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Graviton-electron interactions

H. N. Saif
Physics Department, Concordia University, 1455 de Maisonneuve Boulevard West, Montreal, Quebec, Canada H3G 1M8
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First-order cross sections for the processes of graviton-Compton scattering, $ge^- \to \gamma e^-$, bremsstrahlung, and pair production by gravitons in the Coulomb field, are calculated. The calculations, which are linear in the gravitational coupling κ, are obtained in the extreme relativistic limit.

I. INTRODUCTION

Several processes in which gravitons, the quanta of the gravitational field, interact with other elementary particles are studied in the literature. Since the gravitational coupling strength $\kappa=\sqrt{8\pi G}$ is extremely weak, where G is the Newtonian gravitational constant ($G=6.7\times10^{-39}\text{GeV}^{-2}$), only processes linear in κ are usually considered. Although no real attempt is made to estimate the overall background of gravitational radiation in the Universe, which would clearly be of considerable astrophysical significance, linear processes can be distinctly singled out that contribute to increase the gravitational radiation in the Universe. Some such processes have a direct astrophysical interest; for instance, photoproduction and bremsstrahlung generate sizable amounts of gravitational radiation which can be comparable in magnitude to those of classical processes.

Previous work was done by Weber and Hinds [1], Weinberg [2], Carmeli [3], Boccaletti and Occhionero [4], Boccaletti [5], and Papini [6]. They have shown that in astrophysical applications the gravitational radiation power in quantum processes could be as high as in the classical ones. Papini and Valluri [7] considered the process of photoproduction of gravitons in static magnetic and Coulomb fields in the first- and second-order perturbation theory and applied the results for studying the gravitational radiation from some astrophysical objects. Although the experimental implications of quantum gravity are normally far beyond the range of contemporary experimental physics, some interesting astrophysical objects have recently been observed that emit extremely-high-energy electromagnetic radiation which could be produced by processes involving gravitons. Indeed it is likely that objects such as Cygnus X-3 or neutron stars radiate a significant fraction of their energy in the form of very-high-energy gravitons. Also, there is strong indirect evidence that the rate at which the rotation periods of some massive binary-star systems are slowing down is consistent with the expectation of energy loss due to the emission of gravitational radiation [8].

Part of the motivation for considering linear processes for quantum gravity is due to the question of renormalizability, which now appears as a major obstacle to constructing a complete quantum theory of gravity. For nongravitational radiation, renormalizable quantum field theories exist which seem to describe nature adequately. However, when one considers gravity, because the gravitational coupling constant κ of Einstein's theory of gravity is dimensional, the corresponding quantum field theory is nonrenormalizable. Although in the case of pure gravity the one-loop divergences can be eliminated by field renormalization [9–11], physical divergences remain for the more realistic situation of combined matter interactions. It is implicitly hoped that the first-order terms of perturbation theory are valid for the processes considered.

The main purpose of this paper is to give an estimate of the cross section at the tree level (order of κ^2), for the processes $ge^- \to \gamma e^-$, bremsstrahlung and pair production by gravitons in a Coulomb field. The motivation is to find some process which would enable extremely high-energy gravitons to be detected. In Secs. II–IV, we calculate the cross sections for these processes. Section V contains the conclusions. We use units $\hbar=c=1$.

II. GRAVITON-COMPTON SCATTERING

The first-order contribution to the reaction $ge^- \to \gamma e^-$ is described by the two diagrams of Fig. 1, where k, k', ω, ω' are the four-momenta of the initial graviton, the final photon, the initial electron, and the final electron, respectively. The quantities $\epsilon_{\mu\nu}$, ϵ^*_ρ are the polarization tensor and vector of the initial graviton and the final photon, respectively. The intermediate four-vector momenta are $q'=(k+p)=(k'+p')$, $q=(p-k')=(p'-k)$. The corresponding Feynman rule for the electron-electron-graviton vertex is $\kappa[(p+q')_\mu\gamma_\nu+\gamma_\mu(p+q')_\nu]$ ($p\leftrightarrow p'$ and $q\leftrightarrow q'$ in the second diagram), which has already been discussed in the literature [3,10–13] (more details are given in Appendix A). The matrix element for both diagrams is given by

$$M_{fi}=-e^-\sqrt{8\pi G}\,\epsilon_{\mu\nu}\epsilon^*_\rho\bar{u}(p',s')\times\left[[(p+q')_\mu\gamma_\nu+\gamma_\mu(p+q')_\nu]\frac{\not{q}'+m}{q'^2-m^2}\gamma_\rho+\gamma_\rho\frac{\not{q}+m}{q^2-m^2}[(p'+q)_\mu\gamma_\nu+\gamma_\mu(p'+q)_\nu]\right]u(p,s)\,, \tag{2.1}$$

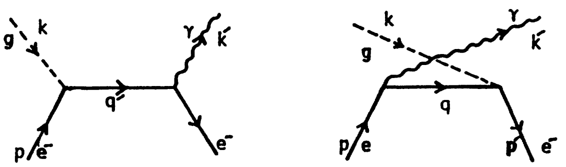

FIG. 1. Graviton-Compton scattering $ge^- \to \gamma e^-$.

