Quantum stabilization of solitonic bubbles

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In a real scalar field model in $1+1$ dimensions with quartic and sextic self-coupling there appears a classically unstable nontopological soliton which is a bubble in the false vacuum. We show that this vacuum is rendered stable by one- and two-loop quantum corrections with appropriate renormalization conditions. In a parameter range of the model, the bubble is stabilized when quantum corrections to its mass are taken into account with a quadratic approximation of the higher-order terms.

I. INTRODUCTION

The real scalar field theory in $1+1$ dimensions with quartic and sextic self-interactions corresponding to a deepest central well and two lateral ones has a static classical solution which takes values in one of these for all the space except for a finite region where it approaches the absolute minimum [1,4]. In a lattice quantum version of the model it has been shown [2] that the condensation of these bubble-type states, together with the kinks, determine the phase diagram which exhibits a tricritical point that may be related to the He^3-He^4 mixture [3]. Equivalent static classical bubbles appear in the nonlinear Schrödinger equation [4] which are unstable but, due to the nonrelativistic nature of the theory, may achieve stability when they move exceeding a critical velocity with respect to the medium [5].

The purpose of this work is to indicate that in the relativistic theory the solitonic bubbles which will be briefly reviewed in Sec. II may be stabilized by quantum corrections. This problem has two aspects. One is that the classical bubble lives mostly in a false vacuum which may tunnel into the true one. The other is that the bubble is classically unstable against small perturbations which produce its evolution into fluctuations around the false vacuum since there is no topological reason to prevent this decay. Regarding the former instability it will be seen in Sec. III that quantum corrections at one and two loops give rise to an effective potential which may exhibit dynamical symmetry breaking turning the false vacuum into a stable one. This is due to the parameter freedom associated to the finite parts of the two counterterms which must be introduced. As for the latter instability, we will indicate in Sec. IV by a semiclassical argument that it may be cured when the mass of the bubble is smaller than the mass of the meson which appears as a quantum excitation around the lateral minimum. A more comprehensive quantum treatment of the bubble shows that all the energies of its excitations turn out to be real, and therefore no decay is possible, for a parameter region qualitatively consistent with the above argument if a quadratic approximation is used for the higher-order terms around the classical contribution.

It must be stressed that these indications for the quantum stability of the bubble are different from those corresponding to other nontopological solitons which are always related to a Noether charge. In the polymer models the scalar real field is coupled to a Dirac field which taken in the mean-field approximation gives way to a classical model of the type we are considering, and the subsequent iterative solution produces the stable polaron [6]. The so-called Q balls [7] are a sort of "negative" of our bubbles, in the sense that they live mostly in the true vacuum of a self-interacting complex scalar field and owe their stability to their time dependence producing a charge which must be above a threshold value.

It might be interesting to see what are the implications of the quantum stabilization discussed in the present paper for $(3+1)$-dimensional models which necessarily must contain two fields to avoid nonrenormalizability, compared with the usual nontopological charged solitons [8] which are largely applied to cosmology [9].

II. SOLITONIC BUBBLE

Given a Lagrangian in $1+1$ dimensions for a real field $\tilde{\phi}$,

$$\tilde{\mathcal{L}}(\tilde{x},\tilde{t})=\tfrac{1}{2}(\tilde{\partial}_\mu\tilde{\phi})^2-\tilde{V}(\tilde{\phi}) , \tag{1}$$

where

$$\tilde{V}(\tilde{\phi})=\frac{K^2}{2}(\tilde{\phi}^2-\rho)^2(\tilde{\phi}^2-A\rho) , \tag{2}$$

the change of variables $\phi=(1/\sqrt{\rho})\tilde{\phi}$, $x_\mu=\rho\tilde{x}_\mu$ allows one to write

$$\mathcal{L}(x,t)=\frac{1}{\lambda}[\tfrac{1}{2}(\partial_\mu\phi)^2-V] \tag{3}$$

with

$$V(\phi)=\frac{K^2}{2}(\phi^2-1)^2(\phi^2-A) \tag{4}$$

and $\lambda=1/\rho^3$.

