Extended inflation from higher-dimensional theories

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We consider the possibility that higher-dimensional theories may, upon reduction to four dimensions, allow extended inflation to occur. We analyze two separate models. One is a very simple toy model consisting of higher-dimensional gravity coupled to a scalar field whose potential allows for a first-order phase transition. The other is a more sophisticated model incorporating the effects of nontrivial field configurations (monopole, Casimir, and fermion bilinear condensate effects) that yield a nontrivial potential for the radius of the internal space. We find that extended inflation *does not* occur in these models. We also find that the bubble nucleation rate in these theories is *time dependent* unlike the case in the original version of extended inflation.

I. INTRODUCTION

Inflation[1] has been widely accepted as the solution to the problems which plague the standard hot big-bang cosmology, namely, the homogeneity problem, the flatness problem, and the structure formation problem. Many realizations of the inflationary scenario have been discussed in the literature.[2] However, all these models themselves seem to give rise to new problems. For example, old inflation has the "graceful exit" problem,[3] while new inflation[4] and variants such as chaotic inflation[5] require fine-tuning of the microphysical parameters of the theory.

Extended inflation[6] has revived, in the context of Jordan-Brans-Dicke[7] gravity, the basic idea of old inflation, namely, that inflation is induced by a field configuration trapped in a metastable state from which it escapes via bubble nucleation. La and Steinhardt[6] showed that because the inflation that resulted from this theory was a power law rather than an exponential, the true vacuum phase could indeed percolate, and thus the graceful exit problem of old inflation could be evaded. Unfortunately, the euphoria of finding such an interesting way of implementing inflation was short lived. Weinberg[8] and La, Steinhardt, and Bertschinger[9] found that the requirements that the energy in the bubble walls be thermalized before any cosmologically sensitive times such as nucleosynthesis or recombination and that a global Robertson-Walker frame be reestablished in the bubble cluster that will become the observable Universe require an *upper* bound on the Brans-Dicke parameter ω. This in itself is not a surprise, since we know that in the $\omega \rightarrow \infty$ limit, Jordan-Brans-Dicke inflation becomes old

inflation. The problem occurs with the actual value of the upper bound. Weinberg shows that this bound is of order 20. However, the experimental lower bound is $\omega > 500$.[10] Thus, if we wish to make use of extended inflation, we must find ways of avoiding this problem. There have already been some attempts in this direction;[11,12] however, it may be that more natural ways of implementing extended inflation exist, and these should be searched for.

One such possibility for successful extended inflation might be multidimensional theories such as Kaluza-Klein[13] theories. After all, the major motivation for the renewed interest in scalar-tensor theories such as the Jordan-Brans-Dicke theory is that an effective low-energy theory of the Jordan-Brans-Dicke form follows naturally in superstring, supergravity, and Kaluza-Klein theories.

Upon reduction to four dimensions, theories originally formulated in higher dimensions take on a Jordan-Brans-Dicke form, with a function of the scale factor of the internal dimensions, $b(t)$, acting as the Jordan-Brans-Dicke field. Thus, it is important to investigate whether these theories can lead to successful models for extended inflation. The aim of the work is to investigate this possibility in some detail via the construction of some Kaluza-Klein models which, we believe, contain all the relevant physics.

The outline of this paper is as follows. Section II contains a description of the time dependence of the bubble nucleation rate, together with a discussion of the percolation problem for a class of models that generalizes the original extended inflation model. In Sec. III we introduce our first model, consisting of higher-dimensional gravity coupled to a scalar field whose potential allows

for a phase transition to occur via bubble nucleation. This theory is then dimensionally reduced, and the inflationary properties of this theory are investigated. In Sec. IV we consider more complicated models, leading to a stabilized potential for the radius of the internal dimensions obtained by means of nontrivial field configurations, such as monopoles, fermion bilinear condensates, or the Casimir effect. Section V contains a summary of this work.

We were not able to successfully implement extended inflation in the models we examined, and our conclusions are that it is very difficult for Kaluza-Klein theories to implement the extended inflation scenario. The problem stems from the fact that the potential for the internal radius $b(t)$ does not allow for enough inflation to occur before $b(t)$ reaches its minimum value.

Before launching into our calculations, we must establish notation. We will assume that there are $N = 4 + D$ dimensions. Quantities in the multidimensional theory will be denoted by tildes ($\widetilde{R}$, $\widetilde{G}$, $\widetilde{g}_{\mu\nu}$, etc.). In the dimensionally reduced four-dimensional world, we will work in either of two conformal frames: the Jordan conformal frame or the Einstein conformal frame. (The definition of these two frames is given in the next section.) Quantities in the Einstein conformal frame will be denoted by overbars ($\overline{R}$, $\overline{G}$, $\overline{g}_{\mu\nu}$, etc.), while quantities in the Jordan conformal frame will not carry any special decoration.

II. BUBBLE NUCLEATION AND PERCOLATION IN GENERALIZED EXTENDED INFLATION

Our first task is to understand how to calculate the bubble nucleation rate in the generalizations of the original extended inflation scenario we shall encounter below. The problem here is that the formalisms developed to perform these calculations (i.e., the Euclidean bounce method of Coleman and Callan[14] and its generalization by Coleman and DeLuccia to include classical gravity[15]) are not immediately applicable to the problem at hand.

The main difference between standard vacuum tunneling calculations and what is required in extended inflation models is due to the time evolution of the Jordan-Brans-Dicke (JBD) field Φ and its nontrivial couplings to the inflaton. The main effect of time evolution is to make the false-vacuum "roll" during the bounce. Some work trying to understand how to generalize the above-mentioned formalisms to this case was done by Accetta and Romanelli,[16] with some success. In this paper, however, we will use a method developed by us[17] which allows us to systematically "freeze-out" gravitational effects in the bounce.

Consider the following generic JBD action coupled to an inflaton scalar field:

$$S = \int d^4x \sqrt{-g} \left[-\Phi R + \omega g^{\mu\nu} \frac{\partial_\mu \Phi \, \partial_\nu \Phi}{\Phi} \right.$$
$$+ (16\pi G_N \Phi)^n \frac{1}{2} g^{\mu\nu} \partial_\mu \sigma \, \partial_\nu \sigma$$
$$\left. - (16\pi G_N \Phi)^m V(\sigma) \right] . \qquad (2.1)$$

Here we have written the action in the *Jordan conformal frame*, where Newton's constant G_N is replaced by the JBD field Φ in the curvature term. The field σ is the inflaton field, and its potential has a metastable (false-vacuum) minimum at $\sigma = \sigma_{FV}$.