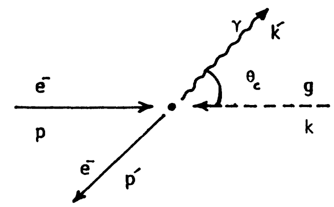

FIG. 2. g-e collision c.m. frame.

where $u(p,s)$, $u(p',s')$ are the Dirac spinors for the initial and final electron and $\not{q}=q_\mu \gamma^\mu=(q\cdot\gamma)$. Disregarding polarization effects, we average the cross section over the initial spins of the graviton and electron, and sum over the polarizations of the final electron and photon. Neglecting the electron masses and squaring Eq. (2.1) one obtains

$$|M_{fi}|^2=8\pi G e^2 \frac{1}{4} \sum_{\text{polar}} \varepsilon_{\mu\nu}\varepsilon^*_{\alpha\beta}\varepsilon_\sigma \varepsilon^*_\rho (\bar{u}\,' T_{\mu\nu\rho} u)(\bar{u}\bar{T}_{\sigma\alpha\beta} u') \tag{2.2}$$

$$= -2\pi G e^2 \mathrm{Tr}[\tfrac{1}{2} g_{\sigma\rho}(g_{\mu\alpha}g_{\nu\beta}+g_{\mu\beta}g_{\nu\alpha}-g_{\mu\nu}g_{\alpha\beta}) \times (p'\cdot\gamma T_{\mu\nu\rho} p\cdot\gamma \bar{T}_{\alpha\beta\sigma})] , \tag{2.3}$$

where we have used the polarization sums of the photon and graviton [3,13,14]:

$$\sum_{\lambda'} \varepsilon_\sigma(k',\lambda')\varepsilon_\rho(k',\lambda')=-g_{\sigma\rho} , \tag{2.4}$$

$$\sum_{\lambda} \varepsilon_{\mu\nu}(k,\lambda)\varepsilon^*_{\alpha\beta}(k,\lambda)=\tfrac{1}{2}(g_{\mu\alpha}g_{\nu\beta}+g_{\mu\beta}g_{\nu\alpha}-g_{\mu\nu}g_{\alpha\beta}) , \tag{2.5}$$

with

$$T_{\mu\nu\rho}=[(2p+k)_\mu \gamma_\nu+\gamma_\mu(2p+k)_\nu]\frac{\not{p}+\not{k}}{(p+k)^2}\gamma_\rho + \gamma_\rho \frac{\not{p}-\not{k}}{(p'-k)^2}[(2p'-k)_\mu\gamma_\nu+\gamma_\mu(2p'-k)_\nu] \tag{2.6}$$

and

$$\bar{T}_{\sigma\alpha\beta}=\gamma_\sigma \frac{\not{p}+\not{k}}{(p+k)^2}[(2p+k)_\alpha\gamma_\beta+\gamma_\alpha(2p+k)_\beta] + [(2p'-k)_\alpha\gamma_\beta+\gamma_\alpha(2p'-k)_\beta]\frac{\not{p}-\not{k}}{(p'-k)^2}\gamma_\sigma . \tag{2.7}$$

We define the kinematical invariants $s=q'^2=(k+p)^2=(k'+p')^2$, $t=(k-k')^2=(p-p')^2$, $u=q^2=(p-k')^2=(p'-k)^2$, and $s+u+t\cong 0$. Equation (2.3) is evaluated in the center-of-mass frame, where the four-vector momentum of the given particles are (Fig. 2) $k^\mu=(\omega,\mathbf{k})$, $p^\mu=(E,-\mathbf{k})$, $k'^\mu=(\omega',\mathbf{k}')$, and $p'^\mu=(E',-\mathbf{k}')$. After very lengthy calculations of traces in Eq. (2.3), we get the differential cross section

$$d\sigma_{\text{Comp}}=\frac{4G\alpha}{s}\left[t+\frac{1}{su}(2t^3+3t^2s+3t^2u + ts^2+2tsu+tu^2)\right]d\Omega' , \tag{2.8}$$

where α is the fine-structure constant and Ω' is the emitted photon solid angle. Setting $s=4k^2$, $t=-2k^2(1-\cos\theta_{\text{c.m.}}), u=-2k^2(1+\cos\theta_{\text{c.m.}})$ and integrating over the angle $\theta_{\text{c.m.}}$ (where $\theta_{\text{c.m.}}$ is the c.m. angle) we get $\sigma_{\text{comp}}=8\pi\alpha G=0.5\times 10^{-66}\ \text{cm}^2$, which no longer depend on the energy of the colliding particles. The radiation of the final particles becomes strongest in the directions of the initial momenta. This result is in agreement with the results of Vladimirov [15] for the collision $e^+e^-\to\gamma g$. Actually this is expected since these two processes are related by crossing symmetry. The calculation by Papini and Valluri [7] of one vertex γ-g interaction also shows that the results are energy independent.

