

Baryogenesis in extended inflation. II. Baryogenesis via primordial black holes

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This is the second of two papers devoted to the study of baryogenesis at the end of extended inflation. Extended inflation is brought to an end by the collisions of bubble walls surrounding regions of true vacuum, a process which produces particles well out of thermal equilibrium. In the first paper we considered baryogenesis via direct production and subsequent decay of baryon-number-violating bosons. In this paper we consider the further possibility that the wall collisions may provide a significant density of primordial black holes and examine their possible role in generating a baryon asymmetry.

I. INTRODUCTION

This is the second of two papers (we shall refer to the first¹ as I throughout) in which we examine whether the out-of-equilibrium conditions automatically generated at the end of extended inflation provide suitable conditions for baryon-number violations to occur. Extended inflation² is a recent revival of the spirit of Guth's original inflationary cosmology³ where the Universe is trapped in a false-vacuum state. In Guth's picture this induces exponential growth in the scale factor and solves various cosmological problems. Inflation ends via the quantum-mechanical formation of bubbles of the true vacuum by tunneling; such bubbles form with a characteristic size determined by microphysics⁴ (provided gravitational corrections are small). The bubbles then grow until they collide with adjacent bubbles, and this disperses the coherent energy in the bubble walls. With exponential inflation, this scenario is flawed because the exponential expansion of the false-vacuum region generically dominates over bubble formation and so inflation never ends.⁵ Extended inflation solves this difficulty by invoking modified gravitational theories in which the gravitational constant may vary; in such theories the inflationary expansion is a rapid power law rather than exponential, and this ensures that the bubble nucleation rate always eventually overcomes the expansion and brings the inflationary era to a satisfactory end. The difficulties of old inflation can also be circumvented in this way in any power-law or slower than exponential inflationary model driven by a suitable phase transition.⁶

However, the extended inflation scenario has difficulties of its own. It was quickly realized by Weinberg⁷ and by La, Steinhardt, and Bertschinger,⁸ that the original (and probably simplest) extended inflation model based on a Jordan-Brans-Dicke theory fails because bub-

bles nucleated early in inflation have time to grow to large sizes. The true vacuum within these large bubbles does not have time to thermalize before radiation decoupling and would create excessively large distortions in the microwave background. In order to avoid this conflict other models have been suggested,⁹⁻¹² with the common theme of arranging that the production of bubbles early in inflation is suppressed. Bubbles formed sufficiently late in the inflationary era do not have time to grow to unacceptable sizes before wall collisions bring inflation to an end. This seems to be an essential feature for any successful extended inflation model, and in this and the preceding paper¹ we have invariably assumed that this requirement is met, although we will not require recourse to a specific model.

In I we outlined the observational status of the baryon asymmetry, and we ask the reader to refer to it for details. In order to explain why the present state of the Universe consists essentially entirely of baryons rather than antibaryons, it is postulated that in the very early Universe a small excess of baryons over antibaryons was created, with the subsequent annihilations leaving the baryons we see today as a residue. This asymmetry is best denoted by a quantity B , called the *baryon number of the Universe*, which is defined as the ratio of the baryon-number density to the entropy density s . This quantity is constant in the late evolution of the Universe and is constrained by primordial nucleosynthesis¹³ to be in the range $B = (3-7) \times 10^{-11}$. Since inflation generates a large thermal entropy it is necessary that the baryon asymmetry be formed after inflation is over.

As discussed in I, there are two standard scenarios for baryogenesis.¹⁴ In I we considered the decays of massive particles (taken to be Higgs bosons) whose decays violate baryon number. These massive bosons were created by the collisions between bubble walls at temperatures low

enough to ensure that no thermal production of Higgs particles occurred, giving a picture different to the conventional one where Higgs bosons leave an original state of thermal equilibrium and then decay. The net baryon asymmetry produced per Higgs-boson decay is parametrized by a fraction ϵ which is in principle derivable from the degree of CP violation in the theory considered; ultimately the value of B can be determined. This paper considers the second scenario, which involves the violation of baryon-number conservation in black-hole decays.

One of the implications of the “no-hair” theorems of black-hole physics is that black holes have an indeterminate baryon number: baryon number is not conserved in black-hole evaporation. In 1975 Hawking suggested that evaporating primordial black holes (PBH’s) might radiate an excess of baryons over antibaryons.¹⁵ This idea was taken up again in the context of grand unified theories (GUT’s) by many people.^{16,17} The violence of bubble-wall collisions may well produce a significant number of black holes as well as relativistic particles, because of the gravitational instability of overdensities generated by the collisions. When such black holes decay by the emission of Hawking radiation, they may emit baryon-number-violating Higgs particles whose decays generate the baryon asymmetry.

The mechanism of baryogenesis by evaporation of primordial black holes divides into two subcategories.¹⁸ In the first version, evaporation occurs while radiation dominates the energy density of the Universe, with the black holes providing the baryon asymmetry but with the entropy arising from the background radiation. Since radiation energy density falls off faster than that of matter, the contribution of the black-hole energy density becomes more important as time goes by. If the time until radiation domination is less than the black-hole lifetime, we get a second version of the mechanism where the black-hole density dominates at the time of evaporation; in such models the black holes provide both entropy and baryon number. This latter class also covers the possibility that so many black holes may be formed that they dominate the energy density of the Universe immediately.

In the next section we shall briefly outline some important parameters relating to extended inflation. For more details concerning these the reader should consult I. We then go on to estimate the baryon asymmetry generated for the different versions of this black-hole-inspired mechanism. The final section demonstrates some typical numbers and compares the results with those obtained via the direct production mechanism.

II. EXTENDED INFLATION PARAMETERS

The details of the end of extended inflation depend primarily on various parameters of the inflaton potential. These determine the duration of inflation, but more important for our purposes they determine the details of the bubble nucleation including the typical bubble size and the energy density of the bubble walls. Without specifying a particular inflationary model, we can identify the important parameters as follows (we use

units $k_B = \hbar = c = 1$, $m_{\text{pl}} = G^{-1/2} = 1.2 \times 10^{19}$ GeV throughout).

(1) σ_0 , the energy scale for spontaneous symmetry breaking (SSB), i.e., the vacuum expectation value (VEV) of the scalar field.

(2) λ , a dimensionless coupling constant of the inflaton potential. We will assume that the potential is proportional to λ .

(3) ξ , a dimensionless number that measures the difference between the false- and the true-vacuum energy density via $\rho_V = \xi \lambda \sigma_0^4$; ξ must be less than unity for sufficient inflation to occur, and this is also precisely the condition that allows the thin-wall approximation (discussed below) to be made.

In terms of these variables, the size of nucleated bubbles (in the thin-wall approximation) is

$$R_C \sim (\xi \lambda^{1/2} \sigma_0)^{-1}, \quad (2.1)$$

the bubble wall thickness is

$$\Delta \sim (\lambda^{1/2} \sigma_0)^{-1}, \quad (2.2)$$

and the energy per unit area of the wall is

$$\eta \sim \lambda^{1/2} \sigma_0^3. \quad (2.3)$$

As shown in I, a typical bubble experiences little growth between nucleation and percolation. The physical size of the bubble will grow with the scale factor; we shall assume, in line with I, that at the end of extended inflation all bubbles are of size αR_C .

