Path-integral derivation of the superconformal anomaly for the Wess-Zumino model

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Fujikawa's method of evaluating the anomalies is extended to the on-shell supersymmetric (SUSY) theories. The supercurrent and the superconformal current anomalies are evaluated for the Wess-Zumino model using the background-field formulation and heat-kernel regularization. We find that the regularized Jacobians for SUSY and superconformal transformations are finite. The results can be expressed in a form such that there is no supercurrent anomaly but a finite nonzero superconformal anomaly, in agreement with similar results obtained using other methods.

Supersymmetry [1] (SUSY) has many interesting properties. One of the most important properties of SUSY theories is the cancellation of divergences between bosons and fermions. If SUSY becomes anomalous this cancellation may not occur. Hence it is important to know whether SUSY is respected by the quantum fluctuations.

It is known that [2] massless SUSY theories also exhibit a much larger superconformal symmetry. Under SUSY transformations, the divergence of the superconformal current, along with the trace of the energy-momentum tensor and the divergence of the chiral current, are members of a supermultiplet [3,2]. For a massless theory, these quantities vanish classically. However the trace of the energy-momentum tensor, and in some cases the divergence of the chiral current, are known to have anomalies. An important question is whether the anomalies in these currents also form a supermultiplet. To understand this issue it is necessary to evaluate the superconformal anomaly (along with the other anomalies, using the same regularization scheme).

Fujikawa, in his seminal papers [4], has shown that the chiral and the trace anomalies are related to the change in the functional measure under the corresponding chiral and dilation transformations respectively. Fujikawa's method has also been used earlier to evaluate the supercurrent and the superconformal current anomalies for some SUSY theories, such as the Wess-Zumino model or the $N=1$ SUSY Yang-Mills theory [5]. These calculations were performed off-shell, using the superfield formulation, with the background fields corresponding to a SUSY Yang-Mills theory and a supergravity theory respectively.

In this paper we work on-shell and the background fields used correspond to the same theory for which the anomalies are being computed. Apart from having the advantage of dealing directly with physical fields, such a procedure is essential for a theory where a complete off-shell formulation is not known. When one works with only physical fields, i.e., on-shell, the SUSY transformation laws become nonlinear. The extension of the background-field techniques and Fujikawa's method to evaluate the on-shell SUSY and superconformal anomaly has been an open problem. These problems are tackled in this paper.

We begin with the evaluation of the supercurrent anomaly for the Wess-Zumino model. All the fields are separated into small fluctuations ϕ around classical background values Φ. The fluctuations are integrated out in the functional integral. The partition function $W[\Phi]$ is then a functional of the background classical fields Φ: $W[\Phi]=\int\mathcal{D}[\phi]e^{S[\Phi+\phi]}$. Using Fujikawa's method we get the Ward identity that the SUSY variation of $W[\Phi]$ equals the Jacobian of the corresponding SUSY transformation of the fluctuation fields. This Jacobian is evaluated in a regularized way. The regulators are obtained by differentiating the quantum action twice. The quantum action is that part of the action which is bilinear in the fluctuations. The quantum action turns out to be nondiagonal in bosons and fermions. This would lead to boson-fermion mixed regulators. To simplify the calculations the quantum action is diagonalized.

It turns out that in terms of the diagonalizing variables the Jacobians depend only on the boson transformation law. Since only the fermion transformation law is nonlinear, the problem of nonlinearity of the on-shell SUSY transformation laws is thus avoided. However the regulator for the new bosonic variables has a nonlocal contribution. Hence the heat-kernel regularization procedure, which has so far been used only for local operators, has to be extended to include nonlocal operators too.

The regularization scheme used here treats bosons and fermions differently, i.e., nonsupersymmetrically. The regularized Jacobian is finite even when the limit $t\to 0$ is taken (here t is the heat-kernel-regulator parameter). All the divergent contributions from bosons cancel with those from fermions. The result can be expressed as the SUSY variation of a (harmless) local counterterm plus a total divergence. These terms can respectively be absorbed in the action to yield a modified effective action and in the supercurrent to yield a modified supercurrent, which is conserved. Thus there is no one-loop supercurrent anomaly for the on-shell Wess-Zumino model. The presence of nonsupersymmetric local counterterms is because of the manifestly nonsupersymmetric nature of our regularization scheme.

The calculation of the supercurrent anomaly is then used to evaluate the superconformal anomaly as follows. The superconformal transformation law for the boson is obtained by replacing the SUSY transformation parameter ε by $(i\not{x}\varepsilon)$. This is not true for the fermion transformation law. However, as mentioned earlier, our Jacobians depend only on the boson transformation law. Thus the superconformal anomaly is obtained by simply making the above substitution in the calculation for the supercurrent anomaly. The result is again finite. However it does have a contribution which cannot be expressed as a total divergence or, as the superconformal variation of a local counterterm. Hence the superconformal symmetry is anomalous. The results obtained here are in agreement with those obtained earlier using other methods [6].

In Sec. I we shall obtain the SUSY Ward identity using the background-field method [7]. The relevant regulators will be obtained in Sec. II and the Jacobians for SUSY transformations evaluated in Sec. III. In Sec. IV we shall derive the superconformal anomaly.

I. THE SUSY WARD IDENTITY

The Wess-Zumino model [8,2] is the simplest interacting supersymmetric model. It consists of a scalar $\mathbb{S}$, a pseudoscalar $\mathbb{P}$ and a Majorana fermion Ψ. The action for this model is given by

$$S=\int d^4x\{\tfrac{1}{2}(\partial_\mu \mathbb{S}_n\partial^\mu \mathbb{S}_n-m^2\mathbb{S}_n\mathbb{S}_n)+\bar{\Psi}[\tfrac{1}{2}(i\not{\partial}-m)-g\not{\mathbb{S}}^\dagger]\Psi-mg\mathbb{S}_1\mathbb{S}_n\mathbb{S}_n-(g^2/2)(\mathbb{S}_n\mathbb{S}_n)^2\}\ , \tag{1}$$

where the scalar and the pseudoscalar fields are compactly denoted by $\mathbb{S}_n=(\mathbb{S},\mathbb{P})$. Other conventions used are

$$g_{\mu\nu}=(+,-,-,-),\quad \{\gamma_\mu,\gamma_\nu\}=2g_{\mu\nu},\quad \gamma_5^\dagger=\gamma_5\ ,$$
$$\gamma_n=(1,-i\gamma_5),\quad \gamma_n^\dagger=(1,+i\gamma_5)\ ,$$
$$\not{\mathbb{S}}=\mathbb{S}-i\gamma_5\mathbb{P},\quad \not{\mathbb{S}}^\dagger=\mathbb{S}+i\gamma_5\mathbb{P}\ .$$

We shall absorb the coupling constant g in the fields and not write it explicitly. The SUSY transformation laws for this model are

$$\delta_\varepsilon \mathbb{S}_n=\bar{\Psi}\gamma_n\varepsilon,\quad \delta_\varepsilon\Psi=-[(i\not{\partial}+m+\not{\mathbb{S}})\not{\mathbb{S}}]\varepsilon\ . \tag{2}$$

$\mathbb{S}_n$ and Ψ are split up into classical background fields S_n and λ and fluctuation fields s_n and ψ as

$$\mathbb{S}_n=S_n+s_n,\quad \Psi=\lambda+\psi\ . \tag{3}$$

The fluctuation fields are to be integrated out in the functional integral. The SUSY transformation laws for these fields are

$$\delta_\varepsilon S_n=\bar{\lambda}\gamma_n\varepsilon,\quad \delta_\varepsilon\lambda=-[(i\not{\partial}+m+\not{S})\not{S}]\varepsilon\ , \tag{4}$$

$$\delta_\varepsilon s_n=\psi\bar{\gamma}_n\varepsilon,\quad \delta_\varepsilon\psi=-[(i\not{\partial}+m+\not{s}+2\not{S})\not{s}]\varepsilon\ . \tag{5}$$

Whereas the SUSY transformations of the background fields involve only the background fields, those of the fluctuation fields involve the fluctuation as well as the background fields.

