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CP asymmetry in top-quark radiative decays

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We analyze the *CP* asymmetry in the top-quark radiative decay $t \to q\gamma$, where $q = c, u$ and γ is an *on-shell* photon, due to the one-loop penguin diagram in the three-generation standard model. This penguin-dominated *CP* asymmetry is a new phenomenon of heavy-quark physics, and corresponds to the onset in the amplitude of the physical thresholds $t \to \text{real}\{Wl\}$, where $l = b, s, d$ is a quark in the loop. A *CP* asymmetry in a radiative decay to an on-shell photon in the light-quark sector (e.g. *B*-meson decays) cannot be generated by the one-loop penguin. The origin of this asymmetry is thus formally of some interest, although the phenomenological application turns out to be rather limited, due to extremely small branching ratios for $t \to q\gamma$, at least to one-loop order (however, our calculations raise an interesting possibility with respect to the role of QCD corrections to this asymmetry). This calculation should also serve as a useful benchmark for the study of possible *CP* asymmetries in the top-quark sector, in the context of new physics.

The origin of *CP* violation represents one of the leading problems in particle physics.[1] Considerable effort has therefore been devoted over the past decade to the analysis of new possibilities for the observation of *CP* violation, particularly in the *B*-meson system.[2]

As new experimental facilities become available, and as our knowledge of the parameters of the standard model becomes more complete, previously unexpected avenues for the exploration of *CP* violation may become available. With the current Collider Detector at Fermilab (CDF) limit on the mass m_t of the top quark passing 90 GeV, and with the expected production of order 10^8 top quarks at the Superconducting Super Collider, it is timely to consider what *CP*-violating effects could arise as a consequence of a heavy top quark.

In this paper, we analyze *CP* asymmetry in the top-quark radiative decay $t \to q\gamma$, where $q = c, u$ and γ is an *on-shell* photon, in the three-generation standard model. To leading order, this decay proceeds through one-loop penguin diagrams (a typical diagram is illustrated in Fig. 1). The occurrence of a *CP* asymmetry in a radiative decay to an on-shell photon, through the one-loop penguin, is a new phenomenon of heavy-quark physics. The absorptive part of the amplitude, which is necessary for a partial-rate asymmetry, corresponds in this case to the onset for sufficiently large m_t of the physical thresholds

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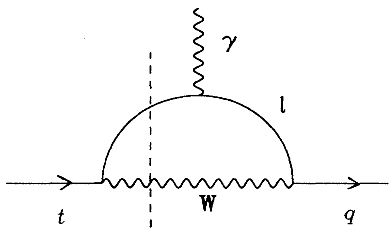

FIG. 1. A typical one-loop penguin diagram contributing to $t \to q\gamma$, where $q = c, u$. The dashed line represents the unitarity cut corresponding to the physical threshold $t \to \text{real}\{Wl\} \to q\gamma$, where $l = b, s, d$ is a quark in the loop.

$t \to \text{real}\{Wl\}$, where $l = b, s, d$ is a quark in the loop, illustrated by the unitarity cut in Fig. 1.

This is in contrast with penguin-dominated asymmetries in the light-quark sector, such as B-meson decays, where the absorptive part of the amplitude can only be generated if the external vector boson V [V = gluon (Ref. 3), photon (Ref. 4), and Z^0 (Ref. 5)] has a sufficiently large timelike four-momentum, above the threshold for $V \to l\bar{l}$. An asymmetry for radiative decays of light mesons to an *on-shell* photon thus cannot be generated from the one-loop penguin.

Given the CDF limit on the top-quark mass, $m_t \gtrsim 90$ GeV, the possibility now arises that a penguin-dominated CP asymmetry in the top-quark radiative decay $t \to q\gamma$ could be realized in experiment (compare with the condition $m_t > M_W + m_d$ for the onset of the first physical threshold in the penguin graph with an on-shell photon). This decay mode would be very attractive experimentally, due to the clean signature provided by the hard photon. The phenomenological application turns out to be rather limited, however, due to extremely small branching ratios for $t \to q\gamma$, at least to one-loop order; however, our calculations raise an interesting possibility with respect to the role of QCD corrections to this asymmetry. This calculation should also serve as a useful benchmark for the study of possible CP asymmetries in the top-quark sector, in the context of new-physics scenarios.

