$I=\frac{1}{2}$ and $\frac{3}{2}$ $K\pi$ scattering in a $qq\bar{q}\bar{q}$ potential model

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We present the results of a study of $I=\frac{1}{2}$ and $\frac{3}{2}$ $K\pi$ scattering based on our earlier analysis of the $I=0$, 1, and 2 pseudoscalar-pseudoscalar systems. While the latter systems formed $K\bar{K}$ molecules in both $I=0$ and 1, here, with the same parameters, we find that neither $K\pi$, $K\eta$, nor $K\eta'$ form bound states. Both $I=\frac{1}{2}$ and $\frac{3}{2}$ phase-shift predictions are found to agree with experimental data. A shift in the mass and width of the "bare" $q\bar{q}$ state K_0^* is induced by the coupled-channel interactions, which also provide a physical source for the low-energy "background" phase shift normally introduced in analyses of $K\pi$ data.

INTRODUCTION

Our early study of the nonstrange scalar $qq\bar{q}\bar{q}$ sectors suggested that the S^* and δ are $K\bar{K}$ molecules and not ordinary $q\bar{q}$ $^3P_0$ mesons,[1] and we have recently elaborated and substantiated this conclusion in a more complete coupled-channel picture.[2] The reassignment of the S^* and δ [also known as the $f_0(975)$ and $a_0(980)$] solves many problems previously associated with their properties, with the $^3P_0$ scalar nonet in general, and with comparisons of the $^3P_0$ nonet to the more firmly established P-wave nonets. The $I=0$ channel has also recently been studied by several other groups: One used a $\pi\pi$-$K\bar{K}$ coupled-channel model based on a separable potential formalism,[3] a second considered a model in which there are both t- and s-channel meson exchanges,[4] and a third carried out a direct K-matrix analysis of the scattering data.[5] These studies all lend support to the notion that the forces in this channel are conducive to the formation of a weakly bound $I=0$ $K\bar{K}$ state of the type predicted in Ref. 1. See Ref. 2 for a discussion of the relationship between the $K\bar{K}$ Molecule Interpretation of S^* and δ and the earlier $qq\bar{q}\bar{q}$ interpretation by Jaffe in the bag model.[6]

In addition to its role in the interpretation of spectroscopic data in the scalar-meson sector, the $K\bar{K}$ molecule picture has been very useful in resolving outstanding problems involving the scattering and production of low-mass S-wave pseudoscalar pairs (P_1P_2). For example, it has been applied to the weak decay $K\to\pi\pi$, where it seems likely to play a role in solving the long-standing problem of understanding the $\Delta I=\frac{1}{2}$ rule.[7] There is also evidence[8] that it may resolve issues in P_1P_2 production experiments[9] such as $J/\psi\to VP_1P_2$, where V is a vector meson or a photon.

In this paper we apply the model to the strange P_1P_2 sector and find that it predicts, in agreement with the data,[10] neither a $K\pi$, $K\eta$, nor $K\eta'$ bound state. In the SU(3) limit a nonet of $K\bar{K}$ moleculelike states would exist, corresponding to the $qq\bar{q}\bar{q}$ nonets that appear in the naive bag model,[6,11] and so this is an important and nontrivial test of the model. When we extract scattering amplitudes from the model, we find $I=\frac{1}{2}$ and $\frac{3}{2}$ $K\pi$ phase shifts in agreement with those seen experimentally.

MODEL

Our analysis is based on solving a multichannel Schrödinger equation in the $J^P=0^+$ sector in which the interaction potentials arise from two distinct mechanisms. The first of these mechanisms is quark exchange [Fig. 1(a)], which leads to $K\pi$, $K\eta$, and $K\eta'$ potentials and channel couplings. (Quark exchange in the P_1P_2 S-wave channels is dominantly driven by the one-gluon-exchange hyperfine interaction.) Our calculations also allow for vector-vector components in the scalar $qq\bar{q}\bar{q}$ system, but they are found to be very weak at low energy.[2]

The techniques developed to extract these quark-exchange potentials have been discussed in Refs. 1 and 2: They lead to effective meson-meson potentials which are obtained in the SU(3) limit in which the $qq\bar{q}\bar{q}$ flavor wave functions can be classified[6] as cryptoexotic ($3\otimes\bar{3}$) and exotic ($\bar{6}\otimes 6$) (c and e, respectively). These potentials are mass dependent and can be determined for constituent quark masses corresponding to four nonstrange quarks,

