RAPID COMMUNICATIONS

PHYSICAL REVIEW D VOLUME 43, NUMBER 1 1 JANUARY 1991

$K_L \to 2\gamma$ and $K_L \to \pi^+\pi^-\gamma$ decays

Pyungwon Ko
Enrico Fermi Institute and Department of Physics, University of Chicago, Chicago, Illinois 60637

Tran N. Truong
*Enrico Fermi Institute and Department of Physics, University of Chicago, Chicago, Illinois 60637 and Centre de Physique Théorique de L'Ecole Polytechnique, 91128 Palaiseau, France**

(Received 21 June 1990)

Under the assumption of pseudoscalar-meson pole dominance for $K_L \to 2\gamma$ and $K_L \to \pi^+\pi^-\gamma$ decays, the $a(K_L\eta)$ and $a(K_L\eta')$ are determined. Experimental consequences on $K_L \to \pi^+\pi^-\gamma$, $K_L \to \pi^0\gamma\gamma$, and $K_L \to \pi^0 e^+e^-$ are discussed.

Recently, there has been interest in calculating the $K_L \to \pi^0\gamma\gamma$ amplitude in connection with the possibility of detecting the *CP*-violation effects in the rare decay $K_L \to \pi^0 e^+ e^-$.[1–7] The *CP*-conserving decay $K_L \to \pi^- e^+ e^-$ which proceeds via two-photon exchange could provide a serious background for this important experiment. It is important then to carry out a careful calculation for the process $K_L \to \pi^0\gamma\gamma$ from which the *CP*-conserving decay $K_L \to \pi^0 e^+ e^-$ amplitude can be calculated. In previous papers,[7,8] one of us showed the equivalence of the chiral perturbation theory for the *P*-wave pion loop, where the unitarity is treated nonperturbatively, and the vector-meson-dominance (VMD) model. In this Rapid Communication we carry out a detailed study of $K_L \to \pi^+\pi^-\gamma$ and $K_L \to 2\gamma$, assuming pseudoscalar-meson pole dominance for these processes, in order to obtain a check on the parameters $a(K_L\eta)$ and $a(K_L\eta')$ which were used in the calculation of $K_L \to \pi^0\gamma\gamma$ and $K_L \to \pi^0 e^+ e^-$. Chiral anomalies[9] and extensive experimental data on $\pi^0,\eta,\eta' \to 2\gamma$ and $\pi^0,\eta,\eta' \to \pi^+\pi^-\gamma$ are used in the following calculation.

Although the assumption of pseudoscalar-meson pole dominance is widely used, there is no firm basis for this hypothesis. It is important then to study the phenomenological consequences of this assumption. We point out that the present experimental data on $K_L \to \pi^+\pi^-\gamma$ and $K_L \to 2\gamma$ rates imply a cancellation of η and η' pole contributions with the π^0 pole giving the major contribution to these decays. This leads possibly to an energy-independent dipion spectrum in $K_L \to \pi^+\pi^-\gamma$, unlike the influence of the vector meson ρ on the dipion spectrum of $\eta \to \pi^+\pi^-\gamma$ decay.

In the pseudoscalar-meson pole-dominance model, the $K_L \to 2\gamma$ and $K_L \to \pi^+\pi^-\gamma$ proceed through the intermediate states of the π^0, η, and η' mesons. The matrix element $a(K_L P) \equiv \langle P|H_w|K_L\rangle$, where P stands for π^0, η, and η', must be determined. The quantity $a(K_L\pi^0)$ can be determined from the $K_S \to \pi^+\pi^-$ rate, using the nonlinear-σ-model Lagrangian for the nonleptonic amplitude with the strong final-state pion-pion interaction taken into account.[10] It was found that $a(K_L\pi^0) = 2.90\times10^{-2}$ MeV^2. This value is fairly reliable with an estimated error of $\pm 10\%$.

