RAPID COMMUNICATIONS

PHYSICAL REVIEW D VOLUME 43, NUMBER 6 15 MARCH 1991

Non-mean-field exponents in strongly coupled quenched QED

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(Received 22 October 1990)

We study quenched quantum electrodynamics (QED) on a lattice in the noncompact formulation. Near the chiral critical point, we avoid critical slowing down using fast-Fourier-transform methods which allow us to obtain critical indices with good accuracy. Using lattices of 16^4 and 24^4 sites and masses between 0.025 and 0.0007 (lattice units) we found that the critical behavior of QED is described by power laws with critical exponents differing from mean-field results. The critical coupling is $\beta_c=0.257\pm0.001$, the exponent δ is 2.2 ± 0.1, and the magnetic exponent β_m is 0.78 ± 0.08. A physical explanation of our results is presented.

Recently, the study of quantum electrodynamics (QED) in the strong-coupling region has attracted considerable attention.[1–3] Lattice simulations of *noncompact* QED with almost massless fermions clearly showed the existence of a critical charge beyond which chiral symmetry is spontaneously broken. The phase transition is continuous (unlike the case of compact QED) opening the possibility of studying the continuum limit of this model near the chiral critical point. The issue of whether the cutoff of QED can be removed in strong coupling without finding the triviality problem that is believed to exist in weak coupling (Landau pole) is one of the main motivations for this analysis. Independent studies using the formalism of Schwinger-Dyson (SD) equations[4–6] in the quenched approximation suggested that interesting phenomena may take place near that critical point. Of particular importance is the possibility of finding *renormalizable* four-Fermi interactions[5] in its vicinity. This would be useful in technicolor models[7] providing an interesting physical application of the strong-coupling phase of QED. Other realizations may exist and have been considered by various authors.

While the SD equations approach involves the arbitrary truncation of Feynman diagrams to the quenched ladder subset, a lattice simulation considers all diagrams in a nonperturbative way and, thus, it can determine to what extent the SD results are accurate. Early exploratory lattice studies of QED showed that if the chiral condensate was extrapolated *linearly* to zero bare mass[8] (m), then the phase transition was described by the essential singularity[2] predicted by Miransky *et al.*[4] in the "collapse of the wave function" scenario. In addition to its economic advantage, a linear extrapolation is correct in the case when the order parameter develops an essential singularity. However, further analysis showed that the assumption of linearity of $\langle\bar{\psi}\psi\rangle$ with m is not correct near the critical point[9–11] for the lattices and masses previously analyzed numerically. Even more, recent additional studies of quenched lattice QED claimed[10] that if the chiral critical coupling is assumed to be $\beta_c=0.2478$, then equally compelling fits to *mean-field* behavior near the critical point could be made suggesting that QED has a trivial continuum limit similar to that of $\lambda\phi^4$ theories in four dimensions. This apparent ambiguity indicated that an accurate estimate of the critical coupling and an unbiased mass extrapolation were needed. The first step in this direction was done in Ref. 12.

However, it is important to note that away from a critical point a mean-field description of the data is usually accurate. This is a general result in critical phenomena and was observed in the first simulations of quenched lattice QED (Refs. 1 and 2) where a mean-field critical coupling $\beta_{\rm MF}\sim0.24$ was reported. The chiral condensate follows a mean-field behavior at strong coupling up to the immediate neighborhood of the critical point. Only in the vicinity of this critical point can genuine asymptotic scaling be observed and the issue of triviality be properly analyzed. The size of the scaling "window" is a dynamical question difficult to determine *a priori* and thus very extensive simulations are needed to fully understand the critical behavior of a theory. As discussed below, and previously in Ref. 12, we believe that noncompact lattice QED in its present form has a very small scaling window and thus it is necessary to work close to the critical coupling β_c and near the chiral limit $m=0$ to observe the actual continuum limit. It is the purpose of this Rapid Communication to provide a numerical study in this region of physical interest. We show below that quenched QED has *non*-mean-field behavior and, thus, it is a strong candidate for a nontrivial nonasymptotically free theory in four dimensions.