Although the Jordan conformal frame is often useful to calculate the classical equations of motion, it may not be the appropriate frame for semiclassical calculations. The problem is that the second term in Eq. (2.1) is not the complete kinetic term for Φ, since an integration by parts of the first term in Eq. (2.1) will make a contribution to the Φ kinetic term. For semiclassical calculations involving the JBD field, it is more appropriate (and often easier) to use the *Einstein conformal frame*, where the gravitational couplings are the standard ones, and then transform back to the Jordan frame. The transformation to the Einstein frame is accomplished with a redefinition of the metric tensor via the conformal transformation

$$g_{\mu\nu} = \Omega^2 \overline{g}_{\mu\nu} ,$$
$$\sqrt{-g} = \Omega^4 \sqrt{-\overline{g}} , \qquad (2.2)$$
$$R = \Omega^{-2} \overline{R} - 6\Omega^{-3} \overline{\Box} \Omega ,$$

where $\Omega^{-2} = 16\pi G_N \Phi$. Defining a new field ψ via the relation

$$\psi \equiv \psi_0 \ln(16\pi G_N \Phi) , \qquad (2.3)$$

with $\psi_0^2 \equiv (3 + 2\omega)/16\pi G_N$, the action of Eq. (2.1) in the Einstein frame is

$$\overline{S} = \int d^4x \sqrt{-\overline{g}} \left[-\frac{\overline{R}}{16\pi G_N} + \frac{1}{2} \overline{g}^{\mu\nu} \partial_\mu \psi \, \partial_\nu \psi \right.$$
$$+ \exp\left[\frac{(n-1)\psi}{\psi_0} \right] \frac{1}{2} \overline{g}^{\mu\nu} \partial_\mu \sigma \, \partial_\nu \sigma$$
$$\left. - \exp\left[\frac{(m-2)\psi}{\psi_0} \right] V(\sigma) \right] . \qquad (2.4)$$

All metric quantities (such as the curvature scalar, etc.) in the Einstein frame will have bars over them to distinguish them from their counterparts in the Jordan conformal frame.

From the action written as above in the Einstein frame, we see that if we want to freeze-out gravitational effects, we must also freeze-out the evolution of the ψ field during the bounce. This is due to the fact that we are taking the $G_N \to 0$ limit and $\psi_0 \propto G_N^{-1/2}$. Corrections to this approximation can also be considered. We expect, using the results of bubble nucleation calculations in standard gravity as a guide, that our approximation will be reliable when the effective Planck mass induced by the JBD field is much greater than the mass scales associated with the σ field. In theories where Φ increases with time, the approximation will work best at late times. We point out that in our first toy model the Φ field rolls to zero (which is one of the reasons why this is a toy model), and so this approximation will break down at late times for this model.

Ignoring gravity and treating ψ as constant during the

bounce yields the following truncated *Euclidean* action for the inflaton σ:

$$\overline{S}_E = \int d^4x \left[\xi^{n-1} \tfrac{1}{2} \partial^\mu \sigma \, \partial_\mu \sigma + \xi^{m-2} V(\sigma) \right] , \qquad (2.5)$$

where $\xi \equiv \exp(\psi/\psi_0)$.

The calculation of the rate of bubble nucleation

$$\lambda = A e^{-B} \qquad (2.6)$$

(λ is the nucleation per time per three volume) involves calculation of the bounce action B and evaluation of the prefactor A.

Let us first consider the bounce action in our truncated action of Eq. (2.5). The bounce action $B = \overline{S}_E(\sigma_B)$ is found by a simple two-step process. First, since ξ is constant in the truncated action, $\overline{S}_E$ can be written as

$$\overline{S}_E = \xi^{2n-m} \int d^4x \left[\xi^{m-n-1} \tfrac{1}{2} \partial^\mu \sigma \, \partial_\mu \sigma + \xi^{2(m-n-1)} V(\sigma) \right] . \qquad (2.7)$$

Now let us rescale the coordinates in Eq. (2.7) to $\hat{x}^\alpha = \xi^{(m-n-1)/2} x^\alpha$. In terms of these coordinates, the action takes the form

$$\hat{S}_E = \xi^{2n-m} \int d^4\hat{x} \left[\tfrac{1}{2} \hat{\partial}^\mu \sigma \, \hat{\partial}_\mu \sigma + V(\sigma) \right] . \qquad (2.8)$$

Evaluating this action for the bounce solution gives the bounce action

$$B(\xi) = \xi^{2n-m} B_0 \quad [\xi = \exp(\psi/\psi_0)] , \qquad (2.9)$$

where B_0 is the (ξ-independent) bounce action calculated for the theory with $\xi = 1$ ($\psi = 0$). Thus the coupling of ψ into the action of Eq. (2.5) leads to a bounce action that is a factor of $\exp[(2n-m)\psi/\psi_0]$ times the bounce action in the absence of the coupling (i.e., $\psi = 0$).

The second part of the calculation of the nucleation rate involves the determination of the prefactor. Recall that the full expression for the prefactor associated with the action of Eq. (2.8) is

$$\hat{A} = \left| \frac{\det'\{\xi^{2n-m}[-\hat{\Box} + V''(\sigma_B)]\}}{|\det\{\xi^{2n-m}[-\hat{\Box} + V''(\sigma_{\mathrm{FV}})]\}|} \right|^{-1/2}$$

$$\times \prod_\mu \left[\frac{\hat{C}_\mu}{2\pi} \right]^{1/2} . \qquad (2.10)$$

Here σ_{FV} is the false-vacuum configuration, σ_B is the bounce solution, and $\det'$ indicates that the functional determinant is to be evaluated in the subspace orthogonal to the four translational zero modes. The $\hat{C}_\mu$ are normalization factors of the zero modes of the operator $\xi^{2n-m}[-\hat{\Box} + V''(\sigma_B)]$, defined so that the properly normalized modes are $\hat{C}_\mu^{-1/2} \hat{\partial}_\mu \sigma_B$ ($\mu = 1, \ldots, 4$). Thus $\hat{C}_\mu = \int d^4\hat{x} (\hat{\partial}_\mu \sigma_B)^2$ (no sum over μ implied), and for an O(4)-symmetric bounce the $\hat{C}_\mu$ are all equal.

To calculate the ξ dependence of $\hat{A}$, we first note that since the bounce configuration satisfies $(\hat{\partial}_\mu \sigma_B)^2 = V(\sigma_B)$, $\hat{C}_\mu = B_0$ does not depend upon ξ. Furthermore, the eigenvalues of the operator $\xi^{2n-m}[-\hat{\Box} + V''(\sigma_B)]$ are ξ^{2n-m} times those of the operator evaluated at $\xi = 1$. This implies that since $\det'\{ \cdots \}$ contains the product of

four fewer eigenvalues than $\det\{ \cdots \}$ (i.e., $\det'\{ \cdots \} = \xi^{-4(2n-m)} \det\{ \cdots \}$) and the ratio of the determinants is taken to the $-\tfrac{1}{2}$ power, the part of $\hat{A}$ involving determinants is a factor of $\xi^{4(2n-m)/2}$ times the value with $\xi = 1$. Putting both these results together, we have

$$\hat{A} = \hat{A}_0 \xi^{4n-2m} , \qquad (2.11)$$

where $\hat{A}_0$ is the (ξ-independent) prefactor calculated for the theory with $\xi = 1$. Therefore, in terms of the rescaled coordinates, the tunneling rate is

$$\hat{\lambda} \equiv \frac{dP}{d^4\hat{x}} = \hat{A}_0 \xi^{4n-2m} \exp(B_0 \xi^{2n-m}) , \qquad (2.12)$$

where P is the bubble nucleation probabilty.

