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Threshold parameters of πK scattering in QCD

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We evaluate the low-energy expansion of the πK scattering amplitude to one-loop order in chiral perturbation theory. We predict the pertinent scattering lengths and effective-range parameters beyond the current-algebra values. We compare to the existing data and point towards the need of more accurate experiments.

The smallness of the current quark masses for the three light flavors u, d, and s allows one to systematically expand the low-energy Green's functions of QCD. This simultaneous expansion in powers of external momenta and quark masses is called chiral perturbation theory.[1-3] The analysis of the Green's functions can be most easily done by use of an effective Lagrangian of the pseudoscalar mesons (the Goldstone bosons of broken chiral symmetry) which depends on a number of low-energy constants that are not fixed by symmetry requirements alone. In this language, the expansion in meson momenta and meson masses amounts to an expansion in the number of loops. For the one-loop approximation, Gasser and Leutwyler have pinned down the ten low-energy constants from experimental information and by invoking large-N_c arguments.[3] To this order, the generating functional is therefore completely determined.

The low-energy expansion of the $\pi\pi$ scattering amplitude has been carried out in Refs. 4 and 5. There, it was found that the data for the threshold parameters agree with the one-loop prediction within 1.5 standard deviations, i.e., within the error bars. Our aim is to present a similar systematic analysis for the πK system. The main interest in studying πK scattering stems from the fact that it is the simplest meson-meson scattering process that involves strangeness and unequal meson (quark) masses. Furthermore, since the low-energy constants have already been determined, comparing the πK scattering amplitude with the data allows one to test the large-N_c predictions which enter into the evaluation of certain constants. Such a test has recently been performed for the K_{l4} decay ($K^+ \to \pi^+\pi^- e^+ \nu_e$) and good compatibility of chiral symmetry and large-N_c arguments with the data was found.[6] Therefore, it is interesting and necessary to additionally investigate the πK system. In fact, pion-kaon scattering experiments have reached a reasonable accuracy to test some general features of symmetry violations in strong interactions.[7] It will, however, become obvious that it would be very helpful to have better and more accurate empirical determinations of the πK threshold parameters. This could and should be a major goal for the proposed kaon factories.

The generating functional which embodies the low-energy structure of QCD to lowest order is a generalized nonlinear σ model. It involves two parameters—the pion decay constant in the chiral limit (F_0) and another constant which measures the strength of dynamical-chiral-symmetry breaking ($B_0 = -\langle 0|\bar{u}u|0\rangle / F_0^2$); i.e., it is related to the vacuum expectation value (VEV) of the scalar quark density. Including the quark mass matrix $\mathcal{M} = \text{diag}(m_u, m_d, m_s)$, one finds familiar relations, e.g., $(M_\pi^0)^2 = (m_u + m_d)B_0$ (and similarly for the K and the η) and these $(M^0)^2$ values together obey the Gell-Mann–Okubo relation (the superscript zero denotes quantities to lowest order which are different from the physical values). We disregard the third constant H_0 related to the singlet and winding-number currents here. At next-to-leading order, the generating functional takes the form

$$Z = Z_T + Z_U + Z_A + O(\Phi^6)\,. \tag{1}$$

Here, Z_T subsumes the tree-level and tadpole contributions, i.e., graphs with no loops or with one loop and one vertex. The so-called unitarity correction Z_U contains all one-loop graphs with two vertices. The functional Z_A correctly reproduces the axial anomaly.[8] It will be of no further relevance for our discussion. The explicit form of the functionals Z_T and Z_U can be found in Ref. 3. To this order (p^4), the effective Lagrangian contains ten low-energy constants $L_1^r, \ldots, L_{10}^r$ plus two parameters related to contact terms which are, however, of no physical significance. The parameters $L_1^r, \ldots, L_{10}^r$ are renormalized quantities which absorb the divergences of the one-loop graphs. Therefore, they depend on a renormalization scale μ, which, however, drops out in all physical observables. At the scale $\mu = M_\eta$, these constants have been determined in Refs. 3 and 6.