The main problem is how to determine the corresponding values for $a(K_L\eta)$ and $a(K_L\eta')$. Because there are two pieces of experimental data on the $K_L \to \pi^+\pi^-\gamma$ and $K_L \to 2\gamma$ decay rates we can determine them in the pseudoscalar-meson pole-dominance model, apart of the sign relative to $a(K_L\pi^0)$ without referring to a specific model such as the nonet scheme with the SU(3)-breaking effect taken into account. In this calculation, only experimental quantities are used for $\pi^0,\eta,\eta' \to 2\gamma$ and $\pi^0,\eta,\eta' \to \pi^+\pi^-\gamma$ amplitudes. Their relative *signs,* but not magnitude of the amplitude, are determined by anomalies and the usual η,η' mixing scheme.

Let us begin with the determination of $P \to 2\gamma$ amplitude from the experimental data, where P denotes π^0, η, η', and K_L. Define $F_{P\gamma\gamma}$ as

$$\mathcal{M}(P \to \gamma_1(k_1,\epsilon_1)\gamma_2(k_2,\epsilon_2)) = \epsilon_{\mu\nu\sigma\tau}\epsilon_1^\mu k_1^\nu \epsilon_2^\sigma k_2^\tau F_{P\gamma\gamma}(m_P^2)\,, \tag{1}$$

where ϵ are the photon polarization vectors. We make the usual approximation $F_{P\gamma\gamma}(m_P^2) = F_{P\gamma\gamma}(0)$. Using the experimental data,[11] $\Gamma(\pi^0 \to 2\gamma) = 7.7 \pm 0.5 \pm 0.5$ eV, $\Gamma(\eta \to 2\gamma) = 0.51 \pm 0.02 \pm 0.04$ keV, $\Gamma(\eta' \to 2\gamma) = 4.7 \pm 0.5 \pm 0.5$ keV, we have

$$F_{\pi\gamma\gamma}(0) = (2.50 \pm 0.05)\times10^{-5}\ \text{MeV}^{-1}\,,$$

$$F_{\eta\gamma\gamma}(0) = (2.49 \pm 0.10)\times10^{-5}\ \text{MeV}^{-1}\,,$$

$$F_{\eta'\gamma\gamma}(0) = (3.28 \pm 0.24)\times10^{-5}\ \text{MeV}^{-1}\,,$$

where we have chosen the phase of $F_{\pi\gamma\gamma}(0)$ to be positive, the remaining ones are determined by chiral anomalies and the standard η,η' mixing. (These amplitudes can be understood by the anomalies, the η,η' mixing angle of $-20°$ and $f_8/f_\pi = 1.25$ and $f_0/f_\pi = 1.04$.)

It is much more complicated to determine the $P \to \pi^+\pi^-\gamma$ amplitudes from the experimental data. This is so because there are three *strongly* interacting particles in the process which make it difficult to apply directly the chiral-anomaly theorems. We must study the chiral-symmetry-breaking effect and the unitarity correction in order to use these theorems in the physical region. This problem was studied in great detail elsewhere,[12] and we give here a brief account of this subject. As was previously shown, the unitarity correction due to the *P*-wave pion loop, when they are treated correctly, would lead to the ρ vector-meson-dominance model for $P \to \pi^+\pi^-\gamma$. We are therefore led to the study of the VMD model. There is, however, the problem of reconciling the chiral limit of the pure VMD model with the low-energy chiral-anomaly theorems.[13] To see this, let us define the $P \to \pi^+\pi^-\gamma$ amplitude by

$$\mathcal{M}(P \to \pi^+(p_+)\pi^-(p_-)\gamma(k)) = \epsilon_{\mu\nu\sigma\tau}\epsilon^{\mu}k^{\nu}p_+^{\sigma}p_-^{\tau}F_{P\pi\pi}(s,t,u)\,, \quad (2)$$

where ϵ is the photon polarization vector, and $s=(p_+ + p_-)^2$, $t=(k+p_+)^2$, and $u=(k+p_-)^2$. Let us denote by $F_{P\pi\pi}(0)$ the chiral limit of $F_{P\pi\pi}(s,t,u)$, i.e., $s=t=u=0$ and the pseudoscalar masses are equal to zero. The discrepancy between the pure VMD and the chiral anomalies can be summarized as