The action of noncompact QED on the lattice is defined as

$$S=-\frac{\beta}{2}\sum_p \theta_p^2+\tfrac{1}{2}\sum_{x,\mu}\eta_\mu(x)(\bar\psi_x e^{i\theta_\mu(x)}\psi_{x+\mu}-\bar\psi_{x+\mu}e^{-i\theta_\mu(x)}\psi_x)+m\sum_x\bar\psi_x\psi_x\ , \tag{1}$$

where the notation is standard.[1,2] The fermionic fields ψ_x are staggered fermions and $\eta_\mu(x)$ is the only remnant of the Dirac matrices. $\theta_\mu(x)$ is the gauge field and β is the inverse of the electromagnetic coupling e_0, i.e., $\beta=1/e_0^2$. In Fig. 1 we show $\langle\bar\psi\psi\rangle$ vs m for different values of the coupling constant β using lattices of 16^4 and 24^4 sites.[13] As a numerical technique we used an algorithm[11,14] where Gaussian fields are generated in momentum space and then transformed to coordinate space by a fast-Fourier-transform (FFT) subroutine (after an appropriate multiplication by the lattice photon propagator).[15] The algorithm avoids correlations between successive configurations and does not suffer from critical slowing down. Of course, this technique works only in the quenched approximation for noncompact actions. Using this method we generated enough independent gauge configurations (typically between 100 and 150) to obtain accurate results for masses as small as $m=0.0007$ (in lattice units) that were not reached before in lattice simulations with fermions (to the best of our knowledge). We have studied the chiral condensate $\langle\bar\psi\psi\rangle$ with the conjugate-gradient (CG) method using a set of random Gaussian numbers as source. For the smallest masses many thousands of CG iterations are needed to achieve convergence. As a criterion for convergence in the CG algorithm we require the parameter r (residual) to be $r_c<0.001\sqrt{V}$ (V is the number of sites) for $m=0.01$, checked explicitly that the results for the chiral condensate were accurate enough at that mass, and then rescaled r_c linearly with m for other masses. We checked in several cases at $m=0.001\,25$ that this criterion produces a very accurate condensate. By explicitly applying the Dirac operator upon the output of the CG subroutine we also checked that the inverse was being produced correctly.

Consider Fig. 1 which shows $\langle\bar\psi\psi\rangle$ vs m for various couplings β. The data at $\beta=0.250$, which extend down to $m=0.0007$, show clearly that $\langle\bar\psi\psi\rangle$ is nonzero in the massless limit and, contrary to claims in Ref. 10, $\beta\approx0.250$ is not the critical point. The data at $\beta=0.250$ cannot be fit by a mean-field power law $Am^{1/3}$. Our accurate small-mass results are crucial for this conclusion—if we only had CG data for $m\geq0.005$ it would be impossible to determine β_c well enough to rule out a mean-field ($\delta=3$) fit at $\beta=0.250$. Note that the 24^4 data at $\beta=0.250$ lie slightly above the results for the 16^4 lattice. Then, even the small finite-size effects in our simulation support the contention that $\beta=0.250$ lies within the chiral-symmetry-broken phase of the model. These conclusions also agree with our past Lanczos calculations of the eigenvalue spectrum of the Dirac operator. In fact, all the results discussed in this Rapid Communication agree with Ref. 12 but rely only on the CG algorithm. Since this technique is essential for the unquenched QED case, this paper lays the groundwork for quantitative simulations of full QED.