Let us now turn to the πK scattering process. In the s channel, there are two independent amplitudes with isospin $I=\frac{1}{2}$ and $I=\frac{3}{2}$. The latter is given by the specific process $\pi^+ K^+ \to \pi^+ K^+$, i.e.,

$$T(\pi^+(p_1) + K^+(p_2) \to \pi^+(p_3) + K^+(p_4)) = T_{\pi K}^{3/2}(s,t,u) \tag{2}$$

with $s=(p_1+p_2)^2$, $t=(p_1-p_3)^2$, and $u=(p_1-p_4)^2$ the conventional Mandelstam variables. For the on-shell scattering amplitude these obey $s+t+u=2(M_\pi^2+M_K^2)$. By crossing symmetry one can relate the reaction $K^+\pi^+ \to K^+\pi^+$ to $K^+\pi^- \to K^+\pi^-$ and $K^-\pi^+ \to K^-\pi^+$ and determine the $I=\frac{1}{2}$ amplitude:

$$T_{\pi K}^{1/2}(s,t,u) = \tfrac{3}{2}\, T_{\pi K}^{3/2}(u,t,s) - \tfrac{1}{2}\, T_{\pi K}^{3/2}(s,t,u)\,. \tag{3}$$

This completely fixes the kinematics and amplitudes we will consider (for a more detailed account of the πK system, the reader is referred to Lang[9]).

It is straightforward but tedious to extract the four-point function related to πK scattering from the generating functional (1). In the general case, it is expressed in terms of the pseudoscalar fields $U=\exp(i\Phi)$ and some external scalar (s), pseudoscalar (p), vector (v_μ), and axial-vector (a_μ) sources. The pertinent N-point functions can then be constructed by expanding around the point $p=v_\mu=a_\mu=0$, $s=\mathcal{M}$. This allows for calculating off- and on-shell amplitudes. Here, we are interested in the on-shell amplitude for four pseudoscalars and can use the recipe given in Ref. 10 to construct the T matrix for on-shell mesons. One notices that the wave-function renormalization of the external legs is given by the physical meson decay constants and consequently one has to use