III. GRAVITON BREMSSTRAHLUNG IN THE COULOMB FIELD

We consider graviton emission in a collision between an electron and a nucleus ($e^-+Z\to e^-+g+Z$). The momentum $k'=p'-p+k$ is the four-vector momentum transfer to the nucleus. Since the recoil of the nucleus is neglected, the time component $k'_0=0$. According to Fig. 3 the matrix element M_{fi} has the form

$$M_{fi}=-e\sqrt{8\pi G}\,\Phi(|\mathbf{k}'|)\varepsilon^*_{\alpha\beta}(k,\lambda)\bar{u}(p',s') \times \left[[(p'+q')_\alpha\gamma_\beta+\gamma_\alpha(p'+q')_\beta]\frac{\not{q}'}{q'^2}\gamma_0+\gamma_0\frac{\not{q}}{q^2} \times[(p+q)_\alpha\gamma_\beta+\gamma_\alpha(p+q)_\beta]\right]u(p,s) , \tag{3.1}$$

where the intermediate four-momenta are $q'=p+k'=k+p'$, $q=p-k=p'-k'$, and $\Phi(|\mathbf{k}'|)$ is the scalar potential of the external field; for a Coulomb field

$$\Phi(|\mathbf{k}'|)=-4\pi Z e^-/|\mathbf{k}'|^2 , \tag{3.2}$$

After averaging the cross section over the initial spin of

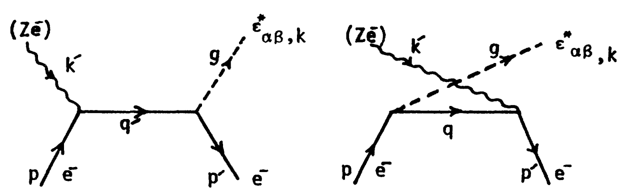

FIG. 3. Graviton bremsstrahlung in the Coulomb field.

the electron and summing over the polarizations of the final electron and graviton, by squaring (3.1) one obtains

$$|M_{fi}|^2=8\pi G\frac{Z^2e^4}{|\mathbf{k}'|^4}\frac{1}{2}\sum_{\text{polar}}\varepsilon_{\mu\nu}\varepsilon^*_{\alpha\beta}(\bar{u}'T_{\mu\nu}u)(\bar{u}\bar{T}_{\alpha\beta}u')\ . \tag{3.3}$$

Inserting (2.5) into (3.3), we get

$$|M_{fi}|^2=4\pi G\frac{Z^2e^4}{|\mathbf{k}'|^4}\mathrm{Tr}[\tfrac{1}{2}(g_{\mu\alpha}g_{\nu\beta}+g_{\mu\beta}g_{\nu\alpha}-g_{\mu\nu}g_{\alpha\beta})\times(p'\cdot\gamma T_{\mu\nu}p\cdot\gamma\bar{T}_{\alpha\beta})]\ , \tag{3.4}$$

where

$$T_{\mu\nu}=[(2p'+k)_\mu\gamma_\nu+\gamma_\mu(2p'+k)_\nu]\frac{\not{p}'+\not{k}}{(p'+k)^2}\gamma_0+\gamma_0\frac{\not{p}-\not{k}}{(p-k)^2}[(2p-k)_\mu\gamma_\nu+\gamma_\mu(2p-k)_\nu] \tag{3.5}$$

and

$$\begin{aligned}\bar{T}_{\mu\nu}&=\gamma_0T^\dagger_{\alpha\beta}\gamma_0\\&=\gamma_0\frac{\not{p}'+\not{k}}{(p'+k)^2}[(2p'+k)_\alpha\gamma_\beta+\gamma_\alpha(2p'+k)_\beta]\\&\quad+[(2p-k)_\alpha\gamma_\beta+\gamma_\alpha(2p-k)_\beta]\frac{\not{p}-\not{k}}{(p-k)^2}\gamma_0\ .\end{aligned} \tag{3.6}$$

The differential cross section for bremsstrahlung is given by [16]

$$d\sigma_{\text{brems}}=\frac{1}{(2\pi)^5}|M_{fi}|^2\frac{E'\omega}{E}\,d\omega\,d\Omega_k\,d\Omega'\ , \tag{3.7}$$

where $d\Omega_k\,d\Omega'$ are the solid angles of the graviton emission and the final electron. Inserting the value of $|M_{fi}|^2$ in Eq. (3.7), we obtain the following expression for bremsstrahlung cross section:

$$\begin{aligned}d\sigma_{\text{brems}}=&\frac{32Z^2\alpha^2G}{|\mathbf{k}'|^4}\frac{E'\omega}{E}\,d\omega\,d(\cos\theta)d(\cos\theta')\\&\times\Bigg\{(p\cdot k)-2(p_0k_0)+\frac{1}{2(p'\cdot k)(p\cdot k)}\\&\times[3(p'\cdot p)^2(p\cdot k)+2(p'\cdot p)^3-3(p'\cdot p)^2(p'\cdot k)-4(p'\cdot p)(p'_0p_0)(p\cdot k)-4(p'_0p_0)(p'\cdot p)^2\\&+4(p'_0p_0)(p'\cdot k)(p'\cdot p)-2(p'\cdot p)(p'\cdot k)(p\cdot k)+(p'\cdot p)(p'\cdot k)^2+(p'_0k_0)(p'\cdot p)(p\cdot k)+2(p'_0k_0)(p'\cdot p)^2\\&-(p'_0k_0)(p'\cdot p)(p'\cdot k)-(p'_0p'_0)(p\cdot k)^2-2(p'_0p'_0)(p'\cdot p)(p\cdot k)+(p'_0p'_0)(p'\cdot k)(p\cdot k)+2(p'_0p_0)(p'\cdot k)(p\cdot k)\\&-(p'_0p_0)(p'\cdot k)^2+(p'\cdot p)(p\cdot k)^2-(p'_0p_0)(p\cdot k)^2+(p_0p_0)(p'\cdot k)(p\cdot k)+2(p_0p_0)(p'\cdot k)(p'\cdot p)\\&-(p_0p_0)(p'\cdot k)^2-(p_0k_0)(p'\cdot p)(p\cdot k)-2(p_0k_0)(p'\cdot p)^2+(p_0k_0)(p'\cdot p)(p'\cdot k)]-(p'\cdot k)+2(p'_0k_0)\Bigg\}\ ,\end{aligned} \tag{3.8}$$

where k_0,p_0,p'_0 are the energy components of the graviton, the initial, and the final electron, respectively; θ,θ' the angles between $\mathbf{k}$ and $\mathbf{p},\mathbf{p}'$, respectively. It is convenient to write, at extremely high energy $E\gg m$ the relations

$$p'\cdot p\cong p'\cdot k-p\cdot k-\mathbf{k}'^2/2\ ,\quad\text{where } p'^2=p^2=m^2,\ \ k^2=0\ , \tag{3.9}$$

$$p\cdot k=E\omega(1-\cos\theta)=\omega\delta,\quad\text{where }\delta=E(1-\cos\theta)\ , \tag{3.10}$$

and

$$p'\cdot k=E'\omega(1-\cos\theta')=\omega\delta'\ ,\quad\text{where }\delta'=E'(1-\cos\theta')\ , \tag{3.11}$$

substituting (3.9)–(3.11) in (3.8). The integration of Eq. (3.8) over the angles θ,θ' is rather lengthy (see Refs. [16–19] for photon bremsstrahlung). We shall give only the final result (a few steps are described in Appendix B):

$$\begin{aligned}d\sigma_{\text{brems}}=&32Z^2\alpha^2G(E'/E)(d\omega/\omega)\\&\times[\tfrac{1}{2}+\beta L(1-3E'/2E+E'^2/E^2)+L(-\tfrac{11}{8}+3E/16E'-5E'/16E)+\beta(\tfrac{15}{8}-9E'/8E-2E/3E')\\&+\beta'(\tfrac{7}{8}-29E/16E'-9E'/16E)+\beta\beta'(1-5E'/4E)]\ ,\end{aligned} \tag{3.12}$$

where $\beta=\ln[(E+p)/(E-p)]\cong2\ln(2E/m)$; $\beta'=\ln[(E'+p')/(E'-p')]\cong2\ln(2E/m)$ and $L=\ln[(EE'+pp'$

$-m^2)/(EE'-pp'-m^2)]\cong 2\ln(2EE'/m\omega)$ as defined by Bethe and Heitler [18]. The presence of the logarithm of a large quantity [the ratio $(2EE'/m\omega)\gg 1$ even if $\omega\cong E$] should be noted, the logarithmic terms become the principal ones in (3.12). Finally, we shall give the limiting formula for the region near the end of the spectrum, when the extreme-relativistic electron radiates almost all its energy $\omega\cong E\gg E'\gg m$, then $L=\beta'$ one can easily find

$$d\sigma_{\text{brems}}=32Z^2\alpha^2G\,d\omega/E(2\beta/3+13\beta'/8)\ . \tag{3.13}$$

Equation (3.13) covers all the range of ω values for extremely relativistic initial electron.

IV. PAIR PRODUCTION BY A GRAVITON IN THE COULOMB FIELD

The process of pair production by a graviton in the field of a nucleus ($g+Z\to e^-+e^++Z$) is very closely related to the process of bremsstrahlung in the previous section. Figure (4) shows the corresponding diagrams, where $E,p\to -E,-p;\omega,k\to-\omega,-k$; and $\varepsilon^*_{\alpha\beta}\to\varepsilon_{\mu\nu}$. The four-vector momentum transfer to the nucleus can be written as $k'=p'+p-k$, and the energy transition is $\omega=E+E'$. The matrix element M_{fi} is, therefore,

$$M_{fi}=-e\sqrt{8\pi G}\,\Phi(|\mathbf{k}'|)\varepsilon_{\mu\nu}\bar{u}(p',s')\times\left\{[(p'+q')_\mu\gamma_\nu+\gamma_\mu(p'+q')_\nu]\frac{\not{q}'}{q'^2}\gamma_0\frac{\not{q}}{q^2}[(-p+q)_\mu\gamma_\nu+\gamma_\mu(-p+q)_\nu]\right\}\bar{v}(-p,s)\ , \tag{4.1}$$