$$\frac{F_{\pi\pi\pi}(0)}{F^{\rm VMD}_{\pi\pi\pi}(0)}=\frac{F_{\eta_8\pi\pi}(0)}{F^{\rm VMD}_{\eta_8\pi\pi}(0)}=\frac{F_{\eta_0\pi\pi}(0)}{F^{\rm VMD}_{\eta_0\pi\pi}(0)}=\frac{2}{3}\,, \quad (3)$$

where $F^{\rm VMD}(0)$ refers to the chiral limit of the VMD amplitude. Chiral-anomaly theorems stated that

$$F_{\pi\pi\pi}=\lambda,\ F_{\eta_8\pi\pi}=\frac{\lambda}{\sqrt{3}},\ F_{\eta_0\pi\pi}=(\tfrac{2}{3})^{1/2}\lambda\,,$$

with $\lambda=F_{\pi\gamma\gamma}(0)/ef_\pi^2=e/4\pi^2f_\pi^3=9.45\times10^{-9}$ MeV^{-3}, where $f_\pi=93$ MeV and e is the electric charge. Fujiwara *et al.*[14] showed that the discrepancy for $F^{\rm VMD}_{\pi\pi\pi}(0)$ can be removed by introducing a contact term in the $\pi^0 \to \pi^+\pi^-\gamma$ amplitude to reduce the VMD amplitude to that given by the chiral-anomaly theorem. The physical $F_{\pi\pi\pi}$ amplitude is now given by

$$F_{\pi\pi\pi}(s,t,u)=\lambda\left[1+\frac{1}{2}\left(\frac{2}{s_\rho-s}+\frac{t}{s_\rho-t}+\frac{u}{s_\rho-u}\right)\right]; \quad (4)$$

i.e., the VMD terms decouple from the $F_{3\pi}$ expression in the chiral limit and hence the low-energy theorem is recovered. Similar to Eq. (4), we have

$$F_{\eta_8\pi\pi}(s,t,u)=\frac{\lambda}{\sqrt{3}}\left(1+\frac{3}{2}\frac{s}{s_\rho-s}\right), \quad (5a)$$

$$F_{\eta_0\pi\pi}(s,t,u)=(\tfrac{2}{3})^{1/2}\lambda\left(1+\frac{3}{2}\frac{s}{s_\rho-s}\right), \quad (5b)$$

where we have set $f_\pi=f_{\eta_8}=f_{\eta_0}$. The prescription of Fujiwara *et al.* applies to a narrow resonance. In the real world where the ρ is unstable, we could try to introduce an imaginary part in the ρ propagator. This prescription is seen to violate the unitarity by calculating the phase of Eqs. (5) and comparing them with the P-wave $\pi\pi$ phase shift.

An alternative approach to this problem which is consistent with unitarity was proposed by one of us (T.N.T) using either dispersion theory or unitarized chiral perturbation theory. By requiring the consistency relations for $\pi^0 \to 2\gamma$, $\eta_8 \to 2\gamma$, $\eta_0 \to 2\gamma$, and $F_{P\pi\pi}$ anomalies, instead of Eqs. (5), one has[12]

$$\begin{aligned}F_{\eta_8\pi\pi}(s,t,u)&=\frac{\lambda}{\sqrt{3}}\frac{(s_R-8\gamma m_\pi^2/\pi)(1+s/2s_\rho)}{s_R-s+\gamma(s-4m_\pi^2)h(s)-i\gamma(s-4m_\pi^2)[(s-4m_\pi^2)s)/]^{1/2}}\\ &\simeq\frac{\lambda}{\sqrt{3}}\frac{s_\rho(1+s/2s_\rho)}{s_\rho-s-i\gamma(s-4m_\pi^2)[(s-4m_\pi^2)/s]^{1/2}}\,,\end{aligned} \quad (6)$$

with $\sqrt{s_R}=710$ MeV, $\gamma=0.185$, and

$$h(s)=\frac{2}{\pi}\left(\frac{s-4m_\pi^2}{s}\right)^{1/2}\ln\left(\frac{\sqrt{s}+(s-4m_\pi^2)^{1/2}}{2m_\pi}\right),$$

and a similar expression for $F_{\eta_0\pi\pi}$. It is seen from Eq. (6) that the phase of $F_{\eta_8\pi\pi}$ is the same as the phase of the P-wave $\pi\pi$ interaction and hence the unitarity is respected. Equation (6) satisfies the low-energy theorem and has the same "residue as Eq. (5a) at $s=s_\rho$.