Our penultimate task is to transform the tunneling rate in the scaled coordinates ($\hat{x}$) to the Einstein frame. This is most easily accomplished:

$$\overline{\lambda} \equiv \frac{dP}{d^4x} = \hat{\lambda} \frac{d^4\hat{x}}{d^4x} = \hat{\lambda} \xi^{2(m-n-1)}$$

$$= A_0 \xi^{2n-2} \exp(-B_0 \xi^{2n-m})$$

$$= A_0 \exp[(2n-2)\psi/\psi_0] \exp\{-B_0 \exp[(2n-m)\psi/\psi_0]\} . \qquad (2.13)$$

Equation (2.13) can be used to obtain directly the tunneling rate in the Jordan frame. Recall that $\exp(\psi/\psi_0) = 16\pi G_N \Phi$, and the nucleation rate in the Jordan frame is related to that in the Einstein frame by

$$\lambda \equiv \frac{dP}{d^4x \sqrt{-g}} = \frac{dP}{d^4x \sqrt{-\overline{g}}} \frac{\sqrt{-\overline{g}}}{\sqrt{-g}} = \Omega^{-4} \overline{\lambda}$$

$$= (16\pi G_N \Phi)^2 A_0 (16\pi G_N \Phi)^{2n-2}$$

$$\times \exp[-B_0 (16\pi G_N \Phi)^{2n-m}]$$

$$= A_0 (16\pi G_N \Phi)^{2n} \exp[-B_0 (16\pi G_N \Phi)^{2n-m}] . \qquad (2.14)$$

A_0 and B_0 are Φ independent and depend only upon the inflaton potential. B_0 is dimensionless, while A_0 has mass dimension 4.

In the original extended inflaton model $m = n = 0$, and in the Jordan frame the nucleation rate is time independent, although it is time dependent in the Einstein frame (as discussed in Ref. 17). However, as we shall see, in dimensionally reduced theories the generic form has m and n different from zero. We see from the above equation that, if $2n - m \neq 0$, the time dependence of the nucleation rate can be *exponentially* strong through the time dependence of Φ (or equivalently, ψ). If $2n - m = 0$, but $n \neq 0$, the nucleation rate is still time dependent in the Jordan frame and time dependent in the Einstein frame if $n \neq 1$.

We emphasize that this calculation is an *approximation* to the actual result that should be good as long as gravitational corrections can be neglected. The correct calculation to do, of course, is one in which both the Brans-Dicke field *and* the inflaton are accounted for in the tun-

neling process. Unfortunately, this problem is still beyond our abilities to solve completely.

The nucleation rate is crucial in extended inflation, since the parameter that determines the percolation properties of the model is[3]

$$\epsilon(t) = \frac{\lambda(t)}{H^4(t)} , \qquad (2.15)$$

where $H(t)$ is the Hubble parameter of the model. This quantity essentially measures the number of bubbles nucleated within a Hubble volume $H^{-3}(t)$ in a Hubble time $H^{-1}(t)$. The graceful exit problem of old inflation can be phrased in terms of ϵ as follows: In order for inflation to occur long enough to be useful, we need $\epsilon \lesssim 4 \times 10^{-3}$, while in order for the Universe to be percolated by bubbles of true vacuum the condition $\epsilon \gtrsim \epsilon_{CR}$ must be satisfied, with ϵ_{CR} lying between 10^{-6} and 0.24. Whereas in old inflation both λ and H are constants, the beauty of extended inflation is that it allows for the possibility that one or both of these quantities can vary in time. Thus ϵ can start small enough to satisfy the inflation requirement and then grow to satisfy the percolation conditions.

III. HIGHER-DIMENSIONAL GRAVITY COUPLED TO A SCALAR FIELD

Here we consider our first model. It consists of higher-dimensional gravity coupled to a scalar field $\tilde{\chi}$ with a potential $\tilde{U}(\tilde{\chi})$ allowing for a metastable vacuum state as well as a completely stable one. The action for this theory can be written as

$$\tilde{S} = \int d^{4+D}w \sqrt{-\tilde{g}}$$
$$\times \left[-\frac{1}{16\pi\tilde{G}}\tilde{R} + \frac{1}{2}\tilde{g}^{MN}\partial_M\tilde{\chi}\partial_N\tilde{\chi} - \tilde{U}(\tilde{\chi}) \right] . \quad (3.1)$$

Here D is the dimension of the internal space (which we take to be a D sphere S^D), $\{w^M\}$ represents the full set of coordinates on the $(4+D)$-dimensional spacetime, and all the quantities with tildes refer to objects living in the full $(4+D)$-dimensional spacetime. We now assume that the spacetime line element $d\tilde{s}^2$ takes the form

$$d\tilde{s}^2 = dt^2 - a^2(t)d\Omega_3^2 - b^2(t)d\Omega_D^2 , \qquad (3.2)$$

where $d\Omega_3^2$ is the line element corresponding to a maximally symmetric three-space and $d\Omega_D^2$ is that of a unit D-sphere. Denoting by $\tilde{\chi}_0$ the zero mode of $\tilde{\chi}$ (i.e., the part of the harmonic expansion of $\tilde{\chi}$ which is independent of the coordinates $\{y^\alpha\}$ of the D-sphere),[18] we can rewrite $\tilde{S}$ as

$$\tilde{S} = \left[\int d^D y \sqrt{\gamma(y)} \right] S , \qquad (3.3)$$

where $\gamma_{mn}(y)$ is the metric tensor of the D-sphere and S is the effective four-dimensional action, given by

$$S = \int d^4x \sqrt{-g(x)}\Omega_D b^D(t)$$
$$\times \left[-\frac{R}{16\pi\tilde{G}} - \frac{D(D-1)}{16\pi\tilde{G}}g^{\mu\nu}\frac{\partial_\mu b \, \partial_\nu b}{b^2} \right.$$
$$\left. + \frac{\rho_D}{16\pi\tilde{G}b^2} + \frac{1}{2}g^{\mu\nu}\partial_\mu\tilde{\chi}_0\partial_\nu\tilde{\chi}_0 - \tilde{U}(\tilde{\chi}_0) \right] . $$

$$(3.4)$$

Here $\rho_D b^{-2}$ is the Ricci scalar of the internal space [i.e., constructed from $\gamma_{mn}(y)$ alone]. [ρ_D has the value $D(D-1)$ for a D-sphere.] Also, $\{\chi^\alpha\}$ are the coordinates in the four-dimensional space, and $g_{\mu\nu}(x)$ is the metric on this space. Note that the kinetic term for the b field has the wrong sign. We will now rewrite S using the definitions

$$\Omega_D \equiv \left[\int d^D y \sqrt{\gamma(y)} \right]$$
$$= 2\pi^{(D+1)/2}/\Gamma[(D+1)/2] \text{ for a } D\text{-sphere} ,$$

$$\frac{\Omega_D b_0^D}{16\pi\tilde{G}} \equiv \frac{1}{16\pi G_N} ,$$

$$\sigma \equiv (\Omega_D b_0^D)^{1/2}\tilde{\chi}_0 ,$$

$$V(\sigma) \equiv (\Omega_D b_0^D)\tilde{U}\left[\frac{\sigma}{(\Omega_D b_0^D)^{1/2}} \right] , \qquad (3.5)$$

where G_N is the four-dimensional Newton's constant. Note that σ is a canonical scalar field and $V(\sigma)$ is its four-dimensional potential. Finally, defining

$$\Phi \equiv \frac{1}{16\pi G_N}\left[\frac{b}{b_0} \right]^D \qquad (3.6)$$

to be the effective JBD field, we have

$$S = \int d^4x \sqrt{-g(x)}$$
$$\times \left[-\Phi R - \omega g^{\mu\nu}\frac{\partial_\mu\Phi \, \partial_\nu\Phi}{\Phi} + \alpha\Phi^{1-2/D} \right.$$
$$\left. + (16\pi G_N\Phi)\left[\frac{1}{2}g^{\mu\nu}\partial_\mu\sigma \, \partial_\nu\sigma - V(\sigma) \right] \right] , $$

$$(3.7)$$

with $\omega \equiv 1 - 1/D$ and $\alpha \equiv \rho_D(16\pi G_N)^{-2/D}b_0^{-2}$. Note that α has mass dimension $2(1+2/D)$. We have thus recast the Kaluza-Klein action into a JBD form as expressed in the Jordan frame. There, are, however, some important differences: (a) Φ has the "wrong" sign for a standard kinetic term, (b) there is a nontrivial self-interaction term for Φ, namely, $\alpha\Phi^{-1/2D}$, and (c) there are also Φ-σ cross terms.

