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Black holes initiate false-vacuum decay

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We consider O(3)-invariant tunneling processes which induce false-vacuum decay in general relativity. We find that in the presence of a black hole in a false vacuum the process of spontaneous nucleation of a bubble around a black hole (with the true phase in between the bubble shell and the black-hole surface) proceeds at a tremendously faster rate than that of an empty O(4)-invariant true phase bubble provided the black-hole mass does not exceed $m_{\max}=M_{\rm Pl}^4/12\sqrt{3}\pi S$. There we also find the spontaneous creation of black holes during first-order vacuum-vacuum phase transitions.

The importance of phase transitions for the evolution of the Universe is now clearly understood and many papers are devoted to the subject (for reviews see, e.g., Refs. 1 and 2). Of particular interest is the problem of a metastable vacuum decay in the early Universe, especially accounting for effects of general relativity. The first considerations of false-vacuum decay in the framework of general relativity[3,4] were carried out in the thin-wall approximation for O(4)-symmetric decay. However, there are a number of O(3)-symmetric processes of interest as well, for instance, nucleation of a true phase in the presence of seeds such as black holes. The probabilities of some O(3)-invariant processes were found in Ref. 5 neglecting gravitational effects, while the first consideration accounting for gravity was carried out in Ref. 6.

The most adequate approach which accounts for quantum gravity effects is the use of the Wheeler-DeWitt equation, say, in the form[7] appropriate for the thin-wall treatment. In some simple cases it is possible to relate the probability of subbarrier transitions to the imaginary-time action for the system. In the present paper we analyze in the thin-wall approximation the Euclidean action for O(3)-invariant processes, and, more specifically, for a new phase bubble nucleation around a black hole.

Convenient expressions for the Euclidean action on O(3)-symmetric solutions to the Einstein equations were found in Ref. 8:

$$I=-\frac{M_{\rm Pl}^2}{2}\int d^2x\sqrt{\det\gamma}\,, \tag{1}$$

where $\det\gamma$ is the determinant of the two-dimensional (2D) metric obtained by the substitution of $\theta=$const, $\phi=$const into the 4D metric, the latter being a solution to the Einstein equations. We denote this 2D surface by π. Thus, the action for O(3)-symmetric solutions is proportional to the area of π. Equation (1) is valid up to integrals over total derivatives. However, surface terms are irrelevant in our case, since we calculate either a bounce, which is the difference of actions for two solutions (with and without the bubble, respectively) with the same boundary conditions, or the action for solutions corresponding to the closed universe.

The probability of a transition in the first WKB approximation is given by

$$p\sim C\exp(-B),\ \ B=I_{\rm new}-I_{\rm old}\,, \tag{2}$$

B being the bounce. For O(4)-invariant decay one may estimate C as follows: $C\sim R^{-4}$, where R is a generic size of a configuration. For O(3)-invariant decay one can take $C\sim R^{-1}$ due to lack of translational invariance. It follows from (1) that the bounce is determined by the difference of areas of two 2D surfaces π before ($\pi_{\rm old}$) and after ($\pi_{\rm new}$) the transition[8]

$$B=\tfrac{1}{2}(S_{\rm old}-S_{\rm new})M_{\rm Pl}^2\,. \tag{3}$$

We emphasize that this formula has nothing to do with the thin-wall approximation, being valid in this approximation as well.

Thus, in order to estimate tunneling probabilities one has to calculate surface areas π for various particular vacuum transitions of interest, using for the Euclidean version of pure vacuum manifolds the metric

$$ds^2=f\,dt^2+f^{-1}dr^2\,, \tag{4}$$

where $f=1-r^2/r_c^2-2M_{\rm Pl}^{-2}m/r$, r_c being given by $r_c^{-2}=8\pi\varepsilon/3M_{\rm Pl}^2$, and ε being the vacuum energy density. The metric coefficients do not depend on time t and the surfaces π are axisymmetric; the time t is an angular variable, $t=T\psi$, where T is the period and ψ is a dimensionless angle. In this paper we consider the case $\varepsilon_{\rm old}=0$ only; therefore we shall obtain the Schwarzschild metric with $T=4mM_{\rm Pl}^2$ as well as the Schwarzschild–anti-de Sitter metric; the mass m and the period T are given by m

$=r_g M_{\rm Pl}^2(r_c^2+r_g^2)/2r_c^2$ and $T=-2r_g r_c^2/(3r_g^2-r_c^2)$, respectively, r_g being the event horizon.

Before the tunneling transition the metric in our case is just the Schwarzschild one. Spontaneous nucleation of a new phase bubble results in a change of the surface. Following Ref. 8, we shall find the form and the area of the world containing a bubble in the thin-wall approximation when a solution with a bubble looks like the initial surface, but with a patch taken from the vacuum metric parameters.