In Fujikawa's method, the regularization of the Jacobian is carried out in Euclidean space. To go to Euclidean space we use the continuation prescription

$$t\to -it,\quad A_\mu B^\mu\to -A_\mu B_\mu,\quad \gamma_\mu\partial^\mu\to -i\gamma_\mu^E\partial_\mu\ , \tag{6}$$

with $(\gamma_\mu^E)^\dagger=+\gamma_\mu^E$. We shall use $\delta_{\mu\nu}$ as the Euclidean metric and drop the superscript E from the Hermitian Euclidean γ matrices. Further, in Euclidean space $\bar{\psi}$ and ψ are to be treated as independent Dirac fermions [9,10]. The Euclidean action and the Euclidean SUSY transformation laws corresponding to Eqs. (1), (4), and (5) are

$$S^E=\int d^4x\{-\tfrac{1}{2}(\partial_\mu \mathbb{S}_n\partial_\mu \mathbb{S}_n+m^2\mathbb{S}_n\mathbb{S}_n)+\bar{\Psi}[\tfrac{1}{2}(\not{\partial}-m)-g\not{\mathbb{S}}^\dagger]\Psi-m\mathbb{S}_1\mathbb{S}_n\mathbb{S}_n-\tfrac{1}{2}(\mathbb{S}_n\mathbb{S}_n)^2\}\ , \tag{7}$$

and

$$\delta_\varepsilon S_n=\tfrac{1}{2}(\bar{\lambda}\gamma_n\varepsilon+\bar{\varepsilon}\gamma_n\lambda)\ ,\quad \delta_\varepsilon\lambda=-[([\not{\partial}+m+\not{S})\not{S}]\varepsilon\ ; \tag{8}$$

$$\delta_\varepsilon s_n=\tfrac{1}{2}(\bar{\psi}\gamma_n\varepsilon+\bar{\varepsilon}\gamma_n\psi)\ ,\quad \delta_\varepsilon\psi=-[(\not{\partial}+m+\not{s}+2\not{S})\not{s}]\varepsilon\ . \tag{9}$$

Here we have written the scalar transformation in this symmetrized way for later convenience. It reduces to the usual SUSY transformation law of the scalar field, Eq. (5), in Minkowski space by the Majorana property of ε, λ, and ψ. To obtain the SUSY Ward identity, we start with the exponential of the Euclidean effective action in the presence of the background fields S_n and λ:

$$W^E=\int \mathcal{D}[s_n]\mathcal{D}[\bar{\psi}]\mathcal{D}[\psi]\exp(S^E[\mathbb{S},\Psi])\ . \tag{10}$$

The change in W^E under the SUSY transformations of the background fields S_n and λ, given by Eqs. (8), is

$$\delta_\varepsilon W^E=\int \mathcal{D}[s_n]\mathcal{D}[\bar{\psi}]\mathcal{D}[\psi]\exp(S^E[\mathbb{S},\Psi])\times\left[\delta_\varepsilon S_n\frac{\delta}{\delta_\varepsilon S_n}+\delta_\varepsilon\lambda\frac{\delta}{\delta_\varepsilon\lambda}\right]S^E\ . \tag{11}$$

On the right-hand side we make a change of integration variables given by $s_n\to s_n+\delta_\varepsilon s_n$, $\bar{\psi}\to\bar{\psi}+\delta_\varepsilon\bar{\psi}$, and $\psi\to\psi+\delta_\varepsilon\psi$, where the variations δ_ε are as defined in Eq. (9), and retain terms only linear in the infinitesimal transformation parameter ε. The variation of the fluctuating fields combines with the already present variation of the background fields to give a total supersymmetry variation of the classical Euclidean action $\delta_\varepsilon S^E[\mathbb{S},\Psi]$. Further the variation of the measure gives a Jacobian $1+J^E(\varepsilon)$. Thus,

$$\delta_\varepsilon W^E=\int \mathcal{D}[s_n]\mathcal{D}[\bar{\psi}]\mathcal{D}[\psi]\exp(S^E[\mathbb{S},\Psi])\times\{\delta_\varepsilon S^E[\mathbb{S},\Psi]+J^E(\varepsilon)\}\ . \tag{12}$$

Since in Euclidean space ψ and $\bar{\psi}$ are not Majorana fermions, the boson-fermion degrees of freedom do not match. Therefore the action S^E is not invariant under rigid SUSY transformations [10]. The variation of the action is proportional to $\bar{\Psi}(\delta_\varepsilon \mathbb{S})\Psi$ which is nonzero in Euclidean space. However, when continued back to Minkowski-space, such a term vanishes by Fierz rearrangement, and we get the required Minkowski-space Ward identity:

$$\delta_\varepsilon W=\int \mathcal{D}[s_n]\mathcal{D}[\bar{\psi}]\mathcal{D}[\psi]\exp(iS[\mathbb{S},\Psi]) \times \left[i\int d^4x\, \bar{Q}_\mu \partial^\mu \varepsilon + J(\varepsilon)\right] . \tag{13}$$

Here

$$W=\int \mathcal{D}[s_n]\mathcal{D}[\bar{\psi}]\mathcal{D}[\psi]\exp(iS[\mathbb{S},\Psi]) \tag{14}$$

is the Minkowski-space vacuum-to-vacuum amplitude and the variation of the Minkowski action S is given by

$$\delta_\varepsilon Si = -i\int d^4x\, \bar{\Psi}\gamma_\mu[m\mathbb{S}+i\partial\!\!\!/\mathbb{S}+(\mathbb{S})^2]\partial^\mu\varepsilon = \int d^4x\, \bar{Q}_\mu \partial^\mu \varepsilon . \tag{15}$$

Here $\bar{Q}_\mu$ is the supercurrent. For rigid SUSY transformations, $\delta_\varepsilon S$ vanishes and $J(\varepsilon)$ gives the supercurrent anomaly.

II. EVALUATION OF THE REGULATORS

As we shall see in the next section, the Jacobian $J^E(\varepsilon)$ in (12) consists of the functional traces of certain quantities. These are ill defined and hence should be regulated. The heat-kernel regulators $h(x,y;t)=\langle y|e^{tR}|x\rangle$, with R as some appropriate negative-definite operator, will be used for this purpose. In Fujikawa's method, R is related to the operator obtained by double functional differentiation of those terms of the Euclidean action which are quadratic in the fluctuation fields. For this purpose we collect all the terms in S^E which are bilinear in the fluctuation fields, s_n, $\bar{\psi}$, and ψ, and denote them by S^Q the quantum action

$$S^Q=\int d^4x\{\tfrac{1}{2}s_n(\Box-m^2)s_n+\bar{\psi}[\tfrac{1}{2}(\partial\!\!\!/-m)-g\mathbb{S}^\dagger]\psi - \bar{\lambda}s\!\!\!/^\dagger\psi-\bar{\psi}s\!\!\!/^\dagger\lambda - mS_1s_n^2-2ms_1S_ns_n - S_n^2s_n^2+2S_ns_nS_ms_m\} . \tag{16}$$

This action is not diagonal in s_n and ψ and hence would lead to boson-fermion mixed regulators, which are not convenient to handle. To simplify calculations, the quantum action should be diagonalized. This is done by the change of variables

$$\begin{aligned} &s_n \to s_n ,\\ &\psi(x)\to\psi'(x)=\psi(x)-\int \tilde{G}(x,y)s\!\!\!/^\dagger(y)\lambda(y)d^4y ,\\ &\bar{\psi}(x)\to\bar{\psi}'(x)=\bar{\psi}(x)-\int \bar{\lambda}(y)s\!\!\!/^\dagger(y)\tilde{G}'(x,y)d^4y , \end{aligned} \tag{17}$$

where the Green's functions $\tilde{G}$ and $\tilde{G}'$ are defined as

$$\begin{aligned} &[\tfrac{1}{2}(\partial\!\!\!/-m)+\mathbb{S}^\dagger]_x\tilde{G}(x,y)=\delta^4(x-y) ,\\ &\tilde{G}'(x,y)[\tfrac{1}{2}(-\overleftarrow{\partial\!\!\!/}+m)+\mathbb{S}^\dagger]_x=\delta^4(x-y) . \end{aligned} \tag{18}$$