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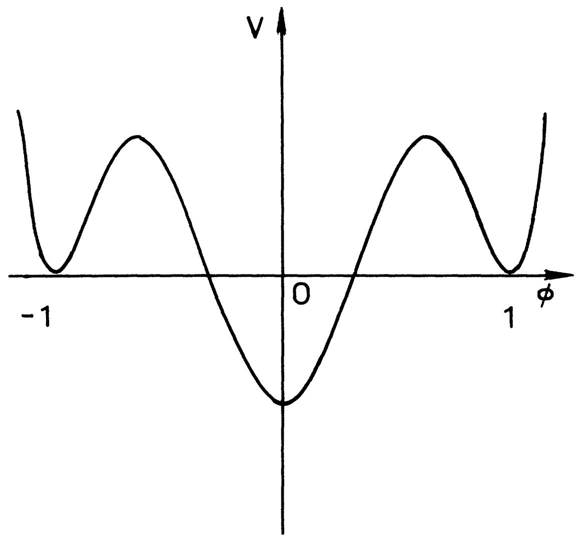


FIG. 1. Classical potential.

If $A<0$ there is spontaneous symmetry breaking and topological solitons of kink type appear.

If instead $0<A<1$ the central minimum is the absolute one, the true vacuum corresponds to $\phi=0$ (see Fig. 1) and there is a static solution which starts for $x=-\infty$ from a lateral maximum of $-V(\phi)$, reaches a point near the central maximum and returns to the lateral maximum for $x=+\infty$ (see Fig. 2). This classical solution which satisfies

$$\tfrac{1}{2}(\phi')^2=V(\phi) \tag{5}$$

is

$$\phi_c^2=\frac{A}{1-(1-A)\tanh^2[K\sqrt{1-A}\,(x-x_0)]} \tag{6}$$

and corresponds to an energy

$$E=\frac{K}{8\lambda^{2/3}}\left[2(2+A)\sqrt{1-A}\right.\\ \left.-A(4-A)\ln\left[\frac{1+\sqrt{1-A}}{1-\sqrt{1-A}}\right]\right] . \tag{7}$$

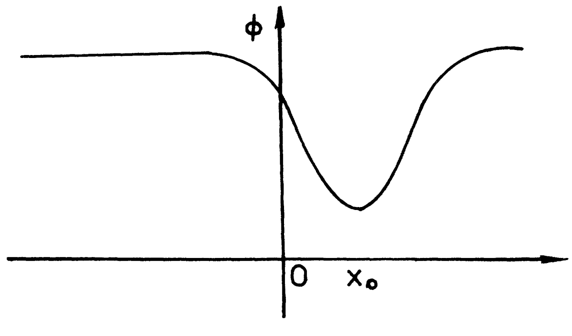


FIG. 2. Solitonic bubble.

The bubble of Eq. (6) is classically unstable since a small perturbation $\psi(x)e^{i\omega t}$ satisfies

$$\left[-\frac{d^2}{dx^2}+V''(\phi_c(x))\right]\psi(x)=\omega^2\psi(x) \tag{8}$$

and being the zero mode $\phi_c'(x)$ a one-node function, the ground state of Eq. (8) corresponds to imaginary ω, which is consistent with the fact that there is no topological reason for the stability.

III. DYNAMICAL SYMMETRY BREAKING

Considering the effective quantum potential

$$V_{\text{eff}}(\phi)=\sum_n \frac{\phi^n}{n!}\Gamma^{(n)}(0,\ldots,0) , \tag{9}$$

where $\Gamma^{(n)}$ is the sum of all irreducible graphs with n legs at zero momentum, for small values of λ it is possible to perform a loop expansion:

$$V_{\text{eff}}(\phi)=\lambda^{-1}V_0(\phi)+V_1(\phi)+\lambda V_2(\phi)+\cdots . \tag{10}$$

V_0 corresponds to the tree graphs and simply gives the classical equation (4).

