Sourceless Abelian gauge string in an expanding universe

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A model consisting of a sourceless Abelian gauge string in an expanding spacetime background is examined. Exact solutions are found which describe a radial inflow of gauge field energy that can be associated with the creation or destruction of a sourceless gauge string. The time rate of change of the string flux depends upon the expansion rate of the universe.

I. INTRODUCTION

It is well known that an Abelian gauge theory characterized by the inclusion of a complex-valued scalar Higgs field along with an assumption of cylindrical symmetry admits a solution describing an infinitely long straight cosmic gauge string centered along the symmetry axis [1–4]. Such a cosmic gauge string acquires structure from the Higgs field and possesses the salient feature that there exists a cylindrical region of nonvanishing "magnetic" flux. On the symmetry axis the phase of the Higgs field becomes undefined, forcing the Higgs field to vanish there. Allowing the modulus of the Higgs field to approach a constant value asymptotically induces a breaking of the U(1) gauge symmetry. The string solution has a relatively large energy density near the core, and describes a localized, nonperturbative, topologically nontrivial soliton state. Enlarging the gauge group can give rise to further interesting solutions, such as superconducting cosmic strings [5].

Various aspects of string solutions have been studied extensively [6–12], but the system of equations describing a single string can be sufficiently complicated that without simplifying assumptions analytical solutions are often difficult to obtain. Thus it is often necessary to resort to numerical studies and simplified scenarios in an attempt to gain some insight regarding the behavior and properties of strings.

The model presented here is that of a sourceless Abelian gauge string with the complex-valued Higgs field being completely removed. The removal of the Higgs field has some obvious effects, which include the removal of structure from the gauge string, making it singular, along with the dismissal of a symmetry-breaking mechanism so that the gauge field remains massless outside of the string, and no topologically conserved winding number exists. Although in the limit of vanishing Higgs fields the model is rendered artificial as a description of realistic cosmic strings, a mathematical advantage is gained in that *exact* mathematical solutions can be obtained. Such a procedure has been used to study non-Abelian solitonic gauge field configurations [13–16].

In Sec. II the model is presented, and solutions are obtained in Sec. III. Both static and time-dependent solutions exist. Associated with the time-dependent solution is the radial inflow of "electromagnetic" radiation which changes the string's "magnetic" field and flux. A summary is presented and conclusions are drawn in Sec. IV. It is concluded that the time rate of change of the string flux depends upon the rate of expansion of the Universe when the cosmic scale factor is assumed to have a power-law behavior, $a(t)\sim t^{\alpha}(0\leq\alpha<1)$. Specifically, it is determined that the smaller the value of α, the greater the time rate of change of the string flux. Such a string could therefore form more rapidly during a period of radiation dominance ($\alpha=\frac{1}{2}$) than during a period of matter dominance ($\alpha=\frac{2}{3}$).

II. THE MODEL

A U(1) gauge field with cylindrical symmetry is assumed to exist within a flat Robertson-Walker (RW) background described by the line element

$$ds^2=dt^2-a^2(t)(dr^2+r^2d\varphi^2+dz^2)\ . \tag{2.1}$$

The Lagrangian for the gauge field is

$$L=\sqrt{-g}\,(-\tfrac{1}{4}F^{\mu\nu}F_{\mu\nu})\ , \tag{2.2}$$

where

$$\sqrt{-g}=a^3r \tag{2.3}$$

and the gauge field tensor is

$$F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu\ . \tag{2.4}$$

The gauge vector field A_μ is parametrized by

$$A_u=\frac{1}{e}[P(r,t)-1]\delta_\mu^\varphi\ , \tag{2.5}$$

where $P(r,t)$ is the gauge field structure function. The equation of motion for the gauge field is given by

$$\nabla_\mu F^{\mu\nu}=\frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}\,F^{\mu\nu})=0\ , \tag{2.6}$$

where ∇_μ is the covariant derivative.

The nonvanishing components of the gauge field tensor are listed for reference:

$$F_{t\varphi}=\frac{\dot{P}}{e},\quad F^{t\varphi}=-\frac{\dot{P}}{ea^2r^2}\ , \tag{2.7a}$$

$$F_{r\varphi}=\frac{P'}{e}, \quad F^{r\varphi}\frac{P'}{ea^4r^2} , \tag{2.7b}$$

$$F^t{}_{\varphi}=\frac{\dot{P}}{e}, \quad F^{\varphi}{}_t=\frac{\dot{P}}{ea^2r^2} , \tag{2.7c}$$

$$F^r{}_{\varphi}=-\frac{P'}{ea^2}, \quad F^{\varphi}{}_r=\frac{P'}{ea^2r^2} , \tag{2.7d}$$

$$F_{\mu\nu}=-F_{\nu\mu}, \quad F^{\mu}{}_{\nu}=-F_{\nu}{}^{\mu} . \tag{2.7e}$$

Combining (2.5)–(2.7) results in the partial differential equation

$$\ddot{P}+H\dot{P}-\frac{1}{a^2}\left[P''-\frac{1}{r}P'\right]=0 , \tag{2.8}$$

where an overdot represents differentiation with respect to t, a prime represents differentiation with respect to r, and $H=\dot{a}/a$ is the Hubble "constant."

