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# Two-loop bosonic contribution to the electron electric dipole moment

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We calculate the complete set of two-loop diagrams in the multi-Higgs-doublet model for the electron electric dipole moment including both the vertices $H\gamma\gamma$ and $HZ\gamma$ induced by the unphysical-charged-Higgs-boson and the $W$ contributions. These additional amplitudes modify the result previously studied by Barr and Zee.

Recently, Barr and Zee[1] pointed out there is a new class of two-loop Feynman diagrams [generically given by Fig. 1(a)] which can lead to a large electric dipole moment (EDM) of the charged leptons or light quarks due to the $CP$ violation in the neutral Higgs propagators.[2] One of the loops in this two-loop mechanism involves a heavy fermion, say the top quark, or the $W$ boson that couples to an external photon line. As a result of integrating out these heavy particles, the effective $H\gamma\gamma$ or $HZ\gamma$ vertices are induced at this first loop. However, the $W$-boson contribution to these effective vertices considered in Ref. 1 was not complete even if we limit ourselves to the scenario that the EDM of the electron is only due to the $CP$ violation in the neutral Higgs scalar and pseudoscalar mixings. We close the gap in this communication. In addition, their quantitative result was restricted to the model with only two Higgs doublets. We also generalize it to the case of an arbitrary number of Higgs doublets.

First of all, for the bosonic loop as the one in Fig. 1(b), a general argument of Ref. 3. shows that it cannot produce a $CP$-violating effective EDM for the $W$ boson and hence gives no contribution here. The argument can be generalized to an arbitrary number of loops to show that without using the fermion in the loop the induced $WW\gamma$ vertex cannot contribute to the EDM of any fermion in any gauge theory of $CP$ violation. This is because, without a fermion, one can find a discrete symmetry, which we shall call $V$ parity, such that it transforms all the gauge particles like ordinary parity $P$ but leaves the spinless particles invariant. $V$ parity forbids any $WW\gamma$ vertex which is $P$ odd. Therefore the only $CP$-violating $WW\gamma$ vertex that can be induced through bosonic loops has to be $P$ even and $C$ odd. To generate the EDM of fermion, the photon field in the $WW\gamma$ vertex has to be in the gauge-invariant form $F^{\mu\nu}$. One can show that, in this case, the $WW\gamma$ vertex is always $C$ even and no EDM of fermion can be induced. As a result, we need the scalar Higgs-boson coupling in the first loop and then the pseudoscalar Higgs-boson coupling to the electron line in the second loop so as to produce the scalar-pseudoscalar mixing which is $CP$ nonconserving.

The amplitudes for the effective $H\gamma\gamma$ and $HZ\gamma$ vertices due to the $W$ loop in the standard model are given in Ref. 4. The result has been confirmed by more than one group. We can easily translate their results into the

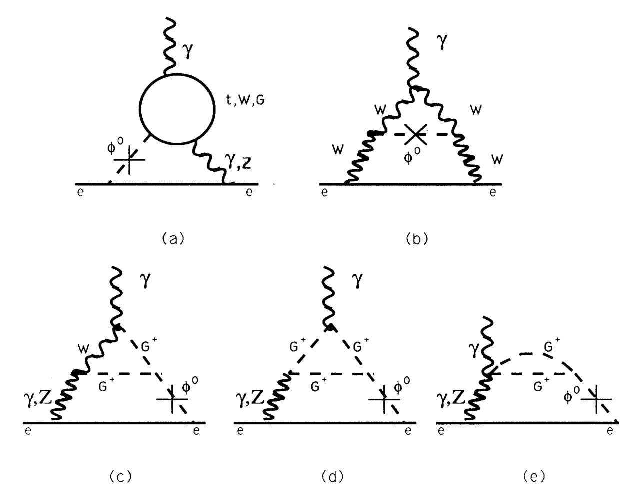


FIG. 1. Feynman diagrams for the EDM of the electron. The generic loop in (a) involves the $t$ quark, the $W$ boson and its ghost, or the charged unphysical Goldstone boson $G^{\pm}$. The contribution via $W$ EDM in (b) is zero. Amplitudes of diagrams (c), (d), and (e) depend on the Higgs-boson coupling to $G^{\pm}$.