$$\Phi = \sum_P \phi_P \lambda_P F_P^{-1} \quad (P=\pi,K,\eta)\,. \tag{4}$$

Keeping this in mind, one finds, for the $I=\frac{3}{2}$ amplitude,

$$\begin{aligned}
T_{\pi K}^{3/2}(s,t,u) = \frac{1}{F_\pi^2 F_K^2} \Bigg[& \frac{F_0^2}{6} \Bigg(2M_\pi^2 + 2M_K^2 + (M_\pi^0)^2 + (M_K^0)^2 - 3s + \frac{\mu_\pi}{8}[66s - 34M_\pi^2 - 54M_K^2 - 15(M_\pi^0)^2 - 21(M_K^0)^2] \\
& + \frac{\mu_K}{4}[30s - 22M_\pi^2 - 18M_K^2 - 11(M_\pi^0)^2 - 9(M_K^0)^2] \\
& + \frac{\mu_\eta}{24}[54s - 54M_\pi^2 - 18M_K^2 - 17(M_\pi^0)^2 - 11(M_K^0)^2] \Bigg) \\
& + 8L_1^r(t-2M_\pi^2)(t-2M_K^2) + 4L_2^r[(s-M_\pi^2-M_K^2)^2 + (u-M_\pi^2-M_K^2)^2] \\
& + 2L_3[(u-M_\pi^2-M_K^2)^2 + (t-2M_\pi^2)(t-2M_K^2)] \\
& + 8L_4^r[(M_\pi^0)^2(t - \tfrac{1}{2}s + \tfrac{1}{3}M_\pi^2 - \tfrac{5}{3}M_K^2) + (M_K^0)^2(t - s - \tfrac{4}{3}M_\pi^2 + \tfrac{2}{3}M_K^2)] \\
& + \tfrac{4}{3}L_5^r[(M_\pi^0)^2(2M_\pi^2 - 3s) + (M_K^0)^2(2M_K^2 - 3s)] + \tfrac{8}{3}L_6^r[(M_\pi^0)^4 + 15(M_\pi^0)^2(M_K^0)^2 + 2(M_K^0)^4] \\
& + \tfrac{8}{3}L_8^r[(M_\pi^0)^4 + 6(M_\pi^0)^2(M_K^0)^2 + (M_K^0)^4] + \frac{t}{2}(u-s)[M_{\pi\pi}^r(t) + \tfrac{1}{2}M_{KK}^r(t)] \\
& + \tfrac{3}{8}\{(s-t)[L_{\pi K}(u) - uM_{\pi K}^r(u)] + (M_K^2 - M_\pi^2)^2 M_{\pi K}^r(u)\} \\
& + \tfrac{3}{8}\{(s-t)[L_{K\eta}(u) - uM_{K\eta}^r(u)] + (M_K^2 - M_\pi^2)^2 M_{K\eta}^r(u)\} \\
& + \tfrac{1}{8}(M_K^2 - M_\pi^2)K_{\pi K}(u)\{5(u - M_\pi^2 - M_K^2) + 3[(M_\pi^0)^2 + (M_K^0)^2]\} \\
& + \tfrac{1}{8}(M_K^2 - M_\pi^2)K_{K\eta}(u)[3(u - M_\pi^2 - M_K^2) + (M_\pi^0)^2 + (M_K^0)^2] + \tfrac{1}{4}J_{\pi K}^r(s)(s - M_\pi^2 - M_K^2)^2 \\
& + \tfrac{1}{32}J_{\pi K}^r(u)\{11(u - M_\pi^2 - M_K^2)^2 + 10(u - M_\pi^2 - M_K^2)[(M_\pi^0)^2 + (M_K^0)^2] + 3[(M_\pi^0)^2 + (M_K^0)^2]^2\} \\
& + \tfrac{3}{32}J_{K\eta}^r(u)\{u - M_\pi^2 - M_K^2 + \tfrac{1}{3}[(M_\pi^0)^2 + (M_K^0)^2]\}^2 \\
& + \tfrac{1}{8}J_{\pi\pi}^r(t)[4M_\pi^2 - 2t - 3(M_\pi^0)^2][2M_K^2 - t - 2(M_K^0)^2] \\
& + \tfrac{3}{16}J_{KK}^r(t)[2M_\pi^2 - t - 2(M_\pi^0)^2][2M_K^2 - t - 2(M_K^0)^2] + \tfrac{1}{8}J_{\eta\eta}^r(t)(M_\pi^0)^2[t - 2M_K^2 + \tfrac{10}{9}(M_K^0)^2] \Bigg]\,.
\end{aligned} \tag{5}$$

TABLE I. Threshold parameters of πK scattering. We give the current-algebra predictions (CA) together with the chiral-perturbation-theory (CHPT) results as well as the size of the corrections beyond the lowest-order (CA) theory. For the empirical values (Expt.), we have taken the mean values listed in Refs. 15–19. These values should be considered indicative (Ref. 20).

	CA	CHPT	Size of correction	Expt.
$a_0^{1/2}$	0.14	0.17 ± 0.02	1.19	0.13, . . . ,0.24
$b_0^{1/2}$	0.07	0.14 ± 0.02	2.12	. . .
$a_0^{3/2}$	−0.07	-0.05 ± 0.02	0.75	−0.13, . . . , −0.05
$b_0^{3/2}$	−0.05	-0.011 ± 0.005	0.23	. . .
$a_1^{1/2}$	0.01	0.013 ± 0.003	1.36	0.017, . . . ,0.018
$a_1^{3/2}$	0	$(7.5\pm3.1)\times10^{-4}$	. . .	. . .