Other interesting features of Fig. 1 are the following. (i) For small masses a linear behavior of the condensate can be observed even very close to β_c. In the bulk limit we expect to find deviations from a linear behavior $\langle\bar\psi\psi\rangle\sim m$ near $m\approx0$ *only* at $\beta=\beta_c$. Then, the assumption of linearity of the condensate made before[1,2] was conceptually correct although not applied in the proper region of parameter space. (ii) Note also that there are no strong finite-size effects in the results. For example, at $\beta=0.250$ only for masses $m=0.001\,25$ and 0.0025 it is possible to distinguish between results for 16^4 and 24^4 lattices. This indicates that the correlation lengths typical of the problem are growing slowly with $\beta_c-\beta$ and only very close to β_c do they become comparable to the lattice size and satisfy asymptotic scaling. We present a physical explanation of this behavior below. In Fig. 1 we observe that for

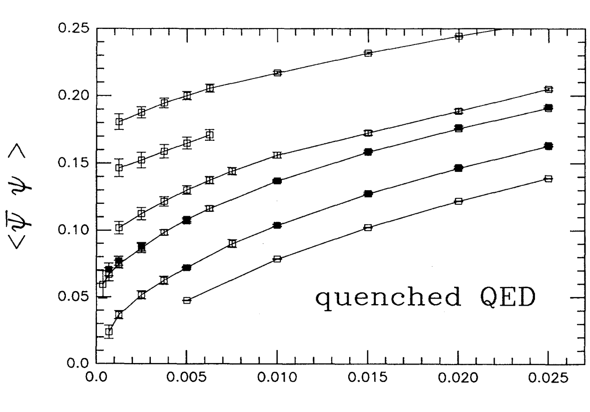

FIG. 1. $\langle\bar\psi\psi\rangle$ vs m at different values of the coupling constant β. □ (■) denote results using a 16^4 (24^4) lattice. The different sets of numerical data correspond to (starting from the top) $\beta=0.23$, 0.235, 0.245, 0.25, 0.26, and 0.27, respectively.

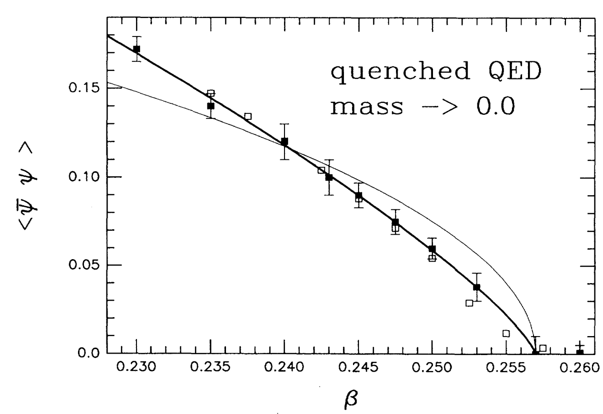

FIG. 2. $\langle\bar\psi\psi\rangle$ (■) extrapolated to zero bare mass vs β. The thick solid line is the best fit of the data $\langle\bar\psi\psi\rangle=2.88\times(0.257-\beta)^{0.78}$. The thin solid line is the best mean-field fit assuming a magnetic exponent $\beta_m=0.5$. □ denote Lanczos results (Ref. 19) from Ref. 12.

$\beta = 0.260$ the chiral condensate extrapolates to zero in the massless limit and thus we have an upper bound on the critical coupling, i.e., $0.250 < \beta_c \leq 0.260$.

The extrapolation to zero mass of the results shown in Fig. 1 can be done safely using (i) an eyeball extrapolation or (ii) a more sophisticated extrapolation using the equation of state of the model.[12,16] Both techniques give similar results. The extrapolated condensate is shown in Fig. 2 as a function of β. We estimate the critical coupling as $\beta_c = 0.257 \pm 0.001$. The thick solid line in Fig. 2 is the best least-squares fit of the form $\langle\bar{\psi}\psi\rangle = A(\beta_c - \beta)^{\beta_m}$ which defines the "magnetic" critical exponent β_m (not to be confused with the coupling constant). The fit is very accurate and from it we obtain $\beta_m = 0.78 \pm 0.08$ where the error bars come mainly from the uncertainty in the determination of β_c. The thin solid line of Fig. 2 is an attempt to fit the data assuming $\beta_m = 0.5$ (i.e., the mean-field exponent) showing that it does not describe the numerical results properly.