Since the scalar fields are unchanged in (17), the Jacobian of the above change of variables is trivially 1. The diagonalized action is

$$S^Q=\int d^4x\left[\tfrac{1}{2}s_n(\Box-m^2)s_n+\bar{\psi}'[\tfrac{1}{2}(\partial\!\!\!/-m)-g\mathbb{S}^\dagger]\psi' - mSs_n^2-2ms_1S_ns_n-S_n^2s_m^2-2S_ns_nS_ms_m - \int d^4y\,\bar{\lambda}(x)s\!\!\!/^\dagger(x)\tilde{G}(x,y)s\!\!\!/^\dagger(y)\lambda(y)\right] . \tag{19}$$

Differentiating S^Q twice with respect to the bosonic variables yields $B_{mn}(x,y)$, the bosonic regulator operator:

$$B_{mn}(x,y)=B'_{mn}(x,y)-\bar{\lambda}(x)\gamma_m^\dagger\tilde{G}(x,y)\gamma_n^\dagger\lambda(y) - \bar{\lambda}(y)\gamma_n^\dagger\tilde{G}(y,x)\gamma_m^\dagger\lambda(x) , \tag{20}$$

where

$$B'_{mn}(x,y)=[(\Box-m^2-2mS-2S^2)\delta_{mn} - 2m(\delta_{1n}S_m+\delta_{1m}S_n)-4S_mS_n]\delta(x-y) \tag{21}$$

is the local part of the bosonic regulator. Now we consider the fermion regulator. Differentiating S^Q with respect to $\bar{\psi}'$ and ψ' yields

$$D\!\!\!\!/=[\tfrac{1}{2}(\partial\!\!\!/-m)-g\mathbb{S}^\dagger] . \tag{22}$$

Further

$$D\!\!\!\!/^\dagger=[\tfrac{1}{2}(-\partial\!\!\!/-m)-g\mathbb{S}] . \tag{23}$$

Since $D\!\!\!\!/$ does not have definite Hermiticity property, $e^{tD\!\!\!\!/^2}$ cannot be used as the fermion regulator. However, in Euclidean space $\bar{\psi}'$ and ψ' are independent fermions and hence can be regulated differently. We choose the regulators for $\bar{\psi}'$ and ψ' to be $e^{t\bar{F}}$ and e^{tF} respectively [10], where

$$F=-4D\!\!\!\!/^\dagger D\!\!\!\!/=\Box-(m^2+4S^2+4mS_1+2\partial\!\!\!/\mathbb{S}^\dagger) \tag{24}$$

and

$$\bar{F}=-4D\!\!\!\!/D\!\!\!\!/^\dagger=\Box-(m^2+4S^2+4mS_1-2\partial\!\!\!/\mathbb{S}) . \tag{25}$$

As desired, all the regulator operators, Eqs. (20), (24), and (25) are diagonal in bosons and fermions. The fermionic regulators F and $\bar{F}$ are completely local; however, the bosonic regulator B_{mn} has a nonlocal part. Further, our regularization procedure is not manifestly supersymmetric. This will lead to nonsupersymmetric counterterms, but the expressions for the supercurrent and the superconformal anomaly are expected to be independent of the regularization procedure used.

The SUSY transformation laws of the new variables are found using (9) and (17) to be

$$
\begin{aligned}
\delta_\varepsilon \psi' = \{ & \delta_\varepsilon \psi - (\delta_\varepsilon \tilde{G}) \not{s}^\dagger \lambda - \tilde{G} \not{s}^\dagger \delta_\varepsilon \lambda \\
& - \tfrac{1}{2} \tilde{G} \gamma_n^\dagger \lambda (\bar{\psi} \gamma_n^\dagger \varepsilon + \bar{\varepsilon} \gamma_n \tilde{G} \not{s}^\dagger \lambda) \} \\
& - \tfrac{1}{2} \tilde{G} \gamma_n^\dagger \lambda (\bar{\varepsilon} \gamma_n \psi') , \\
\delta_\varepsilon \bar{\psi}' = \{ & \delta_\varepsilon \bar{\psi} - (\delta_\varepsilon \bar{\lambda}) \not{s}^\dagger \tilde{G}' - \bar{\lambda} \not{s}^\dagger \delta_\varepsilon \tilde{G}' \\
& - \tfrac{1}{2} (\bar{\varepsilon} \gamma_n \psi + \bar{\lambda} \not{s}^\dagger \tilde{G}' \gamma_n \varepsilon) \bar{\lambda} \gamma_n^\dagger \tilde{G}' \} \\
& - \tfrac{1}{2} (\bar{\psi}' \gamma_n \varepsilon) \bar{\lambda} \gamma_n^\dagger \tilde{G}' , \qquad (26) \\
\delta_\varepsilon s_n = & \tfrac{1}{2} \{ \bar{\psi}' \gamma_n \varepsilon + \bar{\varepsilon} \gamma_n \psi' \} \\
& + \tfrac{1}{2} (\bar{\varepsilon} \gamma_n \tilde{G} \not{s}^\dagger \lambda + \bar{\lambda} \not{s}^\dagger \tilde{G}' \gamma_n \varepsilon) .
\end{aligned}
$$

The SUSY transformations of the new variables contain, in addition to other fields, the transformed variables themselves. This is unlike the SUSY transformations (9) of the old variables. We have collected in the curly brackets all the terms which do not contain the variable being transformed. These terms do not contribute to the Jacobian, as will be shown presently. The form of the Jacobian, as is clear from the SUSY transformation laws (26), is

$$1 + J(\varepsilon) = \mathrm{SDet} \left[1 + \begin{bmatrix} A(\varepsilon) & B(\varepsilon) \\ C(\varepsilon) & D(\varepsilon) \end{bmatrix} \right] ,$$

where A, B, C and D are some functions, linear in ε and SDet is the superdeterminant. The boson and the fermion regulators R_b and R_f can be combinedly written in a diagonal form as

$$R = \begin{bmatrix} R_b & 0 \\ 0 & R_f \end{bmatrix} .$$

The regulated Jacobian is

$$
\begin{aligned}
1 + J^R(\varepsilon) &= \mathrm{SDet} \left[1 + \begin{bmatrix} A(\varepsilon) R_b & B(\varepsilon) R_f \\ C(\varepsilon) R_b & D(\varepsilon) R_f \end{bmatrix} \right] \\
&= 1 + \mathrm{Tr}(A R_b) - \mathrm{Tr}[D R_f] + O(\varepsilon^2) .
\end{aligned}
$$

Here the minus sign in front of the fermion term comes from the fact that the fermionic Jacobian is an inverse determinant as contrasted to the bosonic Jacobian. To lowest order in ε, only the diagonal elements of $J^R(\varepsilon)$ contribute. They arise due to those terms in the transformations (26) which contain the transformed variables themselves. Similarly for the diagonalizing variables (17), the parts of SUSY transformations which do not contain the variables being transformed will not contribute anything to the Jacobian up to order ε. All the terms in the curly brackets in Eqs. (26) are of this form and hence will be ignored.

One of the problems encountered in extending Fujikawa's method to on-shell SUSY theories is the nonlinearity of the on-shell SUSY transformation laws. Note however that the parts of the SUSY transformations (26) that do contribute to the Jacobian depend only on the boson transformation law, and hence are linear in the variable being transformed. Thus we have circumvented the problem of nonlinearity of the SUSY transformation laws. The discussion presented above is quite general and applies to all the SUSY theories in flat spacetime. The fact that the Jacobians are independent of the fermion transformation law will also be useful later. It will enable us to carry over the calculation for the supercurrent Jacobian to that of the superconformal Jacobian.