where the intermediate four-momenta are $q'=-p+k'=-k+p'$, $q=-p+k=p'-k'$, and $\bar{v}(-p,s)$ is the Dirac spinor of the emerging positron. The differential cross section for the pair production is

$$d\sigma_{\text{pair}}=[1/(2\pi)^5]|M_{fi}|^2(EE'/\omega)dE\,d\Omega'\,d\Omega\ ; \tag{4.2}$$

we have multiplied (3.7) by $(E^2/\omega^2)dE/d\omega$ and replaced by $d\Omega_k$ by $d\Omega$, the solid angle of the emerging positron (see Ref. 16, Sec. 91]). By squaring (4.1) one obtains

$$|M_{fi}|^2=8\pi G\frac{Z^2e^4}{|\mathbf{k}'|^4}\frac{1}{2}\sum_{\text{polar}}\varepsilon_{\mu\nu}\varepsilon^*_{\alpha\beta}(\bar{u}'T_{\mu\nu}v)(\bar{v}\bar{T}_{\alpha\beta}u') \tag{4.3}$$

$$=4\pi G\frac{Z^2e^4}{|\mathbf{k}'|^4}\mathrm{Tr}[\tfrac{1}{2}(g_{\mu\alpha}g_{\nu\beta}+g_{\mu\beta}g_{\nu\alpha}-g_{\mu\nu}g_{\alpha\beta})(p'\cdot\gamma T_{\mu\nu}p\cdot\gamma\bar{T}_{\alpha\beta})]\ , \tag{4.4}$$

where

$$T_{\mu\nu}=[(2p'-k)_\mu\gamma_\nu+\gamma_\mu(2p'-k)_\nu]\frac{-\not{k}+\not{p}'}{(-k+p')^2}\gamma_0+\gamma_0\frac{-\not{p}+\not{k}}{(-p+k)^2}[(-p+k)_\mu\gamma_\nu+\gamma_\mu(-p+k)_\nu] \tag{4.5}$$

and

$$\bar{T}_{\alpha\beta}=\gamma_0T^\dagger_{\alpha\beta}\gamma_0$$

$$=\gamma_0\frac{-\not{k}+\not{p}'}{(-k+p')^2}[(2p'-k)_\alpha\gamma_\beta+\gamma_\alpha(2p'-k)_\beta]+[(-p+k)_\alpha\gamma_\beta+\gamma_\alpha(-p+k)_\beta]\frac{-\not{p}+\not{k}}{(-p+k)^2}\gamma_0\ . \tag{4.6}$$

By means of (4.4) and (4.2) the cross section can be now written as

$$\begin{aligned}d\sigma_{\text{pair}}=&-\frac{32Z^2\alpha^2G}{|\mathbf{k}'|^4}\frac{EE'}{\omega}dE\,d(\cos\theta)d(\cos\theta')\\&\times\Big\{(p\cdot k)-2(p_0k_0)-\frac{1}{2(p'\cdot k)(p\cdot k)}\\&\times[3(p'\cdot p)^2(p\cdot k)-2(p'\cdot p)^3+3(p'\cdot p)^2(p'\cdot k)-4(p'\cdot p)(p\cdot k)(p_0'p_0)\\&+4(p'\cdot p)^2(p_0'p_0)-4(p'\cdot p)(p'\cdot k)(p_0'p_0)-2(p'\cdot p)(p'\cdot k)(p\cdot k)\\&-(p'\cdot p)(p'\cdot k)^2+(p_0'k_0)(p'\cdot p)(p\cdot k)-2(p'\cdot p)^2(p_0'k_0)+(p_0'k_0)(p'\cdot p)(p'\cdot k)\\&-(p_0'p_0')(p\cdot k)^2+2(p_0'p_0')(p'\cdot p)(p\cdot k)-(p_0'p_0')(p'\cdot k)(p\cdot k)\\&+2(p_0'p_0)(p'\cdot k)(p\cdot k)+(p_0'p_0)(p'\cdot k)^2-(p'\cdot p)(p\cdot k)^2+(p_0'p_0)(p\cdot k)^2\\&-(p_0p_0)(p'\cdot k)(p\cdot k)+2(p_0p_0)(p'\cdot k)(p'\cdot p)-(p_0p_0)(p'\cdot k)^2+(p'\cdot p)(p\cdot k)(p_0k_0)\\&-2(p'\cdot p)^2(p_0k_0)+(p'\cdot p)(p'\cdot k)(p_0k_0)]+(p'\cdot k)-2(p_0'k_0)\Big\}\ .\end{aligned} \tag{4.7}$$

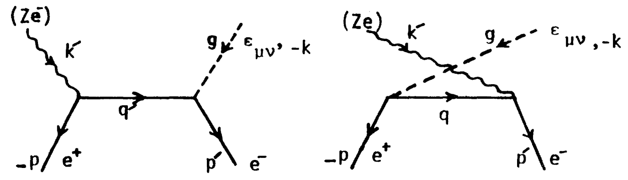

FIG. 4. Pair production by a graviton in the Coulomb field.