The energy-momentum tensor is given by [17]

$$T^{\mu\nu}=-F^{\mu\gamma}F^{\nu}{}_{\gamma}+\tfrac{1}{4}g^{\mu\nu}F_{\alpha\beta}F^{\alpha\beta} \tag{2.9}$$

with the nonvanishing components [18]

$$T^{tt}=\left[\frac{1}{2e^2a^2r^2}\right]\left[\dot{P}^2+\left[\frac{P'}{a}\right]^2\right] , \tag{2.10a}$$

$$T^{rr}=g^{rr}\left[\frac{1}{2e^2a^2r^2}\right]\left[-\dot{P}^2-\left[\frac{P'}{a}\right]^2\right] , \tag{2.10b}$$

$$T^{\varphi\varphi}=g^{\varphi\varphi}\left[\frac{1}{2e^2a^2r^2}\right]\left[\dot{P}^2-\left[\frac{P'}{a}\right]^2\right] , \tag{2.10c}$$

$$T^{zz}=g^{zz}\left[\frac{1}{2e^2a^2r^2}\right]\left[-\dot{P}^2+\left[\frac{P'}{a}\right]^2\right] , \tag{2.10d}$$

$$T^{tr}=\frac{-\dot{P}P'}{e^2a^4r^2} . \tag{2.10e}$$

The "ordinary" "magnetic" field is described by [19]

$$B_z=\frac{1}{\sqrt{g_{rr}g_{\varphi\varphi}}}F_{\varphi r}=\frac{1}{a^2r}F_{\varphi r}=-\frac{P'}{ea^2r} . \tag{2.11}$$

Consider the area element of a small disk of radius r, perpendicular to the z axis,

$$dA=\sqrt{g_{rr}g_{\varphi\varphi}}dr\,d\varphi=a^2r\,dr\,d\varphi , \tag{2.12}$$

and the length element around the disk's periphery

$$dl=\sqrt{-g_{\varphi\varphi}}d\varphi=ar\,d\varphi . \tag{2.13}$$

Aside from the "magnetic" field of (2.11), in general the sourceless U(1) gauge theory possesses an additional singular gauge string with a "magnetic" field B_s, with an associated field tensor component $F^2{}_{\phi r}$, restricted to the z axis. To see this, write the total "magnetic" flux through a circular element of area with radius r, perpendicular to the z axis, as

$$\Phi_T(r,t)=\int_0^{2\pi}\int_0^r F^T{}_{\varphi r}dr\,d\varphi=\oint \mathbf{A}\cdot\hat{\boldsymbol{\varphi}}\,dl$$
$$=-\oint A_{\varphi}d\varphi=\frac{2\pi}{e}[1-P(r,t)] . \tag{2.14}$$

But the flux Φ associated with the field $F_{\phi r}$ in (2.7b) is

$$\Phi(r,t)=\int_0^{2\pi}\int_0^r F_{\varphi r}dr\,d\varphi$$
$$=\frac{2\pi}{e}[P(0,t)-P(r,t)] . \tag{2.15}$$

Since $P(0,t)$ need not be identically equal to unity [20], from (2.14) and (2.15) it follows that $\Phi\neq\Phi_T$ in general, implying the existence of a singular string field $F^s_{\phi r}$ with an associated string flux

$$\Phi_s(t)=\Phi_T(r,t)-\Phi(r,t)$$
$$=\frac{2\pi}{e}[1-P(0,t)] . \tag{2.16}$$

Since

$$\Phi_s(t)=\int_0^{2\pi}\int_0^r F^s{}_{\varphi r}dr\,d\varphi$$
$$=\int_0^{2\pi}\int_0^r \frac{2\pi}{e}[1-P(r,t)]\delta(r)\delta(\varphi)dr\,d\varphi , \tag{2.17}$$

it follows that

$$F^s{}_{\varphi r}=\frac{2\pi}{e}[1-P(r,t)]\delta(r)\delta(\varphi) . \tag{2.18}$$

The total field therefore consists of a string field $F^T{}_{\phi r}$ and an external (nonstring) field $F_{\phi r}$.

