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Gravitational particle production during the formation of global texture

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The production of particles from the vacuum during the rapid variation of the gravitational field accompanying the formation of a knot of global texture is investigated. It is found that the formation of a knot of texture will, in general, be accompanied by a burst of particle production, the intensity of this process being inversely proportional to the fourth power of the distance from the texture.

As recently shown in Refs. 1–3, topological defects known as texture can arise generically in field theories admitting a non-Abelian global symmetry.[1,2] Such topological defects are three-dimensional objects and are related to the existence of a nontrivial third homotopy group of the degenerate vacuum manifold, π_3. (Other nontrivial homotopy groups of the vacuum manifold, π_2, π_1, and π_0, give rise to walls, strings, and monopoles, respectively.[4])

"Knots" of texture form when the Higgs field winds around a three-sphere in a nontrivial manner. This typically occurs on scales greater than the cosmological horizon. On entering the horizon, texture knots begin to collapse at the speed of light and the corresponding space-time metric rapidly approaches the form

$$ds^2=dt^2-dr^2-r^2\left(1-\frac{4\epsilon}{3}\right)(d\theta^2+\sin^2\theta\, d\phi^2)\ , \quad (1)$$

where $8\pi\epsilon/3=\frac{128}{3}\pi^2G\eta^2$. η is the vacuum expectation value of the Higgs field, in grand unified theories (GUT's) its value is directly related to the symmetry-breaking scale.[1-3]

The form of the metric (1) is, in fact, very similar to that induced by a static global monopole.[5] If texture were stable it could create grave problems for viable cosmological scenarios, since then the energy density in texture knots would scale as $\rho\propto a^{-3}$ leading to an overabundance of texture knots today, reminiscent of the monopole problem. Fortunately, a texture knot is known to be unstable and unwinds itself, emitting Goldstone bosons, so that the space-time metric at late times, once more, approaches flat space.

The fact that global texture is unstable has led to its being proposed as a possible mechanism for seeding the large-scale structure of the Universe.[2] Some astrophysical consequences of texture also have been studied in Ref. 3. I propose to examine the possibility that the formation of a knot of global texture is accompanied by an intense burst of particle production which arises because the background space-time metric is no longer flat but is changing rapidly.

It is well known that a rapidly changing gravitational field gives rise to particle production generically.[6] This effect can be very significant during the very early Universe when the energy density of created particles can exceed the energy density of conventional matter and modify the expansion of the Universe.[7] Recent studies, however, indicate that significant particle production can also occur during the formation of intense gravitational fields, which accompany the formation of topological defects such as cosmic strings, domain walls, and global monopoles.[8] Since the formation of global texture is also accompanied by a rapid change in the space-time metric there is every reason to believe that the formation of knots of global texture will, in general, be accompanied by copious particle production. Furthermore, since, as opposed to GUT scale strings and walls, knots of texture will be forming in the Universe today,[2,3] this effect could have interesting observational consequences.

We assume that the formation of texture knots and their subsequent evaporation can be qualitatively described by a space-time metric with a time-varying deficit solid angle:

$$ds^2=dt^2-dr^2-r^2b^2(t)(d\theta^2+\sin^2\theta\, d\phi^2)\ , \quad (2)$$

where b is chosen so that $b(\pm\infty)=1$; i.e., the metric at early and late times becomes flat Minkowski space.[9] At $t=t_0$ we shall require that metric (2) become identical with (1); i.e., $b^2(t_0)=1-\frac{4}{3}\epsilon$.

In order to investigate particle production in this background space-time we shall consider the simplest case of a minimally coupled massless scalar field which satisfies the Klein-Gordon equation:

$$\frac{1}{\sqrt{-g}}\partial_\mu\sqrt{-g}g^{\mu\nu}\partial_\nu\Psi=0\ .$$

The Klein-Gordon equation does not, in general, admit a separation of variables in the background space-time described by the metric (2). However some useful insight can be gained by restricting ourselves to the lowest angular modes for which the variables can be separated giving

$$\Psi=\psi(t)j_0(kr)Y_0^0(\theta,\phi)\ ,$$

where $j_0(kr)$ is a spherical Bessel function of zeroth order, and $Y_0^0(\theta,\phi)=1/\sqrt{4\pi}$ is the zeroth-order spherical harmonic. $\psi(t)$ satisfies the differential equation