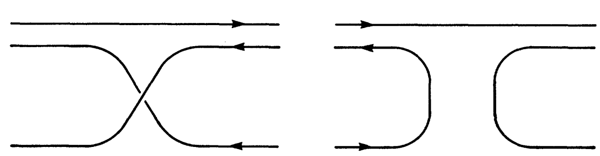

FIG. 1. Two sources of the pseudoscalar-pseudoscalar potentials: (a) the quark-exchange interaction and (b) the $q\bar{q}$ pair annihilation and creation interaction through ordinary s-channel $^3P_0$ $q\bar{q}$ scalar mesons.

four strange quarks, or to their average (as appropriate, e.g., for the $K\bar{K}$ system). They can then be converted to P_1P_2 potentials via the transformation connecting the SU(3)-flavor basis to the physical P_1P_2 basis.

The second mechanism allowing $(q\bar{q})$-$(q\bar{q})$ interactions is the $q\bar{q}$ annihilation process which couples the scalar $qq\bar{q}\bar{q}$ system to the usual $q\bar{q}$ scalar mesons [Fig. 1(b)]. In the case of S-wave P_1P_2 systems, these s-channel mesons are the ${}^3P_0$ scalar mesons; for $I=\frac{1}{2}$ the lightest of these is believed to be the $K_0^*(1430)$ (formerly called the κ).

In order to apply the coupled-channel model to the strange-quark sector, one must find the underlying effective $(n\bar{s})(n\bar{n})$ and $(n\bar{s})(s\bar{s})$ potentials (where n is a u or d quark) with both exotic and cryptoexotic symmetries, determine the unitary transformation between the $\bar{3}\otimes 3$ and $6\otimes\bar{6}$ flavor wave functions and the $K\pi$, $K\eta$, and $K\eta'$ flavor wave functions, and finally calculate the couplings of these meson pairs to the ${}^3P_0$ scalar-meson channel.

In our study of the $I=0$, 1, and 2 P_1P_2 sectors, we found that, within the accuracy of our techniques, the SU(3)-broken potentials could be obtained by taking appropriate linear combinations of SU(3) exotic and cryptoexotic potentials corresponding to the average quark mass in the system. Applying this ansatz here leads us, for the potentials of Ref. 2, to the "square-well equivalent potentials" $V^e_{(n\bar{s})(n\bar{n})}=+0.43$ GeV, $V^e_{(n\bar{s})(s\bar{s})}=+0.24$ GeV, $V^c_{(n\bar{s})(n\bar{n})}=-0.34$ GeV, and $V^c_{(n\bar{s})(s\bar{s})}=-0.20$ GeV with a common square-well range of 0.8 fm.

To convert these SU(3) potentials to meson-meson potentials, we use the basis transformation

$$\begin{bmatrix} |[K\pi]_+^{I=3/2}\rangle \\ |[K\pi]_+^{I=1/2}\rangle \\ |[K\eta_{n\bar{n}}]_+\rangle \\ |[K\eta_{s\bar{s}}]_+\rangle \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -\sqrt{1/10} & -\sqrt{3/20} & \sqrt{3/4} \\ 0 & -\sqrt{3/10} & -\sqrt{9/20} & -\sqrt{1/4} \\ 0 & \sqrt{3/5} & -\sqrt{2/5} & 0 \end{bmatrix} \begin{bmatrix} |27,\frac{3}{2},e\rangle \\ |27,\frac{1}{2},e\rangle \\ |8,\frac{1}{2},e\rangle \\ |8,\frac{1}{2},c\rangle \end{bmatrix}, \tag{1}$$

where the notation is $|$representation, I, exoticity$\rangle$, $[K\pi]_+=\sqrt{(1/2)}(K\pi+\pi K)$, etc., and the physical η and η' are approximately given by $\eta=\sqrt{(1/2)}(\eta_{n\bar{n}}+\eta_{s\bar{s}})$ and $\eta'=\sqrt{(1/2)}(\eta_{n\bar{n}}-\eta_{s\bar{s}})$, where $\eta_{n\bar{n}}=\sqrt{(1/2)}(u\bar{u}+d\bar{d})$ and $\eta_{s\bar{s}}=s\bar{s}$.