The equation of the Euclidean bubble trajectory in which we are interested is[9,10]

$$\sigma_{\rm in}(f_{\rm in}-\dot r^2)^{1/2}-\sigma_{\rm out}(f_{\rm out}-\dot r^2)^{1/2}=4\pi M_{\rm Pl}^{-2}Sr\,. \quad (5)$$

Here $\dot r=dr/d\tau$, τ is the proper time on the shell, and $f_{\rm in}$ and $f_{\rm out}$ are vacuum metric coefficients (4) inside and outside the bubble, respectively. S is the surface energy density of the shell. The sign functions σ take the values $\sigma_{\rm in(out)}=+1$ if the radii r of 2D spheres increase in the direction of an outer normal to the phase separation surface, and $\sigma_{\rm in(out)}=-1$ in the opposite case.

The shell Euclidean trajectories at $m\neq 0$ oscillate in the region $r_a\leq r\leq r_b$ and might be attributed to one of two classes, as identified in Ref. 8. There are always points on the second-class trajectories (such as the point $\psi=\psi_*$ in Fig. 1), where $d\psi/d\tau=0$. The first-class trajectories do not contain such points.

It was conjectured[8] that the meaning of the first- and second-class trajectories might be as follows.

(i) One may guess that in the case of second-class trajectories there appears the possibility of connection by the Euclidean motion of the section $\psi<\psi_*$, which does not contain bubbles and corresponds entirely to the old phase, with the spatial section containing two shells at rest simultaneously. The configuration is that of a double bubble, i.e., a new phase bubble (shell b) contains inside a remnant of the old phase (shell a). Shell a collapses in real time, while shell b expands. The section $\psi=\psi_*$ is distinguished by the feature that those two shells are created from the vacuum at this point. The problems related to the fact that the bubble trajectories do not form closed loops if $m\neq 0$ shall be discussed later on. The O(4)-invariant trajectory also belongs to this class and corresponds to the particular case when shell a is absent: $r_a=0$.

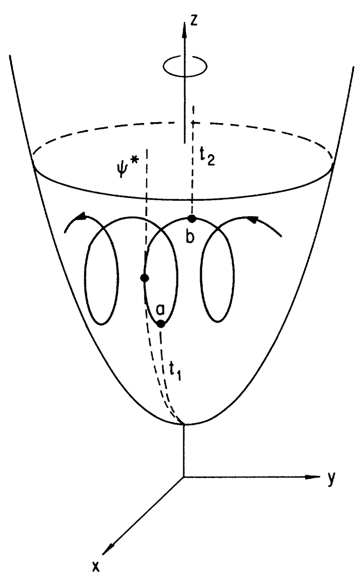

FIG. 1. The schematic view of the second-class bubble trajectory on the surface π corresponding to the Schwarzschild metric.

(ii) The first-class trajectories describe a kind of "thermodynamical" nucleation of new phase bubbles.[8,11,12]

(iii) Trajectories of either class could describe tunneling from the state with a bubble of the size r_a to the state with the bubble of the size r_b (or vice versa) without changing metric parameters. The shell trajectory in real time contains two disconnected pieces $0\leq r\leq r_a$ and $r_b\leq r<\infty$, the Euclidean trajectory describing a subbarrier transition. The spacetime always contains a bubble in one or another state. However, if the trajectory belongs to the first class, then both bubbles are on one and the same side of the Einstein-Rosen bridge. In other words, it is black hole to black hole[5,8] or wormhole to wormhole[13] tunneling, while in the case of a second-class trajectory σ changes its sign during subbarrier tunneling and it would be black hole to wormhole tunneling.[7] It was shown that in both latter cases the sequence of three-dimensional surfaces during the tunneling process does not form a Euclidean four-manifold.[7,13] There was also an attempt[14] to use the action on the specific space with a bubble to describe in the framework of third-quantized gravity the probability of creation of a universe with a cosmological constant tending to zero but with an inflating fluctuation. It was found that it is possible to obtain the action unbounded from below but that the corresponding construction does not describe a four-manifold (in fact, it contains the same singularities which were described in Refs. 7 and 13).