In I, we calculated the baryon asymmetry produced via the production and decay of baryon-number-violating bosons. Using the information about the bubbles given above, we obtained the result that

$$B_0 = \epsilon f_H \alpha^{-1} g_*^{-1/4} \lambda^{-1/4} \xi^{1/4}, \quad (2.4)$$

where g_* is the effective number of degrees of freedom in all species of particles formed during thermalization (g_* would be expected to be of order 100 in a grand unified theory) and f_H is the fraction of baryon-number-violating Higgs particles formed in the wall collisions. If the typical energy of particles formed in the collisions exceeds the Higgs-boson mass then $f_H \sim g_H/g_*$ where g_H is the number of Higgs degrees of freedom. This result is useful for comparison with those we shall derive in the next section for the case where a significant density of black holes are produced in the wall collisions. Note that we shall use different subscripts on B to distinguish the baryon asymmetry obtained in different situations.

III. BARYOGENESIS BY BLACK-HOLE EVAPORATION

We consider the possibility that the formation of primordial black holes may have led to significant baryogenesis. There are two possible sources for the formation of small primordial black holes. First, holes may form via the gravitational instability of inhomogeneities formed during the thermalization phase, particularly during the wall collisions themselves where we can expect

high local densities to prevail, and second, there is the possibility of trapped regions of false vacuum (within their Schwarzschild radii) caught between bubbles of true vacuum.¹⁹

Unfortunately, the technical details of even estimating the typical number density and mass of the black holes formed by these processes are quite difficult. Some progress in this direction was made by Hawking *et al.*,¹⁷ in the context of the original inflationary scenario, and more recently Hsu²⁰ has examined black-hole production from false-vacuum regions in extended inflation. In order to keep our discussion on a more general footing, we shall for now simply assume that some fraction β of the energy after collisions is in black holes, while the remaining $1-\beta$ is in radiation,¹⁸ and later consider the various outcomes implied by the differing values of β .

The stage will be set for baryogenesis at the end of extended inflation. At the end of extended inflation, the expansion rate of the Universe is H_{end} , and from H_{end} we can define a characteristic timescale at reheating, which formally represents a patching of a radiation or matter-dominated Friedmann-Robertson-Walker (FRW) cosmology onto the inflationary one. For a radiation-dominated universe this is $t_{\text{RH}} \simeq H_{\text{end}}^{-1}/2$, while for matter domination we have $t_{\text{RH}} \simeq 2H_{\text{end}}^{-1}/3$; we will refer to t_{RH} as the time at the end of inflation. We shall derive equations below for the case of patching to a radiation-dominated universe, pointing out any differences that matter domination implies.

The total energy density at the end of extended inflation is partitioned between the energy density of radiation ρ_R and black holes ρ_{BH} :

$$\begin{aligned} \rho(t_{\text{RH}}) &= \rho_R(t_{\text{RH}}) + \rho_{\text{BH}}(t_{\text{RH}}), \\ \rho_R(t_{\text{RH}}) &= (1-\beta)\rho(t_{\text{RH}}) = \frac{\pi^2}{30} g_* T_{\text{RH}}^4, \\ \rho_{\text{BH}}(t_{\text{RH}}) &= \beta\rho(t_{\text{RH}}) = M_0 n_{\text{BH}}(t_{\text{RH}}), \end{aligned} \quad (3.1)$$

where T_{RH} is the reheat temperature, M_0 is the initial mass of the black holes formed (for convenience we will assume that they all have the same mass), and n_{BH} is the number density of black holes. The time t_{RH} can also be expressed in terms of $\rho(t_{\text{RH}})$:

$$t_{\text{RH}}^2 \equiv \left[\frac{3}{32\pi} \right] \frac{m_{\text{Pl}}^2}{\rho(t_{\text{RH}})}. \quad (3.2)$$

(For matter domination, the factor $3/32\pi$ is replaced by $1/6\pi$). From H_{end} and ρ we also define a ‘‘horizon mass’’ at the end of inflation:

$$\begin{aligned} M_{\text{hor}} &= \frac{4\pi}{3} \rho(t_{\text{RH}}) (2t_{\text{RH}})^3 \\ &= \left[\frac{3}{32\pi} \right]^{1/2} \frac{m_{\text{Pl}}^3}{\rho^{1/2}(t_{\text{RH}})}. \end{aligned} \quad (3.3)$$

(The right-hand side is the same in the matter-dominated case.) M_{hor} represents the mass within the ‘‘physics horizon,’’ at the end of inflation, and plays the same role as the mass within the horizon in the standard FRW model.

Once formed, the black holes evaporate at a rate given by

$$\dot{M}_{\text{BH}} = - \frac{g_*}{3} \frac{m_{\text{Pl}}^4}{M_{\text{BH}}^2}, \quad (3.4)$$

which leads to a time dependence of the black hole mass of

$$M_{\text{BH}}^3(t) = M_0^3 - g_* m_{\text{Pl}}^4 (t - t_{\text{RH}}). \quad (3.5)$$

It is convenient to define a black-hole lifetime,

$$\tau \equiv M_0^3 / g_* m_{\text{Pl}}^4, \quad (3.6)$$

and the expression for the mass as a function of time becomes

$$M(t) = M_0 [1 - (t - t_{\text{RH}})/\tau]^{1/3}.$$

The evaporation ends at time $t_{\text{BH}} = t_{\text{RH}} + \tau$.

Black holes radiate as blackbodies with temperature $T_{\text{BH}} = m_{\text{Pl}}^2 / 8\pi M_{\text{BH}}$. This allows us to calculate what is, for our purposes, the most important quantity: the number of particles emitted during the course of the evaporation. Let us first calculate the number of particles emitted while the black hole is between the temperatures T and $T+dT$. The change in mass of the black hole dM , which is the amount of energy radiated as particles, is given by

$$dM = \frac{m_{\text{Pl}}^2}{8\pi} \left[\frac{1}{T} - \frac{1}{T+dT} \right]. \quad (3.7)$$

Each emitted particle has energy $3T$ (the mean energy of a particle in a Maxwell-Boltzmann distribution at temperature T), so the number of particles emitted between those temperatures is just

$$dN = \frac{m_{\text{Pl}}^2}{24\pi T} \left[\frac{1}{T} - \frac{1}{T+dT} \right] = \frac{m_{\text{Pl}}^2}{24\pi T^3} dT. \quad (3.8)$$