For completeness we may also express the dimensionally reduced action in the Einstein frame via the conformal transformation $g_{\mu\nu} = (16\pi G_N\Phi)^{-1}\bar{g}_{\mu\nu}$ in terms of the field $\psi = \psi_0\ln(16\pi G_N\Phi)$:

$$\overline{S} = \int d^4x \sqrt{-\overline{g}} \left\{ -\frac{\overline{R}}{16\pi G_N} + \frac{1}{2}\overline{g}^{\mu\nu}\partial_\mu\psi\,\partial_\nu\psi \right.$$

$$+ \frac{\rho_D}{16\pi G_N b_0^2}\exp\left[-\left(1+\frac{2}{D}\right)\frac{\psi}{\psi_0}\right]$$

$$+ \frac{1}{2}\overline{g}^{\mu\nu}\partial_\mu\sigma\,\partial_\nu\sigma$$

$$\left. -\exp\left[-\frac{\psi}{\psi_0}\right]V(\sigma)\right\}, \qquad (3.8)$$

where here, because of the sign of ω in Eq. (3.7), $\psi_0^2 = (3-2\omega)/16\pi G_N$.

We may now use the action of Eq. (3.7) to arrive at the Friedmann-Robertson-Walker (FRW) equations for this system. Setting $\sigma = \sigma_{\rm FV}$, its value in the false vacuum, and defining $V(\sigma_{\rm FV}) \equiv \rho_V$ and $\Lambda \equiv 8\pi G_N\rho_V$, we have the following equations of motion in the flat-space $(k=0)$ limit:

$$\left[\frac{\dot{a}}{a}\right]^2 + \frac{\dot{a}}{a}\frac{\dot\Phi}{\Phi} = -\frac{1}{6}\omega\left[\frac{\dot\Phi}{\Phi}\right]^2 - \frac{1}{6}\alpha\Phi^{-2/D} + \frac{\Lambda}{3},$$
$$\qquad (3.9)$$
$$\ddot\Phi + 3\frac{\dot{a}}{a}\dot\Phi = -\alpha\Phi^{1-2/D} + \frac{2\Lambda}{1+2/D}\Phi.$$

In order to analyze this system, it proves convenient to rescale the variables and fields so as to obtain dimensionless quantities. Thus we define

$$\tau \equiv \sqrt{\Lambda}\,t, \quad a(t) \equiv a(0)A(\tau), \quad \Phi(t) \equiv \Phi(0)P(\tau), \qquad (3.10)$$

where $\Phi(0)$ is arbitrary. If we also define $z \equiv (\alpha/\Lambda)\Phi(0)^{-2/D}$ and $C(\tau) \equiv z^{-D/2}P(\tau) \equiv (\alpha/\Lambda)^{-2/D}\Phi(t)$, then our equations become

$$C'' + 3\frac{A'}{A}C' = -\frac{\partial W}{\partial C},$$
$$\qquad (3.11)$$
$$\left[\frac{A'}{A}\right]^2 + \frac{A'}{A}\frac{C'}{C} = -\frac{1}{6}\omega\left[\frac{C'}{C}\right]^2 - \frac{1}{6}C^{-2/D} + \frac{1}{3},$$

where primes denote τ derivatives and the "potential" $W(C)$ is given by

$$W(C) = \frac{1}{2(1-1/D)}C^{2(1-1/D)} - \frac{1}{1+2/D}C^2. \qquad (3.12)$$

We have assumed that $D \neq 1$ here; we will make this assumption throughout. Figure 1 shows the scalar potential $W(C)$ for the special case $D=6$. Note that for C greater than the value corresponding to the maximum of the potential, defined as C_0, $W(C)$ is unbounded from below.

In general, no power-law solutions to our equations exist, in contradistinction with the original extended inflation scenario. However, since the equation for the rescaled JBD field is just that for the scalar field C in a potential $W(C)$, the usual techniques for analyzing this situation can be applied. It can be seen immediately that an exact solution exists when we set the field C equal to its value at the maximum of the potential, $C_0 = [(D+2)/2D]^{D/2}$. Here the scale factor expands exponentially, $A(\tau) = \exp(H_0\tau)$, with $H_0^2 = 2/3(D+2)$.
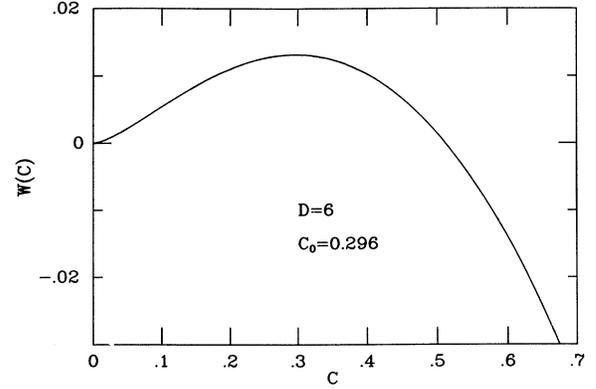

FIG. 1. Graph of the potential $W(C)$ of Eq. (3.12) for $D=6$.

We can perform an analysis of what happens if the C field is placed, not at the exact maximum, but near it. Thus we write

$$C(\tau) = C_0[1-\Delta(\tau)],$$
$$A(\tau) = \exp(H_0\tau)[1+\delta(\tau)]$$
$$\qquad (3.13)$$

[where $\Delta(\tau) > 0$ for $\tau > 0$], and insert these expressions into our equations for A and C. Keeping only those terms linear in the perturbation Δ and δ, we find the following equations for these quantities:

$$\Delta'' = -3H_0\Delta' + \frac{4}{D+2}\Delta,$$
$$\qquad (3.14)$$
$$\delta' + H_0\delta = \frac{1}{2}\Delta' - \frac{1}{3H_0(D+2)}\Delta.$$

These equations are easy to solve and yield exponentially growing and decaying modes $\exp(r_+\tau)$ and $\exp(r_-\tau)$, respectively, where $r_\pm \equiv H_0[(-3\pm\sqrt{33})/2]$. From this we see that we only count on the exponential expansion for $\Delta\tau \sim r_+^{-1}$ or $H_0\Delta\tau \sim 0.729$. But, in order to get all of the required 65 e-folds of inflation during this time, we must require that $H_0\Delta\tau > 65$. We thus see that there is never enough inflation before C is driven to zero. This conclusion is independent of the value of D and cannot be saved by any amount of fine-tuning.

Although this is not a promising model for inflation (extended or otherwise), we can learn some things to guide our thinking for the construction of a successful model. First, a potential of the form of Fig. 1 will not work. There must be a long, flat region in the potential so enough e-folds of inflation may occur before the JBD field is driven to its minimum. Second, the potential is sick for small Φ—there is nothing to prevent the extra dimensions from shrinking to zero—the minimum of Φ is at $\Phi=0$. Both of these problems have been previously recognized, and several solutions have been proposed. These solutions will be discussed in the following section.