The $q\bar{q}$ annihilation contributions, which couple the P_1P_2 channels to the K_0^* channel, are modeled in the same way as the annihilation couplings to the a_0, f_0, and f_0' in Ref. 2. They are of the form

$$V_{P_1P_2\to K_0^*}=C_{P_1P_2K_0^*}\Omega\left[\frac{\mu_{K\bar{K}}}{\mu_{P_1P_2}}\right]^{1/2}\Theta(0.8\ \text{fm}-r)\ , \tag{2}$$

where $\mu_{P_1P_2}$ is the P_1P_2 reduced mass, the flavor coupling factors are

$$C_{K\pi K_0^*}=\sqrt{6}\ , \tag{3a}$$

$$C_{K\eta K_0^*}=(1-\sqrt{2}\lambda)\ , \tag{3b}$$

$$C_{K\eta' K_0^*}=(1+\sqrt{2}\lambda)\ , \tag{3c}$$

$\Omega=0.11$ GeV, and λ (which allows for a possible suppression of $s\bar{s}$ pair creation) is equal to 1 [corresponding to an SU(3)-symmetric pair creation mechanism], as in the $I=0$ and 1 systems.

The final ingredient required to completely specify the interactions of these systems is the $n\bar{s}\ {}^3P_0$ potential or, equivalently, the "bare" mass of the lowest K_0^*. Instead of fixing this potential in terms of the bare masses of the a_0, f_0, and f_0' (the last of which is especially poorly known[2]), we here varied the depth of the $n\bar{s}$ potential to fit the $I=\frac{1}{2}$ phase shift. This leads to a "bare" K_0^* mass, which we will compare to expectations below.

These considerations lead to the potential matrix

$$\mathbf{V}=\begin{bmatrix} -0.148 & 0.236 & 0.236 & 0.406 \\ 0.236 & 0.236 & 0.001 & -0.044 \\ 0.236 & 0.001 & 0.236 & 0.232 \\ 0.406 & -0.044 & 0.232 & -0.070 \end{bmatrix} \times\Theta(0.8\ \text{fm}-r)\ \text{GeV}\ , \tag{4}$$

in the $I=\frac{1}{2}$ basis $(K\pi, K\eta, K\eta', n\bar{s}\,{}^3P_0)$, and to

$$\mathbf{V}=0.43\ \Theta(0.8\ \text{fm}-r)\ \text{GeV}\ , \tag{5}$$

for $I=\frac{3}{2}$. In addition to $\mathbf{V}$, in the $n\bar{s}\ {}^3P_0$ channel we must include the P-wave centrifugal barrier $1/(\mu_{n\bar{s}}r^2)$.

RESULTS

The $I=\frac{3}{2}$ channel has a repulsive $K\pi$ interaction and leads, with *no adjustable parameters*, to the phase shift shown in Fig. 2. The agreement with the data is satisfactory.

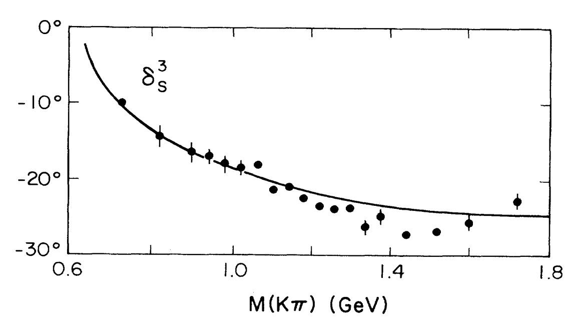

FIG. 2. Comparison of the $I=\frac{3}{2}$ $K\pi$ phase shift due to the effective potential of Eq. (5) and the data of Estabrooks *et al.* in Ref. 10.

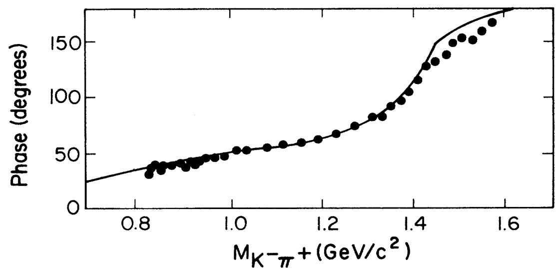

FIG. 3. $I=\frac{1}{2}$ $K\pi$ phase shift from the model compared to the experimental data from Aston *et al.* in Ref. 10.

In the $I=\frac{1}{2}$ sector we find the $K\pi$ phase shift shown in Fig. 3. In Fig. 4 we compare the predicted and experimental Argand diagrams for $K\pi$ elastic scattering. Note in particular that none of the $K\pi$, $K\eta$, or $K\eta'$ systems form bound states. There is, however, a strongly attractive $K\pi$ potential for which there is considerable direct evidence in the form of low-mass $K\pi$ enhancements (see, e.g., Ref. 12). The quality of the agreement shown in Figs. 3 and 4 seems satisfactory in all but perhaps one respect: Although there are indications of significant inelasticity at $K\eta'$ threshold in the data of Estabrooks *et al.* as predicted by the model, the data of Aston *et al.* remain near the unitarity circle over the whole kinematic range considered here. In Fig. 5 we show the effects on the predicted $I=\frac{1}{2}$ Argand diagrams of the replacements $V^e \to 0.5V^e$, $V^c \to 0.8V^c$, $\Omega \to 1.1\Omega$, or $\lambda \to 0.75\lambda$. The stability of the results is clear. The "bare" mass of the lowest $n\bar{s}\,{}^3P_0$ state in the $I=\frac{1}{2}$ channel was found by our fitting procedure to be approximately 1470 MeV. We note that our conclusions on the absence of a $K\pi$ bound state and on the K_0^* are very similar to those of Ref. 4.