III. SOLUTIONS

The partial differential equation (PDE) (2.8), subject to the asymptotic boundary conditions

$$P(r,t)\to\text{const} \quad \text{as } r\to\infty , \tag{3.1a}$$

$$P(r,t)\to\text{const} \quad \text{as } t\to\infty , \tag{3.1b}$$

can be separated into a radial differential equation (DE) and a temporal DE. Let

$$P(r,t)=F(r)G(t) . \tag{3.2}$$

The PDE (2.8) then separates into

$$F''-\frac{1}{r}F'-k^2F=0 , \tag{3.3a}$$

$$\ddot{G}+H\dot{G}-\frac{k^2}{a^2}G=0 , \tag{3.3b}$$

with the boundary conditions

$$F(r)\to\text{const} \quad \text{as } r\to\infty , \tag{3.4a}$$

$$G(t)\to\text{const} \quad \text{as } t\to\infty . \tag{3.4b}$$

The separation constant k^2 may be negative, zero, or positive. Each case can be examined separately.

A. $k^2<0$

For the radial DE (3.3a), define $kr\equiv i\lambda r\equiv i\rho$. Equation (3.3a) then becomes

$$\frac{d^2F}{d\rho^2}-\frac{1}{\rho}\frac{dF}{d\rho}+F=0 . \tag{3.5}$$

Upon defining the function

$$u(\rho)\equiv\frac{F(\rho)}{\rho}\ , \tag{3.6}$$

(3.5) reduces to

$$\frac{d^2u}{d\rho^2}+\frac{1}{\rho}\frac{du}{d\rho}+\left[1-\frac{1}{\rho^2}\right]u=0\ , \tag{3.7}$$

solved by

$$u(\rho)=cJ_1(\rho)+dN_1(\rho)\ , \tag{3.8}$$

where c and d are constants, and J_1 and N_1 are the Bessel and Neumann functions of order one. From (3.6) it follows that (3.5) is solved by

$$F(\rho)=\rho[cJ_1(\rho)+dN_1(\rho)]\ . \tag{3.9}$$

This solution is compatible with the boundary condition (3.4a) only if $c=d=0$. It therefore follows that

$$P(r,t)=0 \quad \text{for } k^2<0\ . \tag{3.10}$$

That is, only the trivial solution is admitted, so that by (2.16) and (2.18) a static string solution is described.

B. $k^2=0$

The radial DE (3.3a) can be solved by integration to give

$$F(r)=Cr^2+F_0\ , \tag{3.11}$$

where C and F_0 are constants. Application of the boundary condition (3.4a) forces $C=0$, so that (3.11) reduces to

$$F(r)=F_0\ . \tag{3.12}$$

The temporal DE (3.3b) can also be solved by integration [using $a(t)\sim t^\alpha$, $0\le\alpha<1$] to yield

$$G(t)=\frac{Dt}{(1-\alpha)a(t)}+G_0\ , \tag{3.13}$$

where D and G_0 are constants. When subjected to the boundary condition (3.4b), (3.13) reduces to

$$G(t)=G_0\ , \tag{3.14}$$

so that by (3.12) and (3.14)

$$P(r,t)=F_0G_0$$
$$\equiv P_0=\text{const} \quad \text{for } k^2=0\ . \tag{3.15}$$

Again, by (2.16) and (2.18) a static string solution is obtained.

C. $k^2>0$

Taking $k>0$ for definiteness the radial DE (3.3a) can be solved by defining the variable $\xi\equiv kr$ and the function

$$v(\xi)\equiv\frac{F(\xi)}{\xi}\ . \tag{3.16}$$

Equation (3.3a) is then transformed into the modified Bessel DE for v,

$$\frac{d^2v}{d\xi^2}+\frac{1}{\xi}\frac{dv}{d\xi}-\left[1+\frac{1}{\xi^2}\right]v=0\ , \tag{3.17}$$

which is solved by the hyperbolic Bessel functions of order one, $I_1(\xi)$ and $K_1(\xi)$. By (3.4a) and (3.16) the physically admissible solution is

$$F(\xi)=\xi K_1(\xi),\quad \xi=kr\ . \tag{3.18}$$

For $a(t)\sim t^\alpha$, $0\le\alpha<1$, (3.3b) is solved (see the Appendix; $\Gamma=$ const, $t_0=$ an "initial" time) by

$$G(t)=\exp\left\{\frac{\Gamma}{1-\alpha}\left[1-\left[\frac{t}{t_0}\right]^{1-\alpha}\right]\right\}\ . \tag{3.19}$$

By (3.18) and (3.19) one therefore has

$$P(r,t)=P_1krK_1(kr)\times\exp\left\{\frac{\Gamma}{1-\alpha}\left[1-\left[\frac{t}{t_0}\right]^{1-\alpha}\right]\right\},\quad k^2>0\ , \tag{3.20}$$

where P_1 is a constant. From (2.16) and (2.18) this solution is seen to describe a time-dependent string solution.