The vertex function V^μ for $t \to q\gamma$ takes the form[6–8]

$$V^\mu = (k^2\gamma^\mu - k^\mu \not{k})[F_1^L L + F_1^R R] + i\sigma^{\mu\nu}k_\nu[F_2^L m_q L + F_2^R m_t R], \quad (1)$$

where $R, L \equiv \frac{1}{2}(1 \pm \gamma^5)$. For real-photon emission ($k^2 = 0$) only the spin-flip term contributes, and since the top-quark mass m_t is much larger than the mass m_q of the produced quark, the width can be reduced to

$$\Gamma(t \to q\gamma) = \frac{\alpha}{128\pi^4} m_t^5 G_F^2 \left| \sum_l \widetilde{F}_2^R(m_t, m_l) V_{tl} V_{ql}^* \right|^2, \quad (2)$$

where V_{tl} is the Kobayashi-Maskawa (KM) matrix,[9] and where we introduce the dimensionless form factor $\widetilde{F}_2^R$:

$$F_2^R \equiv \frac{ieg^2}{32\pi^2 M_W^2} \widetilde{F}_2^R . \quad (3)$$

The expression for $\widetilde{F}_2^R$ to one-loop order (cf. Fig. 1) was derived in Ref. 8, for arbitrary quark masses and photon four-momentum. In the case at hand, some simplification is obtained by neglecting $m_q^2/m_t^2 \ll 1$, with the result[10]

$$\widetilde{F}_2^R(m_t, m_l) = \int_0^1 d\alpha_1\, d\alpha_2\, (\widehat{e}_W \alpha_1 + \widehat{e}_l \overline{\alpha}_1) \frac{1}{Y} \times [2\alpha_1(1 - \overline{\alpha}_1\alpha_2) + \widehat{m}_l^2 \overline{\alpha}_1(1 - \alpha_1\alpha_2)], \quad (4)$$

where $\overline{\alpha}_1 \equiv (1 - \alpha_1)$, and

$$Y = \alpha_1 + \widehat{m}_l^2 \overline{\alpha}_1 - \widehat{m}_t^2 \alpha_1 \overline{\alpha}_1 \alpha_2 - i\epsilon. \quad (5)$$

$\widehat{e}_W$ and $\widehat{e}_l$ denote the charges of the W (directed into the tlW vertex), and the quarks in the loop, in units of the proton charge, and $\widehat{m}_t \equiv m_t/M_W$, $\widehat{m}_l \equiv m_l/M_W$. The $i\epsilon$ prescription [Eq. (5)], coming from the Feynman propagators, provides the appropriate contour of integration for handling the onset in the vertex of the physical thresholds $t \to \text{real}\{Wl\} \to q\gamma$, illustrated by the unitarity cut in Fig. 1.

The absorptive part of the form factor $\widetilde{F}_2^R$ [Eq. (4)] was evaluated analytically in Ref. 10:

$$\text{Im}\, \widetilde{F}_2^R(m_t, m_l) = \frac{\pi}{m_t^4} \Theta(m_t - M_W - m_l) \left[\widehat{e}_W M_W^2 (4E_l m_t - 3m_l^2) \ln\left(\frac{E_W + P}{E_W - P}\right) + \widehat{e}_l m_l^2 (2E_W m_t - 3M_W^2) \ln\left(\frac{E_l + P}{E_l - P}\right) + 4(\widehat{e}_W - \widehat{e}_l)(E_W m_l^2 - 2E_l M_W^2) P \right], \quad (6)$$

where E_W, E_l, and P are the c.m. energies and momentum of the W-quark pair in the decay $t \to \text{real}\{Wl\}$, $E_{W,l} = (m_t^2 \pm M_W^2 \mp m_l^2)/(2m_t)$, $P = \lambda^{1/2}(m_t^2, M_W^2, m_l^2)/(2m_t)$. The real part of $\widetilde{F}_2^R$ can be obtained from the dispersion relation[10]

$$\text{Re}\, \widetilde{F}_2^R(m_t, m_l) = \frac{1}{\pi} \mathcal{P} \int_{(M_W + m_l)^2}^{\infty} d\widetilde{m}^2 \frac{\text{Im}\, \widetilde{F}_2^R(\widetilde{m}, m_l)}{\widetilde{m}^2 - m_t^2}. \quad (7)$$

Detailed numerical results for the dispersive and absorptive parts of the form factor, along with further analytical results of interest,[11] can be found in Ref. 10.