$$\ddot{\psi}+2\frac{\dot{b}}{b}\dot{\psi}+k^2\psi=0\ , \quad (3)$$

which in terms of the new time coordinate $\tau=\int dt/b^2$, assumes the more convenient form

$$\ddot{\psi}+k^2b^4(\tau)\psi=0\ . \quad (4)$$

A physically well-motivated choice for $b(\tau)$ is

$$b^4(\tau)=1-\frac{a}{\cosh^2(\alpha\tau)} \tag{5}$$

so that $b(\pm\infty)=1$. (The free parameter α regulates the rate of change of the metric.) The condition $b^2(0)=1-\frac{4}{3}\epsilon$ then sets the value of the parameter a: $a=\frac{8}{3}\epsilon=\frac{128}{3}\pi G\eta^2$. With this choice of $b(\tau)$, Eq. (4) can be solved exactly. The fact that $b(\pm\infty)=1$ allows us to unambiguously define asymptotic vacuum states at $t=\pm\infty$. A normalized solution of (4) corresponding to an asymptotically positive-frequency mode at $\tau=-\infty$ is

$$\begin{aligned}\psi(\tau)&=\frac{[\frac{1}{4}(1-\xi^2)]^{-ik/2\alpha}}{\sqrt{4\pi k}}F\left(-\frac{ik}{\alpha}-s,1+s-\frac{ik}{\alpha};1-\frac{ik}{\alpha};\tfrac{1}{2}(1+\xi)\right),\\ \psi&\underset{\tau\to-\infty}{\sim}\frac{1}{\sqrt{4\pi k}}e^{-ik\tau},\end{aligned} \tag{6}$$

where $\xi=\tanh(\alpha\tau)$ and $s=\frac{1}{2}[-1+(1-4k^2a/\alpha^2)^{1/2}]$.

From the linear transformation property of the hypergeometric function,[10]

$$\begin{aligned}F(a,b;c;z)=&\frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}F(a,b;a+b+1-c;1-z)\\&+\frac{\Gamma(c)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)}(1-z)^{c-a-b}F(c-a,c-b;c+1-a-b;1-z)\end{aligned} \tag{7}$$

we obtain

$$\frac{1}{\sqrt{4\pi k}}e^{-ik\tau}\underset{\tau\to\infty}{\sim}\frac{1}{\sqrt{4\pi k}}e^{-ik\tau}c_1(k,\alpha,s)+\frac{1}{\sqrt{4\pi k}}e^{ik\tau}c_2(k,\alpha,s), \tag{8}$$

where c_1 and c_2 are the Bogoliubov coefficients:

$$\begin{aligned}c_1&=\frac{\Gamma(-ik/\alpha)\Gamma(1-ik/\alpha)}{\Gamma(-ik/\alpha-s)\Gamma(1-ik/\alpha+s)},\\ c_2&=\frac{\Gamma(ik/\alpha)\Gamma(1-ik/\alpha)}{\Gamma(-s)\Gamma(s)},\end{aligned} \tag{9}$$

so that

$$\begin{aligned}|c_1|^2&=\frac{\sin^2(\pi s)+\sinh^2(\pi k/\alpha)}{\sinh^2(\pi k/\alpha)},\\ |c_2|^2&=\frac{\sin^2(\pi s)}{\sinh^2(\pi k/\alpha)},\end{aligned} \tag{10}$$

and $|c_1|^2-|c_2|^2=1$.

$|c_2|^2$ gives the number density per mode of particles created by the changing space-time metric. For particles with momenta $k/\alpha\gg 1/\sqrt{a}$, we have

$$|c_2|^2=\exp\left(-\frac{2\pi}{\alpha}k(1-\sqrt{a})\right) \tag{11}$$

and we see that the production of particles is exponentially damped for large k. The total number density of created particles is given by

$$n=\frac{1}{2\pi^2}\int_0^\infty |c_2|^2k^2dk \tag{12a}$$

and the corresponding energy density is

$$\epsilon=\frac{1}{2\pi^2}\int_0^\infty k|c_2|^2k^2dk\,. \tag{12b}$$

Evaluating the integrals in (12) we obtain

$$\begin{aligned}n&\simeq\frac{1}{60\pi}\alpha^3a^2,\\ \epsilon&\simeq\frac{315}{8\pi^8}\alpha^4a^2.\end{aligned} \tag{13}$$