The one-loop correction has been calculated in Ref. [10] and, including the counterterms necessary to cancel the divergences which appear for $n=0$, 2, 4, it gives

$$V_1(\phi)=\frac{V''(\phi)}{8\pi}\left[1+\ln\left[\frac{\Lambda^2}{V''(\phi)}\right]\right]+a_1+b_1\phi^2+c_1\phi^4 . \tag{11}$$

Separating the $\Lambda\to\infty$ divergent contribution,

$$V_{\text{eff}}=\frac{1}{\lambda}V(\phi)+\frac{V''(\phi)}{8\pi}\left[1-\ln\left[\frac{V''(\phi)}{\mu^2}\right]\right]\\ +\frac{V''(\phi)}{8\pi}\ln\left[\frac{\Lambda^2}{\mu^2}\right]+a_1+b_1\phi^2+c_1\phi^4 \tag{12}$$

we choose the infinite part of a_1, b_1, and c_1 to cancel it. The finite part of a_1 is irrelevant since it merely adds a constant. The finite parts of b_1 and c_1, together with $\ln\mu^2$, determine the coefficients of the terms ϕ^2 and ϕ^4 of the potential. Two of these parameters are therefore independent, which replaced by $\ln\mu_1^2$ and $\ln\mu_2^2$ allow one to write

$$V_{\text{eff}}=\frac{1}{\lambda}V(\phi)+\frac{V''(\phi)}{8\pi}\left[1-\ln\left[\frac{V''(\phi)}{\mu_1^2}\right]\right]\\ +\frac{15}{8\pi}\phi^4\ln\left[\frac{\mu_2^2}{\mu_1^2}\right] . \tag{13}$$

The approximation to the effective potential equation (13) is not defined when V'' is negative but it is valid close to its minima. Therefore we may obtain the shift ε for the position of the lateral minimum from $V'_{\text{eff}}(\phi=1+\varepsilon)=0$:

$$\varepsilon=\frac{\lambda}{8\pi}\left[\frac{3(3-A)}{1-A}\ln\left[\frac{4(1-A)}{u_1^2}\right]-\frac{15}{1-A}\ln\left[\frac{u_2^2}{u_1^2}\right]\right] \tag{14}$$

in terms of dimensionless parameters $u_i^2=\mu_i^2/K^2$. Moreover we may define a critical parameter λ_c equating the values of the two minima at $\phi=0$ and $\phi=1+\varepsilon$:

$$\frac{A}{2\lambda_c}=\frac{1+2A}{8\pi}\left[1-\ln\left[\frac{1+2A}{u_1^2}\right]\right]-\frac{4(1-A)}{8\pi}\left[1-\ln\left[\frac{4(1-A)}{u_1^2}\right]\right]+\frac{15}{8\pi}\ln\left[\frac{u_1^2}{u_2^2}\right]. \tag{15}$$

Of the five parameters of V_{eff}, i.e., u_1^2, u_2^2, A, λ, K^2, the first two may be chosen for fixed A so that $\varepsilon=0$ and λ_c is sufficiently small to ensure the validity of the loop expansion (see Fig. 3). Once this is done, the last two parameters are determined, e.g., by the renormalized mass and quartic coupling at the symmetry-breaking vacuum:

$$m_R^2=V''_{\text{eff}}|_{\phi=1},\quad \alpha_R=V^{iv}_{\text{eff}}|_{\phi=1}\ . \tag{16}$$

The bubble will then live in a stable vacuum if these values are such as to make $\lambda>\lambda_c$.

If we go on to the two-loop correction one has to consider the insertion of the previous counterterms into the one-loop graphs, the two-loop diagrams, and the new order of the counterterms [10]. The net result is

$$V_2(\phi)=\frac{1}{8\pi}\left[b_1+\frac{c_1}{2}\phi^2\right](\ln\Lambda^2-\ln V'')-\frac{K(V''')^2}{192\pi^2V''}+\frac{V^{iv}}{128\pi^2}(\ln\Lambda^2-\ln V'')^2+a_2+\frac{b_2}{2}\phi^2+\frac{c_2}{4!}\phi^4\ , \tag{17}$$

where b_1 and c_1 must be fixed by the renormalization conditions for the one-loop effective potential and b_2 and c_2 will be determined by the second-order conditions.