The absorptive part of the form factor above threshold [Eq. (6)], and the coherent sum over amplitudes corresponding to different quark flavors in the loop (with complex KM matrix elements), lead to interference terms which generate a CP asymmetry A_q in the radiative decay:

$$A_q \equiv \frac{\Gamma(\bar{t} \to \bar{q}\gamma) - \Gamma(t \to q\gamma)}{\Gamma(\bar{t} \to \bar{q}\gamma) + \Gamma(t \to q\gamma)}. \tag{8}$$

The asymmetry can be expressed in the familiar form

$$A_q = \frac{2\,\mathrm{Im}(v_b v_s^*)\mathrm{Im}(\Delta_{bd}\Delta_{sd}^*)}{|v_b|^2|\Delta_{bd}|^2 + |v_s|^2|\Delta_{sd}|^2 + 2\,\mathrm{Re}(v_b v_s^*)\mathrm{Re}(\Delta_{bd}\Delta_{sd}^*)}, \tag{9}$$

where

$$v_i \equiv V_{ti}V_{qi}^*,$$
$$\Delta_{ij} \equiv \widetilde{F}_2^R(m_t, m_i) - \widetilde{F}_2^R(m_t, m_j). \tag{10}$$

We used the unitarity of the three-generation KM matrix, $v_b + v_s + v_d = 0$, to arrive at Eq. (9).

The combination of KM angles appearing in the numerator in Eq. (9) is, up to a sign, the unique three-generation rephasing-invariant measure of CP violation[12]

$$\mathrm{Im}(v_b v_s^*) = \pm J, \quad q = c, u, \tag{11}$$

where, in KM notation,

$$J \equiv c_1 c_2 c_3 s_1^2 s_2 s_3 \sin\delta. \tag{12}$$

To one-loop order, Eq. (9) can be simplified, using the fact that $\Delta_{sd} = O((m_s/m_b)^2)\Delta_{bd}$ (Refs. 10 and 11). We also note that $\mathrm{Re}(v_b v_s^*)/|v_b|^2 \approx 1$, and $|v_s| \approx |v_b|$. To a good approximation, we thus obtain, to one-loop order,

$$A_q \cong \frac{\pm 2J\,\mathrm{Im}(\Delta_{bd}\Delta_{sd}^*)}{|V_{qb}|^2|\Delta_{bd}|^2}, \quad q = c, u, \tag{13}$$

where we also used the fact that $|V_{tb}| \approx 1$. It is clear from Eq. (13) that the asymmetry is largest for the decay to the final-state u quark, due simply to the smaller width ($\propto |V_{qb}|^2$) appearing in the denominator in that case.

We plot the value of the CP asymmetry for $t \to u\gamma$ in Fig. 2. We use the typical current quark masses $m_b = 5$ GeV, $m_s = 150$ MeV, and $m_d \approx 0$. We saturate the (approximate) experimental bound[13] on J ($|J| \lesssim 10^{-4}$), and we take $|V_{ub}| \simeq 0.006$ [these values for J and $|V_{ub}|$ are self-consistent (Ref. 14)]. We note that the asymmetry to one-loop order depends strongly on the strange-quark mass, $A_q = O((m_s/m_b)^2)$ [see comments above Eq. (13)]. The asymmetry is largest for m_t near the first threshold in the Penguin diagram ($m_t = M_W + m_d$), and falls off slowly at large m_t [$A_q = O(1/\ln(m_t^2))$ for $m_t \to \infty$], once threshold effects have died off. We point out in this connection that the sharp dips in Fig. 2, occurring near the thresholds for the internal s and b quarks, are smoothed out if one includes a width $\Gamma_W \approx 2$ GeV for the W propagators in the vertex function. The effect on the asymmetry of including Γ_W is negligible above all thresholds ($m_t \gtrsim 90$ GeV).