The requirement that the peculiar velocity field predicted by texture agree with current observations sets the value of a at $a=\frac{128}{3}\pi G\eta^2\sim(16/3\pi)10^{-5}h^{-1}$, h being the value of the Hubble parameter in units of 100 km/sec Mpc[3]. It seems reasonable to assume that $\alpha\simeq 1/r$, where r is the distance to the texture knot—such a choice of α is supported by the self-similar form of the texture metric recently obtained by Notzold[11] and Barriola and Vachaspati.[12] One then obtains

$$\epsilon=3\times10^{82}\left(\frac{r_{\rm GUT}}{r}\right)^4\ \text{ergs/cm}^3, \tag{14}$$

where $r_{\rm GUT}$ is the GUT scale length $r_{\rm GUT}=3\times10^{-28}$ cm. From (14) one immediately sees that the energy density of created particles is very high in the immediate vicinity of the knot, and drops off rapidly at increasing distances, so that at a distance of just 1 cm from the texture knot, the energy density of created particles has already dropped to $\sim2.4\times10^{-28}$ ergs/cm^3. It is important to mention that although the particle creation rate has been explicitly calculated only for massless minimally coupled particles, an effect of roughly the same order of magnitude will also exist for other massless particles—including photons. One would therefore expect to see radiation being emitted from the precise location of the texture knot at the time of its formation.

As seen from (10), the main contribution to the spectrum of particles emitted from the vicinity of the knot comes from the GUT frequency range. In the case of photons, it will be difficult to observe such high-frequency γ rays directly, since most photons will scatter off the lower-energy cosmic-microwave-background quanta, leading to pair production enroute to the observer.

It may be noted that the mean number of particles created during the collapse of a texture knot is less than unity, since $N=\int n dV \sim 10^{-8}$. This means that one in every 10^8 knots of texture will emit a very energetic particle. (A similar result for cosmic strings was obtained by Parker and by Mendell and Hiscock,[8] who found that the mean number of particles created during the formation of a cosmic string is $N \leq 10^{-6}$, for a GUT-scale string.)

Integrating (14) for the total energy in massless particles created by a collapsing and unwinding knot of texture, we obtain $E \simeq 10$ ergs, which is much smaller than the total energy radiated by Goldstone bosons during the unwinding of a texture knot.[2,3] Strictly speaking, this result presents a lower bound on the particle production process since we have restricted ourselves to modes carrying the lowest angular momentum. However, it is well known that modes with the lowest quantum numbers also contribute predominantly to the particle creation rate, the creation of particles with higher quantum numbers being adiabatically suppressed.[7] One therefore expects (14) to provide the main contribution to the particle production process, and to give a correct order-of-magnitude estimate of the energy and number density of created particles, during the collapse and subsequent unwinding, of a texture knot.

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[9] Notzold (Ref. 11) recently shows that the texture metric will, in general, depend upon both the time and space variables, so that

$$ds^2=dt^2-dr^2-r^2b^2(t/r)(d\theta^2+\sin^2\theta\, d\phi^2)\,.$$

It is easy to show that a change of variables will then bring this metric to the form

$$ds^2=(1-x^{-2})dt^2+\frac{2t}{x^3}dt\,dx-x^{-2}\left(\frac{t}{x}\right)^2dx^2 - \left(\frac{t}{x}\right)^2 b^2(x)(d\theta^2+\sin^2\theta\, d\phi^2)\,,$$

where $x=t/r$. However, it is not clear how one can quantitatively study quantum gravitational effects in this space-time, since the metric is quite complicated and also the spacelike hypersurfaces of constant time are no longer perpendicular to the time coordinate. To circumvent this difficulty we study the problem of particle creation in metric (2), assuming $b=b(\alpha,t)$, α being an adiabatic parameter which governs the rate of change of the metric. The spatial dependence of the metric will be introduced explicitly later, when the value of α is set to be inversely proportional to the distance from the texture knot.

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