Here we are interested in the second-class solutions which might describe the spontaneous creation of a spherical layer of a new phase due to vacuum fluctuations. In the sense of the Wheeler-DeWitt equation it is possible to match such Euclidean solutions to those which describe (in real time) the collapse of an inner shell and the expansion of an outer shell if there is some 3D hypersurface in the total Euclidean four-space at which the shell momenta vanish as well as momenta corresponding to the evolution of the three-geometry. In order for the shell momenta to vanish, this hypersurface should pass through points a and b (see Fig. 1) at which the shell is at rest. It may be shown that momenta corresponding to a three-geometry vanish in Schwarzschild–de Sitter space if (and only if) this hypersurface corresponds to the section of constant Schwarzschild time t in the metric (4). In the thin-wall case this requirement must be satisfied in both new and old phases.

It is easy to see that there is essentially only one free parameter in the problem, just $m_{\rm new}$. Indeed, of the whole set of input parameters, $\varepsilon_{\rm old},\varepsilon_{\rm new},S,m_{\rm old},m_{\rm new}$, the first three are, in principle, determined somehow by a field-theory model, although their values are in practice unknown. The fourth parameter $m_{\rm old}$ does actually fix the initial conditions.

By examining shell trajectories in a wide enough range of all the input parameters (in fact, we probed parameters differing from the corresponding Planckian values up to several orders of magnitude) we have found (see also Refs. 11 and 12) that rest points of the shell are situated, in general, at different Euclidean Schwarzschild times. That is, in both new and old metrics the second-class trajectory looks like that in Fig. 1. Nevertheless, there is a slicing of the old metric possessing turning points for the corresponding parts of the three-geometry. These are slices of $t_{1,\text{old}}=\text{const}$ in the region $r<r_a$ and $t_{2,\text{old}}=\text{const}$ in the region $r>r_b$ (see Fig. 2). We do not think that the fact that the times $t_{1,\text{old}}$ and $t_{2,\text{old}}$ are different is of any basic significance. On the contrary, both rest points of the shell should belong to one and the same section $t_{\text{new}}=\text{const}$. There are only two (if any) values of m_{new}, M_1 and M_2, $M_1<M_2$, at which this might be achieved. It is just the case $r_a=r_{g,\text{new}}$. It does not mean at all that at these values of m_{new} the shell trajectory becomes closed in a new metric. Rather it reflects the fact that hypersurfaces of any $t=\text{const}$ meet at the horizon. M_1 and M_2 correspond to the points of intersection of two curves

$$y=x^3-\sigma_{\text{in}}\frac{x^2}{A}\left(1-\frac{1}{x}\right)^{1/2} \tag{6}$$

and

$$y^+=x^3(\xi-1)/(\xi+1)\,, \tag{7}$$

where

$$\begin{gathered} y=(m_{\text{old}}-m_{\text{new}})M_{\text{Pl}}^2/8\pi^2(1+\xi)S^2r_g^3\,,\\ x=r/r_g,\ \ r_g\equiv r_{g,\text{old}}\,,\\ \xi=(\varepsilon_{\text{new}}-\varepsilon_{\text{old}})M_{\text{Pl}}^2/6\pi S^2\,,\\ A=r_g/R_\kappa,\ \ R_\kappa=R_0/(1-R_0^2/4|r_c|^2),\ \ R_0=3S/|\varepsilon_{\text{new}}|\,, \end{gathered} \tag{8}$$

R_κ being the radius of the bubble nucleated in the O(4)-invariant way; σ_{in} takes both values $+1$ and -1. If

$$m_{\text{old}}<m_{\text{old,max}}=M_{\text{Pl}}^4/12\sqrt{3}\pi S\,, \tag{9}$$

then these curves intersect just at two points and, correspondingly, there exist two different values of m_{new} for a given m_{old}, see Fig. 3. At both $m_{\text{new}}=M_1$ and $m_{\text{new}}=M_2$ the inner shell nucleates just on the new black-hole horizon, $r_a=r_{g,\text{new}}$. These solutions are absent if $m_{\text{old}}>m_{\text{old,max}}$.

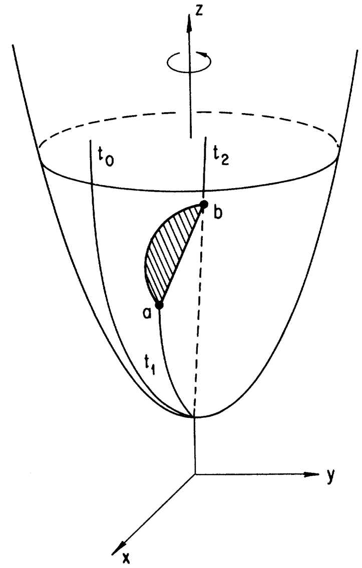

FIG. 2. The Euclidean manifold with boundaries at which all momenta equal zero. This manifold determines a semiclassical contribution to the new phase bubble nucleation around a black hole.