Integrating this, we find that the number of particles emitted as the black-hole temperature increases from its initial temperature $T_0 \rightarrow \infty$ is

$$N = \frac{4\pi M_0^2}{3m_{\text{Pl}}^2}. \quad (3.9)$$

Note that this gives the total number of particles emitted. A fraction f_H of these will be Higgs particles. To determine the appropriate form for f_H , the initial temperature of the black hole at formation may be important. If it is less than the mass of the Higgs boson, m_H , then the thermal spectrum in the initial phase of the evaporation will not include the Higgs boson as the typical energy is not high enough to produce so massive a particle. Only when the black hole temperature has increased to m_H will the thermal radiation include a significant fraction of Higgs bosons. This can lead to an overall suppression in the number of Higgs bosons produced during the complete course of the evaporation. Discussion of such a suppression will mostly be reserved for the conclusions. Once the temperature is high enough to radiate Higgs bo-

sons, we expect that the energy of radiated particles will be distributed evenly amongst all radiated species, so that f_H is a constant given by g_H/g_* as discussed in Sec. II

Black-hole evaporation affects the evolution of both components of the total mass density. Since the hole mass is decreased by evaporation, the evolution of the black-hole energy density, which in the absence of evaporation would be that of nonrelativistic matter ($\rho_{\text{NR}} \propto a^{-3}$, where a is the scale factor), is altered. The production of radiation from the hole evaporation also modifies the evolution of radiation energy density, which normally scales as a^{-4} . Of course, the departure of the energy densities from the normal evolution is most pronounced around the time $t \sim t_{\text{RH}} + \tau$. An exact treatment of this effect is given in the Appendix, where a network of equations is derived describing the evolution of the different components of the energy density and also the evolution of the baryon asymmetry. In order to understand the general results, let us for the moment ignore the complication resulting from the decrease of the hole mass. In Sec. IV we will discuss the inclusion of this effect.

Two different situations arise, depending on whether black holes or radiation dominates the energy density of the Universe at the time the holes evaporate.¹⁹ If $\beta < \frac{1}{2}$, then the evolution of the scale factor is that appropriate to a radiation-dominated universe, i.e., $a(t) \sim t^{1/2}$, and the energy density of black holes goes as $a^{-3} \propto t^{-3/2}$, while that of radiation goes as $a^{-4} \propto t^{-2}$. Therefore, provided their lifetime is sufficiently long, black holes will come to dominate the Universe at a time $t_* = t_{\text{RH}}(1-\beta)^2/\beta^2$, and hence if $\tau > t_* - t_{\text{RH}}$, they will come to dominate before their evaporation. If $\beta > \frac{1}{2}$, black holes dominate even initially.

A. Evaporation before domination

We first examine the case where black-hole evaporation occurs before domination. This corresponds to small β and initially light black holes, with

$$\frac{\tau}{t_{\text{RH}}} < \frac{1-2\beta}{\beta^2}. \quad (3.10)$$

Since the black holes never dominate, the Universe expands like a radiation-dominated universe, with $a \propto t^{1/2}$. If the black holes evaporate before domination, their radiation will not significantly change the background entropy density.

The number density of black holes will be diluted by the effects of expansion, scaling as a^{-3} . Notice that this result is exact regardless of whether or not the holes are losing mass through evaporation, which leads to the energy density in holes falling off somewhat faster than this. At the time of evaporation t_{BH} , the number density of holes is

$$\begin{aligned} n_{\text{BH}}(t_{\text{BH}}) &= n_{\text{BH}}(t_{\text{RH}}) \left(\frac{a(t_{\text{RH}})}{a(t_{\text{BH}})} \right)^3 \\ &= n_{\text{BH}}(t_{\text{RH}}) \left(\frac{t_{\text{RH}}}{t_{\text{BH}}} \right)^{3/2}. \end{aligned} \quad (3.11)$$

Equation (3.9) gives us the number of Higgs particles produced during the evaporation of a single hole (we leave consideration of a suppression due to the black holes being initially too cool to radiate Higgs bosons for the conclusions). Hence the number density of Higgs bosons produced in the evaporation is

$$\begin{aligned} n_H(t_{\text{BH}}) &= f_H N n_{\text{BH}}(t_{\text{BH}}) \\ &= f_H \frac{4\pi M_0^2}{3m_{\text{Pl}}^2} \frac{\rho_{\text{BH}}(t_{\text{RH}})}{M_0} \left(\frac{t_{\text{RH}}}{t_{\text{BH}}} \right)^{3/2}. \end{aligned} \quad (3.12)$$

Notice here that we have assumed all the particles are produced at the end of the evaporation; however, if the baryon number has the same scaling with time as the black-hole energy density then this assumption gives exactly the correct result.

With the assumption that each Higgs-boson decay generates a net baryon number ϵ as mentioned earlier (see I for a definition of ϵ),

$$n_B(t_{\text{BH}}) = \epsilon f_H \frac{4\pi}{3m_{\text{Pl}}^2} M_0 \rho_{\text{BH}}(t_{\text{RH}}) \left(\frac{t_{\text{RH}}}{t_{\text{BH}}} \right)^{3/2}. \quad (3.13)$$

The radiation density meanwhile has been dropping as $1/t^2$, so we have

$$\rho_R(t_{\text{BH}}) = \rho_R(t_{\text{RH}}) \left(\frac{t_{\text{RH}}}{t_{\text{BH}}} \right)^2, \quad (3.14)$$

from which we obtain the radiation temperature at the evaporation time as

$$T^4(t_{\text{BH}}) = \frac{30}{g_* \pi^2} \rho_R(t_{\text{BH}}). \quad (3.15)$$

The entropy density in the Universe at $t = t_{\text{BH}}$ is

$$\begin{aligned} s(t_{\text{BH}}) &= \frac{2\pi^2}{45} g_* T^3(t_{\text{BH}}) \\ &= \frac{2\pi^2}{45} g_* [\rho_R(t_{\text{BH}})]^{3/4} \left(\frac{30}{g_* \pi^2} \right)^{3/4}, \end{aligned} \quad (3.16)$$

which ultimately leads to a baryon asymmetry of

$$\begin{aligned} B_A \equiv \frac{n_B}{s} &= \frac{1}{2} \epsilon f_H \left(\frac{45\pi}{g_*} \right)^{1/4} \left(\frac{M_0}{m_{\text{Pl}}} \right)^{1/2} \\ &\quad \times \left[\frac{M_0}{M_{\text{hor}}} \right]^{1/2} \frac{\beta}{(1-\beta)^{3/4}}, \end{aligned} \quad (3.17)$$

where we have used Eq. (3.3). Note that the penultimate factor gives the initial black-hole mass as a fraction of the horizon mass.

In the Appendix, we demonstrate how this result may be obtained from the evolution network of Eq. (A13). The approximations of this subsection are equivalent to ignoring the last term in the \bar{B}' equation and keeping $R_R = 1$. Simple integration of the network equation for B leads directly to Eq. (3.17).