Before proceeding onto a more promising model, we can ask if the first-order phase transition will be completed before Φ evolves to its minimum. In order to do that, we must calculate $\lambda(t)$ and $H(t)$, and determine $\epsilon(t) = \lambda(t)/H^4(t)$.
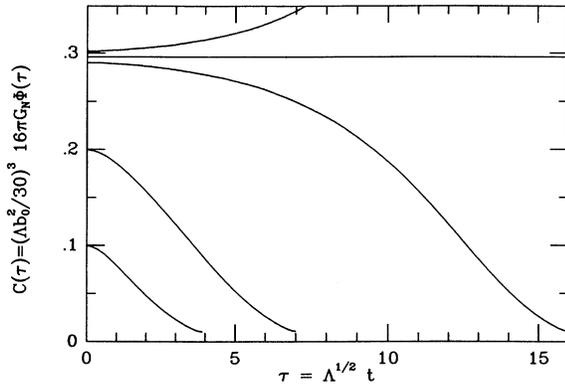
FIG. 2. Evolution of the Jordan-Brans-Dicke field as a function of time for the model of Sec. II for $D = 6$ for various initial conditions for the field. Note that $C = 0.296$, denoted by the straight line, corresponds to the maximum of the potential of Fig. 1. For initial values larger than this value, the field grows without limit and, hence, are not physically allowed.

We may calculate the nucleation rate in this model following the analysis in Sec. II. We see from a comparison of Eqs. (2.1) and (3.7) that the parameters m and n in Eq. (2.14) are both unity, so that the nucleation rate is given by

$$\lambda = A_0 (16\pi G_N \Phi)^2 \exp(-B_0 16\pi G_N \Phi) . \tag{3.15}$$

In the above equation, B_0 is the bounce action calculated from $V(\sigma)$, and $P = \Phi/\Phi(0)$ is the dimensionless JBD field.

Plots of $C(\tau)$, $H(\tau)$, and $\epsilon(\tau)$ for $D = 6$ ($C_0 = 0.296$) are given in Figs. 2–4, respectively, for the numerical solutions to the equations of motion. Figures 2 and 3 show $C(\tau)$ and $H(\tau)$ for the initial conditions $C(0) = 0.302$, 0.29, 0.2, and 0.1. Of course, if $C(0) > C_0$, the field will grow without limit. For $C(0)$ exactly equal to C_0, the field is classically static, but unstable to small
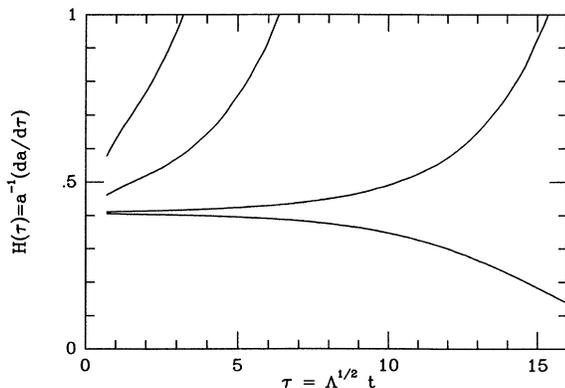

FIG. 3. Evolution of the Hubble expansion rate for the different initial conditions of Fig. 2. From top to bottom, the initial conditions are $C(0) = 0.1$, 0.2, 0.29, and 0.302. Note that for the physically allowed solutions, H increases in time.
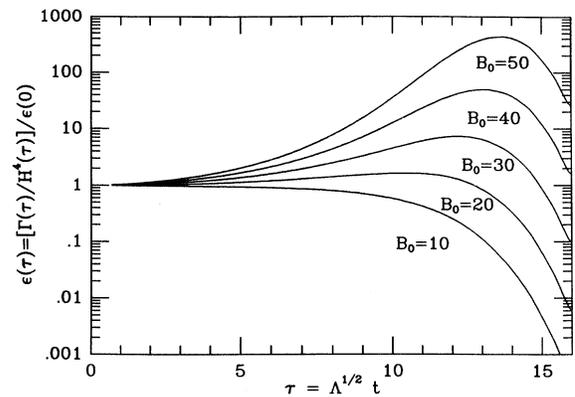

FIG. 4. Evolution of the efficiency of nucleation, $\epsilon = \lambda/H^4$ as a function of time for the initial condition $C(0) = 0.29$ for various values of B_0 [see Eq. (3.15)].

perturbations. Note that even for $C(0) = 0.29$ (very near C_0), insufficient inflation occurs before $C(\tau)$ is driven to zero. Figure 3 shows the evolution in time of the expansion rate H. Note that it increases in time, which is opposite to other extended inflation models. Of course, as we have emphasized, the relevant parameter is not H, but rather $\epsilon = \lambda/H^4$. In order for extended inflation to work, ϵ must increase in time. Figure 4 shows the evolution of $\epsilon(\tau)$ for the specific case $C(0) = 0.29$ for various values of B_0. It can be seen that even though H increases with time, ϵ can in fact increase with time because of the time dependence of λ. Hence it is in principle possible to have successful models of extended inflation, even if H increases with time.

IV. STABILIZED POTENTIAL MODELS

The model considered in the previous section, while simple, suffered from two fatal flaws. First, the potential for the internal radius only had a minimum at *zero*. Clearly, some additional effect(s) must stabilize the internal space. Proposals have included models where an N-dimensional cosmological constant, together with the curvature stress from the compact internal space, is balanced against a stress either due to a classical background field in the internal space[19] or due to the Casimir effect of fields on the curved internal space.[20] We will shortly study such a model and discover that it does not mitigate the second flaw in our model. This flaw can be seen in the potential for the scalar field illustrated in Fig. 1. Even if one imagines that somehow a minimum is developed before $C = 0$, the potential is unstable in the sense that for large initial values of C the field will grow without limit. Although this is not necessarily a fatal flaw for constructing a sensible particle physics model, we will see that it does not allow for enough inflation. Therefore, for extended inflation to have a chance to work, we must find models with a minimum at $C \neq 0$ which will allow enough inflation to occur before C classically evolves to the ground state.

Before embarking upon the analysis of the baroque

model discussed below, it is useful to study a trivial extension of the model of the previous section that results in a minimum away from $\Phi = 0$. This will illustrate some computational procedures we will use and demonstrate the problem of insufficient inflation we will encounter. The extension involves considering a six-dimensional model that contains a Maxwell field, in addition to the fields in the action of Eq. (3.1).[19] Anticipating that the only dynamical role the multidimensional scalar field $\widetilde{\chi}$ plays in the equations of motion is through its contribution to the cosmological constant, the action for our new model is most simply expressed as

$$\widetilde{S}_M = -\int d^6 w \sqrt{-\widetilde{g}} \left[\frac{1}{16\pi\widetilde{G}} \widetilde{R} + \frac{\Lambda}{8\pi\widetilde{G}} \right.$$
$$\left. - \frac{1}{4} \widetilde{g}^{MP} \widetilde{g}^{NQ} \widetilde{F}_{PQ} \widetilde{F}_{MN} \right] . \quad (4.1)$$

Having already established that upon dimensional reduction the theory will resemble a JBD model and learned the prescription for identifying the JBD field with the radius of the internal space, we may now proceed to find the equations of motion directly from Einstein's equation:

$$\widetilde{R}_{MN} = 8\pi\widetilde{G} \left[T_{MN} - \frac{1}{D+2} \widetilde{g}_{MN} T_p^p \right.$$
$$\left. - \frac{2}{D+2} \frac{\Lambda}{8\pi\widetilde{G}} \widetilde{g}_{MN} \right] . \quad (4.2)$$