Since we are trying to obtain an effective potential with features different from those of the classical potential it is not reasonable to fix the same conditions at all orders except the one that the lateral minimum occurs always at $\phi=1$, i.e., $\varepsilon=0$. We may establish, e.g., that the difference between the two minima reduces at each order to a half of that of the previous one (see Fig. 4) as a tendency toward symmetry breaking. Once the counterterms are determined in agreement with these conditions, the renormalized mass and quartic coupling will be obtained from Eq. (16). For an indication of the convergence of the loop expansion it will be important that each correction is smaller than the one for the previous order. In Fig. 5 the corresponding values for mass and quartic coupling are shown indicating that for small enough values of λ, and A not too close to 1, the expansion seems to converge.

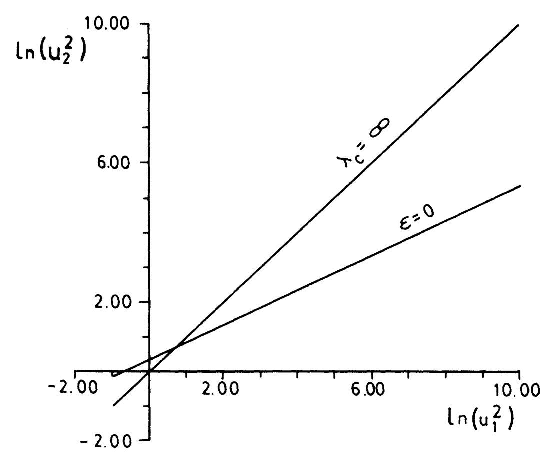


FIG. 3. Renormalization parameters for fixed $A=0.5$ at a one-loop approximation. On the line $\varepsilon=0$, the critical value λ_c to reach symmetry breaking is inversely proportional to the distance to the line $\lambda_c=\infty$.

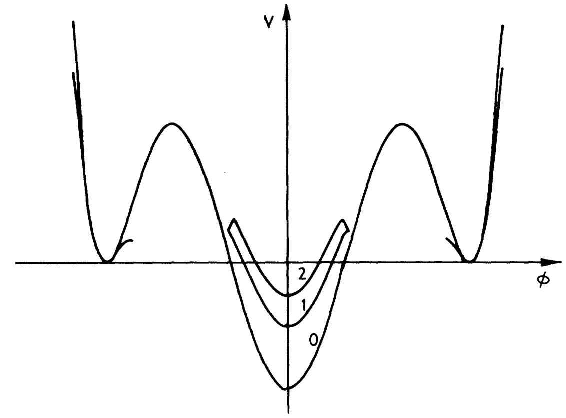


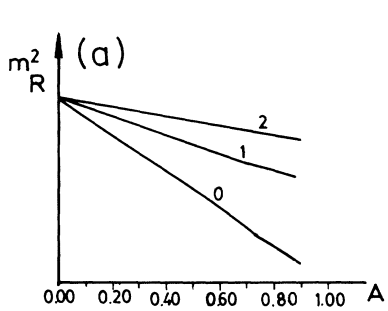
FIG. 4. Effective potential at zero-, one-, and two-loop approximations fixing a tendency toward symmetry breaking. It is not defined around the classical maxima and all almost coincide for the lateral minima.

It is interesting to compare, in the framework of the loop expansion, the previous renormalization at $\phi=1$ with the renormalization done in Ref. [10] at the origin $\phi=0$.

With this latter choice the five parameters of V_{eff} are more conveniently denoted by u_1^2, u_2^2, m_0^2, α_0, ξ_0 where the last three are the coefficients of quadratic, quartic, and sextic terms in the classical potential. Fixing the conditions that the physical mass and quartic coupling are equal to the initial ones

$$m^2=V''_{\text{eff}}|_{\phi=0}=m_0^2,\quad \alpha=V^{iv}_{\text{eff}}|_{\phi=0}=\alpha_0\ , \tag{18}$$

the parameters m_0^2 and α_0 are therefore determined and Eq. (18) together with the one defining the sextic coupling ξ as the sixth derivative allow one to obtain the values of the other three. The height of the lateral minimum is not defined by the renormalization conditions and turns out to increase compared with the central one both in our case $0<A<1$ and when $A<0$ restoring the symmetry in this latter case. It must be noted that when the central minimum is assumed to be the absolute one, one may order normally the Lagrangian eliminating the divergent tadpoles, so that there is no need of counterterms. The