B. Evaporation after domination

We now consider the second possibility, that holes evaporate after they dominate the energy density. This divides into two further cases; in the former, black holes come to dominate at time t_* as defined earlier, while in the latter black holes dominate immediately after formation

In the first of these cases, once $t > t_*$ the scale factor evolves as appropriate for a matter-dominated universe, $a(t) \sim t^{2/3}$, and so $\rho_{\text{BH}}(t) = \rho_{\text{BH}}(t_*) / (t_*/t)^2$ and $\rho_R(t) = \rho_R(t_*) / (t_*/t)^{8/3}$, with the energy densities equal at t_* .

As before, the evaporation of a single black hole gives a baryon number

$$n_B = \epsilon f_H N n_{\text{BH}}(t_{\text{BH}}). \quad (3.18)$$

This time, though, the entropy is also determined by the other black-hole evaporation products, as they provide the dominant contributions. Here we must make an additional assumption that all the black-hole energy density is transformed to radiation at the evaporation time. In reality, radiation will be produced throughout the evaporation, and because radiation dilutes more rapidly than black holes our approximation will tend to overestimate the entropy density and hence underestimate the baryon number. However, in the light of Eq. (3.4) we can see that most energy is transferred near the evaporation time and so this approximation should give fairly accurate results. Assuming that all the black-hole density goes into entropy, we obtain

$$s = \frac{2\pi^2}{45} g_*^{1/4} \left[\frac{30}{\pi^2} \right]^{3/4} \rho_{\text{BH}}^{3/4}(t_{\text{BH}}), \quad (3.19)$$

leading to a baryon asymmetry of

$$\frac{n_B}{s} = \frac{4\pi}{3} \frac{45}{2\pi^2} \left[\frac{\pi^2}{30} \right]^{3/4} \epsilon f_H \frac{M_0}{m_{\text{pl}}^2} g_*^{-1/4} \rho_{\text{BH}}^{1/4}(t_{\text{BH}}). \quad (3.20)$$

Substituting for ρ_{BH} and t_* leads to

$$B_{B1} = \frac{1}{2} \epsilon f_H \left[\frac{45\pi}{g_*} \right]^{1/4} \left[\frac{M_0}{m_{\text{pl}}} \right]^{1/2} \left[\frac{M_0}{M_{\text{hor}}} \right]^{1/2} \times (1-\beta)^{1/4} \left[1 + \frac{\tau}{t_{\text{RH}}} \right]^{-1/2}. \quad (3.21)$$

This expression is very similar to that obtained in the ‘‘evaporation before domination’’ scenario; in particular, the black-hole mass appears in the same functional form, and the prefactors are all the same with the exception of the β term, which naturally has changed as we move to a different physical situation. The last factor demonstrates how a long black-hole lifetime dilutes the baryon asymmetry obtained; if τ is very small this factor is just equal to one, while for $\tau \gg t_{\text{RH}}$ we get a reduction in the baryon asymmetry by a factor of about $\sqrt{M_0^3/M_{\text{hor}} m_{\text{pl}}^2 g_*}$. Clearly, this factor can be important for long-lived (initially massive) black holes. These are also exactly the type of holes that one might expect to

survive long enough to come to dominate even if β is originally substantially less than $\frac{1}{2}$.

We note here that in the Appendix we demonstrate that the result of Eq. (3.17) gives an absolute upper bound on the baryon asymmetry for a given β , and M_0 that may be obtained when we consider the full network evolution equations. [Of course, having chosen β and M_0 we have determined which physical situation we are in, so Eq. (3.17) may not be applicable; nevertheless, it still gives the upper bound for those parameter values.] This is consistent with the last factor in the above expression always being less than 1, and is easily understood by realizing that producing the entropy later from the black-holes means that up until evaporation the energy density representing what will become entropy has fallen off only as a^3 , whereas if it were in the background it would be falling as a^4 . Therefore, models where the black holes provide entropy lead to a greater entropy, and hence smaller baryon number, than models where the entropy is associated with the background. We also remind the reader that we have had to make approximations to obtain Eqs. (3.17) and (3.21). Despite this, they match on the border where domination occurs ($\beta = \frac{1}{2}$) in the case of very fast evaporation ($\tau \simeq 0$), which is precisely what one would expect of exact results.

We now examine the second case of black-hole domination, that in which the black holes dominate even initially. The black-hole energy density is now given by $\rho_{\text{BH}}(t) = \rho_{\text{BH}}(t_{\text{RH}})(t_{\text{RH}}/t)^2$. Equation (3.20) still holds, and now the substitution gives

$$B_{B2} = \frac{1}{2} \epsilon f_H \left[\frac{45\pi}{g_*} \right]^{1/4} \left[\frac{M_0}{m_{\text{pl}}} \right]^{1/2} \left[\frac{M_0}{M_{\text{hor}}} \right]^{1/2} \times \beta^{1/4} \left[1 + \frac{\tau}{t_{\text{RH}}} \right]^{-1/2}. \quad (3.22)$$

which is just Eq. (3.21) multiplied by $[\beta/(1-\beta)]^{1/4}$. This factor represents the dilution of the black-hole energy density up to domination. As expected, Eqs. (3.21) and (3.22) match in the case of marginal domination where $\beta = \frac{1}{2}$. The β dependence in Eq. (3.22) simply reflects the fraction of the horizon mass contributed by black holes. It differs from Eq. (3.21) because here there is no evolution in the initial radiation-dominated phase, and, hence, no era of dilution before domination. In the case of Eq. (3.21) an extra multiplier of $[(1-\beta)/\beta]^{1/4}$ is needed to account for the evolution in the initial radiation-dominated phase.

We also draw the reader’s attention to one slight subtlety relating to this final answer; for this final case we must patch a matter-dominated rather than radiation-dominated Friedmann universe onto the end of extended inflation. As discussed around Eq. (3.2), we must then use a slightly different formula to obtain t_{RH} from the energy density. The expression for the horizon mass is however the same.

This completes the set of results for the different regions of domination, and is summarized in Table I.

Note that to obtain the results of Table I we have not yet assumed that an era of extended inflaton has oc-

TABLE I. Results for the baryon number produced by black-hole evaporation depend upon β (the fraction of the energy of the Universe in black holes at $t = t_{\text{RH}}$, where t_{RH} is taken to be the end of inflation), t_* (the time at which the black holes dominate the mass of the Universe), and $\tau = M_{\text{BH}}^3 / g_* m_{\text{Pl}}^4$ (the lifetime of a black hole of mass M_{BH}).