With the choice of $R^1 \times R^3 \times S^2$ for the symmetry of the vacuum state, the relevant components of the Ricci tensor in the six-dimensional model are (m,n are indices in the internal space and i,j are indices in the external space)

$$R_{00} = -3\frac{\ddot{a}}{a} - 2\frac{\ddot{b}}{b} ,$$

$$R_{ij} = -\left[\frac{\ddot{a}}{a} + 2\frac{\dot{a}^2}{a^2} + 2\frac{\dot{a}}{a}\frac{\dot{b}}{b} \right] g_{ij} , \quad (4.3)$$

$$R_{mn} = -\left[\frac{\ddot{b}}{b} + \frac{\dot{b}^2}{b^2} + 3\frac{\dot{a}}{a}\frac{\dot{b}}{b} + \frac{1}{b^2} \right] g_{mn} .$$

With the choice of the metric in the form of Eq. (3.2), the stress tensor may be expressed in terms of an energy density ρ, an external pressure p and an internal pressure $p^{(2)}$: $T_N^M = \text{diag}(\rho, -p, -p, -p, -p^{(2)}, -p^{(2)})$.

For the Maxwell field we take the "Freund-Rubin" ansatz[19,21] $\widetilde{F}_{MN} = \sqrt{\gamma}\epsilon_{MN}f(t)$ for M,N in the internal space and zero otherwise. This choice automatically satisfies the field equation for $\widetilde{F}_{MN}$, and the Bianchi identities can be used to relate $f(t)$ to the radius of the internal space b: $f(t) = f_0/b^2(t)$, where f_0 is a constant.

For the Maxwell field, $T_{MN} = \widetilde{F}_{MQ}\widetilde{F}_N^Q$ $-(1/4)\widetilde{g}_{MN}\widetilde{F}_{PQ}\widetilde{F}^{PQ}$, and the equations of motion for a and b are

$$3\frac{\ddot{a}}{a} + 2\frac{\ddot{b}}{b} = -2\pi\widetilde{G}\frac{f_0^2}{b^4} + \frac{\Lambda}{2} ,$$

$$\frac{\ddot{a}}{a} + 2\frac{\dot{a}^2}{a^2} + 2\frac{\dot{a}}{a}\frac{\dot{b}}{b} = -2\pi\widetilde{G}\frac{f_0^2}{b^4} + \frac{\Lambda}{2} , \quad (4.4)$$

$$\frac{\ddot{b}}{b} + \frac{\dot{b}^2}{b^2} + 3\frac{\dot{a}}{a}\frac{\dot{b}}{b} = 6\pi\widetilde{G}\frac{f_0^2}{b^4} + \frac{\Lambda}{2} - \frac{1}{b^2} .$$

We can find the static value of b, defined as b_0, directly by setting the right-hand side (RHS) of Eq. (4.4) equal to zero. In doing so we find $8\pi\widetilde{G}f_0^2 = b_0^2$ and $\Lambda = 1/2b_0^2$.

We may now make direct connection with the exercise of the previous section by taking linear combinations of Eqs. (4.4) and recalling the definitions of Φ, ω, and α from the previous section [cf. Eq. (3.9)]:

$$\left[\frac{\dot{a}}{a} \right]^2 + \frac{\dot{a}}{a}\frac{\dot{\Phi}}{\Phi} = -\frac{1}{6}\omega \left[\frac{\dot{\Phi}}{\Phi} \right]^2 - \frac{1}{6}\alpha\Phi^{-2/D} + \frac{\Lambda}{3}$$
$$+ \frac{\alpha}{12}\Phi^{-1}(16\pi G_N \Phi)^{-1} ,$$

$$\ddot{\Phi} + 3\frac{\dot{a}}{a}\dot{\Phi} = -\alpha\Phi^{1-2/D} + \frac{2\Lambda}{1+2/D}\Phi$$
$$+ \frac{3\alpha}{4}(16\pi G_N \Phi)^{-1} . \quad (4.5)$$

The effect of the Maxwell field is completely contained in the last terms of Eqs. (4.5). This new term has little effect upon the evolution of the system for $16\pi G_N \Phi \gg 1$. For instance, if we form the dimensionless potential $W(C)$ as in Eq. (3.12), we find

$$W(C) = C - \frac{1}{2}C^2 - \frac{3}{16}\ln(C) , \quad (4.6)$$

where the last term from the Maxwell field has little effect at large C where inflation occurs. This potential is shown in Fig. 5. Clearly, it is similar to the potential of Fig. 1 except for the fact that there is a (local) minimum at $C = \frac{1}{4}$. The system will evolve to the ground state without sufficient inflation as the simple model of the previous section. The problem is that at large C (large Φ) the potential becomes negative.
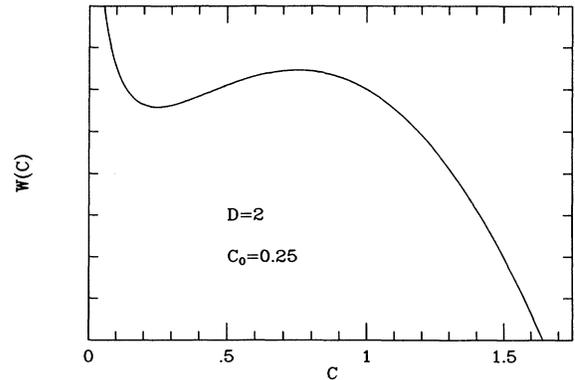

FIG. 5. Graph of the potential $W(C)$ of Eq. (4.6).

Next, we examine a model that has been proposed to lead to both a static ground state and yields a potential that does not turn over for large Φ.[22] The models starts with the (bosonic part of the) ten-dimensional Chapline-Manton action.[23] This action is believed to have many of the features expected in superstring models. A crucial difference between this model and the one just discussed will be the role of the dilaton field.

Taking G_{MN} as the Yang-Mills field strength, H_{MNP} as defined in terms of the Kalb-Ramond and the Yang-Mills and Lorentz Chern-Simons three-forms, and χ and λ as the gluino and subgravitino fields, respectively, we can write the bosonic part of the ten-dimensional action as (setting $8\pi\tilde{G}=1$)

$$\tilde{S}=-\tfrac{1}{2}\int d^{10}w\sqrt{-\tilde{g}^{(10)}(w)}\{-\tilde{R}+\tfrac{3}{2}\exp(-\sigma)[H_{MNP}-\exp(\sigma/2)(\mathrm{Tr}\bar{\chi}\Gamma_{MNP}\chi)]^2$$
$$+\tfrac{1}{2}\exp(-\sigma/2)(\mathrm{Tr}G_{MN}G^{MN})+\tfrac{1}{2}\partial_M\sigma\,\partial^M\sigma+(\mathrm{Tr}\bar{\chi}\Gamma_{MNP}\chi)(\bar{\lambda}\Gamma^{MNP}\lambda)\}\ . \tag{4.7}$$