In Fig. 3 we present $\langle\bar{\psi}\psi\rangle$ vs m at $\beta_c = 0.257$. The continuous thick line denotes the best power-law fit of the numerical data using a standard least-squares subroutine assuming $\langle\bar{\psi}\psi\rangle = Am^{1/\delta}$. The fit is very good (it provides stable results in all the range of masses we studied) and the best value for δ is 2.2. Considering the error bars in the determination of β_c we conclude that $\delta = 2.2 \pm 0.1$. Note also in Fig. 3 that a mean-field attempt to fit the data fails. The crucial point in the proper calculation of critical exponents is the accurate determination of β_c. Once that is achieved (and very small masses are necessary for that) the critical exponent δ can be obtained accurately even using results for larger masses.

An interesting check of the numerical results presented in this paper consists in the search for universal "scaling" plots. As in any second-order phase transition, we expect that results obtained in the "vicinity" of the critical point of QED (i.e., inside the scaling window) will satisfy universal relations. In particular, we know that in the presence of a symmetry-breaking external field m, the chiral condensate and m are related by

$$\frac{\langle\bar{\psi}\psi\rangle}{m^{1/\delta}} = f\left[\frac{\Delta\beta}{\langle\bar{\psi}\psi\rangle^{1/\beta_m}}\right], \qquad (2)$$

where δ and β_m are the critical exponents, $\Delta\beta = (\beta_c - \beta)$, and f is a universal function.[17] In Fig. 4(a) we present results assuming $\beta_c = 0.257$, $\delta = 2.2$, and $\beta_m = 0.78$. They clearly show that numerical data obtained in a wide range of values of couplings and masses can be collected together in one single scaling plot if the above-mentioned parameters are assumed. For fixed β_c we observed that changing the critical exponents the good scaling behavior of the data disappears. At this point it is important to remark that in Fig. 4(a) we have *not* assumed any hyperscaling relation[12,16] among the critical exponents. If these relations are satisfied, then $\beta_m = \gamma/(\delta - 1)$, where γ is the susceptibility exponent which is predicted to equal one by the SD analysis.[12] From our numerical data where δ and β_m have been obtained *independently*, we find that this re-

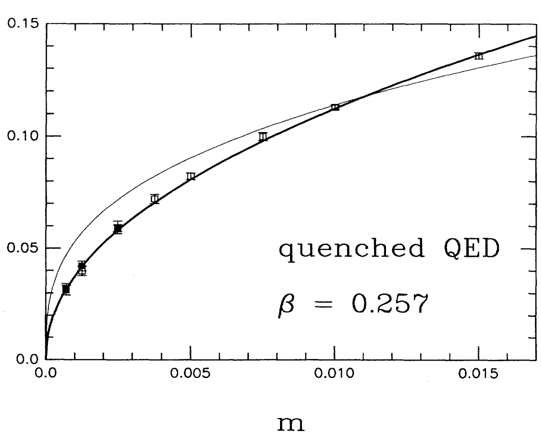

FIG. 3. $\langle\bar{\psi}\psi\rangle$ vs m at $\beta_c = 0.257$. □ (■) denote results using a 16^4 (24^4) lattice. The thick solid line corresponds to the best power-law fit of the numerical data using $\delta = 2.2$. The thin solid line denotes the best fit using the mean-field exponent $\delta = 3.0$.