D. General solution

The general solution to the PDE (2.8) and the boundary conditions (3.1) is a linear superposition of the particular solutions (3.10), (3.15), and (3.20), and can be written as

$$P(r,t)=P_0+P_1krK_1(kr)\exp\left\{\frac{\Gamma}{1-\alpha}\left[1-\left[\frac{t}{t_0}\right]^{1-\alpha}\right]\right\}$$
$$\equiv P_0+P_1F(r)G(t) \tag{3.21}$$

where P_0 and P_1 are constants. Using the fact [21] that $\xi K_1(\xi)\to1$ as $\xi\to0$, one finds

$$P(0,t)=P_0+P_1\exp\left\{\frac{\Gamma}{1-\alpha}\left[1-\left[\frac{t}{t_0}\right]^{1-\alpha}\right]\right\}$$
$$=P_0+P_1G(t) \tag{3.22}$$

so that with the aid of (3.22), (2.16) gives a string "magnetic" flux

$$\Phi_s(t)=\frac{2\pi}{e}\{1-[P_0+P_1G(t)]\}\ . \tag{3.23}$$

Thus for $P_1=0$ the gauge string is static, while for $P_1\neq0$, the gauge string is nonstatic with a time-dependent flux. Upon noting that $G(t_0)=1$ and $G(\infty)=0$, then in the limit that $t\to\infty$ one has $\Phi_s(t)\to\Phi_s(\infty)$, where

$$\Phi_s(\infty)=\frac{2\pi}{e}(1-P_0)\ . \tag{3.24}$$

Depending upon the values of the constants P_0 and P_1, Φ_s can be an increasing or a decreasing function of time.

For example, for $P_0=0$ and $P_1=1$, where t_0 is some initial time at which cosmic expansion begins with the behavior $a(t)\sim t^\alpha$, then $P(0,t_0)=1$ and $P(0,\infty)=0$, so that

$$\Phi_s(t_0)=0, \quad \Phi_s(\infty)=\frac{2\pi}{e}, \tag{3.25}$$

corresponding to the creation of a gauge string with one quantized unit of flux.

On the other hand, for $P_0=1$ and $P_1=1$, then $P(0,t_0)=0$ and $P(0,\infty)=1$, so that

$$\Phi_s(t_0)=\frac{2\pi}{e}, \quad \Phi_s(\infty)=0, \tag{3.26}$$

corresponding to the annihilation of a preexisting string with one quantized unit of flux.

It is also interesting to note that by (2.10e) and (3.21) the radial component of the Poynting vector $[S_r=(-g_{rr})^{1/2}T^{tr}=aT^{tr}]$ is proportional to [22]

$$T^{tr}=-\frac{\dot{P}P'}{e^2a^4r^2} = -\frac{k^4P_1^2}{e^2a^5}K_0(kr)K_1(kr)G^2(t), \tag{3.27}$$

indicating that "electromagnetic" gauge field energy flows radially inward toward the string. [The fact that $d(\xi K_1(\xi))/d\xi=-\xi K_0(\xi)$ has been used [21] in obtaining (3.27).] The physical interpretation is that the radial inflow of gauge field energy can create a string, cancel a preexisting string, or in some intermediate fashion change the flux of the string.

IV. SUMMARY AND CONCLUSIONS

A model of a sourceless gauge string in an expanding spacetime has been examined where, as a first approximation, the singular gauge string is embedded within a fixed space-time geometry described by a flat Robertson-Walker metric, with no back reaction of the gauge field upon the spacetime being considered. In this way an effort has been made to study the effects of the space-time expansion on an Abelian gauge field. By removing the Higgs field from consideration and holding the space-time geometry fixed, the equation of motion for the gauge field can be solved exactly. It is hoped that the exact analytical nature of this model might illuminate the understanding of some aspects of gauge field behavior in more realistic models, although the simplified sourceless U(1) model has its own interesting characteristics.