III. EVALUATION OF THE JACOBIANS

As mentioned earlier, the Jacobians are ill defined and have to be regulated. To do so, we start by regulating the transformations themselves. This will simultaneously regulate the Jacobian and the divergence of the supercurrent. Thus we directly get the regularized Ward identity. The fields s_n, ψ', and $\bar{\psi}'$ are regulated using the heat kernels h_{mn}, h, and $\bar{h}$, respectively, where

$$
\begin{aligned}
h_{mn}(x,y;t) &= \langle y | \exp(t B_{mn}) | x \rangle , \\
h'(x,y;t) &= \langle y | \exp(t F) | x \rangle , \qquad (27) \\
\bar{h}'(x,y;t) &= \langle y | \exp(t \bar{F}) | x \rangle .
\end{aligned}
$$

The operators B_{mn}, F, and $\bar{F}$ are as defined in Eqs. (20), (24), and (25) respectively. Using these heat kernels we get the regularized transformation laws corresponding to Eqs. (26) to be

$$
\begin{aligned}
\delta_\varepsilon \psi'(x) &= -\tfrac{1}{2} \int d^4y \, \tilde{G}(x,y) \gamma_n^\dagger \lambda(y) \left[\bar{\varepsilon}(y) \gamma_n \int d^4y' h'(y,y';t) \psi'(y') \right] , \\
\delta_\varepsilon \bar{\psi}'(x) &= -\tfrac{1}{2} \int d^4y \left[\int d^4y' \bar{\psi}'(y') \bar{h}'(y',y;t) \gamma_n \varepsilon(y) \right] \bar{\lambda}(y) \gamma_n^\dagger \tilde{G}'(y,x) , \qquad (28) \\
\delta_\varepsilon s_n(x) &= \tfrac{1}{2} \int d^4y \, \bar{\lambda}(y) \int d^4y' s_1(y') h_{lm}(y',y;t) \gamma_m^\dagger \tilde{G}(y,x) \gamma_n \varepsilon(x) \\
&\quad + \tfrac{1}{2} \int d^4y \, \bar{\varepsilon}(x) \gamma_n \tilde{G}(x,y) \gamma_m^\dagger \int d^4y h_{ml}(y,y';t) s_l(y') \lambda(y) .
\end{aligned}
$$

Since, in the limit $t \to 0$ the heat kernel tends to the δ function, (28) reduce to (26) in this limit.

The regularized Jacobian for the SUSY transformation of the scalar fields is

$$
\begin{aligned}
\mathrm{Det} \left[\frac{\delta(s_n(x) + \delta_\varepsilon s_n(x))}{\delta s_m(x')} \right] &= 1 + \tfrac{1}{2} \int d^4x \int d^4y [\bar{\lambda}(y) h_{mn}(x,y;t) \gamma_m^\dagger \tilde{G}(y,x) \gamma_n \varepsilon(x) \\
&\quad + \bar{\varepsilon}(x) \gamma_n \tilde{G}(x,y) \gamma_m^\dagger h_{mn}(y,x;t) \lambda(y)] + O(\varepsilon^2) . \qquad (29)
\end{aligned}
$$

Similarly we have the ψ' and $\bar{\psi}'$ Jacobians as

$$\left[\mathrm{Det}\left[\frac{\delta(\psi'(x)+\delta_\varepsilon\psi'(x))}{\delta\psi'(x')}\right]\right]^{-1}=1-\tfrac{1}{2}\int d^4x\int d^4y\bar{\varepsilon}(y)\gamma_n h'(y,x;t)\tilde{G}(x,y)\gamma_n^\dagger\lambda(y)+O(\varepsilon^2) \tag{30}$$

and

$$\left[\mathrm{Det}\left[\frac{\delta(\bar{\psi}'(x)+\delta_\varepsilon\bar{\psi}'(x))}{\delta\bar{\psi}'(x')}\right]\right]^{-1}=1-\tfrac{1}{2}\int d^4x\int d^4y\bar{\lambda}(y)\gamma_n^\dagger\tilde{G}'(y,x)\bar{h}'(x,y;t)\gamma_n\varepsilon(y)+O(\varepsilon^2)\ . \tag{31}$$

Being fermion Jacobians, these are inverse determinants, in contrast with the bosonic Jacobian (29) above. The product of all these Jacobians leads to the following contribution $J^E(\varepsilon)$ in Eq. (12):

$$J^E(\varepsilon)=\tfrac{1}{2}\int d^4x\int d^4y\{[\bar{\varepsilon}(y)\gamma_n h_{mn}(x,y;t)-\delta_{mn}\bar{\varepsilon}(x)\gamma_n h'(x,y;t)]\tilde{G}(y,x)\gamma_m^\dagger\lambda(x)$$
$$+\bar{\lambda}(x)\gamma_m^\dagger\tilde{G}'(x,y)[h_{nm}(y,x;t)\gamma_n\varepsilon(y)-\delta_{mn}\bar{h}'(y,x;t)\gamma_n\varepsilon(x)]\}\ . \tag{32}$$

Our task now is to evaluate these Jacobians. This is done using the short-distance expansion [11]. Since eventually we will take the limit $t\to 0$, the heat kernels will contribute only when their arguments x and y are close to each other. In this coincident point limit the heat kernels can be evaluated as follows. The regulator operators F, $\bar{F}$, and B'_{mn}—the local part of the B_{mn}—have a generic form:

$$R=\Box+2X\cdot\partial+\partial\cdot X+X^2+Y\ . \tag{33}$$

Here X_μ and Y are matrix valued and they do not involve any differential operators. From Eqs. (20), (24), and (25), we see that $X_\mu=0$ for all the regulator operators, and

$$y=-[(m^2+2mS_1+2S^2)\delta_{mn}$$
$$+2m(\delta_{1n}S_m+\delta_{1m}S_n)+4S_mS_n]\ , \tag{34}$$

$$y=-(m^2+4S^2+4mS_1+2\partial\!\!\!/ \mathcal{S}^\dagger)\ , \tag{35}$$

$$y=-(m^2+4S^2+4mS_1-2\partial\!\!\!/ \mathcal{S})\ , \tag{36}$$

for B'_{mn}, F, and $\bar{F}$, respectively. The generic heat kernel corresponding to (33) is

$$h(y,x,t)=\langle x|e^{tR}|y\rangle \tag{37}$$

which satisfies the heat equation

$$\frac{\partial}{\partial t}h=Rh\ , \tag{38}$$

subject to the boundary condition

$$\lim_{t\to 0}h(y,x,t)=\delta^4(x-y)\ . \tag{39}$$

For small t the heat kernel has an asymptotic expansion [11]

$$h(y,x,t)=\frac{e^{-(x-y)^2/4t}}{16\pi^2t^2}\sum_{k=0}^{\infty}a_k(y,x)t^k\ , \tag{40}$$

where a_k are the expansion coefficients called the Meenakshisundaram-DeWitt-Hadamard heat coefficients. These will now be evaluated. Substituting (40) in the heat equation (38) yields the recursion relation

$$[k+z_\mu(\partial_\mu+X_\mu)]a_k(y,x)=Ra_{k-1}(y,x). \tag{41}$$

Here $z_\mu=(y_\mu-x_\mu)$. The boundary condition (39) translates into

$$a_0(y,x)|_{y=x}=1\ . \tag{42}$$

Using the form of R, Eq. (42), and successive differentiation of Eq. (41), various coefficients a_k and their derivatives can be evaluated in the coincident point $(y\to x)$ limit. This is done in Appendix A.

Let us define a heat kernel from the local part of the bosonic regulator B'_{mn} (21) as

$$h'_{mn}(x,y;t)=\langle y|\exp(tB'_{mn})|x\rangle\ . \tag{43}$$

h'_{mn}, h', and $\bar{h}'$ can be expanded in the same manner as (40). The corresponding heat coefficients are denoted by $(a'_k)_{mn}$, a'_k, and $\bar{a}'_k$ respectively. As will be explained, at most the third derivative of a_0 and the first derivative of a_1 are needed for our purpose. The derivative with respect to the first or the second argument of a_k will be denoted by ∂_μ or $\bar{\partial}_\mu$ respectively. The relevant heat coefficients, as given in Eq. (A11) of Appendix A are listed below:

$$[a'_0(x,y)]_{mn}|_{y=x}=\delta_{mn},\quad a'_0|=\bar{a}'_0=1\ ,$$

all derivatives of $(a'_0)_{mn}$, a'_0, and $\bar{a}'_0$ are equal to zero, and

$$
\begin{aligned}
&(a_1')_{mn}| = -[(m^2+2mS_1+2S^2)\delta_{mn}+2m(\delta_{1n}S_m+\delta_{1m}S_n)+4S_mS_n] , \\
&(\partial_\mu a_1')_{mn}| = (\bar{\partial}_\mu a_1')_{mn}| = -[(mS_1+S^2)\delta_{mn}+m(\delta_{1n}S_m+\delta_{1m}S_n)+2S_mS_n] , \\
&a_1'| = -(m^2+4S^2+4mS_1+2\not{\partial}\not{S}^{\dagger}), \quad (\partial_\mu a_1')| = (\bar{\partial}_\mu a_1')| = -\partial_\mu(2S^2+2mS_1+\not{\partial}\not{S}^{\dagger}) , \\
&\bar{a}_1'| = -(m^2+4S^2+4mS_1-2\not{\partial}\not{S}), \quad (\partial_\mu \bar{a}_1')| = (\bar{\partial}_\mu \bar{a}_1')| = -\partial_\mu(2S^2+2mS_1-\not{\partial}\not{S}) .
\end{aligned}
\tag{44}
$$