Taking only one branch of the shell trajectory in between $r_a=r_{g,\text{new}}$ and r_b for the junction of the old and new metrics and taking slices for the second turning point of the whole three-geometry, as described above, then cutting the old metric at any $t=\text{const}<T\psi_*$ where the shell does not exist yet to get the slice for the first turning point of the whole three-geometry, one can construct as a result the Euclidean space which (at least formally) fits requirements for the semiclassical spontaneous nucleation of the ring of the new vacuum around a black hole in a false vacuum. This space is shown by the solid lines in Fig. 2. The hatched region in Fig. 2 corresponds to the new vacuum, its area being given by (for more details on construction, see Refs. 8 and 12)

$$S_{\text{new}}=-2\pi M_{\text{Pl}}^{-2}S(\xi-1)\int_{r_{g,\text{new}}}^{r_b}dr(r-r_{g,\text{new}})(r^3-r_{g,\text{new}}^3)/r^2f_{\text{new}}|\dot r|\,. \tag{10}$$

In order to find the value of B, one has to extract the area of the space shown in Fig. 2 from the one corresponding to the vacuum-vacuum transition amplitude (without the bubble). The natural assumption seems to be to take for this amplitude just the area of the sector in between two lines $t=t_0$ and $t=t_2$, the latter one being extended to $r=r_{g,\text{old}}$. (In order to justify this assumption one has actually to solve the corresponding Wheeler-DeWitt equation). One can see that under this subtraction the part of the old surface in between two branches of the bubble trajectory at $t<t_1$ and its part which is in between the bubble trajectory and the horizon at $t_1<t<t_2$ do not have counterparts in the space shown in Fig. 2 and are not canceled. The area of these parts of π_{old} is

$$S_{\text{old}}=-2\pi M_{\text{Pl}}^{-2}S(\xi+1)\int_{r_a}^{r_b}dr(r-r_{g,\text{old}})(r^3-r_+^3)/r^2f_{\text{new}}|\dot r|\,. \tag{11}$$

According to (3) the bounce is the difference between S_{old} given by (11) and S_{new} given by (10). We calculated it in the whole range $m<m_{\text{old,max}}$ and at several values of ξ and B_4, B_4 being the O(4)-invariant bounce. B scales linearly with $B_4, B\approx B_4/3$ for $|\xi|>10$. The bounce for the solution $m_{\text{new}}=M_2$ is always much larger than that one correspond-

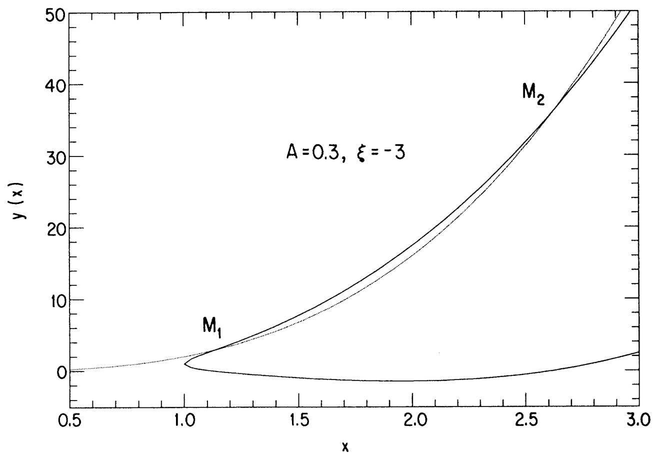

FIG. 3. The functions $y = y^+(x)$ (shown as a monotonic line) and $y = y(x)$ (a double-valued one).

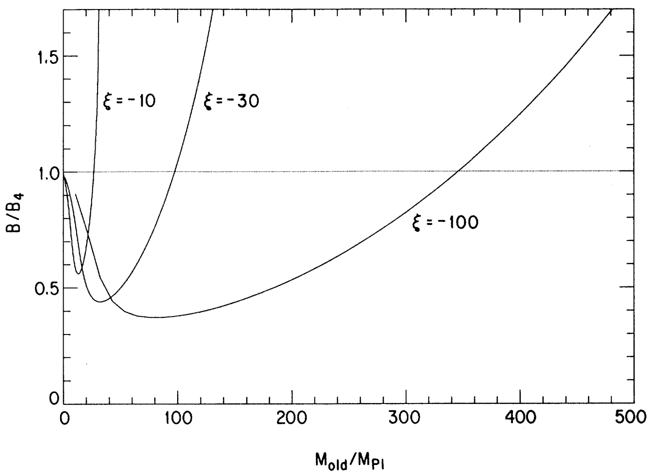

FIG. 5. $B = B(m_{\rm old})$ at different ξ.