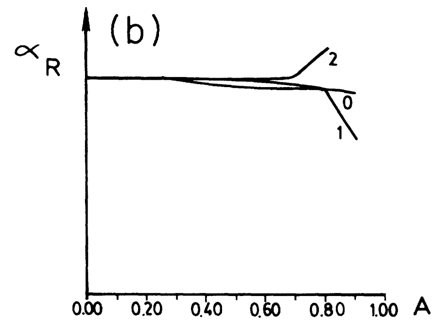


FIG. 5. (a) Renormalized mass and (b) quartic coupling at lateral minimum with $\lambda=0.0001$ for zero-, one-, and two-loop approximations defined according to a tendency toward symmetry breaking.

physical mass and couplings will be different from the initial ones so that fixing the former will allow to determine the latter.

It appears therefore that due to the existence of two coupling constants, the ϕ^6 theory allows one to obtain through loop expansion either symmetry breaking or symmetry restoring according to the chosen conditions.

It is noteworthy to give a glance in the above spirit to the renormalizable ϕ^4 theory in $3+1$ dimensions, where the normal ordering does not eliminate all the divergent diagrams. If the classical potential shows a single central minimum, the loop expansion requires two counterterms so that there are in all four parameters. The renormalization conditions at the origin equating physical mass and coupling to the initial ones as in Eq. (18) will supply two equations for the remaining two parameters. For a classical potential with symmetry breaking, the normal ordering for ϕ is not sensible since the vacuum does not correspond to $\phi=0$, so that the conditions for the values of $\langle\phi\rangle$ and of mass and coupling calculated as derivatives of V_{eff} at the asymmetric minimum allow to fix the four parameters with the choice [11], e.g., $\mu_1=\mu_2$.

Looking back at Eq. (15) for the sextic theory in $1+1$ dimensions, it may be observed that the gap between the two minima may be compensated not only by the intuitive meson mass contributions around each classical minimum which correspond to the terms independent on μ_1, but also by those depending on the renormalization parameters due to UV divergences which make the zero-point energies less simple.

IV. BUBBLE STABILIZATION

The bubble is classically unstable in the sense that the imaginary frequency ω solution of Eq. (8) produces its evolution into excitations around the lateral vacuum. A semiclassical argument suggests how it may be stabilized, corresponding to the situation when the mass of the meson on this vacuum

$$m=\left[\frac{\partial^2\tilde{V}}{\partial\tilde{\phi}^2}\bigg|_{\tilde{\phi}=\rho^{1/2}}\right]^{1/2}=\frac{2K}{\lambda^{1/3}}\sqrt{1-A} \tag{19}$$

is larger than the bubble energy Eq. (7). The values of λ for which both masses are equal are shown in Fig. 6 indicating that the bubble stabilization may be obtained with values of λ small enough to be consistent with the previous loop expansion. It is clear that this stabilization argument should be improved by a quantum treatment which takes into account both zero-point energies above the soliton and the vacuum and renormalization effects.

Another argument corresponds to see in which way Eq. (8) is modified when corrections higher than the quadratic ones in an expansion of energy around the soliton mass are included.

We consider the field operator in terms of excitations around the classical bubble solution ϕ_c:

$$\phi(x,t)=\phi_c(x)+\hat{\phi}(x,t) \tag{20}$$

with

$$\hat{\phi}(x,t)=\sum_n\frac{1}{\sqrt{2\omega_n}}[a_n\psi_n(x)e^{-i\omega_n t}+a_n^\dagger\psi_n^*(x)e^{i\omega_n t}]\ , \tag{21}$$