To evaluate (32) we also need to know the Green's functions $\tilde{G}$ and $\tilde{G}'$ in the limit $y \to x$. This can be done as follows. Define a Green's function $G(x,y)$ such that

$$G(x,y)\overleftarrow{F}_y = \delta^4(x-y) ,$$

where F_y is defined in Eq. (24). G can be expressed as

$$G(x,y) = \int_0^\infty dt\, e^{tF} = \lim_{\tau\to\infty} \int_0^\tau h'(x,y;t)dt .$$

Substituting the asymptotic expansion (40) of h gives

$$G(x,y) = \frac{1}{4\pi^2}\left[\frac{a_0'(x,y)}{z^2} + \frac{1}{4}\ln\left[\frac{z^2}{4\tau}\right]a_1'(x,y) + \cdots\right] ,$$

where the limit $\tau \to \infty$ is not written explicitly. The Green's function $\tilde{G}$, defined in (18), also satisfies [using (22)] $\tilde{G}(x,y)\overleftarrow{\not{D}}_y = \delta(x-y)$. ($\overleftarrow{\not{D}}$ is obtained from $\not{D}$ by a partial integration.) Using $\overleftarrow{F} = -4\overleftarrow{\not{D}}^{\dagger}\overleftarrow{\not{D}}$ gives

$$
\begin{aligned}
\tilde{G}'(x,y) &= -4G(x,y)\overleftarrow{\not{D}}_y^{\dagger} \\
&= \frac{1}{\pi^2}\left[-\frac{\not{z}}{z^4} - \frac{1}{z^2}\left[\frac{m}{2}+\not{S}(y)\right] - \frac{a_1'(x,y)}{4z^2}\not{z}\right] .
\end{aligned}
\tag{45}
$$

Here the expression (23), for $\overleftarrow{\not{D}}_y^{\dagger}$, and (44) have been used. The other Green's function $\tilde{G}$, defined in (16), can be evaluated using a similar procedure to yield

$$\tilde{G}(y,x) = \frac{1}{\pi^2}\left[\frac{\not{z}}{z^4} - \frac{1}{z^2}\left[\frac{m}{2}+\not{S}(y)\right] + \frac{\not{z}}{4z^4}\bar{a}_1'(y,x)\right] . \tag{46}$$

Since the coefficients a_1' and $\bar{a}_1'$ are known in the coincident point limit, so are $\tilde{G}$ and $\tilde{G}'$.

The bosonic regulator (19) has a nonlocal contribution. To begin with, the contribution of the nonlocal part has to be separated from that of the local part. This is done in Appendix B. Using the equation (B11), the Jacobian (32) can be written as

$$
\begin{aligned}
J^E(\varepsilon) = {}& \tfrac{1}{2}\int d^4x \int d^4y \{[\varepsilon(y)\gamma_n h_{mn}'(y,x;t) - \delta_{mn}\bar{\varepsilon}(x)\gamma_n h'(x,y;t)]\tilde{G}(y,x)\gamma_m^{\dagger}\lambda(x) \\
&+ \bar{\lambda}(x)\gamma_m^{\dagger}\lambda(y)\tilde{G}'(x,y)[h_{nm}'(y,x;t)\gamma_n\varepsilon(y) - \delta_{mn}\bar{h}'(y,x;t)\gamma_n\varepsilon(x)]\} \\
&- \frac{t}{2}\int d^4x \int d^4y \Big[\bar{\lambda}(x)\gamma_m^{\dagger}\tilde{G}'(x,y)\gamma_n\varepsilon(y)\int d^4x'[\bar{\lambda}(y)\gamma_m^{\dagger}\tilde{G}(y,x')\gamma_l^{\dagger}\lambda(x') + \bar{\lambda}(x')\gamma_l^{\dagger}\tilde{G}(x',y)\gamma_m^{\dagger}\lambda(y)]h_{ln}'(n',x;t) \\
&+ \bar{\varepsilon}(x)\gamma_m^{\dagger}\tilde{G}(x,y)\gamma_n\lambda(y)\int d^4x'[\bar{\lambda}(y)\gamma_m^{\dagger}\tilde{G}(y,x')\gamma_l^{\dagger}\lambda(x') + \bar{\lambda}(x')\gamma_l^{\dagger}\tilde{G}(x',y)\gamma_m^{\dagger}\lambda(y)]h_{ln}'(x',x;t)\Big] .
\end{aligned}
\tag{47}
$$

We are now in a position to evaluate the Jacobian (47). The contributions from the fermionic Jacobian and the local part of the bosonic Jacobian, i.e., the term in the first set of curly brackets in Eq. (47), will be evaluated first. To make a short-distance expansion, we expand all functions of y around x as

$$y = x + z = x + 2\sqrt{t}\,z' .$$

The new integration variables are x and z'. The limit $t \to 0$ will be taken only at the end. Hence only the contributions proportional to t^n, $n \le 0$, will survive. The terms with negative powers of t or with $\ln(z^2/4\tau)$ as a coefficient will correspond to divergent contributions. The Green's functions $\tilde{G}$ and $\tilde{G}'$ have the most singular contribution proportional to $t^{-3/2}$ coming from the first terms in Eqs. (45) and (46). Hence we need to evaluate only the first three derivatives of the heat coefficient a_0 and the first derivative of a_1, at coincident points (these are proportional to t^n, $0 \le n \le \frac{3}{2}$).

We now substitute (44)–(46) in the first curly brackets in Eq. (47), and collect all the possible terms with coefficients t^n, $n \le 0$. The terms with odd powers of z will vanish because the rest of the integrand in (47) is an even function of z. We also have

$$\int_{-\infty}^{\infty} e^{-z^2} d^4 z = \int \frac{e^{-z^2}}{z^2} d^4 z = \pi^2$$

and

$$\int e^{-z^2} \frac{z_\mu z_\nu}{z^2} d^4 z = \frac{\pi^2 \delta_{\mu\nu}}{4} .$$

Further, $\gamma_n = (1, -i\gamma_5)$, therefore $\gamma_n^\dagger \gamma_n = 2$ and $\gamma_n^\dagger \gamma_\mu \gamma_n = 0$. Using all this information and going through long but straightforward algebra gives

$$J^E(\varepsilon) = \frac{1}{2\pi^2} \int d^4x \left[\left\{ -\frac{1}{8} \bar{\varepsilon} \not\partial (\mathcal{S}^\dagger(\mathcal{S}^\dagger + m) - \not\partial \mathcal{S}) + \frac{1}{4} \partial_\mu \bar{\varepsilon} \left[-\overleftarrow{\partial}_\mu \left[\frac{m}{2} + \mathcal{S} \right] - 2\partial_\mu \mathcal{S} - \gamma_\mu (-\mathcal{S}^\dagger(\mathcal{S}^\dagger + m) + \not\partial \mathcal{S}) \right] \right\} \lambda \right.$$
$$\left. + \frac{1}{8} \bar{\lambda} \not\partial [(\mathcal{S} + m + \not\partial)\mathcal{S}] \varepsilon + \frac{1}{4} \bar{\lambda} \left[-2\partial_\mu \mathcal{S} \partial_\mu + (\mathcal{S}^\dagger + m - \not\partial)\mathcal{S}^\dagger \not\partial - \left[\frac{m}{2} + \mathcal{S} \right] \Box \right] \varepsilon \right] + (3\lambda \text{ contribution}) . \quad (48)$$