case of the multi-Higgs-doublet models. To do the translation, one notes that the diagrams can be separated into two sets. The first set involves loops containing the $W$ boson or its ghost while the second set involves specifically a Higgs boson coupling to the unphysical charged Higgs boson $G^{\pm}$ associated with the $W$ boson as shown in Figs. 1(c)–1(e). In particular, since the amplitudes of Figs. 1(d) and 1(e) for $H\gamma\gamma$ or $HZ\gamma$ sum up to zero when contracted with the momentum of the corresponding gauge boson (i.e., a kind of gauge invariance), we group these two diagrams as the $G$-loop contribution. Then, Fig. 1(c) is combined with the first set as the $W$-loop contribution.

For multi-Higgs-doublet models, there are more than two $CP$ violating mixings[2]. They can be parametrized as

$$
\begin{aligned}
A_i &= \frac{1}{\lambda_i^2}\langle \phi_i^0 \phi_i^0 \rangle = -\sum_n \frac{\sqrt{2}G_F Z_i^n}{q^2 - M_{H_n}^2}\,, \\
A_{ij} &= \frac{1}{\lambda_i \lambda_j^*}\langle \phi_i^0 \phi_j^{0*} \rangle = -\sum_n \frac{\sqrt{2}G_F Z_{ij}^n}{q^2 - M_{H_n}^2}\,, \\
\tilde{A}_{ij} &= \frac{1}{\lambda_i \lambda_j}\langle \phi_i^0 \phi_j^0 \rangle = -\sum_n \frac{\sqrt{2}G_F \tilde{Z}_{ij}^n}{q^2 - M_{H_n}^2}\,,
\end{aligned}
\tag{1}
$$

where $\lambda_i = \langle \phi_i^0 \rangle$. The sums above do not include the contributions from the neutral Goldstone boson as it will not participate in the $CP$ violating amplitudes. Such exemption makes these definitions independent of the gauge parameter $\xi$ in the $R_\xi$ gauge. Note that $A_{ij}$ is Hermitian and $\tilde{A}_{ij}$ is symmetric. The unitarity gauge condition leads to the relations

$$
\mathrm{Im}\, A_i = \sum_{k\neq i}^{N} \left|\frac{\lambda_k}{\lambda_i}\right|^2 \left(\mathrm{Im}\, A_{ik} - \mathrm{Im}\, \tilde{A}_{ik}\right)\,. \tag{2}
$$

This equation is independent of the gauge parameter $\xi$. In particular, we have

$$
\mathrm{Im}\, A_1 = -|\lambda_1|^{-2} \sum_{k=2}^{N} |\lambda_k|^2 (\mathrm{Im}\, A_{k1} + \mathrm{Im}\, \tilde{A}_{k1})\,,
$$

where $N$ is the number of Higgs doublets. For $N = 2$ this equation was first obtained in Ref. 2. This is enough to translate the set of diagrams with bosonic loops which do not depend on the Higgs-boson mass in the couplings. For the $H\gamma\gamma$ case, one obtains the electron EDM

$$
(d_e/e)^{H\gamma\gamma}_{W\ \mathrm{loop}} = \frac{G_F m_e \alpha}{8\sqrt{2}\pi^3} \sum_n \eta_n \Big[\, 3f(z_{H_n}) + 5g(z_{H_n}) + \tfrac{3}{4}g(z_{H_n}) + \tfrac{3}{4}h(z_{H_n}) \Big]\,, \tag{3}
$$

where we closely follow the notation of Refs. 1 and 2 with $\eta_n = \Lambda^{-2}\sum_{k=2}^{N}|\lambda_k|^2 \mathrm{Im}\, Z_{k1}^n$, $\Lambda^{-2} = \sum_k |\lambda_k|^2$, and $z_{H_n} = M_W^2/M_{H_n}^2$. The function $h(z)$ is defined to be

$$
h(z) = \frac{z}{2}\int_0^1 \frac{dx}{z - x(1-x)} \times \left(1 + \frac{z}{z - x(1-x)} \ln \frac{x(1-x)}{z}\right)\,. \tag{4}
$$

