity span δy and the center of a rapidity window y_c ($|y - y_c| < \delta y/2$). The parameters of the partially coherent and fully chaotic representations correspond to those of Fig. 1.

	δy	y_c	$\langle n \rangle$	f_2	ν_2
Partial coherence		0.5	0.44	1.16	0.16
		1.0	0.38	1.11	0.11
	0.5	1.5	0.33	1.07	0.07
		2.0	0.23	1.02	0.02
		2.5	0.12	1.01	0.01
		3.0	0.07	1.01	0.01
		0.5	0.82	1.15	0.15
		1.0	0.77	1.11	0.11
		1.5	0.65	1.06	0.06
	1.0	2.0	0.45	1.03	0.03
		2.5	0.26	1.01	0.01
		3.0	0.13	1.01	0.01
		0.5	1.61	1.12	0.12
		1.0	1.48	1.10	0.10
		1.5	1.22	1.07	0.07
	2.0	2.0	0.92	1.04	0.04
		2.5	0.59	1.02	0.02
		3.0	0.30	1.01	0.01
Chaotic	0.5	0.5	0.44	1.17	0.17
		3.0	0.07	1.17	0.17
	1.0	0.5	0.82	1.15	0.15
		3.0	0.13	1.16	0.16
	2.0	0.5	1.61	1.12	0.12
		3.0	0.30	1.14	0.14

of this problem can only be obtained through supplementary dynamical considerations beyond the level of identical particles. On the other hand, because of the lack of identical-particle data at high energies and the lack of an adequate theory, one is tempted to apply the usual scheme of QS to charged data as well. This means that one describes the multiplicity distributions, i.e., its cumulants μ_j, by some effective parameters p' and ξ' which satisfy formally the same equations as identical-particle data, and one expects that the qualitative behavior of p' and ξ' with respect to δy and $\sqrt{s}$ is similar to that of p and ξ of the identical particles [3,5]. That such an expectation is not too far fetched can be seen from the fact that if the two-particle correlations are independent of the sign of the charge of the individual particles this expectation is fulfilled [26].

V. DISCUSSION

It is clear from the above investigation that the $\mathbf{q}_T$ distribution introduces substantial modifications of the various moments of multiplicity distributions. Correlations in $\mathbf{q}_T$ always introduce a damping factor in the rapidity correlation. It is, however, possible to accommodate the experimental data in the new formulation with "renormalized" values of the relevant correlation parameter ξ_y

TABLE II. B_{T2}, $\bar{B}_{T2}$, B_{T3}, $\bar{B}_{T3}$, and B_{T4}, $\bar{B}_{T4}$, as functions of β_T. A denotes analytical, G denotes Gaussian, and E denotes exponential.

β_T		0.5	1.0	2.0	3.0
	A	0.67	0.50	0.33	0.25
B_{T2}	G	0.66	0.48	0.32	0.24
	E	0.71	0.57	0.42	0.33
	A	0.80	0.67	0.50	0.40
$\bar{B}_{T2}$	G	0.79	0.66	0.49	0.39
	E	0.82	0.71	0.57	0.48
	A	0.53	0.33	0.16	0.095
B_{T3}	G	0.55	0.36	0.19	0.12
	E	0.58	0.41	0.25	0.17
	A	0.65	0.46	0.27	0.18
$\bar{B}_{T3}$	G	0.66	0.48	0.29	0.20
	E	0.68	0.52	0.35	0.26
	A	0.43	0.22	0.083	0.040
B_{4T}	G	0.45	0.26	0.11	0.065
	E	0.50	0.32	0.17	0.11
	A	0.52	0.31	0.14	0.078
$\bar{B}_{T4}$	G	0.54	0.34	0.17	0.10
	E	0.58	0.40	0.23	0.15

and the chaoticity parameter $p_{\rm chao}$. The net effect is an increase of the chaoticity $p_{\rm chao}$, and a slight decrease in ξ_y. The importance of $\mathbf{q}_T$ for the analysis of multiplicity distribution was emphasized for the first time by Gyulassy [19] and we confirm his conclusions in this respect.