β	τ	$B \equiv n_B / s$
$\beta < \frac{1}{2}$	$\tau < t_* - t_{\text{RH}}$	Eq. (3.17)
$\beta < \frac{1}{2}$	$\tau > t_* - t_{\text{RH}}$	Eq. (3.21)
$\beta > \frac{1}{2}$	Independent of τ	Eq. (3.22)

curred; all we have assumed is that at some time t_{RH} a fraction β of the energy density is in black holes. Because we are assuming that this occurs after extended inflation, one further piece of information can be used—the energy density at the end of inflation is known in terms of the inflation parameters. (However, to get the reheat temperature we need to know β as well, as only the energy density in radiation contributes to T_{RH} .) This gives us an expression for the horizon mass M_{hor} which can be substituted into the expressions we obtained above for the baryon asymmetry. Recalling that thermalization distributes the energy in the bubble walls throughout the volume of a bubble, we have (using the parameters of Sec. II)

$$\rho(t_{\text{RH}}) \sim \xi \lambda \sigma_0^4 \quad (3.23)$$

and hence M_{hor} is given by

$$M_{\text{hor}} \simeq \sqrt{3/32\pi} \xi^{-1/2} \lambda^{-1/2} m_{\text{Pl}}^3 / \sigma_0^2. \quad (3.24)$$

Although we included numerical factors in the preceding discussion, the quantities derived from extended inflation are less well known and hence some of the expressions we shall use henceforth are approximate. Substitution of Eq. (3.24) into the various answers, Eqs. (3.17), (3.21), and (3.22), gives us the baryon asymmetry obtained at the end of extended inflation for the differing physical situations.

IV. DISCUSSION, COMPARISONS, AND CONCLUSIONS

Here we examine typical numbers for the baryon asymmetry. For ease of comparison, we shall express the various results from the black-hole mechanism as ratios of the result expressed in Eq. (2.4) for the baryon asymmetry B_0 produced by the direct production mechanism.¹ For a typical GUT theory $B_0 \sim 10^{-2} \epsilon (\xi/\lambda)^{1/4}$. For sample parameters this implies a small ϵ , perhaps of order 10^{-8} , in order to give the observed asymmetry $B \sim 10^{-10}$. Note that here we assume that the f_H are the same in all cases; i.e., we have not yet incorporated any suppression of Higgs-boson production.

To aid comparison, we define the quantity \hat{B} :

$$\hat{B} \equiv \frac{1}{2} \epsilon f_H \left[\frac{45\pi}{g_*} \right]^{1/4} \frac{M_0}{m_{\text{Pl}}^{1/2} M_{\text{hor}}^{1/2}}. \quad (4.1)$$

This combination appears in each of the formulas for the

baryon asymmetry obtained in the previous section, excluding only the β factors, and the dependence on the black-hole lifetime (itself dependent on the initial mass). We introduce parameter $\mu = M_0 / M_{\text{hor}}$. We expect μ to be less than 1, though nothing prevents it from being much smaller. Using the formula for M_{hor} , Eq. (3.24), we have

$$\hat{B} \sim \frac{1}{2} \epsilon f_H g_*^{-1/4} \xi^{-1/4} \lambda^{-1/4} \mu \frac{m_{\text{Pl}}}{\sigma_0}. \quad (4.2)$$

We can now compare the differing black-hole cases in turn, via the expression

$$\frac{\hat{B}}{B_0} \simeq \frac{1}{2} \xi^{-1/2} \alpha \mu \frac{m_{\text{Pl}}}{\sigma_0}. \quad (4.3)$$

First consider the case where black-hole evaporation occurs before domination. This corresponds to $\beta < \frac{1}{2}$ and a short black-hole lifetime. We obtain from Eq. (3.17) the simple expression

$$\frac{B_A}{B_0} \sim \frac{\beta}{(1-\beta)^{3/4}} \xi^{-1/2} \alpha \mu \frac{m_{\text{Pl}}}{\sigma_0}. \quad (4.4)$$

The “domination then evaporation” cases allow a similar comparison, e.g.,

$$\frac{B_{B1}}{B_0} \sim (1-\beta)^{1/4} \xi^{-1/2} \alpha \mu \frac{m_{\text{Pl}}}{\sigma_0} \left[1 + \frac{\tau}{t_{\text{RH}}} \right]^{-1/2}. \quad (4.5)$$

for the first case and the same expression with $(1-\beta)^{1/4}$ replaced by $\beta^{1/4}$ for the second.

To get a better feel for the meaning of μ , we now examine when μ is of such a value as to give holes of an interesting lifetime. As we know the horizon mass, we can determine both t_{RH} and the evaporation time τ for the black holes, the former being a function solely of the inflation parameters, the latter being a function of μ . Equations (3.3), (3.6), and (3.23) lead to the ratio

$$\frac{\tau}{t_{\text{RH}}} \sim \frac{1}{32\pi g_*} \xi^{-1} \lambda^{-1} (\sigma_0 / m_{\text{Pl}})^{-4} \mu^3. \quad (4.6)$$

For simplicity of discussion, we shall insert some plausible values for the various inflaton parameters; results for other values can be obtained by a suitable scaling. We choose

$$g_* = 100, \quad \xi = 10^{-2}, \\ \lambda = 10^{-2}, \quad \sigma_0 = 10^{-3} m_{\text{Pl}}. \quad (4.7)$$

These values give, for Eq. (4.3),

$$\frac{\hat{B}}{B_0} \sim 10^4 \mu \alpha. \quad (4.8)$$

Although it seems from this that the black-hole mechanism has the possibility of generating a much greater baryon asymmetry than the direct production mechanism's B_0 (by choice of a sufficiently large μ), recall that we must use the β and μ values appropriate to each regime. These will contribute to reduce the actual asym-

metry obtained; for example, for B_A we much choose $\beta < 0.5$, but then also we must choose μ small enough so that the black holes do not come to dominate.

Using the sample values from above, we obtain

$$\frac{\tau}{t_{\text{RH}}} \sim 10^{12} \mu^3. \quad (4.9)$$

Hence only when black holes have masses such that $\mu > 10^{-4}$ are the lifetimes sufficiently long that the final factor in the expression for B_B becomes important for those choices of parameters.

We can also calculate the black-hole lifetime required in order for black holes to dominate, which requires $\tau > t_* - t_{\text{RH}}$. We obtain

$$\frac{\tau}{t_* - t_{\text{RH}}} = \left[\frac{\beta^2}{1 - 2\beta} \right] \frac{\tau}{t_{\text{RH}}}. \quad (4.10)$$

That this ratio must be greater than one gives a lower bound on τ , and hence μ , which must be satisfied in order for black holes to come to dominate. Equally, it gives an upper bound on μ which must obey for the ‘‘evaporation before domination’’ result B_A to be applicable.

These bounds on μ for a given β allow us to calculate the maximum baryon asymmetry that can be obtained by each of the expressions within their range of validity; we do this for our sample parameters. The value of μ corresponding to the bound in the above expression is just

$$\mu = 10^{-4} \left[\frac{1 - 2\beta}{\beta^2} \right]^{1/3}. \quad (4.11)$$

In the ‘‘evaporation before domination’’ scenario, for a given β the maximum asymmetry is obtained when μ saturates this bound. Hence the maximum asymmetry obtained from B_A is at the value of β which maximizes

$$\frac{B_A}{B_0} \simeq (1 - 2\beta)^{1/3} \beta^{1/3} (1 - \beta)^{-3/4} \alpha, \quad \beta \in [0, \frac{1}{2}]. \quad (4.12)$$

The maximum value of the β factors is 0.652, obtained for

$$\beta = (-11 + \sqrt{153})/4 \simeq 0.342,$$

which implies for all the above that at best $B_A \sim \alpha B_0$. For small β we just get $B_A \sim \beta^{1/3} \alpha B_0$, provided we choose μ to optimize the asymmetry (for a given β). If μ is smaller than the optimizing value given above the asymmetry obtained becomes yet smaller.