The loop functions J^r_{PQ}, K_{PQ}, L_{PQ}, and M^r_{PQ} (with $P,Q=\pi,K,\eta$) as well as the functions μ_P are explicitly given in Ref. 3. The superscript r denotes the occurrence of the renormalization scale μ in J^r_{PQ} and M^r_{PQ}. As a nontrivial check, one verifies that the πK amplitude (5) is indeed independent of the scale μ—the μ dependence of J^r_{PQ} and M^r_{PQ} is balanced by the scale dependence of the low-energy constants $L^r_1,\dots,L^r_8$ (note that L^r_3 and L^r_7 are scale independent). One can also write the amplitude (5) in a more compact form by eliminating the lowest-order quantities in favor of the physical observables, the L_i's and the functions μ_P; i.e., the lowest-order quantities are not free parameters. This form will be discussed in detail in Ref. 11. It only differs from the one given here by terms of the order p^6.

Let us now turn to the results. For the numerical evaluation, we use $F_\pi=93.1$ MeV,[12] $F_K=1.22F_\pi$,[13] $M_\pi=139.57$ MeV, $M_K=493.65$ MeV, $M_\eta=548.8$ MeV, and the renormalized coupling constants L^r_1,L^r_2,L_3 from Ref. 6 and $L^r_4,\dots,L^r_8$ from Ref. 3 (at the scale $\mu=M_\eta$). We work in the isospin limit $m_u=m_d=\hat{m}$. This leads, e.g., to $F_0=87$ MeV. The numerical values for the various scattering lengths a^I_l and the effective range parameters b^I_l (here, l denotes the angular momentum and I the total isospin) are given in Table I, in appropriate units of the inverse charged-pion mass. The current-algebra predictions are given in the column CA.[14] We also give the size of the corrections to the current-algebra results (column 3) and the experimental numbers are from Ref. 15. One notices that the corrections to the soft-meson theorems are substantial (20–70%). Also, one would need more precise data to test the QCD predictions. This latter point is visualized in the figure, where we have plotted the empirical information on the S-wave scattering lengths (Refs. 16–19) together with the current-algebra (CA) and QCD [chiral perturbation theory (CHPT)] predictions. One observes that the CHPT point is closer to the main cluster of data than the current-algebra value. In fact, while current algebra predicts $R=a_0^{1/2}/a_0^{3/2}=-2$, the improved chiral representation gives $R=-3.2$ and the mean value of the data[15] gives $R_{\rm expt}=-3.0$. The errors on the CHPT predictions given in Table I reflect the uncertainties in the determination of the low-energy constants $L_1,\dots,L_8$.

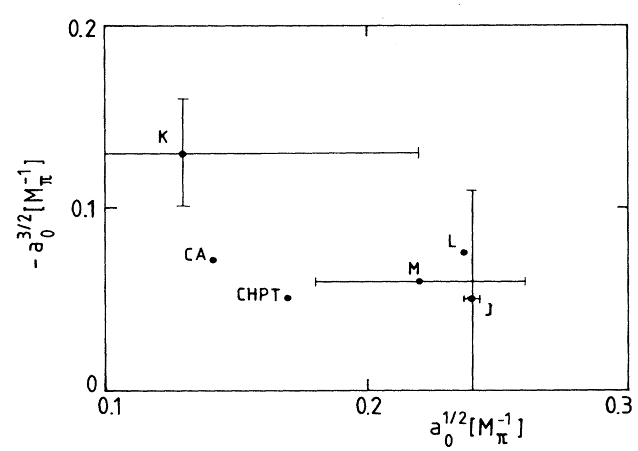

FIG. 1. Theoretical predictions and empirical data on the S-wave scattering lengths. We show the current-algebra point (CA) and the one-loop chiral-perturbation-theory (CHPT) result together with the data of Refs. 16 (M), 17 (L), 18 (J), and 19 (K).

There are many topics to be discussed in more detail.[11] First, since there are strong nonlinearities in πK scattering, a direct comparison with a dispersion theoretical analysis of the available πK data might be preferable. This is most easily done if one expands around the symmetry point $\nu=(s-u)/4M_K=0$, $t=0$. Also, since the minimal energy to be considered here is $(S_0)^{1/2}=M_\pi+M_K\approx633$ MeV one might question the validity of the one-loop approximation.[21] These theoretical problems can, however, be controlled. What is more urgent is to get more precise empirical information on the threshold parameters of πK scattering. This would be a crucial test of our understanding of the low-energy structure of QCD.

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