In either this approach [Eq. (6)] or that given by Fujiwara *et al.*, Eqs. (5), the dipion spectrum in $\eta \to \pi^+\pi^-\gamma$ is proportional to $(1+3s/s_\rho)$, which is slightly larger than the experimental data[15] suggesting a smaller slope $(1+2s/s_\rho)$. There are probably systematic errors in the experimental data. Elsewhere, one of us (T.N.T.) shows that it is possible to reduce the slope of the spectrum to $(1+2.5s/s_\rho)$. Below we give results for $F_{P\pi\pi}(0)$ by using Eq. (6) and also in parentheses for the case when the numerator of Eq. (6) is replaced by $s_\rho[1+s/(s_{\rho'}-s)]$ where $s_{\rho'}=2s_\rho$. Using the experimental data $\Gamma(\eta \to \pi^+\pi^-\gamma)=0.064\pm0.005$ keV, $\Gamma(\eta' \to \pi^+\pi^-\gamma)=62\pm6$ keV, and the anomaly equation for $F_{\pi\pi\pi}$, we have

$$\begin{aligned}F_{\eta\pi\pi}(0)&=(6.47\pm0.25)\times10^{-9}\,\text{MeV}^{-3}\ (6.8\times10^{-9})\,,\\ F_{\eta'\pi\pi}(0)&=(5.45\pm0.38)\times10^{-9}\,\text{MeV}^{-3}\ (5.4\times10^{-9})\,.\end{aligned} \quad (7)$$

We are now in a position to calculate the $K_L \to \pi^+\pi^-\gamma$ amplitude $F_{K_L\pi\pi}(s,t,u)$ defined by Eq. (2). Using Eqs. (5)–(7), and *assuming* that Eq. (4) is also valid for the π^0 off its mass shell,

$$F_{K_L\pi\pi}(s,t,u)=\frac{a(K_L\pi^0)}{m_K^2}\left[\frac{F_{\pi\pi\pi}(0)}{1-r_\pi}\left(1+\frac{m_K^2+2m_\pi^2}{2s_\rho}\right)+\left(\frac{\alpha F_{\eta\pi\pi}(0)}{1-r_\eta}+\frac{\beta F_{\eta'\pi\pi}(0)}{1-r_\eta'}\right)\left(1+\frac{3}{2}\frac{s}{s_\rho-s}\right)\right], \quad (8)$$

where $a(K_L\pi^0)=\langle\pi^0|H_w|K_L\rangle$, $\alpha=a(K_L\eta)/a(K_L\pi^0)$, $\beta=a(K_L\eta')/a(K_L\pi^0)$, and $r_\pi=m_\pi^2/m_K^2$, etc. Because of the small energy available in the K_L decay, we have expanded the ρ propagator in Eq. (4) to use it in Eq. (8) and set $s+t+u=m_K^2+2m_\pi^2$.

It is seen from Eq. (8) that only the η and η' pole contributions can give rise to the s dependence for the $K_L \to \pi^+\pi^-\gamma$ amplitude. This result would be invalid if there were large corrections when the π^0 is off its mass shell. Within the spirit of the pseudoscalar pole dominance, we assume that this correction is small.