We comment here on the role of α , the expansion factor of a typical bubble. In calculating B_0 , we made the conservative though plausible assumption that only the potential energy in the walls, a factor $1/\alpha$ of the total energy, participated in baryogenesis. In black-hole formation, gravitational effect, all the bubble energy can participate.

A similar comparison can be carried out for the ‘‘domination then evaporation’’ scenarios. Let us first consider the second case where the black holes dominate immedi-

ately; here no bound on μ arises, since the black holes no longer have to survive long enough to come to dominate. In this case μ can, in principle, be as small or large (up to its maximum value of 1) as we like. Notice that there is a trade-off between the terms in the expressions for B_B . We can write $B_B \propto \mu(1 + c\mu^3)^{-1/2}$ for some constant c . Asymptotically, $B_B \propto \mu$ and $B_B \propto \mu^{-1/2}$ for small and large μ , respectively. In fact, baryon number production is most efficient at an intermediate value of $\mu \equiv \mu_*$ where $\tau/t_{\text{RH}} = 2$ (true for any model parameters); for our sample parameters this once more corresponds to μ of around 10^{-4} and hence we find, as in the previous case, that at best $B_{B2} \sim \alpha B_0$. Lighter or heavier holes will lead to a smaller asymmetry, particularly in the latter case, as we shall shortly see there is an additional temperature suppression. The β factor plays little role here as it is simply $\beta^{1/4}$ where $\beta \in [\frac{1}{2}, 1]$.

Similar criteria also apply to the remaining case, where black holes come to dominate. Again the β factor is unimportant; the remaining terms are exactly as in the domination immediately case, and hence the upper limit on the baryon asymmetry is the same. However, we have to take one more thing into account, for in order for the B_{B1} equation to apply μ must exceed the lower limit from Eq. (4.10). If μ_* is greater than that bound, then the analysis is just as before. However, if the bound is larger than μ_* then the maximum asymmetry that may be obtained will occur when this bound is just met, and will be smaller than that obtained if $\mu = \mu_*$ were allowed. Again temperature suppression may also be important, as we now discuss.

The initial temperature of the black hole depends on the value of μ , with more massive holes being cooler. As stated earlier, if this temperature is below the mass of the Higgs boson then the initial phase of evaporation will not feature Higgs particles. The black-hole temperature is given by

$$T_{\text{BH}} = \frac{m_{\text{Pl}}^2}{8\pi M_{\text{BH}}}, \quad (4.13)$$

which leads to a ratio

$$\frac{T_{\text{BH}}}{m_H} \sim \left[\frac{\lambda}{\lambda_H} \right]^{1/2} \xi^{1/2} \left[\frac{\sigma_0}{m_{\text{Pl}}} \right] \mu^{-1}, \quad (4.14)$$

where we have written the Higgs-boson mass as $m_H = \lambda_H^{1/2} \sigma_0$ (guided by GUT). This gives a critical value μ_{crit} , which μ must be less than in order for Higgs-boson radiation to occur. Using the sample parameters of Eq. (4.7) and assuming $\lambda_H \sim \lambda$ we get

$$\frac{T_{\text{BH}}}{m_H} \sim 10^{-4} \mu^{-1}. \quad (4.15)$$

Hence only when $\mu < 10^{-4}$ is the black hole hot enough to be radiating Higgs particles immediately. For larger μ , one can expect an initial evaporation phase (during which no Higgs bosons are radiated) until μ reaches μ_{crit} . Equation (3.9) tells us that $N \propto M_0^2 \propto T_0^{-2}$. Hence if μ is

greater than the critical value which allows the radiation of Higgs bosons, then there will be a suppression of the baryon number formed by a factor of $(\mu_{\text{crit}}/\mu)^2$. Such a suppression will occur in all versions of the black-hole scenario, including the “evaporation before domination” result. The number 10^{-4} given above is of course dependent on the particular choice of parameters; the general form of the suppression factor can also be written as $[M(T=m_H)/M_0]^2$. It is coincidental that for our choice of parameters μ_{crit} is approximately the value of μ required to make $\tau \sim t_{\text{RH}}$.

The two different scenarios we have described also lead to qualitatively different non-uniformities in the density distribution of the Universe. In the case of “domination then evaporation” the initial inhomogeneities in the black-hole number distribution will lead to both non-uniformity in the photon and baryon number distribution following black-hole evaporation because both are determined by the black-hole evaporation products. The resulting density perturbations will therefore be of a quasiadiabatic nature. In the second case of “evaporation before domination” evaporation products determine only the baryon number irregularity, and hence if the radiation distribution were initially smooth, the resulting density perturbations would be of a quasi-isothermal character.

One other feature of this model worth mentioning is that a fraction of the rest mass of the black holes will evaporate as gravitons. For black holes in the range $10^{14} - 10^{15}$ g one finds that about 2% of this rest mass is emitted as gravitons.²¹ In versions of the model where the black holes have dominated the energy density we would therefore create an initial graviton abundance of perhaps between 0.01 and 0.1 of that residing in photons. Both gravitons and photons scale as a^{-4} as the Universe expands, leaving the ratio effectively constant; however, the gravitons will remain collisionless after they form, and hence their abundance will not be exactly thermal (rather, it will be a superposition of different Planck spectra with $T \sim T_{\text{BH}}$ with a Bose-Einstein form). Because gravitons are collisionless their temperature will not keep pace with that of the thermal sea of interacting particles, such as photons, into which massive particle-antiparticle pairs will annihilate. Assuming the evolution is entropy conserving then $g_{\text{int}} T^3$ will stay constant through annihilation thresholds, where g_{int} is the number of degrees of freedom interacting with the photons. This will give the photon an enhanced temperature over the gravitons by a factor $(g_{\text{int}}/2)^{1/3}$ where the 2 represents the photon degrees of freedom. Hence the fraction of the energy density in gravitons relative to photons will be down by a further factor of $(g_{\text{int}}/2)^{4/3}$ over and above that at formation. The characteristic wavelength of such gravitons at formation is expected to be the Schwarzschild radius of the hole, so that $\lambda_* \sim 2M_{\text{BH}}/m_{\text{Pl}}^2$; they will then be re-shifted by the expansion to a wavelength today of $\lambda_0 = \lambda_* (1+z_{\text{evap}})$.