The gauge field structure function $P(r,t)$, given in general form in Sec. III, indicates that the singular gauge string has an associated "magnetic" flux which can be either static or time dependent. For the time-dependent solution the radial component of the Poynting vector, proportional to T^{tr}, is nonvanishing and directed radially inward, allowing gauge field energy to collapse upon the symmetry axis. In this way a string can gradually be created, or a preexisting string can gradually be destroyed. The fact that T^{tr} is nonzero also shows that the spacetime becomes anisotropic at a higher level of approximation wherein back reactions upon the spacetime are considered (or in a fully general-relativistic theory), as pointed out by the authors of Ref. [23].

It has been assumed that the scale factor $a(t)$ has a power-law behavior $a\sim t^\alpha$, $0\le\alpha<1$. By (3.23) the time dependence of the string flux depends strongly on α. The smaller the value of α, the faster Φ_s changes at small times in an approach to its asymptotic value. Therefore a sourceless Abelian gauge string would be created at a faster rate during an epoch of radiation dominance ($\alpha=\frac{1}{2}$) than during an epoch of matter dominance ($\alpha=\frac{2}{3}$), and would be created at its fastest rate in the Minkowski limit ($\alpha=0$).

APPENDIX

Equation (3.3b) for the function $G(t)$ is

$$\ddot{G}+H\dot{G}-\frac{k^2}{a^2}G=0 \tag{A1}$$

and can be solved for the $k^2>0$ case by defining a function $q(t)$ so that $G(t)$ can be formally written as

$$G(t)=\exp\left[\int_{t_0}^{t}\frac{q(x)}{a(x)}dx\right], \tag{A2}$$

where $a(x)\sim x^\alpha$, $0\le\alpha<1$. Then (A1) becomes

$$\dot{q}(t)-\frac{k^2-q^2(t)}{a^2(t)}=0\,. \tag{A3}$$

If $\dot{q}=0$, then one obtains the simple solution

$$q=-k \quad (k>0), \tag{A4}$$

allowing $G(t)$ to conform to the boundary condition (3.4b). For $\dot{q}\neq 0$ (A3) is solved by

$$\int\frac{dx}{a(x)}=\int\frac{dq}{k^2-q^2} = \begin{cases} \dfrac{1}{k}\ \text{arctanh}\ \dfrac{q}{k}-\dfrac{c_1}{k}, & k^2>q^2, \\[2ex] \dfrac{1}{k}\ \text{arccoth}\ \dfrac{q}{k}-\dfrac{c_2}{k}, & k^2<q^2, \end{cases} \tag{A5}$$

where c_1 and c_2 are integration constants. Upon defining

$$I(x)=\int\frac{k}{a(x)}dx, \tag{A6}$$

the solution to (A3) can be written as

$$q(x)=\begin{cases} k\ \tanh\,[I(x)+c_1], & k^2>q^2, \\ k\ \coth\,[I(x)+c_2], & k^2<q^2. \end{cases} \tag{A7}$$

$G(t)$ is then given by (A2) and (A7).

Computer-aided evaluations of $G(t)$ for particular values of parameters allowed the inference of a general solution expressible as

$$G(t)=A\sinh\left[\frac{bkt}{a(t)}\right]+B\cosh\left[\frac{bkt}{a(t)}\right], \tag{A8}$$

which indeed solves (A1) if $b=(1-\alpha)^{-1}$. Implementation of (3.4b) demands that $A=-B\equiv-G_1$, so that, from (A8),

$$G(t)=G_1 \exp\left[\frac{-kt}{(1-\alpha)a(t)}\right] . \quad \text{(A9)}$$

Note that this coincides with the simple solution given by (A2) and (A4), with the provision that

$$G_1=\exp\left[\frac{kt_0}{(1-\alpha)a(t_0)}\right] . \quad \text{(A10)}$$

Writing (with t_0 an "initial" time)

$$a(t)=a(t_0)\left[\frac{t}{t_0}\right]^{\alpha} \quad \text{(A11)}$$

and defining

$$\Gamma\equiv\frac{kt_0}{a(t_0)} , \quad \text{(A12)}$$

$G(t)$ can be displayed as

$$G(t)=\exp\left\{\frac{\Gamma}{1-\alpha}\left[1-\left[\frac{t}{t_0}\right]^{1-\alpha}\right]\right\} . \quad \text{(A13)}$$

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$$\nabla_{\mu}T^{\mu}{}_{t}=(\dot{P}/e^2a^2r^2)[\ddot{P}+H\dot{P}-a^{-2}(P''-P'/r)]=0 .$$

[19] See, for example, S. Weinberg, *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity* (Wiley, New York, 1972), Chap. 4, Sec. 8.
[20] The linear energy density is not required here to be finite at $r=0$.
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