It was shown previously [7] that the observed KNO violation with energy increasing from the CERN ISR to the CERN $Sp\bar{p}S$ Collider regime [27] can be interpreted in the QS formulation by an increase of $p_{\rm chao}$ and ξ_y with energy. Provided the dependence of ξ_T with energy is not opposite to that of ξ_y and that the factorization between y and $\mathbf{q}_T$ is not too bad an approximation (both these assumptions appear reasonable), it seems likely that the above conclusion still remains valid.

One of the major factors that we have not included is the additional complication due to charge correlations. It is known that if the $+/-$ correlation is the same as the $+/+$ and $-/-$ correlation, the present formulation in terms of a single species is correct. Investigations along this direction are in progress [26].

To summarize, the quantum statistical formulation can account for the available identical-particle data of NA22. The application of this formalism to higher energies is still hampered by the lack of identical-particle data. Given the generality of the QS formalism and the important physical implications linked with its fundamental parameters, chaoticity and coherence length, a remedy of this inadequate situation is urgently needed.

ACKNOWLEDGMENTS

R.M.W. was supported in part by the German Minister for Research and Technology (BMFT) under Contract No. 06 MR 777. E.M.F. was supported by the Director of the Office of Energy Research, Office of High Energy and Nuclear Physics, Nuclear Physics Division of the U.S. Department of Energy under Contract No. DE-AC03-76SF00098. X.C.H. and C.C.S. were supported in part by the National Science Foundation and the Science Alliance of Tennessee. C.C.S. was also supported by the Deutsche Forschungsgemeinschaft (DFG).

APPENDIX A

This section deals with simple expressions of B_{Tj} and $\bar{B}_{Tj}$ for $\mathbf{q}_T$. Given a Gaussian profile for $\mathbf{q}_T$, they can be easily calculated. For example, the lower orders are

$$B_{T4}=1/[(1+\beta_T^2/2)^2(1+\beta_T^2)] , \tag{A1}$$

$$B_{T5}=1/(1+5\beta_T^2/4+3\beta_T^4/16)^2 , \tag{A2}$$

$$B_{T6}=1/[(1+\beta_T^2)(1+\beta_T^2/4)^2(1+3\beta_T^2/4)^2] , \tag{A3}$$

$$\bar{B}_{T4}=1/[(1+\beta_T^2/2)(1+\beta_T^2+\beta_T^4/8)] , \tag{A4}$$

$$\bar{B}_{T5}=1/[(10+3\beta_T^2/4+\beta_T^4/16)(10+5\beta_T^2/4+5\beta_T^4/16)] , \tag{A5}$$

$$\bar{B}_{T6}=1/[(1+\beta_T^2/4)(1+\beta_T^2/2)(10+3\beta_T^2/4)(1+\beta_T^2+\beta_T^4/16)] . \tag{A6}$$

APPENDIX B

This appendix compares an exponential $\mathbf{q}_T$ profile with the Gaussian one used above. Given the correlation function

$$\gamma(\mathbf{q}_{T1},\mathbf{q}_{T2})=\exp(-|\mathbf{q}_{T1}-\mathbf{q}_{T2}|/\xi_T) , \tag{B1}$$

we estimated $2j$-dimensional overlapping integrals through a Monte Carlo simulation. Here the single inclusive q_T distribution is simulated through the inversion of the probability density of $\mathbf{q}_T$ in terms of uniform variable x by

$$e^{-|\mathbf{q}_T|/\Lambda}|\mathbf{q}_T|\,d|\mathbf{q}_T|=dx , \tag{B2}$$

or explicitly,

$$x(|\mathbf{q}_T|)=\Lambda^2-\Lambda(\Lambda+|\mathbf{q}_T|)e^{-|\mathbf{q}_T|/\Lambda} , \tag{B3}$$

where Λ is the distribution width of $\mathbf{q}_T$. The overlapping integrals are then identified as the ensemble averages of the products of the two-body correlations. The resultant values of the overlapping integrals are shown in Table II as a function of the scaling variable $\beta_T=[\langle(|\mathbf{q}_T|-\langle|\mathbf{q}_T|\rangle)^2\rangle]^{1/2}/\xi_T$. They are to be compared with the alternative choice of the Gaussian profile with the same values of $[\langle(|\mathbf{q}_T|-\langle|\mathbf{q}_T|\rangle)^2\rangle]^{1/2}$. It is clear that the differences between the two choices are not important at the level of accuracy demanded in our present investigation.