One might hope that black holes can produce a baryon asymmetry after the electroweak phase transition and thus avoid wash-out by the electroweak anomalous baryon-number-violating current.²² Unfortunately, it can

be shown that this is not possible; in order to last until the electroweak era, the black holes must be sufficiently massive that their initial temperature is well below any conceivable Higgs mass, and so for most of their evolution Higgs bosons are not radiated. This leads to a large temperature suppression in B , and an unacceptable value of $\epsilon > 10^2$ would be needed to match observations. Hence, other methods of avoiding wash-out, such as generating a nonzero $B - L$ as discussed in I, must be implemented.

To conclude then, we list the typical outcomes of the mechanisms we have discussed, in comparison to the direct production model. At best these models can generate an asymmetry of the same order of magnitude as the direct production mechanism. However, one must remember that for a range of inflaton parameters the direct production mechanism will not work;¹ for example the wall collisions may not be sufficiently energetic to produce Higgs particles directly. In such cases the black-hole mechanisms we have outlined may be the only way in which to generate an asymmetry, especially in cases where the reheat temperature is substantially less than the Higgs-boson rest mass. We illustrate the outcomes for the specific choice of inflation parameters given in Eq. (4.7), though our methods as illustrated in this section can be applied to any choice of parameters with ease.

The simplest version is “evaporation before domination,” with $\beta < \frac{1}{2}$. The holes must have a mass such that the ratio given by Eq. (4.10) is less than one. Such holes are probably light (and hence hot) enough for there to be no suppression of radiated Higgs bosons, and hence the asymmetry formed is very similar to that of direct production. The asymmetry is substantially less, though, in case where β or μ are very small. In the first of the “domination then evaporation” scenarios, $\beta < \frac{1}{2}$ but now the holes are massive enough to last until domination, with μ greater than about 10^{-4} . Here there is the possibility that the holes initially cannot radiate Higgs particles and there may be some suppression of baryon asymmetry because of this. Thus the baryon asymmetry is likely to be a few orders of magnitude less than direct production, and hence if the model parameters allow direct production, this mechanism operating on the remaining $1-\beta$ of the energy density will be the dominant contributor. Finally, there is the version where black holes dominate even initially. If the black holes have μ greater than about 10^{-4} the picture will be very similar to that of the first “domination then evaporation” scenario. However, here the black holes can be much lighter, allowing them to radiate Higgs bosons immediately. If their initial mass is around $\mu \sim 10^{-4}$ a baryon asymmetry of similar magnitude to that of the direct production mechanism may be obtained. Note though that the case of a long-lasting hole leads to a small asymmetry in either of the “domination then evaporation” cases. Finally, throughout this paragraph a reduction in the baryon asymmetry in one model as compared to another can be interpreted as simply requiring a larger ϵ .

For different model parameters the details may be somewhat different when the constraints of lifetime and temperature have been taken into account; in general for

instance the critical values of μ governing the temperature and lifetime behavior need not be as close as in the case we have illustrated. However, the principles of estimating the asymmetry remain exactly the same as discussed in the preceding paragraph. This concludes our investigation of baryogenesis after extended inflation, in which we have outlined methods of estimating the baryon asymmetry formed in wall collisions for a variety of different mechanisms. Each of the models we have outlined appears to have prospects for generating a baryon asymmetry of the correct order of magnitude to match observations, depending of course on the degree of baryon-number violation in the particle theory under consideration. We have found here that in cases where direct production of Higgs particles in the wall collisions may occur, the asymmetry generated is generically greater than that via the black-hole mechanism, so if direct production is allowed this will be the dominant contributor to the asymmetry. However, it is possible that the inflaton parameters may not allow direct production, in which case if there is a substantial production of black holes they may provide a route to a baryon asymmetry of the appropriate magnitude. For a discussion of further relevant points such as the role of sphalerons and of methods of avoiding monopole production, we refer the reader to the final section of I.

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APPENDIX A: DYNAMICAL EVOLUTION OF BARYON NUMBER

This appendix includes a derivation of the rate equations which determine the evolution of the baryon number during black-hole evaporation. An examination of the limiting cases of these network equations allows us to regain the results outlined in Sec. III. To cast the equations in their simplest form several new forms of notation will be introduced.

At t_{RH} we start with energy densities $\rho_{BH}(t_{RH}) = \beta\rho(t_{RH})$ in black holes and $\rho_R(t_{RH}) = (1-\beta)\rho(t_{RH})$ in radiation. We denote the initial black-hole mass as M_0 . As discussed in Sec. III, the black-hole mass as a function of t is

$$M_{BH}^3(t) = M_0^3 [1 - (t - t_{RH})/\tau], \quad (A1)$$

where, as before, $\tau = M_0^3 / (g_* m_{pl}^4)$ is the black-hole lifetime. Now the black-hole energy density is $\rho_{BH}(t) = n_{BH}(t)M_{BH}(t)$. Since the number density of black holes scales as a^{-3} and the mass as a function of time is given in Eq. (A1), the black-hole energy density is

$$\rho_{BH}(t) = [\beta\rho(t_{RH})] [1 - (t - t_{RH})/\tau]^{1/3} [a(t_{RH})/a(t)]^3. \quad (A2)$$

Of course, the factor $\beta\rho(t_{RH})$ is simply the black-hole energy density at t_{RH} . One can check that if the black holes are given a thermal velocity then they still contribute a negligible pressure. This confirms that the black-hole number density scales as matter, justifying the form used above. We have here neglected accretion onto the black holes from the background; in principle this may be an important effect at early times before the expansion dilutes the radiation. The amount of accretion presumably will be proportional to the square of the Schwarzschild radius of the holes, multiplied by some capture cross section of order 1 and by the density of the background. Rough calculations indicate that accretion would be negligible at late times.

For convenience we introduce a set of dimensionless variables

$$\begin{aligned} \alpha &\equiv a(t)/a(t_{RH}), \\ x &\equiv (t - t_{RH})/\tau, \\ R_{BH} &\equiv \rho_{BH}\alpha^3 / [\beta\rho(t_{RH})], \\ R_R &\equiv \rho_R\alpha^4 [(1-\beta)\rho(t_{RH})]. \end{aligned} \quad (A3)$$

Note that during evaporation the new time variable x simply goes from 0 to 1. The purpose behind the new variables should be obvious. Until evaporation starts in earnest, the evolution of the energy densities is simple: $\rho_R \propto a^{-4}$ and $\rho_{BH} \propto a^{-3}$. By defining R_R and R_{BH} we isolate the deviation from these simple scalings: R_{BH} and R_R have been defined so as to be constant in the absence of black-hole evaporation.