ing to the solution $m_{\rm new} = M_1$, see Fig. 4. However, this is of considerable interest since it determines the probability of the spontaneous creation of a new phase bubble with the remnant of the old phase inside (which would be a black hole after the collapse) starting with the state with no black holes *ab initio*, $m_{\rm old} = 0$. It is seen that this probability is always smaller than the O(4)-invariant creation of an empty bubble. However, the probability of the spontaneous creation of black holes is by no means negligible since one has to take care not to overpopulate the Universe with black holes. The solution $m_{\rm new} = M_1$ corresponds to the limiting case $B \to B_4$ when $m_{\rm old} \to 0$, reproducing the Coleman-DeLuccia result.[3] The values of bounce $B = B(m_{\rm old})$ for the solution $m_{\rm new} = M_1$ at different values of ξ are presented in Fig. 5. It is seen that there always exists a range of values of $m_{\rm old}$ for which the presence of a black hole in a system heavily stimulates bubble nucleation, and that there is a kind of a "resonance" value of $m_{\rm old}$ at which the probability of bubble nucleation is maximum. The ratio $r_{g,\rm old}/R_\kappa$ as a function of ξ for the resonance black hole is presented in Fig. 6. Note that $A > 1$ for $|\xi| > 10$ while $A < 1$ for smaller values of $|\xi|$ corresponding to the nonvanishing probability of spontaneous creation of black holes.

A few comments are in order. A thin-wall treatment is obviously an approximation to a real field-theoretical problem and there should exist an analog of a complete solution to any thin-wall result. From the uniqueness theorem for solutions to the elliptic differential equations one may conclude that there should be only one (or a discrete set of) value(s) of $m_{\rm new}$ at any fixed value of other parameters which provide a solution to the tunneling problem. It is quite encouraging that we have found unique solutions. However, we are not aware of what might be a field-theoretical analog to the thin-shell configuration passing through the horizon. Further, the very range of

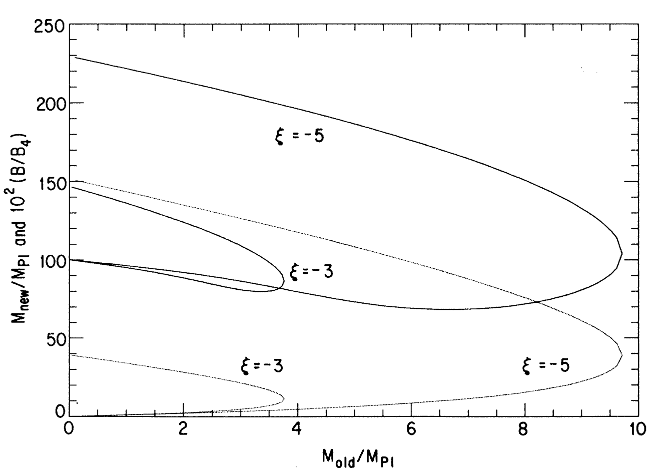

FIG. 4. The bounces $B = B(m_{\rm old})$ (upper curves) and masses of black holes after the transition $m_{\rm new}(m_{\rm old})$ (lower curves) for both $m_{\rm new}$ solutions.

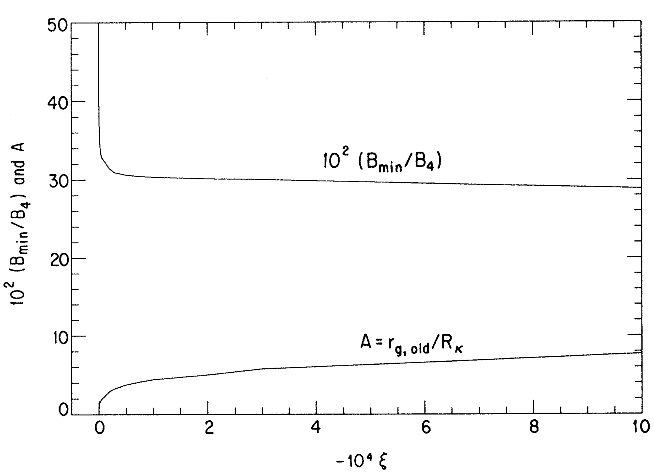

FIG. 6. The ratio $A = r_{g,\rm old}/R_\kappa$ and minimum values of bounces as functions of ξ for "resonance" black holes.

validity of the thin-wall approach is still not quite clear. Nevertheless, we think that we have found some evidence that new physical processes of importance, namely the stimulation of new phase bubble nucleation by black holes and the spontaneous creation of black holes, could take place. It might have had an impact on both the evolution of the early Universe and the dark-matter problem.

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