We note that the first two terms in Eq. (3) agree with those in the paper of Barr and Zee.[1] For the $HZ\gamma$ case, one has

$$
(d_e/e)^{HZ\gamma}_{W\ \mathrm{loop}} = \frac{1 - 4\sin^2\theta_W}{4\sin^2\theta_W} \frac{G_F m_e \alpha}{8\sqrt{2}\pi^3} \sum_n \eta_n \Big[\, \tfrac{1}{2}(5 - \tan^2\theta_W)\tilde{f}(z_{H_n}, z_Z) + \tfrac{1}{2}(7 - 3\tan^2\theta_W)\tilde{g}(z_{H_n}, z_Z) + \tfrac{3}{4}g(z_{H_n}) + \tfrac{3}{4}h(z_{H_n})\Big]\,. \tag{5}
$$

Here $\tilde{f}(x,y) = yf(x)/(y-x) + xf(y)/(x-y)$ and similarly for $\tilde{g}$; $z_Z = M_W^2/M_Z^2$. Note that only the vector part of the $Z$-$\bar{e}$-$e$ vertex contributes to the $CP$ violating EDM operator and thus produces the suppression factor of $(1 - 4\sin^2\theta_W)$ in Eq. (5). If one, following Refs. 1 and 2, assumes that the lightest Higgs boson $H_0$ dominates and the other heavier Higgs boson can be neglected, then the numerical result due to the contributions from Eqs. (3)-(5) is shown in Fig. 2 with $\eta_0 = \frac{1}{2}$. The $W$-loop contribution of $HZ\gamma$ is about 10% of that of $H\gamma\gamma$ and they have the same sign.

In Eqs. (3)-(5), we have already used the coupling of the physical Higgs boson to the unphysical Higgs pair $G^+G^-$ as required in Fig. 1(c). The coupling can be shown to be

$$
\mathcal{L} = -\Lambda^{-2}\sum_{i,j}\lambda_i M_{ij}^2 G^+ G^- \phi_j^0 + \cdots\,, \tag{6}
$$

where $M_{ij}^2$ is the $N \times N$ submatrix of the neutral-Higgs-boson mass matrix associated with $\phi_i\phi_j$. Using this coupling, we also derive the following contribution from

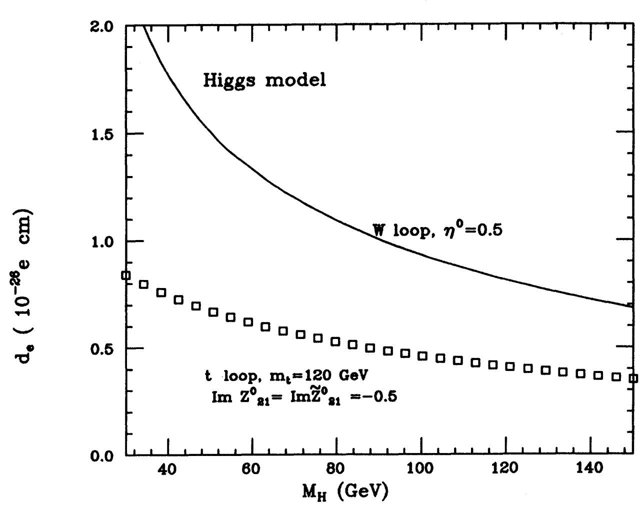


FIG. 2. Numerical estimate of the $d_e/e$ via the $W$ loop when $\eta_n = \frac{1}{2}$. The data points show the contribution due to the top-quark loop for the case $\mathrm{Im}Z_{21}^0 = \mathrm{Im}\tilde{Z}_{21}^0 = -\frac{1}{2}$ and $m_t = 120$ GeV.