The evolution of the black-hole energy density now has the simple form

$$R_{BH} = (1-x)^{1/3}. \quad (A4)$$

The energy density of radiation is diluted by the expansion but is increased by energy fed in from the black-hole evaporation according to

$$\dot{\rho}_R = -4 \left[\frac{\dot{a}}{a} \right] \rho_R - \frac{\dot{M}_{BH}}{M_{BH}} \rho_{BH}, \quad (A5)$$

which after some manipulation gives the evolution equation for radiation as (where primes denote derivatives with respect to x)

$$(R_R)' = \alpha \frac{\beta}{3(1-\beta)} (1-x)^{-2/3}. \quad (A6)$$

To complete this set, we need the equation for α , which is just the Friedmann equation

$$\left[\frac{\dot{a}}{a} \right]^2 = \frac{8\pi}{3m_{pl}^2} (\rho_R + \rho_{BH}), \quad (A7)$$

which after some manipulation leads to

$$\begin{aligned} (\alpha')^2 &= \frac{8\pi}{3} \frac{M_0^6}{g_*^2 m_{pl}^{10}} \frac{(1-\beta)\rho(t_{RH})}{\alpha^2} \\ &\times \left[R_R + \alpha \frac{\beta}{1-\beta} (1-x)^{1/3} \right]. \end{aligned} \quad (A8)$$

Having the equations governing the evaporation of the black hole, we must now calculate the baryon number produced during the evaporation. The baryon number is generated by the decay of Higgs particles produced during the evaporation, with a baryon asymmetry of ϵ produced per Higgs boson. We shall assume that the only source of Higgs bosons is in primary production from the hole evaporation, and neglect any Higgs bosons later produced as the emitted particles thermalize. Further, when the hole is at a temperature T we assume that the mean energy of particles produced is just $\langle E \rangle = 3T = 3m_{\text{Pl}}^2 / 8\pi M_{\text{BH}}$. The fraction of Higgs particles produced will depend on this temperature, as at low energies there is insufficient energy to create a Higgs boson. A typical form for this thermal suppression may be $f_H = (g_H/g_*) \exp(-m_H/T_{\text{BH}})$, where g_H is the number of Higgs-boson degrees of freedom. This just says that at high temperatures Higgs-boson production matches that of other species, with Boltzmann suppression at low temperatures.

We note here that in Sec. III we demonstrated that the number of particles produced in the course of evaporation from a temperature T is proportion to T^2 . Hence if we consider the particles emitted from when the temperature matches the Higgs-boson mass, we find that $\frac{3}{4}$ of them will have energies between m_H and $2m_H$. This reinforces the impression that secondary production will not be important as most particles produced with energies above m_H have energies not far above the Higgs-boson mass and hence their thermalization is unlikely to prompt much secondary production.

The rate of particle production *per hole* is

$$\begin{aligned} \alpha' &= \left[\frac{m_0}{m_{\text{Pl}}} \right]^2 \left[\frac{M_0}{M_{\text{hor}}} \right] \frac{(1-\beta)^{1/2}}{2\alpha g_*} \left[R_R + \frac{\alpha\beta}{1-\beta} (1-x)^{1/3} \right]^{1/2}, \\ R_R' &= \frac{\alpha\beta}{3(1-\beta)} (1-x)^{-2/3}, \\ \tilde{B}' &= \frac{\epsilon f_H}{12} \left[\frac{M_0}{m_{\text{Pl}}} \right]^{1/2} \left[\frac{M_0}{M_{\text{hor}}} \right]^{1/2} \frac{\beta}{(1-\beta)^{3/4}} \left[\frac{32\pi}{3} \right]^{3/4} (1-x)^{-1/3} R_R^{-3/4} - \frac{3}{4} \frac{(R_R)'}{R_R} \tilde{B}. \end{aligned} \quad (\text{A13})$$

The equations are to be integrated from $x=0$ to $x=1$, with initial conditions $\alpha(0) = R_R(0) = 1$, and $\tilde{B}(0) = 0$.

Assuming f_H to be constant (approximately true for hot holes $T_{\text{BH}} > m_H$) this equation has an immediate first integral via

$$(\tilde{B} R_R^{3/4})' = C(1-x)^{-1/3}, \quad (\text{A14})$$

where C is a constant as seen from the preceding equation. This leads to

$$\tilde{B}(x) = \frac{3C}{2R_R^{3/4}(x)} [1 - (1-x)^{2/3}]. \quad (\text{A15})$$

The baryon number at the end of evaporation is obtained simply by substituting $x=1$ into the equation to get

$$\dot{N} = -\frac{\dot{M}}{3T_{\text{BH}}} = \frac{8\pi g_*}{9} \frac{m_{\text{Pl}}^2}{M_{\text{BH}}}, \quad (\text{A9})$$

from which we obtain the rate of Higgs-boson production, and then the baryon-number production as

$$\dot{N}_B = \epsilon f_H \frac{8\pi g_*}{9} \frac{m_{\text{Pl}}^2}{M_{\text{BH}}}. \quad (\text{A10})$$

Converting from number per hole to number density and letting the baryon number density evolve in an expanding universe leads to the expression

$$n_B' = \epsilon f_H \frac{8\pi}{9} \beta \rho(t_{\text{RH}}) \alpha^{-3} m_{\text{Pl}}^{-2} M_0 (1-x)^{-1/3} - 3 \frac{\alpha'}{\alpha} n_B. \quad (\text{A11})$$

For convenience we define the quantity $\tilde{B} = n_B / \rho_R^{3/4}$ which is related to the baryon number B via $B = \frac{3}{4} (30/\pi^2 g_*)^{1/4} \tilde{B}$. Rewriting the evolution of baryon number in terms of \tilde{B} leads to

$$\begin{aligned} \tilde{B}' &= \epsilon f_H \frac{8\pi}{9} \frac{\beta}{(1-\beta)^{3/4}} \rho^{1/4}(t_{\text{RH}}) m_{\text{Pl}}^{-2} M_0 \\ &\times (1-x)^{-1/3} R_R^{-3/4} - \frac{3}{4} \frac{(R_R)'}{R_R} \tilde{B}. \end{aligned} \quad (\text{A12})$$

Equations (A4), (A6), (A8), and (A12) form a closed set of equations to integrate to give the baryon number. The input parameters are M_0 , β , and $\rho(t_{\text{RH}})$. Rather than input $\rho(t_{\text{RH}})$, it is more physical to input the horizon mass at t_{RH} from Eq. (3.3). The set of equations becomes

$\tilde{B} = 3C/2R_R^{3/4}(x=1)$ and using the equation for B given above. Note that R_R can only increase from its initial value of 1, so putting in $R_R=1$ gives an upper limit on the baryon number obtainable for a given set of parameters. Notice further that this limit coincides with Eq. (3.17) obtained in Sec. III for the case of ‘‘evaporation before domination.’’

We have been unable to reproduce analytically the results for either of the ‘‘domination then evaporation’’ cases from the network equations, a task made complex because at the end of the evaporation we go back into a radiation-dominated region from the era of black-hole domination. Hence we cannot consistently neglect either of the terms in the equation for α for the entire evolution, though perhaps good answers can be obtained by assuming that the majority of the baryon asymmetry is pro-

duced during the era of black-hole domination. A further problem may be that f_H can no longer be regarded as a constant if there is the possibility that the holes are initially too cool to radiate Higgs bosons. Numerical

evolution of the network is another method of obtaining results for this case, though this is hampered by the large number of free parameters to be chosen (e.g., ϵ , M_0 , β , etc).

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