We can similarly write

$$F_{K_L\gamma\gamma} = \frac{a(K_L\pi^0)}{m_K^2}\left[\frac{F_{\pi\gamma\gamma}}{1-r_\pi} + \alpha\frac{F_{\eta\gamma\gamma}}{1-r_\eta} + \beta\frac{F_{\eta'\gamma\gamma}}{1-r_{\eta'}}\right]. \quad (9)$$

Using the experimental data $B(K_L \to \gamma\gamma) = (4.41 \pm 0.32)\times 10^{-5}$ we get $|F_{K_L\gamma\gamma}| = (3.4 \pm 0.1)\times 10^{-12}$ MeV^{-1}. Using it in Eq. (9), we have

$$1 - 4.30\alpha - 0.45\beta = \pm(1.073 \pm 0.032). \quad (10)$$

Using the experimental data on $F_{P\pi\pi}$ in Eq. (8), we have

$$F_{K\pi\pi}(s,t,u) = (1.49\times 10^{-15}\ \mathrm{MeV}^{-3}) \times \left[1 - (2.37\alpha + 0.158\beta)\left(1 + \frac{3}{2}\frac{s}{s_\rho - s}\right)\right]. \quad (11)$$

Solving for β in Eq. (10), substituting it in Eq. (11), and imposing the condition for the observed direct-emission rate $\Gamma_{\mathrm{DE}}(K_L \to \pi^+\pi^-\gamma) = (3.67 \pm 0.37)\times 10^{-19}$ MeV corresponding to a branching ratio $(2.84 \pm 0.28)\times 10^{-5}$, we have four possible solutions for α. For $a(K_L\pi) = 2.9\times 10^{-2}\ \mathrm{MeV}^2$, we have

(i) $\alpha = 0.08$, $\beta = -0.92$, $\Delta m = 0.66$,

(ii) $\alpha = 1.59$, $\beta = -15.35$, $\Delta m = -97.05$,

(iii) $\alpha = 0.67$, $\beta = -1.80$, $\Delta m = -2.13$,

(iv) $\alpha = -0.84$, $\beta = 12.63$, $\Delta m = -61.06$,

where Δm is the total pseudoscalar pole construction to $\Delta m = m(K_L) - m(K_S)$ in the unit of the experimental mass difference $\Delta m = 3.5\times 10^{-6}$ eV calculated from the formula

$$\Delta m \simeq 1 - 4.3\alpha^2 - 0.37\beta^2.$$

Solutions (i) and (iv) give rise to a $\pi^+\pi^-$ distribution $|1 - 0.05s/(s_\rho - s)|^2$, (ii) and (iii) to a distribution $|1 + 6.5s/(s_\rho - s)|^2$. Solutions (ii) and (iii) are excluded on the dipion energy distribution, in addition to the ridiculously large answer for Δm. Solution (iv) is also excluded due to a large value of Δm.

It is interesting also to study the case where $a(K_L\pi) = 4.0\times 10^{-2}\ \mathrm{MeV}^2$ which is frequently used in the literature under the incorrect assumption that the pion-pion S-wave interaction can be neglected in the $K_S \to 2\pi$ amplitude. In this case the four solutions are

(i) $\alpha = 0.16$, $\beta = -1.04$, $\Delta m = 0.93$,

(ii) $\alpha = 1.25$, $\beta = -11.46$, $\Delta m = -103.19$.

(iii) $\alpha = 0.60$, $\beta = -1.78$, $\Delta m = -3.27$,

(iv) $\alpha = -0.49$, $\beta = 8.64$, $\Delta m = -52.54$.

(i) and (iv) give rise to a $\pi^+\pi^-$ distribution $|1 - 0.4s/(s_\rho - s)|^2$ and (ii) and (iii) $|1 + 13s/(s_\rho - s)|^2$ which are not consistent with the present experimental data. Furthermore, the solution (iv) now gives a too large value of Δm, which should also be excluded.