The electron and the positron are emitted at angles θ,θ' relative to the direction of the incident graviton. The integration over the angles θ,θ' is completely analogous to the bremsstrahlung case [Eq. (3.8)]. We assume the electron and positron share equally the energy of the graviton $\omega=E+E'\cong 2E$, then $\beta=\beta'\cong 2\ln(2E/m)$ and $L\cong 2\ln(E/m)$. We find the differential cross section is

$$d\sigma_{\text{pair}}=Z^2\alpha^2G(dE/E)(7\beta^2/2+201\beta/5-31/5)\ . \tag{4.8}$$

Integration of Eq. (4.8) over E from $m\to\omega$ gives the total cross section for pair production by a graviton having a given energy $\omega\gg m$:

$$\begin{aligned}\sigma_{T\,\text{pair}}=Z^2\alpha^2G\{&\tfrac{7}{3}\ln^3(\omega/m)+\ln^2(\omega/m)\\&\times[7\ln(2/m)+\tfrac{201}{10}]+\ln(\omega/m)\\&\times[7\ln^2(2/m)+\tfrac{201}{5}\ln(2/m)]-\tfrac{31}{5}\}\ .\end{aligned} \tag{4.9}$$

As in bremsstrahlung, the logarithmic terms in pair production become the principal one in (4.8) and (4.9). Taking $\omega=10^4$ TeV, $Z=1$, then $\sigma_{T\,\text{pair}}\cong 10^{-66}$ cm^2. Near threshold, i.e., $\omega\to 2m$, Eq. (4.9) can be reduced to $\sigma_{\text{Th}}=(370)Z^2\alpha^2G\cong 0.05\times 10^{-66}$ cm^2.

V. CONCLUSIONS

We have calculated the first-order cross sections for the processes graviton-Compton scattering ($ge^-\to\gamma e^-$), bremsstrahlung, and pair production by a graviton in the Coulomb field at extremely high energies. In the process $ge^-\to\gamma e^-$ the final result is energy independent, which is in agreement with the results obtained in Refs. [7] and [15]. We have treated the graviton bremsstrahlung and pair production in a manner similar to photon bremsstrahlung and pair production [16–18]. Since the bremsstrahlung and pair production depend on the logarithmic terms, one might hope, at least from the astrophysical point of view, that these processes are significant.

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APPENDIX A

Lagrangian and the Feynman rule

The Lagrangian density for a spin-$\frac{1}{2}$ fermion in a gravitational field is given by the sum of the Dirac and Einstein Lagrangian densities [10]:

$$\mathcal{L}(\bar e,\psi)=-\tfrac{1}{2}\bar e\kappa^{-2}R(\bar g)-\bar e\bar\psi\gamma^a e_a^\mu D_\mu\psi,\quad \kappa^2=8\pi G\ , \tag{A1}$$

where $\bar g$ is expressed in terms of $\bar e$ by the relation

$$e_\mu^a e_\nu^b\eta_{ab}=g_{\mu\nu}\ . \tag{A2}$$

The matrix e_a^μ (the inverse of e_μ^a) is a set of vierbein fields or tetrad fields (which is defined as the matrix square root of the metric tensor $g_{\mu\nu}$), and e is given by

$$e\equiv\det(e_\mu^a)=[\det(e_a^\mu)]^{-1}=[-\det(g_{\mu\nu})]^{1/2}\ . \tag{A3}$$

η_{ab} is the Minkowski metric tensor; R is the curvature scalar; $\bar\psi$ and ψ are fermion fields, which can be introduced into general relativity [20] by describing them with respect to local Lorentz frames; they are defined to be world scalars and transform as ordinary spinors under local Lorentz transformations of the vierbein frames (Lorentz spinors). The covariant derivatives D_μ can be introduced as a covariant world vector and a Lorentz spinor,

$$D_\mu=\partial_\mu+\tfrac{1}{2}\sigma^{ab}\omega_{\mu ab}\ , \tag{A4}$$

where $\sigma^{ab}=\frac{1}{4}(\gamma^a\gamma^b-\gamma^b\gamma^a)$ and

$$\omega_{\mu ab}=[e_a^\nu(\partial_\mu e_{b\nu}-\partial_\nu e_{b\mu})+\tfrac{1}{2}e_a^\rho e_b^\sigma(\partial_\sigma e_{c\rho}-\partial_\rho e_{c\sigma})e_\mu^c]_{(ab)}\ .$$

The last symbol denotes antisymmetrization in (ab), $\omega_{\mu ab}$ is a covariant vector; under local Lorentz transformations it is not a tensor, but acquires an inhomogeneous term which is needed to make $D_\mu\psi$ a Lorentz spinor. The fields $\bar e,\psi$ can be written as a sum of background and quantum fields:

$$\bar e_\mu^a=e_\mu^a+\kappa c_\mu^a,\quad \psi\to\kappa^{-1}\zeta+\psi\ . \tag{A5}$$