It is worth noting that all the divergent contributions have canceled off and expression (48) is finite. We now turn to evaluation of the 3λ terms, i.e., the term in the second set of curly brackets in (47), coming from the nonlocal contribution of the bosonic regulator. These terms contain products of Green's functions. The generic form of such a term is

$$N(y,x) = \int d^4u [\tilde{G}(y,u)][\tilde{G}(u,x)][f(u)] , \quad (49)$$

where $f(u)$ is some nonsingular function involving λ, ε, and S. The spinor indices have been suppressed for convenience. The entire 3λ contribution is multiplied by t and hence only the terms proportional to t^n, with $n \le 1$, will contribute to the Jacobian. Since $\tilde{G}$ is of dimension 3, the product of two $\tilde{G}$ is of dimension 2; hence the most singular term in N is proportional to t^{-1}, and only this term will contribute to the Jacobian. Such a term comes from the most singular term in $\tilde{G}$. For this purpose $\tilde{G}$, given by (46), can be written as

$$\tilde{G}(y,x) = G_\mu[\gamma_\mu] + (\text{less singular terms}) ,$$

where

$$G_\mu = \frac{1}{\pi^2} \frac{z_\mu}{z^4} .$$

Thus

$$N(y,x) = \int d^4u [\tilde{G}(y,u) G_\mu(u,x)][\gamma_\mu][f(u)] . \quad (50)$$

The general form of the most singular, i.e., t^{-1} or equivalently z^{-2}, term in N is

$$N(y,x) = \frac{1}{\pi^2} \left[\frac{a\gamma_\mu}{z^2} + b \frac{z_\mu \not z}{z^4} \right] [\gamma_\mu][f(y)] , \quad (51)$$

where a and b are arbitrary coefficients to be fixed as follows. Since $\not D \tilde{G}(y,x) = \delta^4(x,y)$, operating on Eq. (50) with $\not D$ yields

$$\not D N(y,x) = \tilde{G}(y,x) f(y)$$
$$= \frac{\not z}{\pi^2 z^4} f(y) + (\text{less singular terms}) . \quad (52)$$

Operating on (51) by $\not D$ and comparing the result with (52) gives $a = -\frac{1}{2}$ and $b = 1$. Using similar arguments, the other combinations of products of Green's functions appearing in (47) can be evaluated. Thus

$$\int [\tilde{G}(y,u)\tilde{G}(u,x)][f(u)] d^4u$$
$$= N(y,x)$$
$$= \frac{1}{\pi^2} \left[-\frac{\gamma_\mu}{2z^2} + \frac{\not z z_\mu}{z^4} \right] [\gamma_\mu] f(y) ,$$
$$\int [\tilde{G}(y,u)][\tilde{G}(x,u)] f(u) d^4u = -N(y,x) ,$$
$$\int [\tilde{G}(u,y)][\tilde{G}(u,x)] f(u) d^4u = -N(y,x) , \quad (53)$$
$$\int [\tilde{G}(u,y)][\tilde{G}(x,u)] f(u) d^4u = +N(y,x) .$$

Note that the most singular terms in $\tilde{G}$ and $\tilde{G}'$, i.e., the first terms in (45) and (46), are identical. Hence the results listed above are unchanged if $\tilde{G}$ is replaced by $\tilde{G}'$.

Substituting these expansions in (42) gives the total 3λ contribution:

$$-\frac{t}{2} \int d^4x \int d^4z \frac{e^{-z^2/4t}}{16\pi^2 t^2} \frac{2}{\pi^2} \left[\left\{ \bar{\lambda} \gamma_m^\dagger \left[-\frac{\gamma_\mu}{2z^2} \right] \gamma_n \varepsilon \bar{\lambda} (\gamma_n^\dagger \gamma_\mu \gamma_m^\dagger - \gamma_m^\dagger \gamma_\mu \gamma_n^\dagger) \lambda \right. \right.$$
$$\left. \left. + \left[\bar{\varepsilon} \gamma_n \frac{\not z}{z^4} \gamma_m^\dagger \lambda \right] \bar{\lambda} [\gamma_m^\dagger \not z \gamma_n^\dagger - \gamma_n^\dagger \not z \gamma_m^\dagger] \lambda \right\} + \{\varepsilon \to \bar{\varepsilon}\} \right] . \quad (54)$$

Since $\gamma_n=(1,-i\gamma_5)$, it is easy to see that both terms in curly brackets vanish. Thus there is no 3λ contribution to the Jacobian. The calculation of the Jacobian (47), in Euclidean space is now complete.

To obtain the Minkowski-space Ward identity, (48) is analytically continued using the prescription $t\rightarrow it$, $A_\mu B_\mu \rightarrow -A_\mu B^\mu$, $\gamma_\mu^E\partial_\mu \rightarrow i\partial\!\!\!/$. The Majorana property of the fermions can now be used to see that the λ and $\bar{\lambda}$ contributions are identical and hence they add up. The resultant Jacobian can be cast in a convenient form:

$$J(\varepsilon)=\frac{i}{\pi^2}\int d^4x\left[-\frac{i}{8}\lambda\overleftarrow{\partial\!\!\!/}(\mathcal{S}+m+i\partial\!\!\!/)\mathcal{S}+\bar{C}_\mu\partial^\mu\right]\varepsilon\ , \qquad (55)$$

where

$$\bar{C}_\mu=\tfrac{1}{8}\bar{\lambda}[-2\partial_\mu(\mathcal{S}+m/2)+i(\mathcal{S}^\dagger+m+3i\partial\!\!\!/)\gamma_\mu\mathcal{S}+4\partial_\mu\mathcal{S}]\ . \qquad (56)$$

Evidently, the first term in (55) can be written as the SUSY variation of a local term, and the second term has only derivatives of ε. Thus

$$J(\varepsilon)=\frac{i}{\pi^2}\int d^4x\left[-\frac{i}{16}\delta_\varepsilon(\bar{\lambda}\partial\!\!\!/\lambda)+\bar{C}_\mu\partial^\mu\varepsilon\right]\ . \qquad (57)$$

Substituting for $J(\varepsilon)$ in (13) gives the supercurrent Ward identity. In terms of the modified effective action, defined as

$$W'[S,\lambda]=W[S,\lambda]\exp\left[\frac{i}{16\pi^2}\int d^4x\ i\bar{\lambda}\partial\!\!\!/\lambda\right]\ , \qquad (58)$$

and the modified supercurrent defined as [$\bar{Q}_\mu$ is given by (15)]

$$\bar{Q}'_\mu=\bar{Q}_\mu+\frac{1}{\pi^2}\bar{C}_\mu\ , \qquad (59)$$

the Ward identity (13) can be written as

$$\delta_\varepsilon W'[S,\lambda]=\int\mathcal{D}[s_n]\mathcal{D}[\bar{\psi}]\mathcal{D}[\psi]\exp(iS[\mathrm{S},\Psi])\times\left[i\int\bar{Q}'_\mu\partial^\mu\varepsilon\right]\ . \qquad (60)$$

Thus the entire contribution from the Jacobian of the SUSY transformations can be absorbed into an improvement of the effective action and the supercurrent. Therefore there is no one-loop supercurrent anomaly for the on-shell Wess-Zumino model. This is in agreement with the results obtained using other methods [6]. The structure of the counterterm in (57) is not supersymmetric. This is not surprising because the regulators are not manifestly supersymmetric.

IV. THE SUPERCONFORMAL ANOMALY

Let us now evaluate the superconformal anomaly for the Wess-Zumino model. It can be calculated using the supercurrent anomaly as follows. The superconformal transformation laws for the Wess-Zumino model are [2]

$$\delta_\varepsilon^c\lambda=-\{[(i\partial\!\!\!/+m+\mathcal{S})\mathcal{S}\}(-ix\!\!\!/)+2\mathcal{S}^\dagger]\varepsilon\ , \qquad (61)$$

$$\delta_\varepsilon^c S_n=\tfrac{1}{2}[\bar{\lambda}\gamma_n(-ix\!\!\!/\varepsilon)+(\bar{\varepsilon}ix\!\!\!/)\gamma_n\lambda]\ . \qquad (62)$$

The superconformal Ward identity, similar to the SUSY Ward identity (13), is

$$\delta_\varepsilon^c W[S,\lambda]=\int\mathcal{D}[s_n]\mathcal{D}[\bar{\psi}]\mathcal{D}[\psi]\exp(iS[\mathrm{S},\Psi])\times[i\delta_\varepsilon^c S+k(\varepsilon)]\ , \qquad (63)$$

where $\delta_\varepsilon^c S$ is the superconformal variation of the action and $k(\varepsilon)$ is the corresponding Jacobian:

$$\delta_\varepsilon^c S=-i\int d^4x\{\bar{\Psi}\gamma_\mu[(i\partial\!\!\!/+m+\mathcal{S})\mathcal{S}(-ix\!\!\!/)+2\mathcal{S}^\dagger]\partial^\mu\varepsilon-2im\bar{\Psi}\mathcal{S}^\dagger\varepsilon\}$$
$$=\int d^4x(\bar{S}_\mu\partial^\mu\varepsilon-2m\bar{\Psi}\mathcal{S}^\dagger\varepsilon)\ . \qquad (64)$$