There is only one possible solution corresponding to the corrected value $a_{K_L\pi^0} = 2.9\times 10^{-2}\ \mathrm{MeV}^2$ and $\alpha \simeq 0.08 \pm 0.05$ and $\beta = -0.92^{+0.5}_{-0.3}$. (If we choose $a_{K_L\pi^0} = 4.0 \times 10^{-2}\ \mathrm{MeV}^2$, we have $\alpha \simeq 0.16$ and $\beta = -1.04$. This choice will be given in the parentheses, when we discuss $K_L \to \pi^0\gamma\gamma$.) These solutions correspond to a small η,η' contribution in $K_L \to \pi^+\pi^-\gamma$ and $K_L \to 2\gamma$. If we take, for example, $\alpha = 0.12$ and $\beta = 1.7$, then there is a large cancellation between η and η' pole contribution; it corresponds roughly to the nonet scheme of $K_L\pi^0$, $K_L\eta$, and $K_L\eta'$ mixing.

In conclusion, there is only one acceptable solution, namely the dominance of the π^0 pole. This dominance gives rise to a $K_L \to \pi^+\pi^-\gamma$ matrix element which is almost independent of s, i.e., much less than that observed in $\eta \to \pi^+\pi^-\gamma$ decay. This constitutes a test of our model. There could be one possible flaw in our result, if there was a large correction for off the mass shell π^0 effect due to the multiple pion-pion rescattering which destroyed the symmetry between the s and t,u dependence of Eq. (4).

The result for $K_L \to \pi^+\pi^-\gamma$ obtained in this paper differs from the analysis of $K_L \to \pi^+\pi^-\gamma$ given in Ref. 7 which is a generalization of the chiral perturbation theory. An assumption was made there to take into account only the singularity in the dipion channel. This result is valid if the η and η' contributions dominate over the π^0 pole or that the direct emission of the vector meson ρ from the weak vertex is important.

We end this paper by discussing the $K_L \to \pi^0\gamma\gamma$ and $K_L \to \pi^0 e^+e^-$ decays. Using the standard notation in the analysis of $K_L \to \pi^0\gamma\gamma$, we have new $G_\rho m_K^2 = -0.16$

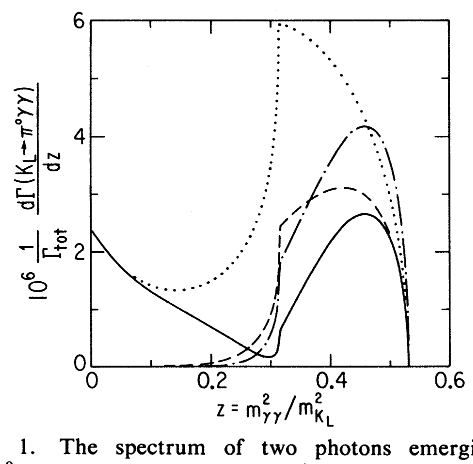

FIG. 1. The spectrum of two photons emerging from $K_L \to \pi^0\gamma\gamma$: destructive interference (result of Ref. 6, with some changes made in the coupling constants, G_ρ and G_ω), solid curve; prediction by chiral perturbation theory to $O(p^4)$, dashed curve; prediction by the pion rescattering model, dash-dotted curve; constructive interference, in case there exists direct emission of vector mesons at the weak vertex, as considered in Ref. 16, dotted curve.

$\times 10^{-8}$ (-0.04×10^{-8}) and $G_\omega m_K^2 = -1.6\times 10^{-8}$ (-2.04×10^{-3}), and $B(K_L \to \pi^0\gamma\gamma) = 0.74\times 10^{-6}$ (0.87×10^{-6}). We fixed the relative sign between the chiral amplitude and the vector-meson amplitude, and found that those two amplitudes gave destructive interference. (The same conclusion was discussed in Ref. 16.) Note that the branching ratio is comparable with those predicted by the chiral perturbation theory[11] and the pion-rescattering model.[5] However, the spectra of two photons are quite different as noted in Ref. 6.

Our prediction for $K_L \to \pi^0\gamma\gamma$ is shown in Fig. 1 (solid curve), in comparison with the chiral perturbation prediction[1] (dashed curve) and the pion rescattering model[5] (dash-dotted curve). The low-$m_{\gamma\gamma}^2$ region is enhanced by vector-meson exchange, but the region between $0.3 \le m_{\gamma\gamma}^2 \le 0.4$ is suppressed so that the total decay rate does not change very much.