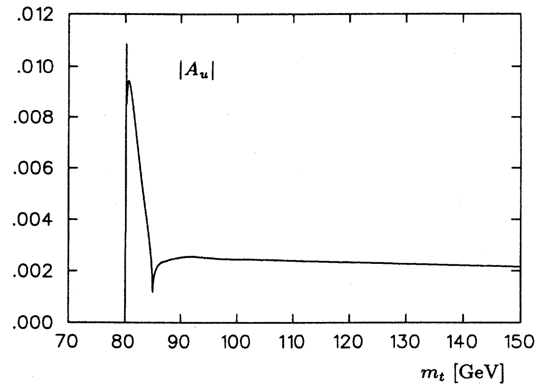

FIG. 2. One-loop CP asymmetry A_u for $t \to u\gamma$ [Eq. (13)] versus top quark mass m_t. We use the typical current quark masses $m_b = 5$ GeV, $m_s = 150$ MeV, and $m_d \approx 0$ [$M_W = 80$ GeV]. Note that A_u to one-loop depends strongly on m_s, $A_u = O((m_s/m_b)^2)$. We saturate the experimental limit on the CP-nonconservation rephasing-invariant J ($|J| \lesssim 10^{-4}$), and take $|V_{ub}| \simeq 0.006$ (Ref. 14). The sharp dips (which occur near the thresholds for the internal s and b quarks) are smoothed out if one includes a width $\Gamma_W \approx 2$ GeV for the W propagators in the vertex function (this effect is negligible above all thresholds, $m_t \gtrsim 90$ GeV).

Although the CP asymmetry is reasonably large, the phenomenological application is rather poor, due to the extremely small branching ratio for $t \to u\gamma$, at least to one-loop order. For example, $\Gamma_{1\mathrm{loop}}(t \to q\gamma) \approx 0.2\,|V_{bq}|^2$ eV, for $m_t \approx 100$ GeV, despite a significant enhancement of the width due to threshold effects;[10,11] this corresponds to a branching ratio for $t \to u\gamma$ of only $\approx 10^{-13}$, given $\Gamma_{\mathrm{tot}} \approx \Gamma(t \to bW) \approx 90$ MeV at $m_t = 100$ GeV. (Although the asymmetry for $t \to c\gamma$ is smaller than for $t \to u\gamma$, we note that the branching ratio to one-loop order is correspondingly larger, $B(t \to c\gamma) \approx 5 \times 10^{-12}$ for $m_t = 100$ GeV. In the standard-model calculation to one-loop order, the CP asymmetry for the $t \to u\gamma$ mode is nevertheless favored, since $A_q B(t \to q\gamma)$ is approximately equal in either case $q = c, u$ [cf. Eq. (13)]. We observe that, in principle, these two modes could thereby serve to reveal non-standard-model couplings.)

On the other hand, it is possible that the inclusion of QCD corrections to the $tq\gamma$ vertex will lead to substantially larger branching ratios,[10] since the QCD-corrected penguin may suffer from a less severe Glashow-Iliopoulos-Maiani suppression than the one-loop diagram, especially if the form-factor at high energies is similar to the known result for the low-energy QCD-corrected penguin for *light* internal quarks.[15] It should be understood that the Penguin graph for $t \to q\gamma$ contains only light internal quarks (b, s, and d quarks), since the top quark is on an *exter-*

nal line in this case. This is very different from $b \to s\gamma$, where the heavy top quark is an internal particle.

A rough estimate based on the low-energy QCD-corrected form factor[15] suggests an enhancement to the branching ratio of as much as four orders of magnitude, e.g., $B_{\rm QCD}(t \to c\gamma) \approx 10^{-8}$ at $m_t \approx 100$ GeV (using $\alpha_s \approx 0.1$, and $m_d \approx 10$ MeV).[10] The absorptive part of the QCD-corrected penguin (for on-shell photons) would be generated in basically the same way as in the one-loop diagram we have analyzed here, with the internal W and quark going on shell for $m_t > M_W + m_l$. The magnitude of the asymmetry with QCD corrections, and the preferred mode (i.e. $t \to u\gamma$, or $t \to c\gamma$) will depend on the details of the QCD-corrected form factors.

This raises the interesting possibility that the *CP* asymmetry to two-loop order might be of a similar size to our result obtained from the one-loop Penguin (with the absorptive part to an on-shell photon arising in a similar way), but with a much larger branching ratio for the radiative decay.

We also note that our calculation of the one-loop *CP* asymmetry for top quark radiative decays in the three-generation standard model should serve as a useful benchmark for the study of possible *CP* asymmetries in the top-quark sector in the context of new physics scenarios. In view of recent conjectures that a heavy top quark may play a central role in physics beyond the standard model, the search for significant manifestations of *CP* violation in this new arena merits further attention.

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