The factor κ has been inserted to give the quantum fields canonical dimension (units $\hbar=c=1$ are used). The quantum field c_μ^a can be considered just like other matter fields, such as photons and fermions, and ζ is a fermion field. We expand $\mathcal{L}(e+\kappa c,\ \kappa^{-1}\zeta+\psi)$ in quantum fields (c,ψ) around the background fields (e,ζ). For the first variational derivative (neglecting the other Lagrangians) one has [10]

$$\begin{aligned}\mathcal{L}_1(e,c;\zeta,\psi)=\kappa^{-1}e\{&c_\mu^a[2G_a^\mu+T_a^\mu(e,\zeta)]\\&-\bar\zeta\gamma^\mu D_\mu\psi-\bar\psi\gamma^\mu D_\mu\zeta\}\ ,\end{aligned} \tag{A6}$$

where $G_a^\mu=e_{a\nu}G^{\nu\mu}$ and $G^{\mu\nu}$ is the symmetric Einstein tensor $R^{\mu\nu}-\frac{1}{2}g^{\mu\nu}R$ ($R^{\mu\nu}$ is the Ricci tensor). $\mathcal{L}_1$ vanishes if and only if the classical field equations are satisfied by the background field, namely,

$$G_{\mu\nu}=-\tfrac{1}{2}T_{\mu\nu},\quad \gamma^\mu D_\mu\zeta=(D_\mu\bar\zeta)\gamma^\mu=0\ . \tag{A7}$$

The Einstein equations consist of a symmetric part

$G_{\mu\nu}=-\frac{1}{2}(T_{\mu\nu}+T_{\nu\mu})$ and an antisymmetric part $T_{\mu\nu}-T_{\nu\mu}=0$. The fermion stress tensor $T_a^\mu=-\delta\mathcal{L}^D/\delta e_\mu^a$ is

$$T_\mu^a=-\bar{\zeta}\gamma^a D_\mu\zeta+\delta_\mu^a\bar{\zeta}\gamma^\nu D_\nu\zeta+\tfrac{1}{2}D_\lambda(\bar{\zeta}\gamma^\lambda\sigma^{\mu a}\zeta)+\tfrac{1}{2}D_\alpha[\bar{\zeta}(\gamma^\mu\sigma^{a\alpha}+\gamma^a\sigma^{\mu\alpha})\zeta] \tag{A8}$$

and is *a priori* nonsymmetric, but it becomes symmetric, conserved, and traceless as a consequence of the Dirac equation $\gamma^\mu D_\mu\zeta=0$ [for convenience, we have neglected the fermion mass in Eq. (A1)], which reduces it to the usual expression $T_{\mu\nu}=\bar{\zeta}(\gamma_\mu\overleftrightarrow{D}_\nu+\gamma_\nu\overleftrightarrow{D}_\mu)\zeta$. Thus, the corresponding Feynman rule for a spin-$\frac{1}{2}$ fermion-graviton vertex is $\kappa(\gamma_\mu P_\nu+\gamma_\nu P_\mu)$, $P=p+q$ (where p and q are the momentum components of the initial and final fermion states).

APPENDIX B

Integrated cross section

The integrals to be evaluated have the general form

$$I_{m,n}=\int d(\cos\theta')k'^{-2m}\delta'^{-n}\ , \qquad m=-1,0,1,2,\quad n=-2,-1,0,1\ . \tag{B1}$$

where $\delta'=E'(1-\cos\theta')$, if one writes $\mathbf{k}'^2=(\mathbf{p}-\mathbf{p}'-\mathbf{k})^2=(T^2+p'^2)(1-\mathbf{p}'\cdot\mathbf{c})$, with $\mathbf{c}=2\mathbf{T}/(T^2+p^2)$, $\mathbf{T}=p-k$, and $\delta'=E'(1-\mathbf{p}'\cdot\mathbf{d})$, with $\mathbf{d}=\mathbf{k}/\omega E'$, then the integrals reduce to the form

$$I_{m,n}=(p'^2+T^2)^{-m}E'^{-n}\times\int d(\cos\theta')(1-\mathbf{p}'\cdot\mathbf{c})^{-m}(1-\mathbf{p}'\cdot\mathbf{d})^{-n}\ . \tag{B2}$$

We find the following cases: $I_{1,0}$, $I_{2,0}$, $I_{0,1}$, $I_{0,2}$, $I_{1,1}$, $I_{1,-1}$, $I_{-1,1}$, $I_{2,1}$, $I_{2,-1}$, $I_{2,-2}$. The integrals for $m=0$ or $n=0$ may be easily evaluated by choosing $\mathbf{c}$ or $\mathbf{d}$ as the polar axis for the integration over θ'; then

$$\begin{aligned}I_{1,0}&=(p'^2+T^2)^{-1}\int_{-1}^{1}d(\cos\theta'_{p'c})(1-\mathbf{p}\cdot\mathbf{c})^{-1}\\&=(p'c)^{-1}(p'^2+T^2)^{-1}\ln[(1+p'c)/(1-p'c)]\\&=(p'T)^{-1}\beta_T\ ,\end{aligned}$$