where $[a_n,a_n^\dagger]=\delta_{nn}$, and $\{\psi_n(x)\}$ is a complete set of

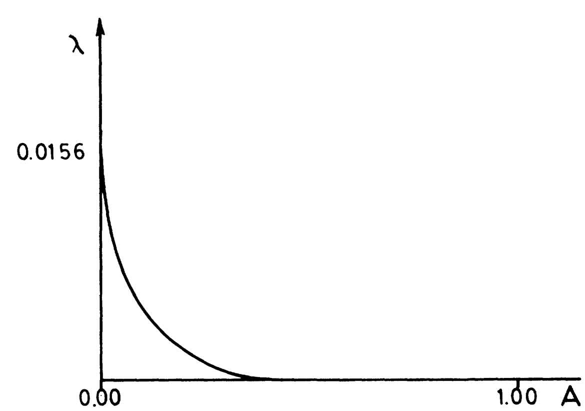


FIG. 6. Values of λ for which the masses of bubble and meson are equal.

functions so that the conjugate momentum is

$$\hat{\pi}(x,t)=-i\sum_n\left[\frac{\omega_n}{2}\right]^{1/2}(a_n\psi_n e^{-i\omega_n t}-a_n^\dagger\psi_n^* e^{i\omega_n t}) .$$

The Hamiltonian keeping only the second-order terms in $\hat{\phi}$ turns out to be

$$H^{(2)}=E_c+\sum_n\omega_n(a_n^\dagger a_n+\tfrac{1}{2}) \tag{22}$$

if ψ_n satisfy Eq. (8). In this quadratic approximation the existence of an imaginary frequency formally produces the instability of the quantum states, though the strict treatment Eq. (21) requires ω_n to be real. When the potential has a structure richer than two wells, as happens in our case with central and lateral minima, terms higher than the quadratic ones are likely to be important. Inspired by recent suggestions [12] we keep the cubic plus quartic contribution and approximate it as a quadratic expansion around its minimum $\hat{\phi}_{\min}=-3V'''/V^{iv}$: i.e.,

$$\frac{1}{3!}V'''(\phi_c)\hat{\phi}^3+\frac{1}{4!}V^{iv}(\phi_c)\hat{\phi}^4 \approx\frac{45}{8}\frac{(V''')^4}{(V^{iv})^3}+\frac{9}{2}\frac{(V''')^3}{(V^{iv})^2}\hat{\phi}+\frac{3}{4}\frac{(V''')^2}{V^{iv}}\hat{\phi}^2 \tag{23}$$

which will be valid if $\hat{\phi}$ is small.

With this new correction, instead of Eq. (22) the Hamiltonian takes a form which can again be easily diagonalized:

$$H^{(4)}=E_c+\int dx\frac{45}{8}\frac{(V''')^4}{(V^{iv})^3}+E(\hat{\phi}) , \tag{24}$$

where

$$E(\hat{\phi})=\int dx[\tfrac{1}{2}\dot{\hat{\phi}}^2+\tfrac{1}{2}\hat{\phi}'^2+f(x)\hat{\phi}+\tfrac{1}{2}g(x)\hat{\phi}^2] \tag{25}$$

with

$$f(x)=\frac{9}{2}\frac{(V''')^3}{(V^{iv})^2},\quad g(x)=V''(\phi_c)+\frac{3}{2}\frac{(V''')^2}{V^{iv}} . \tag{26}$$

Separating from $\hat{\phi}$ a time-independent part

$$\hat{\phi}(x,t)=\chi(x,t)+\eta(x) \tag{27}$$

such that $-\eta''+g\eta+f=0$, the operator term of Eq. (24) becomes

$$E(\hat{\phi})=\int dx\left[\tfrac{1}{2}\dot{\chi}^2+\tfrac{1}{2}\chi'^2+\tfrac{1}{2}g\chi^2+\tfrac{1}{2}\eta'^2+\tfrac{1}{2}g\eta^2+f\eta\right] . \tag{28}$$

The last η-dependent contribution to Eq. (28) simply adds a real constant to the energy whereas the expansion of the operator χ into a complete set of the type of Eq. (21) produces for the new ψ_n a Schrödinger equation analogous to Eq. (8) but with $V''(\phi_c)$ replaced by $g(x)$.