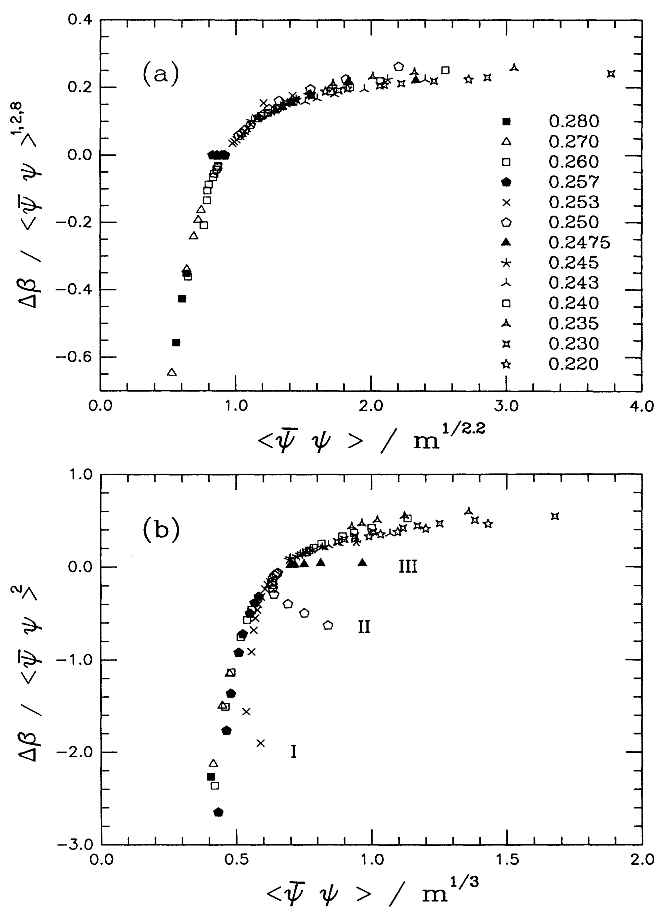

FIG. 4. (a) $\langle\bar{\psi}\psi\rangle/m^{1/\delta}$ vs $\Delta\beta/\langle\bar{\psi}\psi\rangle^{1/\beta_m}$ ($\delta = 2.2$, $\beta_m = 0.78$). The different symbols correspond to different values of the coupling constant β and for each one we plot the available masses in the interval $0.0007 \leq m \leq 0.025$. (b) $\langle\bar{\psi}\psi\rangle/m^{1/3}$ vs $\Delta\beta/\langle\bar{\psi}\psi\rangle^2$, i.e., similar to (a) but assuming mean-field exponents. The notation is the same as for (a). I denotes results for $\beta = 0.253$, II for $\beta = 0.250$, and III for $\beta = 0.2475$.

lation is correct within statistical errors. In fact, if the exponent β_m is assumed to be $\beta_m = 1/(2.2-1) = 0.833$, then the new universality plot is very close to that presented in Fig. 4(a). In Fig. 4(b) we show the same universality plot but now assuming mean-field exponents $\delta = 3.0$ and $\beta_m = 0.5$ and a critical coupling[10] $\beta_c = 0.2478$. Note that there are clear deviations from the universal curve for couplings close to the critical point and small masses. As emphasized before, this region is the *crucial* one to distinguish between mean-field and non-mean-field behavior.

After presenting this numerical evidence it is reasonable to ask why QED has a power-law behavior rather than the essential singularities of Miransky scaling. Consider the lattice photon propagator we use in the generation of configurations, i.e., $4 - \sum_\mu \cos(ak_\mu)$, where a is the lattice spacing. For small ak_μ we recover the correct continuum propagator, but for $ak_\mu \sim \pi$ the propagator becomes flat and lattice fermions exchange heavy modes of mass $\sim 1/a$. In the continuum formulation, high-momentum terms in the lattice action would be represented by four-Fermi interactions so the lattice theory critical behavior would have to be parametrized in the two-dimensional space consisting of a bare fine-structure constant α_0 and a bare four-Fermi term of strength G_0. SD studies of the physics in this plane have shown a fixed line of critical points extending from a Nambu–Jona-Lasinio point at $\alpha_0 = 0$, $G_0 = 4$ to a Bardeen-Leung-Love-Miransky point[4,5,18] at $\alpha_0 = \alpha_c$, $G_0 = 1$ ($\alpha_c = \pi/3$). The theory's critical indices and anomalous dimensions of composite fermionic operators vary continuously along the line.[12,16] Comparing the critical exponents we have found numerically with those calculated with the SD formalism[12,16] we conclude that our results correspond to the point $P = (\alpha_0, G_0) = (0.44\alpha_c, 3.06)$. In Ref. 12 it was found that the scaling window around P is very small in agreement with our numerical results. It is also important to remark that in addition to *explicit* contact interactions in the lattice action, we may have *dynamically* generated four-Fermi interactions in the low-energy effective theory of the model producing the same effect as described above.