Here, as before, capital Roman letters take on values in the entire space, $\tilde{R}$ is the ten-dimensional curvature scalar, and σ is the dilaton field. For simplicity, the Yang-Mills field will be set to zero. Using the σ equation of motion (with $\sigma=\mathrm{const}$), $\exp(-\sigma)H_{MNP}H^{MNP}=\exp(-\sigma/2)H_{MNP}(\mathrm{Tr}\bar{\chi}\Gamma^{MNP}\chi)$, and redefining a new effective Kalb-Ramond field $\mathcal{H}_{MNP}=\exp(-\sigma/2)H_{MNP}$, Einstein's equations become

$$\tilde{R}_{AB}=-\tfrac{9}{2}\mathcal{H}_{ANP}\mathcal{H}_B^{NP}-\tfrac{3}{8}\mathcal{H}_{MNP}\mathcal{H}^{MNP}g_{AB}+9\mathcal{H}_A^{NP}(\mathrm{Tr}\bar{\chi}\Gamma_{BNP}\chi)+\tfrac{3}{8}(\mathrm{Tr}\bar{\chi}\Gamma_{MNP}\chi)^2g_{AB}+\tfrac{1}{4}(\mathrm{Tr}\bar{\chi}\Gamma_{MNP}\chi)(\bar{\lambda}\Gamma^{MNP}\lambda)g_{AB}$$
$$-\tfrac{9}{2}(\mathrm{Tr}\bar{\chi}\Gamma_{ANP}\chi)(\mathrm{Tr}\bar{\chi}\Gamma_B^{NP}\chi)-3(\mathrm{Tr}\bar{\chi}\Gamma_{ANP}\chi)(\bar{\lambda}\Gamma_B^{NP}\lambda)\ . \tag{4.8}$$

In addition to the classical background-field configurations, we wish to include the quantum Casimir effects of the curved internal space. The inclusion of Casimir effects is known for odd-dimensional spaces. Hence the ansatz will be made that the extra six dimensions will consist of two S^D-dimensional spheres (where $D=3$) of radius b_1 and b_2. The spacetime line element $d\tilde{s}^2$ takes on the form $d\tilde{s}^2=dt^2-a^2(t)d\Omega_3^2-b_1^2(t)d\Omega_D^2-b_2^2(t)d\Omega_D^2$.

For a particular case of odd-dimensional internal spaces, in the limit where a approaches infinity, and zero temperature, the Casimir effect on the free energy takes the simple form

$$U=V_3\left[\frac{A^{(1)}}{b_1^4}+\frac{A^{(2)}}{b_2^4}\right]\ , \tag{4.9}$$

where $A^{(i)}$ are constants depending upon the field content of the theory. With the choice of two three-spheres for the internal space, the stress-energy tensor is of the form

$$T_{MN}=\mathrm{diag}(-\rho,pg_{\mu\nu},p^{(1)}g_{m_1n_1},p^{(2)}g_{m_2n_2})\ . \tag{4.10}$$

Thus the energy densities and pressures due to the Casimir effect are

$$\rho_c=\frac{U}{V_3V_D^{(1)}V_D^{(2)}}=\frac{1}{V_D^{(1)}V_D^{(2)}}\left[\frac{A^{(1)}}{b_1^4}+\frac{A^{(2)}}{b_2^4}\right]\ ,$$

$$p_c=-\frac{1}{3V_D^{(1)}V_D^{(2)}}a\frac{\partial U}{\partial a}$$

$$=-\frac{1}{V_D^{(1)}V_D^{(2)}}\left[\frac{A^{(1)}}{b_1^4}+\frac{A^{(2)}}{b_2^4}\right]=-\rho_c\ , \tag{4.11}$$

$$p_c^{(i)}=-\frac{1}{DV_3V_D^{(1)}V_D^{(2)}}b_i\frac{\partial U}{\partial b_i}=\frac{4}{DV_D^{(1)}V_D^{(2)}}\frac{A^{(i)}}{b_i^4}$$

(no sum over i implied), where $i=1,2$ for each of the internal spheres, and $V_D^{(1)}$ and $V_D^{(2)}$ are the volumes of the two internal D-spheres.

We will also employ the generalization of the "monopole" ansatz discussed above for $\mathcal{H}_{MNP}$, χ_{MNP}, and λ_{MNP}. For $\mathcal{H}_{MNP}$, this is

$$\mathcal{H}_{MNP}=\begin{cases}\sqrt{g^{(d)}}\epsilon_{mnp}h(t)&\text{for each internal }D\text{-sphere },\\0,&\text{otherwise },\end{cases}$$
$$\tag{4.12}$$

where ϵ_{mnp} is the totally antisymmetric Levi-Civita tensor which takes on the values ±1. Similar expressions can be written for χ_{MNP} and λ_{MNP}. Using this with the Bianchi identities $\mathcal{H}_{MNP;Q}+\mathcal{H}_{QMN;P}+\mathcal{H}_{PQM;N}=0$ (and similarly for χ_{MNP} and λ_{MNP}), we can express the h's as a function of the internal radii $b_i(t)$:

$$h(t)=h_0/b^D(t),\quad h_0=\mathrm{const}\ . \tag{4.13}$$

Similarly, the associated constants for χ_{MNP} and λ_{MNP} are χ_0 and λ_0, respectively. Taking the simplest case of $b(t)\equiv b_1(t)=b_2(t)$ and adding in the Casimir effects (with $A\equiv A^{(1)}=A^{(2)}$), we obtain the equations of motion

$$3\frac{\ddot{a}}{a}+6\frac{\ddot{b}}{b}=-\left[\frac{2A'}{b^{10}}-\frac{c}{b^6}\right]\ ,$$

$$\frac{\ddot{a}}{a}+2\left[\frac{\dot{a}}{a}\right]^2+6\frac{\dot{a}}{a}\frac{\dot{b}}{b}=-\left[\frac{2A'}{b^{10}}-\frac{c}{b^6}\right]\ , \tag{4.14}$$

$$\frac{\ddot{b}}{b}+5\left[\frac{\dot{b}}{b}\right]^2+3\frac{\dot{a}}{a}\frac{\dot{b}}{b}=-\frac{2}{b^2}+\frac{4A'}{3b^{10}}+\frac{c'}{b^6}\ ,$$

where $A'=A/(4\pi^4)$, and the constants c and c' are given by

$$c \equiv 3(\tfrac{3}{2}h_0^2 - \tfrac{3}{2}\chi_\rho^2 - \chi_0\lambda_0) \,,$$

$$c' \equiv 3(\tfrac{9}{2}h_0^2 - 6h_0\chi_0 + \tfrac{3}{2}\chi_0^2 + \chi_0\lambda_0) \,.$$
(4.15)

Again, substituting in the effective JBD field and using the definition of α (remembering that we have a product of two three-spheres so that the effective $\rho_D = 12$), we obtain the equations of motion

$$\left[\frac{\dot{a}}{a}\right]^2 + \frac{\dot{a}}{a}\frac{\dot{\Phi}}{\Phi} = -\frac{1}{6}\omega\left[\frac{\dot{\Phi}}{\Phi}\right]^2 - \frac{1}{6}\alpha\Phi^{-1/3}$$
$$+ \frac{2A'}{3b_0^{10}(16\pi G_N)^{5/3}}\Phi^{-5/3}$$
$$+ \frac{(c'+c/3)}{b_0^6 16\pi G_N}\Phi^{-1} \,,$$
(4.16)

$$\ddot{\Phi} + 3\frac{\dot{a}}{a}\dot{\Phi} = -\alpha\Phi^{2/3} + \frac{8A'}{b_0^{10}(16\pi G_N)^{5/3}}\Phi^{-2/3}$$
$$+ \frac{6c'}{b_0^6 16\pi G_N} \,,$$

where $\omega = 1 - 1/2D = \tfrac{5}{6}$. We can see that the equations of motion are nearly the same as in the simpler Kaluza-Klein model [cf. Eqs. (3.9) and (4.5)], except that now we do not have the cosmological constant term, but rather have two extra terms which stabilize the potential for small Φ.