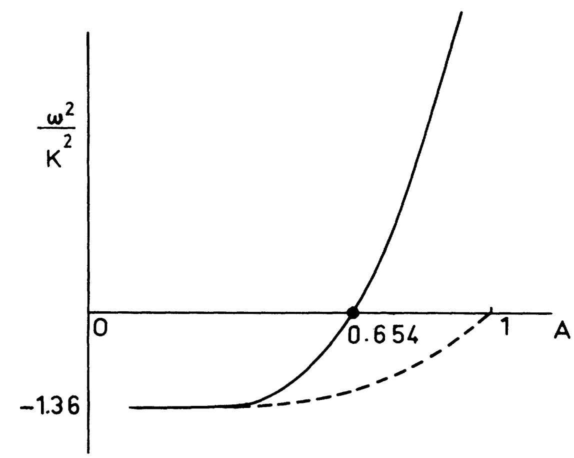


FIG. 7. Lower bound of ground eigenvalue of Schrödinger equation for excitations above soliton with (continuous curve) and without (dashed curve) cubic quartic corrections.

To see whether this equation has a negative eigenvalue we use the semiquantum method [13] which provides a lower bound to the ground state. This method is particularly simple for a Gaussian approximation to the ground wave which may be used when $A>0.5$ because the potential $g(x)$ has a single well. As shown in Fig. 7, for $A>0.654$ the eigenvalue lower bound is positive indicating a stabilization of the soliton. This result is somehow in agreement with the previous argument regarding the equality of masses of the soliton and meson since there the corresponding value of λ decreases for increasing A.

From our analysis, to obtain dynamical breaking it was preferable to have a small A since then the finite effects of the necessary counterterms in the loop expansion are smaller. However, one must note that for λ sufficiently small both dynamical breaking and soliton stabilization are compatible for a certain range of A.

Finally, it must be remarked that the eventual stability of our solitonic bubbles does not depend on the existence of a charge as occurs for the Q balls [7] and the original nontopological solitons [8].

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[1] D. Boyanovsky and L. Masperi, Phys. Rev. D **21**, 1550 (1980).
[2] L. Masperi, Phys. Rev. D **41**, 3263 (1990).
[3] M. Blume, V. Emery, and R. Griffith, Phys. Rev. A **4**, 1071 (1971).
[4] I. Barashenkov and V. Makhankov, Phys. Lett. A **128**, 52 (1988).
[5] I. Barashenkov, T. Boyadjev, I. Puzynin, and T. Zhanlav,

Phys. Lett. A **135**, 125 (1989); I. Barashenkov, A. Gocheva, V. Makhankov, and I. Puzynin, Physica D **34**, 240 (1989).
[6] C. Arãgao de Carvalho, C. Bonato, and G. Costamilan, J. Phys. A **22**, L1153 (1989).
[7] S. Coleman, Nucl. Phys. **B62**, 263 (1985); A. Safian, S. Coleman, and M. Axenides, *ibid.* **B297**, 498 (1988).
[8] R. Friedberg, T. D. Lee, and A. Sirlin, Phys. Rev. D **13**, 2739 (1976); Nucl. Phys. **B115**, 1 (1976); **B115**, 32 (1976).
[9] J. A. Frieman, G. B. Gelmini, M. Gleiser, and E. W. Kolb, Phys. Rev. Lett. **60**, 2101 (1988); J. A. Frieman, A. Olinto, M. Gleiser, and C. Alcock, Phys. Rev. D **40**, 3241 (1989).
[10] R. Rajaraman and M. Raj Lakshmi, Phys. Rev. D **23**, 2399 (1981).
[11] K. Huang, *Quarks, Leptons and Gauge Fields* (World Scientific, Singapore, 1982).
[12] C. Bonato, M. Thomaz, and A. Malbouisson, Phys. Rev. D **41**, 1939 (1990); C. Bonato, M. Laucas, and M. Thomaz, presented at XI Encontro Nacional de Física de Partículas e Campos of Caxambú, 1990 (unpublished).
[13] C. T. Sachrajda, H. A. Weldon, and R. Blankenbecler, Phys. Rev. D **17**, 507 (1978).