Our previous fits to $I=0$ and 1 P_1P_2 scattering within this model led to corresponding masses of the "bare" a_0, f_0, and f_0' of approximately 1300, 1300, and 1500 MeV, respectively, which imply that the bare K_0^* mass should have been about 1400 MeV. This discrepancy is not

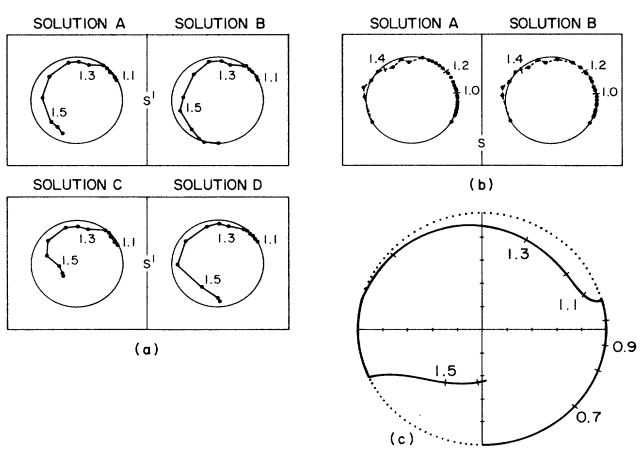

FIG. 4. Comparison of (a) the four experimental solutions presented in Estabrooks *et al.* and (b) the two solutions found in Aston *et al.* in Ref. 10 with (c) the $I=\frac{1}{2}$ $K\pi$ Argand diagram found in the model.

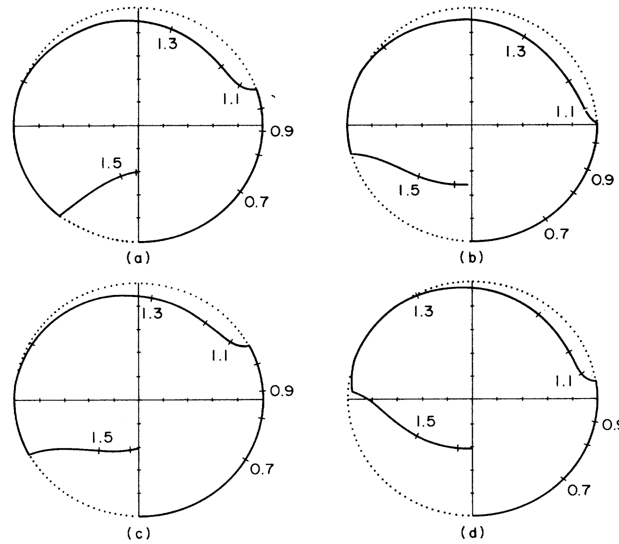

FIG. 5. Variations of the $I=\frac{1}{2}$ $K\pi$ Argand diagrams due to the changes (a) $V^e \to 0.5V^e$, (b) $V^c \to 0.8V^c$, (c) $\Omega \to 1.1\Omega$, (d) $\lambda \to 0.75\lambda$, with respect to the canonical values given in the text.

surprising in view of some of the systematic uncertainties in our calculations, but in Fig. 6 we show the effect of setting the K_0^* mass to the nominal bare value of 1400 MeV. It is important to note that coupled-channel effects have had the effect of significantly reducing the apparent K_0^* mass from its bare value to the experimentally determined value of about 1430 MeV.[13] A similar change occurs in the width: The "bare" K_0^* would have a width in the narrow resonance approximation of about 500 MeV versus the experimental value[13] of about 290 MeV