[1] *Coherent States,* edited by J. Klauder and B. Skagerstam (World Scientific, Singapore, 1985).

[2] P. Carruthers and C. C. Shih, J. Mod. Phys. A **2**, 1447 (1987).

[3] G. N. Fowler *et al.*, Univ. of Marburg Report, 1986 (unpublished); R. M. Weiner, in *Hadronic Matter in Collisions,* edited by P. Carruthers and D. Strottman (World Scientific, Singapore, 1986), p. 106.

[4] R. M. Weiner, in *Multiparticle Production,* Perugia Workshop, edited by R. Hwa, G. Pancheri, and Y. Srivas-

tava (World Scientific, Singapore, 1989), p. 231.
[5] G. N. Fowler and R. M. Weiner, Phys. Rev. D **17**, 3118 (1978).
[6] UA5 Collaboration, G. J. Alner *et al.*, Phys. Rep. **154**, 247 (1987).
[7] G. N. Fowler *et al.*, Phys. Rev. D **37**, 3127 (1988); G. N. Fowler *et al.*, J. Phys. G **16**, 1439 (1990); C. C. Shih, Phys. Rev. D **42**, 3025 (1990).
[8] G. N. Fowler, E. M. Friedlander, and R. M. Weiner, Phys. Lett. **104B**, 239 (1981).
[9] A. Bialas and R. Peschanski, Nucl. Phys. **B273**, 703 (1986); Phys. Lett. B **207**, 59 (1988).
[10] P. Carruthers, R. Friedlander, C. C. Shih, and R. M. Weiner, Phys. Lett. B **222**, 487 (1989).
[11] G. N. Fowler *et al.*, Phys. Rev. Lett. **57**, 2119 (1986).
[12] A. Bialas and R. Hwa, Phys. Lett. B **253**, 436 (1991).
[13] W. Ochs, Phys. Lett. B **247**, 101 (1990).
[14] A. Breakstone *et al.*, Europhys. Lett. **7** (2), 131 (1988).
[15] A. H. Mueller, Phys. Rev. D **14**, 150 (1971).
[16] A. K. Jaiswal and C. L. Mehta, Phys. Rev. A **2**, 168 (1970).
[17] C. C. Shih, Phys. Rev. A **40**, 1456 (1989); Phys. Rev. D **42**, 3025 (1990).
[18] P. Carruthers and I. Sarcevic, Phys. Rev. Lett. **63**, 1562 (1989).
[19] M. Gyulassy, LBL Report No. 26831 (unpublished).
[20] F. Meijers, Ph.D thesis, University of Nijmegen, 1987; EHS/NA22 Collaboration, M. Adamus *et al.*, Phys. Lett. B **177**, 239 (1986).
[21] We require the slope parameter of the forward-backward correlations to be approximately 0.1 through an approximation assuming a linear relationship between $\langle n_B \rangle$ and the fixed n_F. This is consistent with data observed in [20]. Better solutions can be found through more detailed parametrizations.
[22] NA22 Collaboration, M. Adamus *et al.*, Z. Phys. C **37**, 215 (1988).
[23] EHS/NA22 Collaboration, M. Adamus *et al.*, Z. Phys. C **37**, 347 (1988).
[24] C. C. Shih and P. Carruthers, Phys. Rev. D **38**, 56 (1988).
[25] N. N. Biswas, Phys. Rev. Lett. **35**, 1059 (1975).
[26] C. C. Shih, in *Proceedings of Santa Fe Workshop on Intermittency in High Energy Collisions,* Santa Fe, NM, edited by F. Cooper, R. Hwa, and I. Sarcevic (World Scientific, Singapore, in press); in *Proceedings of the Correlations and Multiparticle Production (CAMP) Workshop,* Marburg, Germany, 1990, edited by M. Plümer, S. Raha, and R. M. Weiner (World Scientific, Singapore, in press); C. C. Shih, Phys. Lett. B **259**, 393 (1991).
[27] UA5 Collaboration, R. E. Ansorge *et al.*, Z. Phys. C **37**, 191 (1988).