Of course, we have not excluded the possibility that the SD equations results are incomplete and that, in fact, we have found an isolated fixed point of the theory rather than one particular point on a line of fixed points. This hypothesis can be verified if we obtain the same critical exponents in simulations using different forms of the lattice action (keeping the naive continuum limit unchanged).

What are the consequences of our study for unquenched simulations? We have found that in quenched simulations it is necessary to work very close to the critical point and with small masses to observe the actual critical behavior of the model. Introducing fermionic loops it is reasonable to assume that the scaling window will be even smaller due to screening effects. Then, simulations performed for masses larger than $m \geq 0.025$ cannot determine the critical behavior of QED with $N_f \neq 0$. If data for $m \geq 0.025$ are used then we expect that mean-field exponents will fit the numerical results but will *not* correspond to the asymptotic scaling regime.

Summarizing, we have presented numerical evidence that quenched QED on a lattice has non-mean-field critical exponents. Although Miransky scaling was not found, the correspondence of our results with SD analyses suggests that the dynamics is qualitatively similar to that originally proposed by Miransky and co-workers.[4,16] By varying the lattice action the Miransky point may yet be accessible to lattice simulations. To study this possibility very exact and quantitative work is required in the small-mass region near the chiral critical point of the theory.

E.D. thanks R. Sugar for useful discussions. This project was supported in part by NSF Grants No. PHY89-04035 and No. DE-FG02-85ER40213, and supplemented by funds from NASA. The computer simulations were done on the Cray-2 at the National Center for Supercomputing Applications (NCSA), Urbana, Illinois. We thank NCSA for their support.

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[8]Note that working on a finite lattice the exact massless limit can only be recovered by extrapolating results obtained at finite (but small) masses (otherwise tunneling between different chiral sectors will make the chiral condensate average to zero).

[9]A. M. Horowitz, in *Lattice '88*, proceedings of the International Symposium, Batavia, Illinois, 1988, edited by A. S. Kronfeld and P. B. Mackenzie [Nucl. Phys. B (Proc. Suppl.) **9**, 403 (1989)]; S. P. Booth, R. D. Kenway, and B. J. Pendleton, Phys. Lett. B **228**, 115 (1989).

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[12]A. Kocić, S. Hands, J. B. Kogut, and E. Dagotto, Nucl. Phys. **B347**, 217 (1990).

[13]The numerical values of the condensate will be presented in an extended version of the paper.

[14]S. Hands, J. B. Kogut, and E. Dagotto, Nucl. Phys. **B333**, 551 (1990).

[15]In practice we need to fix a gauge (we use the Feynman

gauge) to avoid a trivial zero mode in the Gaussian matrix but we always concentrate on gauge-invariant quantities and thus our results below are gauge *independent*.

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[17]A more conventional definition of the equation of state in the presence of an external field (Ref. 12) is $m/\langle\bar{\psi}\psi\rangle^{\delta} = g(\Delta\beta/\langle\bar{\psi}\psi\rangle^{1/\beta_m})$, where g is a universal function. In this form it appears after minimizing the free energy. Inverting both members of this equation and raising to the power $1/\delta$ we obtain Eq. (2) with a new universal function f.

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[19]Also shown in Fig. 2 are Lanczos results from Ref. 12 obtained by extrapolating to 0 the density of eigenvalues of the Dirac operator assuming a power-law form $\rho(\lambda) = B\lambda^{1/\delta}$. The good agreement between Lanczos and CG results shows that this procedure is correct.