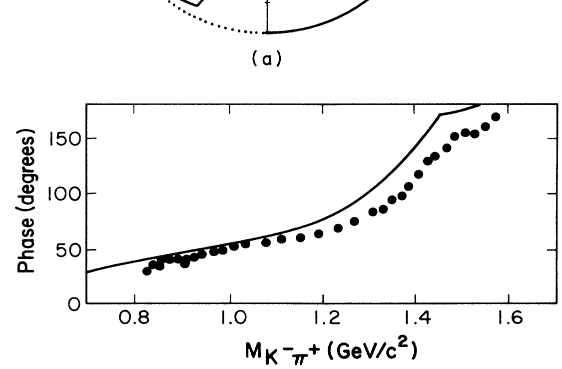

FIG. 6. (a) $I=\frac{1}{2}$ $K\pi$ Argand diagram and (b) phase shift found with the canonical potentials and a bare K_0^* mass of 1400 MeV.

that results from analysis of the data. (Such effects appear to be particularly strong in this channel since the K_0^* mass is near the $K\eta'$ threshold.) This shows the danger inherent in relating the bare meson masses and widths as predicted by the quark model to their experimental values and suggests that a coupled-channel "interface" is required to relate them. In particular, one should exercise some caution in comparing the *bare* $^3P_0$ masses quoted here with the *experimental* masses of, e.g., $^3P_2$ states: This comparison should be made to the bare masses that would emerge from an analogous coupled-channel analysis of the $J^P=2^+$ sector.

BRINGING BACKGROUNDS INTO THE FOREGROUND

Figure 7 shows the separate effects of the two main components of this calculation on the $I=\frac{1}{2}$ phase shift. Figure 7(a) shows the effect of setting all the P_1P_2 potentials due to quark exchange to zero, thereby isolating the effects of the $^3P_0$ resonances, and showing that the P_1P_2 potentials are an essential ingredient of the physics. Figure 7(b) shows the phase shift due to the quark-exchange potentials alone. We see that the very substantial $K\pi$ attraction from the potentials is "half the physics" at low $K\pi$ masses, but it is usually lumped into an "effective-range-approximation" polynomial background amplitude which places the spotlight on resonance physics. We wish to emphasize that, according to the model used here, the $I=0$ and 1 S-wave $K\bar{K}$ molecule states owe their existence to this "background." Moreover, forces analogous to these "background" forces would be expected to play a similarly important role in the understanding of nuclear forces to the role they play here.

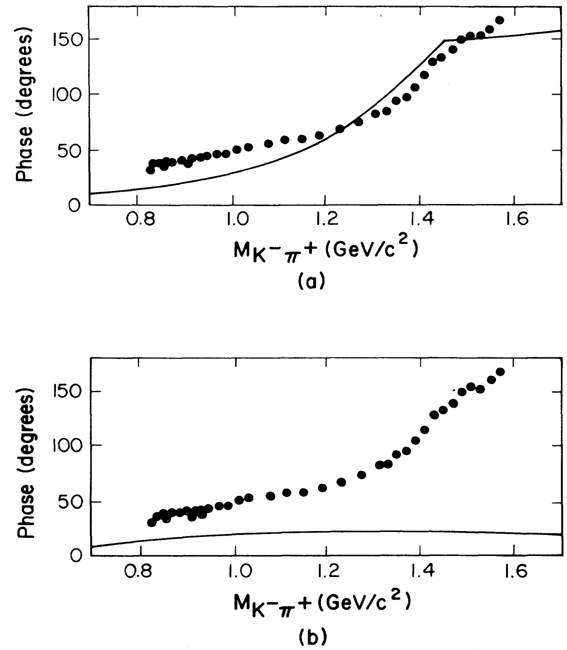

FIG. 7. (a) Phase shift resulting from only the $P_1P_2\leftrightarrow K_0^*$ couplings (the P_1P_2 potentials are set equal to zero), and (b) the opposite situation in which the $P_1P_2\leftrightarrow K_0^*$ couplings are set equal to zero and only the P_1P_2 potentials are allowed.

CONCLUSIONS

We have demonstrated that the model which we previously applied to the nonstrange $I=0$, 1, and 2 S-wave P_1P_2 sectors is equally successful in the $I=\frac{1}{2}$ and $\frac{3}{2}$ sectors. This is important not only for understanding these strange sectors themselves, but also as confirmation that the model gives a reasonable picture of the underlying physics: With a single set of input parameters, it describes all S-wave pseudoscalar-meson–pseudoscalar-meson scattering channels.[14]

We have also emphasized the need for incorporating coupled-channel effects when trying to relate the "bare" masses of the quark model to the measured masses of resonances, even in channels where the interactions do not produce bound states. Finally, we have suggested that the physics of the threshold scattering-length-type "backgrounds" employed in fits to mass distributions have their origins in quark-exchange processes, the study of which may ultimately have much to teach us about interhadron forces.

ACKNOWLEDGMENTS

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