When $m=0$, $\delta_\varepsilon^c\mathcal{S}$ equals ε times the divergence of the superconformal current $\bar{S}_\mu$. As remarked earlier, the Jacobian $k(\varepsilon)$ does not depend on the fermion transformation law. The superconformal transformation law for the boson is similar to the SUSY transformation with the parameter of transformations ε replaced by $-ix\!\!\!/\varepsilon$. Hence the Jacobian of the superconformal transformations is obtained from (55) by replacing ε by $-ix\!\!\!/\varepsilon$. Thus

$$k(\varepsilon)=\frac{i}{\pi^2}\int d^4x\left[-\frac{i}{8}\bar{\lambda}\overleftarrow{\partial\!\!\!/}(\mathcal{S}+m+i\partial\!\!\!/)\mathcal{S}+\bar{C}_\mu\partial^\mu\right](-ix\!\!\!/\varepsilon)$$
$$=\frac{i}{\pi^2}\int d^4x\left[-\frac{i}{16}\delta_\varepsilon^c(\bar{\lambda}\partial\!\!\!/\lambda)+\left[\bar{C}_\mu(-ix\!\!\!/)-i\frac{\bar{\lambda}}{4}(5\mathcal{S}+m)\gamma_\mu\right]\partial^\mu\varepsilon+\frac{1}{2}\bar{\lambda}(-i\overleftarrow{\partial\!\!\!/}+m+\mathcal{S}^\dagger)\mathcal{S}^\dagger\varepsilon\right]\ , \qquad (65)$$

where we have used the superconformal transformation law for the fermion and $\bar{C}_\mu$ is as defined in (56). In terms of the improved effective action (58) and the improved superconformal current $\bar{S}'_\mu$ given by

$$\bar{S}'_\mu=\bar{S}_\mu+\frac{1}{\pi^2}\left[\bar{C}_\mu(-ix\!\!\!/)-i\frac{\bar{\lambda}}{4}(5\mathcal{S}+m)\gamma_\mu\right]\ , \qquad (66)$$

the superconformal Ward identity (63) can be written as

$$\delta_\varepsilon^c W'[S,\lambda]=\int\mathcal{D}[s_n]\mathcal{D}[\bar{\psi}]\mathcal{D}[\psi]\exp(iS'[\mathrm{S},\Psi])\int d^4x\left[-2im\bar{\Psi}\mathcal{S}^\dagger\varepsilon+i\bar{S}'_\mu\partial^\mu\varepsilon-\frac{1}{\pi^2}\bar{\lambda}(-i\overleftarrow{\partial\!\!\!/}+m+\mathcal{S}^\dagger)\mathcal{S}^\dagger\varepsilon\right]\ . \qquad (67)$$

Thus, unlike the case of SUSY, Ward identity (61) the last term in (67) cannot be expressed as the superconformal variation of a local counterterm or as a total divergence. Hence the superconformal symmetry is anomalous, with the corresponding anomaly given by the last term in (67). Unlike the Jacobian of the dilatation transformations [4], the Jacobian of the superconformal transformation is finite. The anomaly obtained here agrees with the result obtained elsewhere using a different method [6].

Fujikawa's method has been used earlier to derive the superanomalies, see for, e.g., Ref. [12]. However our method differs from these calculations in a number of ways. The model discussed in Ref. [12] is the Wess-Zumino multiplet in two dimensions interacting with a *background* supergravity multiplet, unlike our calculation where the background fields also belong to the Wess-Zumino multiplet. The regulators in Ref. [12] were provided by the background theory and hence they did not face the problem of (diagonalizing the action and hence) nonlocality of the bosonic regulator. Further, in our calculation, due to diagonalization, the bosonic and fermionic Jacobians could be evaluated separately and the Jacobians were independent of the nonlinear fermionic transformations. Whereas in Ref. [12] the Wess-Zumino multiplet was off shell and the regulators were not diagonal in bosons and fermions, the resultant Jacobian was a superdeterminant.

CONCLUSIONS

We have extended Fujikawa's method of evaluating anomalies to an on-shell SUSY theory, namely, the Wess-Zumino model. The main difficulty in using Fujikawa's method for on-shell theories is the nonlinearity of the on-shell SUSY transformation laws. However in our calculation we found that, due to the diagonalization of the fluctuation action, the regularized Jacobians of SUSY transformations depended only on the transformation law of the bosons which is linear. Thus the problem of nonlinearity of the transformation laws was circumvented. However the bosonic regulator for the diagonalizing variables turned out to be nonlocal. The heat kernel regularization was then extended to include nonlocal operators.

The resultant Jacobian of the SUSY transformation was finite, and could be expressed as a total divergence plus the SUSY variation of a local counterterm. Thus, although our regularization scheme was not manifestly supersymmetric, there was no one-loop supercurrent anomaly for the on-shell Wess-Zumino model. The nonsupersymmetric nature of the regularization scheme manifested itself as the nonsupersymmetric local counterterms. The calculation for the SUSY anomaly was then used to evaluate the superconformal anomaly. The Jacobian for the superconformal transformations also turned out to be finite however, there was a nonzero superconformal anomaly. These results are in agreement with similar results obtained using other methods [6].

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APPENDIX A

In this appendix the heat coefficients and their derivatives are evaluated at coincident points.

Consider the recursion relations (41) and the boundary condition (42) for the heat coefficients:

$$\left[n+z_\mu\left[\frac{\partial}{\partial x_\mu}+X_\mu(x)\right]\right]a_n(x,y)=R_x a_{n-1}(x,y)\ , \qquad (A1)$$

where $z_\mu=(y-x)_\mu$ with $n\geq 0$, and

$$a_0(x,x)=1\ . \qquad (A2)$$

There are two types of derivatives of a_n, i.e., with respect to x or y, denoted by ∂_μ and $\bar{\partial}_\mu$ respectively. To evaluate the derivative of a_0 with respect to x, we differentiate the recursion relation for a_0 with respect to x and take the coincident point limit, to yield

$$\partial_\mu a_0(x,y)|_{y=x}=-X_\mu(x)\ . \qquad (A3)$$

Similarly, to evaluate $\partial_\mu\partial_\nu a_0(x,y)$, we differentiate (A1) twice with respect to x and then take the coincident point limit, yielding

$$\partial_\mu\partial_\nu a_0(x,y)|_{y=x}=\tfrac{1}{2}(-\partial_\mu X_\nu-\partial_\nu X_\mu+\{X_\mu,X_\nu\})\ . \qquad (A4)$$

The recursion relation for a_1, obtained from (A1), is

$$(n+z\cdot(\partial+X))a_1(x,y)=R_x a_0(x,y)\ . \qquad (A5)$$

Using (33), (A1), (A3), and (A4) in this equation and taking the coincident point limit gives

$$a_1(x,y)|_{y=x}=Y\ . \qquad (A6)$$

The first derivative of a_1 with respect to x involves $\partial_\mu\Box a_0$, i.e., the third derivative of a_0. Evaluation of this is a tedious task. However, it is observed that all such contributions cancel off between different Jacobians and hence we need not calculate such a term explicitly. Denoting $[\partial_\mu(\Box+2X\cdot\partial)a_0]|=Y_0$, the first derivative of a_1 with respect to x can be written as

$$\partial_\mu a_1(x,y)|=\tfrac{1}{2}(\partial_\mu Y-\{Y,X_\mu\}+\partial_\mu Y_0)\ . \qquad (A7)$$

Having obtained the coincident point expressions for the relevant heat coefficients and their derivatives with respect to x, the derivatives of the heat coefficients with respect to y can now be evaluated as follows. We use $a_0(x,y)=a_0(x,x+z)$, with $z=y-x$, and make a Taylor series expansion around $z=0$. Thus

$$a_0(x,x+z)=1+z\cdot(\bar{\partial}a_0)|_{z=0}+\tfrac{1}{2}z_\mu z_\nu(\bar{\partial}_\mu\bar{\partial}_\nu a_0)|_{z=0}+\cdots\ .$$