purely the $G$ loops in Figs. 1(d) and 1(e):

$$(d_e/e)^{H\gamma\gamma}_{G\ \text{loop}} = \frac{G_F m_e \alpha}{16\sqrt{2}\pi^3} \sum_n \frac{\eta_n}{z_{H_n}} \left[ f(z_{H_n}) - g(z_{H_n}) \right] \tag{7}$$

and

$$\begin{aligned}(d_e/e)^{HZ\gamma}_{G\ \text{loop}} = {} & \frac{1-4\sin^2\theta_W}{8\sin^2\theta_W} \frac{G_F m_e \alpha}{16\sqrt{2}\pi^3} \\ & \times \sum_n \frac{\eta_n}{z_{H_n}} (1 - \tan^2\theta_W) \\ & \times \left[ \tilde{f}(z_{H_n}, z_Z) - \tilde{g}(z_{H_n}, z_Z) \right] . \end{aligned} \tag{8}$$

The amplitude for each Higgs boson increases logarithmically with the Higgs boson mass. In this case, the lightest-Higgs-boson contribution may no longer be the most important one. This makes a reliable estimate of this type of contribution difficult. However, the coefficients are small enough that these contributions may not be so significant as compared to the $W$-loop contribution discussed earlier except for the case of very heavy Higgs boson.

We can also generalize the Barr-Zee result[5] of the top-quark loop [in Eq. (1) of Ref. 1] to the case of more than two Higgs doublets. Through the $H\gamma\gamma$ vertex, we find

$$(d_e/e)^{H\gamma\gamma}_{t\ \text{loop}} = -\frac{G_F m_e \alpha}{6\sqrt{2}\pi^3} \sum_n \left( [f(x_{H_n}) + g(x_{H_n})] \text{Im}\, Z^n_{21} - [f(x_{H_n}) - g(x_{H_n})] \text{Im}\, \tilde{Z}^n_{21} \right) . \tag{9}$$

Through the $HZ\gamma$ vertex, we have

$$\begin{aligned}(d_e/e)^{HZ\gamma}_{t\ \text{loop}} = {} & -\frac{(1-4\sin^2\theta_W)(3-8\sin^2\theta_W)}{32\sin^2\theta_W \cos^2\theta_W} \frac{G_F m_e \alpha}{6\sqrt{2}\pi^3} \\ & \times \sum_n \left\{ [\tilde{f}(x_{H_n}, x_Z) + \tilde{g}(x_{H_n}, x_Z)] \text{Im}\, Z^n_{21} - [\tilde{f}(x_{H_n}, x_Z) - \tilde{g}(x_{H_n}, x_Z)] \text{Im}\, \tilde{Z}^n_{21} \right\} , \end{aligned} \tag{10}$$

with $x_{H_n,Z} = m_t^2/M^2_{H_n,Z}$. Numerically the contribution from the top quark is generally smaller than that from the $W$ boson. We demonstrate this point in Fig. 2 by choosing typical values of the $CP$ violating parameters $\text{Im}Z^0_{21} = \text{Im}\tilde{Z}^0_{21} = -\frac{1}{2}$ with $m_t = 120$ GeV. It is worth mentioning that the $t$-quark loop contribution involves linearly independent combination of $CP$ violating parameters, $\text{Im}Z^0_{k1}$, as compared to the $W$-loop or $G$-loop contributions. To conclude, with all the generalizations we did, the basic picture is still the same as pointed out by Barr and Zee. That is, the $W$ loop, ignoring the $G$-loop subset that involves the Higgs boson mass in the vertex, may still provide the majority of the contribution. The $G$-loop subset may become important when the Higgs-boson mass is very heavy; in that case, the Higgs sector may be strongly interacting and a reliable estimate is very difficult.

While finishing this manuscript, we became aware of a paper[6] which studied the same topics. Our result agrees with theirs for the special case of the two-doublet model. They have also shown other small contributions from additional two-loop diagrams not arising from the $H\gamma\gamma$ effective vertex. We also learned that J. Gunion and R. Vega have also done similar work. We thank Professor R. Oakes for bringing the first work[6] to our attention.

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[3] See, for example, R. N. Cahn, M. S. Chanowitz, and N. Fleishon, Phys. Lett. **82B**, 113 (1979); A. Barroso, J. Pulido, and J. C. Romão, Nucl. Phys. **B267**, 509 (1986); we use the result of this paper in the Feynman gauge.

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