Setting the RHS of the second equation in Eq. (4.16) equal to $-\partial V/\partial\Phi$, we obtain the driving potential for $\Phi(t)$:

$$V(\Phi) = V(0) + \frac{36}{5b_0^2(16\pi G_N)^{1/3}}\Phi^{5/3}$$
$$- \frac{24A'}{b_0^{10}(16\pi G_N)^{5/3}}\Phi^{1/3} + \frac{6c'}{b_0^6 16\pi G_N}\Phi \,.$$
(4.17)

The minimum for this potential occurs for $\Phi_0 = 1/(16\pi G_N)$, which implies that

$$b_0^4 = \frac{c'}{4} + \frac{1}{4}\left[c'^2 + \frac{32A'}{3}\right]^{1/2} \,.$$
(4.18)

Taking the second derivative of $V(\Phi)$, we see that the potential is stable for $b_0^8 + 2A'/3 > 0$. And since there can be no effective cosmological constant at the minimum, the RHS of the first equation in Eq. (4.16) must be equal to zero. Therefore, we have the condition that $-6b_0^8 + 2A' + 3(c'+c/3)b_0^4 = 0$. Combining this with Eq. (4.18), we find that

$$2A' = cb_0^4 \,.$$
(4.19)

A plot of the potential for $\Phi(t)$ is shown in Fig. 6 for $c' = 2c/3 = b_0^4$. Thus, if we start at large values of Φ, Φ will eventually decrease to its minimum value. The idea then is to have enough inflation occur before Φ reaches Φ_0.

We can now ask if these equations will lead to inflationary solutions. We will consider two regimes, determined by the value of Φ. For large Φ
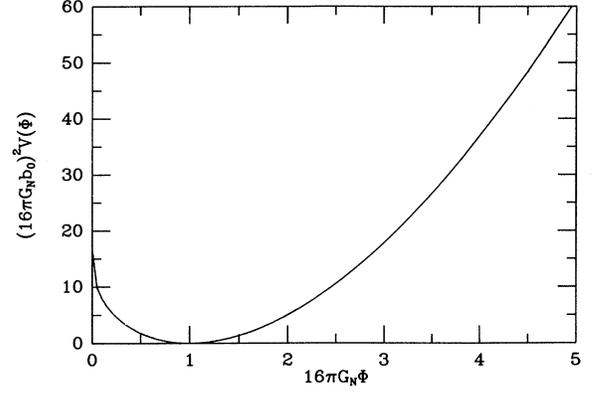

FIG. 6. Graph of the potential of Eq. (4.17).

$(16\pi G_N\Phi \gg 1)$, the equations of motion become

$$\left[\frac{\dot{a}}{a}\right]^2 + \frac{\dot{a}}{a}\frac{\dot{\Phi}}{\Phi} + \frac{5}{36}\left[\frac{\dot{\Phi}}{\Phi}\right]^2 \sim 0 \,,$$
$$\frac{\ddot{\Phi}}{\Phi} + 3\frac{\dot{a}}{a}\frac{\dot{\Phi}}{\Phi} \sim 0 \,.$$
(4.20)

Letting $a \sim \exp(H_a t)$ and $b \sim \exp(H_b t)$, the second equation gives $H_b = -3H_a$, and putting this into the first equation, we get that $-\tfrac{3}{4}H_a^2 = 0$, which implies that $H_a = 0$. Thus no exponential solutions exist for $a(t)$. Letting $a \sim t^n$ and $\Phi \sim t^m$, we find that consistent solutions occur for $n = \tfrac{5}{9}$ and $m = -\tfrac{2}{3}$ or for $n = -\tfrac{1}{3}$ and $m = 2$. (The first describes an expanding universe, while the second describes a shrinking universe.) Since it is necessary for $n > 1$ for inflation to occur, neither solution works. Hence, for large $\Phi(t)$, there are no inflationary solutions.

Now consider the possibility for inflation at intermediate values of Φ, $\Phi \gtrsim 1$. Combining Eqs. (4.18) and (4.19), we find that $b_0^4 = c/3 + c'/2$. Since both c and c' can be as large as b_0^4, we will set $c \sim c'$. Then, for $16\pi G_N\Phi \gtrsim 1$,

$$\left[\frac{\dot{a}}{a}\right]^2 + \frac{\dot{a}}{a}\frac{\dot{\Phi}}{\Phi} + \frac{\omega}{6}\left[\frac{\dot{\Phi}}{\Phi}\right]^2 \sim -\frac{2}{b_0^2(16\pi G_N\Phi)^{1/3}} \,,$$
$$\frac{\ddot{\Phi}}{\Phi} + 3\frac{\dot{a}}{a}\frac{\dot{\Phi}}{\Phi} \sim -\frac{12}{b_0^2(16\pi G_N\Phi)^{1/3}} \,.$$
(4.21)

By inspection no exponential solutions work. Looking then for power-law solutions, we see that power-law solutions exist with $\Phi \propto t^6$, but they do not correspond to inflating solutions for $a(t)$. In fact, simple numerical integration confirms the suspicion that no inflationary solutions exist; i.e., the increase in the scale factor is subluminal, and the evolution of Φ to the ground state is rapid.

Suppose we explicitly break the supersymmetry of the model and try adding a potential for the dilaton field (or equivalently, a cosmological constant). This would allow for inflationary solutions for $a(t)$ when $\Phi(t)$ is large. However, as we saw before in Sec. III, adding a cosmological constant destabilizes the potential for large Φ. One might attempt to adjust the cosmological constant

(Λ) to be very tiny so that the potential would only be destabilized for very large $\Phi > 1/(b_0^6 16\pi G_N \Lambda^3)$. In this case, however, we would only have a small amount of inflation occurring when $\Phi \sim 1/(b_0^6 16\pi G_N \Lambda^3)$; when Φ decreased from this value, this cosmological constant term would again become unimportant, and the curvature term would quickly come to dominate in the second equation of Eq. (4.21). As we have seen already, however, no inflation occurs in this regime.

V. CONCLUSIONS

What conclusions can we draw from our analysis? It is clear from Secs. II and III that the percolation parameter can be made such that the true vacuum phase will, in fact, percolate. In fact, this quantity tends to have an *exponential* dependence on the Jordan-Brans-Dicke Φ, which leads to interesting behavior for ϵ. Note that this is quite different from what happens in the original extended inflation model. We note again that the calculation performed in Sec. II was an approximation to the correct result, valid only when gravitational corrections can be neglected.

The main problem with the higher-dimensional models studied here, in terms of their extended inflation properties, is that they cannot be made to inflate *enough*. The generic situation, even after the potential for the internal radius has been stabilized, is that $b(t)$ is driven to its minimum far too quickly for any significant inflation to occur. It is not clear to us whether or not a potential can be designed that is stable (i.e., $dV/db > 0$) at large b, has a minimum at a nonzero value of b, and yet is flat enough to allow for sufficient inflation to occur. Even if such a potential can be constructed, such "designer potentials" are reminiscent of the unnatural adjustment of parameters needed in most models of rollover inflation. The motivation of extended inflation is to remove these unnatural parameters. Our final conclusion must then be that, unlike various claims in the literature,[6] and despite the fact that higher-dimensional theories do yield effective four-dimensional theories that are similar to Jordan-Brans-Dicke, these models do not seem to be suitable candidates for extended inflationary models. Other models, perhaps involving extra fields such as the dilaton of string theories, may fare better.

ACKNOWLEDGMENTS

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