Here the overbar on ∂ implies differentiation with respect to the second argument of a_n. Differentiating this equation with respect to x, and taking the coincident point limit, gives

$$\bar{\partial}_\mu a_0(x,y)|_{y=x} = -\partial_\mu a_0| = +X_\mu(x) \ . \qquad \text{(A8)}$$

Similarly we find, for the second derivative of a_0 with respect to the second argument, i.e., y, to be

$$[\bar{\partial}_\mu \bar{\partial}_\nu a_0(x,y)]|_{y=x} = \tfrac{1}{2}(\partial_\mu X_\nu + \partial_\nu X_\mu + \{X_\mu, X_\nu\}) \ . \qquad \text{(A9)}$$

The derivative of a_1 with respect to the second argument can be found using identical procedure to be

$$[\bar{\partial}_\mu a_1(x,y)]|_{y=x} = \tfrac{1}{2}(\partial_\mu Y + \{Y, X_\mu\} - \partial_\mu Y_0) \ . \qquad \text{(A10)}$$

In some calculations the heat coefficients of the form $a_n(y,x)$ are needed. The corresponding heat equation and recursion relations are found by interchanging x with y in equations (37) to (42). Repeating the above procedure, it is easy to verify that the derivatives with respect to the first or the second argument of $a_n(y,x)$ are equal to those of $a_n(x,y)$. Hence we will tabulate the coincident point values of only $a_n(x,y)$ and its derivatives.

Substituting for X_μ and Y from Eqs. (34)–(36) in the results obtained above, the following coincident point values are found for the heat coefficients for the Wess-Zumino model:

$$[a'_0(x,y)]_{mn}|_{y=x} = \delta_{mn}, \quad a'_0| = \bar{a}'_0 = 1 \ ,$$

and all derivatives of $(a'_0)_{mn}$, a'_0, and $\bar{a}'_0$ are equal to zero,

$$\begin{aligned}
&(a'_1)_{mn}| = -[(m^2+2mS_1+2S^2)\delta_{mn} + 2m(\delta_{1n}S_m + \delta_{1m}S_n) + 4S_mS_n] \ ,\\
&(\partial_\mu a'_1)_{mn}| = (\bar{\partial}_\mu a'_1)_{mn}| = -[(mS_1+S^2)\delta_{mn} + m(\delta_{1n}S_m + \delta_{1m}S_n) + 2S_mS_n] \ ,\\
&a'_1| = -(m^2+4S^2+4mS_1+2\partial\!\!\!/ \mathcal{S}^\dagger), \quad (\partial_\mu a'_1)| = (\bar{\partial}_\mu a'_1)| = -\partial_\mu(2S^2+2mS_1+\partial\!\!\!/ \mathcal{S}^\dagger) \ ,\\
&\bar{a}'_1| = -(m^2+4S^2+4mS_1-2\partial\!\!\!/ \mathcal{S}), \quad (\partial_\mu \bar{a}'_1)| = (\bar{\partial}_\mu \bar{a}'_1)| = -\partial_\mu(2S^2+2mS_1-\partial\!\!\!/ \mathcal{S}) \ .
\end{aligned} \qquad \text{(A11)}$$

APPENDIX B

In this appendix we show how to separate the contribution of the nonlocal part of a regulator, from the rest.

To begin with, consider a completely local regulator operator R' of the generic form (33):

$$R' = \Box + 2X\cdot\partial + \partial\cdot X + X^2 + Y = R^0 + Y \ . \qquad \text{(B1)}$$

Here X_μ and Y are matrix-valued local functions and they do not involve any derivatives. The corresponding generic heat kernel

$$h'(x,y;t) = \langle x|e^{tR}|y\rangle \qquad \text{(B2)}$$

has the usual asymptotic expansion

$$h'(x,y;t) = \frac{e^{-(x-y)^2/4t}}{16\pi^2 t^2} \sum_{n=0}^{\infty} a'_n(x,y)t^n \ . \qquad \text{(B3)}$$

Let us define another heat kernel:

$$h^0(x,y;t) = \langle x|e^{tR^0}|y\rangle \ . \qquad \text{(B4)}$$

The heat kernel h' can be expressed in terms of h^0 as

$$\begin{aligned}
h'(x,y;t) &= \langle x|e^{tR^0}[1+tY+O(t^2)]|y\rangle \qquad \text{(B5)}\\
&= h^0(x,y;t) + [Y(x)h^0(x,y;t)\\
&\quad + h^0(x,y;t)Y(y)] + O(t^2) \qquad \text{(B6)}
\end{aligned}$$

where the right-hand side has been symmetrized with respect to x and y. The heat kernel h^0 has an asymptotic expansion similar to (B3), with the corresponding heat coefficients denoted by $a_n^0(x,y)$. Substituting the asymptotic expansion for h and h^0 on the left- and the right-hand sides of Eq. (B5), respectively, and comparing the coefficients of equal powers of t yields the following relation between a'_n and a_n^0:

$$a'_0(x,y) = a_0^0(x,y) \qquad \text{(B7)}$$

and

$$a'_1(x,y) = a_1^0(x,y) + \tfrac{1}{2}[Y(x)a_0^0(x,y) + a_0^0(x,y)Y(y)] \ . \qquad \text{(B8)}$$

The coincident point values for a'_n and a_n^0 can be found using the general formulas in Appendix A. The correctness of Eqs. (B6)–(B8) can be easily verified by substituting for a_n^0 in (B7) and (B8) and comparing the expressions for a_n so obtained with those obtained in Appendix A.

The expressions (B5)–(B8) can now be generalized to include nonlocal operators. Consider a regulator operator

$$R(x,y) = R'\delta(x-y) + N(x,y) \ , \qquad \text{(B9)}$$

where N is the nonlocal contribution and R' is as in (B1). The heat kernel corresponding R is denoted by h. Replacing h' by h, h^0 by h', and Y by N in (B5) yields

$$\begin{aligned}
h(x,y;t) &= h'(x,y;t)\\
&\quad + \frac{t}{2}\int d^4x'[N(x,x')h(x',y;t)\\
&\quad + h'(x,x';t)N(x',x)] + O(t^2) \ .
\end{aligned} \qquad \text{(B10)}$$

Thus the contribution from the nonlocal part, given by the term in square brackets in the above equation, is separated from the local part's contribution. We will presently see that the terms of $O(t^2)$ and higher will not contribute anything to the Jacobians and, hence, are irrelevant.

Substituting in (B10) for h, h', and N from Eqs. (27), (43), and (20), respectively, yields

$$h_{mn}(x,y;t)=h'_{mn}(x,y;t)-\frac{t}{2}\int d^4x'\{[\bar{\lambda}(x)\gamma_m^\dagger\tilde{G}(x,x')\gamma_l^\dagger\lambda(x')+\bar{\lambda}(x')\gamma_l^\dagger\tilde{G}(x',x)\gamma_m^\dagger\lambda(x)]h_{ln}(x',y;t)$$
$$+h'_{ml}(x,x';t)[\bar{\lambda}(x')\gamma_l^\dagger\tilde{G}(x',y)\gamma_n^\dagger\lambda(y)+\bar{\lambda}(y)\gamma_n^\dagger\tilde{G}(y,x')\gamma_l^\dagger\lambda(x')]\}+O(t^2) \quad \text{(B11)}$$

The order-t^2 term in the above equation (which contains two $\tilde{G}$), when substituted in the first term in (32), gives the resultant nonlocal contribution proportional to

$$t^2\int d^4x\int d^4y\int d^4u[\tilde{G}(x,y)][\tilde{G}(y,u)][\tilde{G}(u,x)]F_1(x)F_2(y)F_3(u)\ . \quad \text{(B12)}$$

Here F are some nonsingular functions. Since $\tilde{G}$ has mass dimension 3,

$$\int d^4y\int d^4u[\tilde{G}(x,y)][\tilde{G}(y,u)][\tilde{G}(u,x)]$$

is of mass dimension $+1$, i.e., proportional to $t^{-1/2}$. Hence the entire contribution (B12) is of order $t^{3/2}$, which vanishes in the limit $t\rightarrow 0$.

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