If one allows an increase of 20% over the preferred value of $a(K\pi)$ which is not excluded as discussed above, the branching ratio of $K_L \to \pi^0\gamma\gamma$ is essentially unchanged due to the destructive interference. If the experiment data is much larger than this value, then our model based on the assumption of pseudoscalar-meson pole dominance is not valid. We could have, for example, a direct vector meson emission from the weak vertex which interferes with the pole amplitude to enhance the calculated rate. The dipion spectrum in $K_L \to \pi^+\pi^-\gamma$ could, in this case, be distorted by the ρ meson. In Fig. 1 (dotted curve), we show also the constructive interference of the chiral amplitude and the vector-meson amplitude where the last one has the opposite sign of that given by Eq. (8) in Ref. 16. In this case $B(K_L \to \pi^0\gamma\gamma) = 1.57\times 10^{-6}$ (1.85×10^{-6}). [Considering the validity of the phenomenological analysis we are using here, it would be impossible to distinguish $a(K_L\pi) = 2.9\times 10^{-2}$ MeV^2 from $a(K_L\pi) = 4.0\times 10^{-2}$ MeV^2.] The absorptive contribution to $B(K_L \to \pi^0 e^+ e^-)$ is $\sim 3\times 10^{-12}$, independent of the sign of the vector-meson amplitude in $K_L \to \pi^0\gamma\gamma$.

As a further check on the reliability of our model, we recalculate the $\eta \to \pi^0\gamma\gamma$ rate which was originally done by Cheng.[17] Define

$$\mathcal{M}(\eta(k) \to \pi^0 + \gamma(\epsilon_1, q_1) + \gamma(\epsilon_2, q_2)) = \epsilon_1^\mu \epsilon_2^\nu M_{\mu\nu},$$

with the standard decomposition into the invariant amplitudes $A(s,t,u)$ and $B(s,t,u)$, the ρ,ω exchange diagrams give

$$\begin{aligned} A(s,t,u) &= \tfrac{2}{9} g_{\omega\pi^0\gamma}^2 \left[\frac{t+M^2}{m_\rho^2 - t} + \frac{u+M^2}{m_\rho^2 - u} \right] \left[\frac{f_\pi}{f_8}\sqrt{3}\cos\theta - \frac{f_\pi}{f_0}\sqrt{6}\sin\theta \right], \\ B(s,t,u) &= \tfrac{2}{9} g_{\omega\pi^0\gamma}^2 \left[\frac{s}{m_\rho^2 - t} + \frac{s}{m_\rho^2 - u} \right] \left[\frac{f_\pi}{f_8}\sqrt{3}\cos\theta - \frac{f_\pi}{f_0}\sqrt{6}\sin\theta \right], \end{aligned} \tag{12}$$

where for simplicity we have set $m_\omega = m_\rho$. Using $f_8/f_\pi = 1.25$, $f_0/f_\pi = 1.04$, $\theta = -20°$, and $g_{\omega\pi\gamma} = 7.14 \times 10^{-4}$ MeV^{-1} corresponding to $\Gamma(\omega \to \pi\gamma) = 720$ keV, the $\eta \to \pi^0\gamma\gamma$ width is

$$\Gamma(\eta \to \pi^0\gamma\gamma) = 0.96\ \text{eV}, \tag{13}$$

which compares favorably with the experimental value $\Gamma(\eta \to \pi^0\gamma\gamma) = 0.93 \pm 0.20$ eV. [We used the accurate measurements of $B(\eta \to \pi^0\gamma\gamma)$, $B(\eta \to \gamma\gamma)$, and $\Gamma(\eta \to \gamma\gamma)$.] The vector-meson-dominance model as applied to the $\eta \to \pi^0\gamma\gamma$ calculation is therefore fairly reliable. The uncertainty in the $K_L \to \pi^0\gamma\gamma$ calculation can therefore be attributed to the additional assumption of the pseudoscalar pole dominance.