where $\beta'_T=\ln[(T+p')/(T+p')]$, at $E\gg m$ $\beta'_T\cong(L+\beta')/2$, with

$$L=\ln\frac{EE'+pp'-m^2}{EE'-pp'-m^2}\cong 2\ln\frac{2EE'}{m\omega}\ .$$

and

$$\beta'=\ln[(E'+p')/(E'-p')]\cong 2\ln(2E'/m)\ ,$$

$$\begin{aligned}I_{2,0}&=(p'^2+T^2)^{-2}\int_{-1}^{1}d(\cos\theta'_{p'c})(1-\mathbf{p}'\cdot\mathbf{c})^{-2}\\&=2(p'^2+T^2)^{-2}(1-p'^2c^2)^{-1}=2(T^2-p'^2)^{-2}\ ,\end{aligned}$$

$$I_{0,1}=(E'p'b)^{-1}\ln[(1+p'd)/(1-p'd)]=\beta'/p'\ ,$$

$$I_{0,2}=2E'^{-2}(1-p'^2b^2)^{-1}=2(E'^2-p'^2)^{-1}\ .$$

The integrals such as $I_{1,-1}$, $I_{-1,1}$, $I_{2,-1}$, and $I_{2,-2}$ can be expressed in terms of the others by choosing $\mathbf{c}$ as the polar axis for the integration over θ'. For example,

$$\begin{aligned}I_{1,-1}&=(p'^2+T^2)^{-1}E'\int_{-1}^{1}d(\cos\theta'_{p'c})(1-\mathbf{p}'\cdot\mathbf{c})^{-1}(1-\mathbf{p}'\cdot\mathbf{d})\\&=(p'^2+T^2)^{-1}E'\int_{-1}^{1}d(\cos\theta'_{p'c})(1-\mathbf{p}'\cdot\mathbf{d})^{-1}[1-(\mathbf{p}'\cdot\mathbf{c})(\mathbf{c}\cdot\mathbf{d})/c^2]\\&=(p'^2+T^2)^{-1}E'I_{1,0}[1-\mathbf{c}\cdot\mathbf{d}/c^2]\ ;\end{aligned}$$

using $\mathbf{c}=2\mathbf{T}/(p'^2+T^2)$; $\mathbf{d}=\mathbf{k}/\omega E'$, and $2\mathbf{k}\cdot\mathbf{T}=p^2-T^2-\omega^2$, one finds

$$I_{1,-1}=\frac{E'}{2Tp'}\beta'_T[1-(p^2-T^2-\omega^2)(T^2+p'^2)/4\omega ET^2]\ ,$$

$$\begin{aligned}I_{-1,1}&=(p'^2+T^2)E'^{-1}I_{0,1}[1-\mathbf{c}\cdot\mathbf{d}/d^2]\\&=\frac{p'^2+T^2}{p'}\beta'-\frac{p^2-T^2-\omega^2}{\omega}\ .\end{aligned}$$

The integrals $I_{1,1}$,$I_{2,1}$ may be easily evaluated by using the Feynman integral

$$(\alpha\gamma)^{-1}=\int_{-1}^{1}\frac{dx}{[\alpha x+\gamma(1-x)]^2}\ , \tag{B3}$$

and those find by differentiating (B3) with respect to α and γ. For example

$$\begin{aligned}I_{1,1}&=(p'^2+T^2)^{-1}E'^{-1}\int d(\cos\theta'_{p'c})(1-\mathbf{p}'\cdot\mathbf{c})^{-1}(1-\mathbf{p}'\cdot\mathbf{d})^{-1}\\&=(p'^2+T^2)^{-1}E'^{-1}\int_0^1 dx\int_{-1}^{1}d(\cos\theta'_{p'g})(1-\mathbf{p}'\cdot\mathbf{h})^{-2}\ ,\end{aligned}$$

where $\mathbf{h}=\mathbf{c}x+\mathbf{d}(1-x)$. If $\mathbf{h}$ is now as the polar axis, the integration over θ' may be performed, giving $I_{1,1}=2(p'^2+T^2)^{-1}E'^{-1}\int_0^1\xi^{-1}dx$, where ξ is given as a quadratic in x:

$$\xi=1-p'^2h^2=1-p'^2c^2-2xp'^2(\mathbf{c}\cdot\mathbf{d}-b^2)-x^2p'^2(\mathbf{c}-\mathbf{d})^2\ ,$$

then one can obtain $I_{1,1}=\omega(p'p)^{-1}(T^2-p'^2)^{-1}L$. By a similar method one also gets

$$I_{2,1}=4(p'^2+T^2)^{-2}E'^{-1}\int_0^1 x\xi^{-2}\,dx$$
$$=\frac{-32p'^4p^2}{\omega^3E'^4}\left[\frac{p^2-T^2-\omega^2}{(p'^2+T^2)^3}-\frac{4\omega E'T^2}{(p'^2+T^2)^4}\right]-\frac{\omega^2}{(p'p^3)(T^2-p'^2)^3}[E'(p^2-T^2-\omega^2)-\omega(p'^2+T^2)]L\ .$$

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