Note added. After submitting this manuscript, we received a paper[18] on the measurement of $K_L \to \pi^0\gamma\gamma$. The data on $K_L \to \pi^0\gamma\gamma$ from CERN seems to indicate that there may be direct emission of a vector meson from the weak vertex, in addition to the pseudoscalar pole terms which were analyzed in this paper. By integrating the theoretical spectrum under the experimental cut, we find the vector-meson contribution is about 16%, instead of being less than 12%. This in turn implies that $B(K_L \to \pi^0 e^+ e^-) \le 3\times 10^{-12}$ for the *CP*-conserving two-photon contribution. The best fit to the experimental data may be achieved by reducing the vector-meson amplitude by 30% with constructive interference. This amounts to $B(K_L \to \pi^0 e^+ e^-) = 1.5\times 10^{-13}$. This is larger than the claim in Ref. 18. The possibility of direct emissions of vector mesons from the weak vertices could also be clarified by measuring the dipion spectrum in $K_L \to \pi^+\pi^-\gamma$. Also, we need more precise measurements of the two-photon spectrum and the branching ratio of $K_L \to \pi^0\gamma\gamma$, to understand the vector-meson contribution more quantitatively. In conclusion, recent measurement of $K_L \to \pi^0\gamma\gamma$ seems to indicate that pseudoscalar pole dominance may not be sufficient to explain the observed spectrum and the branching ratio of $K_L \to \pi^0\gamma\gamma$.

We would like to thank J. Rosner for useful discussions. One of us (T.N.T.) would like to thank the Enrico Fermi Institute and the Physics Department for hospitality. This work was supported in part by Department of Energy Grant No. DE-AC0280ER10587.

*Permanent address.

[1] G. Ecker, A. Pick, and E. de Rafael, Phys. Lett. **B 189**, 363 (1987); Nucl. Phys. **B303**, 665 (1988).

[2] L. M. Sehgal, Phys. Rev. D **38**, 808 (1988).

[3] T. Morozumi and H. Iwasaki, KEK Report No. TH-206, 1988 (unpublished).

[4] J. Flynn and L. Randall, Phys. Lett. **B 216**, 221 (1989).

[5] P. Ko and J. L. Rosner, Phys. Rev. D **40**, 3775 (1989).

[6]P. Ko, Phys. Rev. D **41**, 1531 (1990).

[7]T. N. Truong, University of Chicago Report No. E.F.I. 89-57 (unpublished).

[8]T. N. Truong, Phys. Rev. Lett. **61**, 2526 (1988).

[9]S. L. Adler, Phys. Rev. **117**, 2426 (1966); J. S. Bell and R. Jackiw, Nuovo Cimento **60A**, 47 (1969); W. A. Bardeen, Phys. Rev. **184**, 1848 (1969); J. Wess and B. Zumino, Phys. Lett. **37B**, 95 (1971).

[10]T. N. Truong, Phys. Lett. B **207**, 495 (1988).

[11]Particle Data Group, G. P. Yost *et al.*, Phys. Lett. B **204**, 1 (1988); and Crystal Ball data.

[12]T. N. Truong, lectures given at "Ettore Majorana" International School on Low Energy Antiproton Physics, 1990 (unpublished).

[13]M. Gell-Mann, D. Sharp, and W. G. Wagner, Phys. Rev. Lett. **8**, 261 (1962); Murray Gell-Mann and Fredrik Zachariasen, Phys. Rev. **124**, 953 (1961); O. Kaymakcalan *et al.*, Phys. Rev. D **30**, 594 (1984); S. Rudaz, Phys. Lett. **145B**, 281 (1984).

[14]T. Fujiwara *et al.*, Prog. Theor. Phys. **73**, 926 (1985).

[15]J. G. Layter *et al.*, Phys. Rev. D **7**, 2565 (1973).

[16]G. Ecker, A. Pick, and E. de Rafael, Phys. Lett. B **237**, 481 (1990).

[17]T. P. Cheng, Phys. Rev. **162**, 1734 (1967).

[18]G. D. Barr *et al.*, Phys. Lett. B **237**, 481 (1990).